Endogenous labour supply, endogenous lifetime and economic development

Luca Gori*• Mauro Sodini†

November 6, 2019

Abstract

This article develops a theory to explain economic development in a neoclassical growth model. The (co)existence of endogenous labour supply and endogenous lifetime (affected by public and private expenditures on health) allows to effectively observe multiple development scenarios under the assumption of gross substitutability between consumption and leisure. The initial condition of a macro-economy (history) and expectation-driven coordination failures (indeterminacy) become alternative explanations of long-term demo-economic outcomes of nations. The novelty of this study is to link the theoretical literature on endogenous lifetime and economic growth with the theoretical literature on endogenous labour supply and indeterminacy by concentrating on the *global* analysis. The model also allows to explain the coexistence of least-developed, developing and developed countries in the same setting.

Keywords Economic development; Endogenous labour supply; Endogenous lifetime; Local and global indeterminacy; OLG model

JEL Classification C61; C62; J1, J22; O41

1 Introduction

"We should continue to use simple models where they capture enough of the core underlying structure and incentives that they usefully predict outcomes. When the world we are trying to explain and improve, however, is not well described by a simple model, we must continue to improve our frameworks and theories so as to be able to understand complexity and not simply reject it."

*L. Gori (corresponding author), Department of Law, University of Pisa, Via Collegio Ricci, 10, I–56126 Pisa (PI), Italy, e-mail: luca.gori@unipi.it or dr.luca.gori@gmail.com, tel.: +39 050 22 12 847.

†M. Sodini, Department of Economics and Management, University of Pisa, Via Cosimo Ridolfi, 10, I–56124 Pisa (PI), Italy, e-mail: mauro.sodini@unipi.it, tel.: +39 050 22 16 234, fax: +39 050 22 10 603.

This is a challenging theoretical study that belongs to three strands of economic literature: the literature on poverty trap, the literature on health and development and the literature on local and global (in)determinacy. There are plenty of pieces of research on these issues. Then, the article begins by discussing the main motivations and the links to the related contributions.

Demographic and macroeconomic outcomes were recognised to be thoroughly correlated in the process of economic growth and development (de la Croix and Doepke, 2003, 2004; Fogel, 2004; Galor, 2005, 2011; Acemoglu and Johnson, 2007; Hall and Jones, 2007; Weil, 2007; Lorentzen et al., 2008; Livi-Bacci, 2017; Fiaschi and Fioroni, 2019), and both demographers and macroeconomists inquiring into the causes of poverty or prosperity of nations are currently wondering the nature of the causal relationship between demographic and economic variables, which is often bidirectional.

Since Leibenstein (1957), Becker (1960) and the origin of the new home economics, the study of the interaction amongst fertility, mortality and income has become a pillar of the macroeconomic theory, representing also an empirical challenging open question. Including fertility and mortality as endogenous variables in an economic model has opened the route to causes for reflection giving birth to several works with the aim of explaining the demographic and economic transitions (e.g., Galor and Weil, 1999, 2000; Blackburn and Cipriani, 2002; Galor and Moav, 2002; Kalemli-Ozcan, 2002; Cervellati and Sunde, 2005, 2011; Galor, 2005, 2011; Fioroni, 2010), i.e. the long-term process describing the transition from high birth and death rates to low birth and death rates as a country develops, moving from a pre-industrial to an industrialised environment. Emphasis was placed on the role the individual state of health can play on life expectancy and long-term outcomes at the economy-wide level. As was pointed out by Weil (2007, p. 1265), "People in poor countries are, on average, much less healthy than their counterparts in rich countries. How much of the gap in income between rich and poor countries is accounted for by this difference in health?" This and other similar questions are the object of a growing body of studies aiming at explaining the relationship amongst mortality decline, economic growth and the historical patterns of the demographic transition (e.g., Ehrlich and Lui, 1991; Kalemli-Ozcan et al., 2000; Kalemli-Ozcan, 2002, 2008; Cervellati and Sunde, 2005, 2015; Cervellati et al., 2017).

It is widely accepted that adult mortality has a negative impact on economic growth because the individual (life)time horizon reduces, physical and human capital accumulation reduce and fertility increases (Lorentzen et al., 2008; Kalemli-Ozcan and Turan, 2011; Kalemli-Ozcan, 2012; Juhn et al., 2013; Gori et al., 2019a, 2019b).[1] In the words of Lorentzen et al. (2008, pp. 81-82), "Development occurs only if people make provision for the future. If they see no future, there is no growth." Improving health conditions, therefore, represents a challenge for several governments around the world, especially in developing and least-developed countries. From theoretical grounds, the overlapping generations (OLG) model with production à la Diamond (1965) has become a natural basis where studying this issue. Within this framework, some studies consider life expectancy as an endogenous variable trying to give an answer to questions related to poverty or prosperity of nations. Endogenous life expectancy may take the form of (private) choices of individuals about health investments that contribute to improve education, the state of health and labour productivity (Blackburn and Cipriani, 2002; Chakraborty and Das, 2005). The awareness that comes from being educated improves the knowledge of the benefits of investing in health. The increase in labour productivity favours the increase in human capital accumulation that in turn allows a reduction in both adult mortality and fertility eventually determining a virtuous circle to escape from poverty. The state of health of individuals can also be improved by specific public interventions (Chakraborty, 2004; Fanti and Gori, 2014), with the aim of furnishing health services to increase life expectancy, savings (individuals save more because they live longer) and economic growth. Private and public health expenditures may also be considered together in the same setting, as private and public healthcare systems often coexist in several actual economies. There are only a few studies accounting for both these wings in a general equilibrium context. We recall here the works of Bhattacharya and Qiao (2007), Varvarigos and Zakaria (2013) and Agénor (2015). Bhattacharya and Qiao (2007), BQ henceforth, were the first to analyse the simultaneous role of private health expenditure (in terms of resources) and public health expenditure in an OLG macroeconomic model. The main motivation of their work was to provide a framework where explaining how the coexistence of these two wings of the health service may cause (long-term) fluctuations in demo-economic outcomes. In this regard, the authors considered a set up where private health spending are incurred when individuals are young and the public expenditure complements the private one in the same period with the purpose of improving the quantity of life (adult survival), which indirectly provides utility as individuals live longer. Non-monotonic dynamics occur because the provision of public health services causes two counterbalancing effects: a reduction in the disposable income of agents (through taxation) that in turn implies a reduction in saving and capital accumulation, and an increase in private health care that contributes to increase the health status, life expectancy and capital accumulation.[2] Their

[1] Hazan and Zoabi (2006) criticised this view finding that increases in health and longevity have not been the main source of human capital accumulation when countries moved from stagnation to modern growth promoting the quantity/quality trade-off. Recently, however, by combining theoretical and empirical assessments, Fiaschi and Fioroni (2019, p. 2) found that "Technological progress plays the major role before the mid-nineteenth century, while adult mortality has a negligible impact; thereafter, the decline in adult mortality and factor accumulation emerge as the most important sources of growth" of Great Britain during the period 1541-1914.

[2] This is essentially because of a health technology capturing changes in the elasticity of longevity with respect

model was used to interpret the different behaviours of countries such as South Korea (high performance) and the Philippines (under performance) - as was mentioned in Lucas (1993) - about the different economic development trajectories they followed in the last decades though their conditions were similar at the beginning of the sixties and they came from a reversed situation. Unfortunately, the model of BQ cannot support the explanation provided by the authors of this phenomenon. This is because the dynamics of their economy is characterised by a one-dimensional (uni-modal) map with a *unique* regular or chaotic attractor. When an attractor is chaotic, there exist non-regular trajectories related to distinct (but similar) initial conditions implying fluctuations in the variables get involved. However, this scenario *does not allow explaining the existence of distinct regimes of development* as the statistical properties of the resulting different time series are the same. This means that if one considers the same set of parameters and imagines two economies that differ regarding their initial conditions, they will intend to experience *the same development scenario though this may happen at different dates.* In other words, the model of BQ cannot be used to explain the reasons why distinct economies starting close to each other eventually experience different levels of real activity and different values of life expectancy.[3] Later, Varvarigos and Zakaria (2013) modified BQ's model by including endogenous fertility. Unlike BQ, they considered that private effort to better health is incurred by old individuals with the aim of improving the (quality of their) health status (which directly provides utility) instead of spending on health when they are young to lengthen their life span (which indirectly provides utility). Their model was capable to provide an additional interpretation of the demographic transition in a context where initial conditions (history) matter(s). Finally, Agénor (2015) enriched the setting by adding *endogenous labour supply* and using three ingredients that may contribute to determine the long-term outcome of poor countries. These assumptions are the "interactions between public capital in infrastructure and health outcomes; the dependence of health in adulthood on health in childhood; and a cross-generation effect, in the sense of parental health affecting directly the health of their children, possibly in utero." (Agénor, 2015, p. 124). The third assumption is crucial for the author to describe an inter-generational health effect that can to characterise a poverty trap environment in the health variable. In this scenario, Agénor (2015) stressed the importance of initial conditions for determining long-term outcomes (history matters, again). By assuming an *exogenous longevity*,[4] the health status of an individual may be improved privately depending on how much time an agent chooses to spend on its accumulation (health is viewed as a durable stock). When he came to extend the model by including *endogenous longevity*, he did not link

to private investments when public investments vary.

[3] In addition, nonlinear dynamics in their model can be generated only when the public input in the longevity technology is a convex function of the public health expenditure, so that the returns of public investments are very large. Indeed, this is quite questionable from an empirical point of view, especially if one wants to explain the process of economic growth of least developed or developing countries. Differently, the results of the present work hold when *all* the functions are concave.

[4] Agénor assumed exogenous longevity in the first part of his work for analytical tractability.

individual decisions about the labour supply with the individual's own health status, i.e. agents did not internalise the effects of their labour/leisure decisions on health outcomes. Then, there are two critical modelling assumptions in Agénor's setting. (1) Individuals do not internalise the implications of their labour supply decisions on their own health status. (2) The dynamics of the labour supply is sterilised because preferences are represented by a log-utility function.

The present research takes BQ seriously and builds on a theory to effectively explain economic development by including endogenous labour supply. This assumption comes from the strong evidence that health investments affect individual decisions about how much time to spend at work and investments in longevity (Garthwaite et al., 2013).[5] Under gross substitutability between consumption and leisure (de Vilder, 1996; Grandmont et al., 1998), the present article shows that the complementarity between public and private expenditures on health is responsible for the existence of different development regimes. This result is impossible with private expenditure alone as there exists only one stationary state (saddle point) in that case. Underdevelopment becomes a matter of coordination failures (*global indeterminacy*) so that history alone may not be sufficient to determine the long-term demo-economic outcome of an economy. The work enters directly into the debate about history versus expectations as discussed in the seminal articles of Krugman (1991) and Matsuyama (1991).

Although there exists an important tradition of works dealing with (in)determinacy and/or (in)stability of stationary equilibria in OLG economies with endogenous labour supply (Cazzavillan, 2001; Nourry, 2001; Cazzavillan and Pintus, 2006; Nourry and Venditti, 2006; Sorger, 2018), the existing literature has essentially tackled out these issues from a *local* perspective by neglecting the global analysis.[6] However, focusing just on the local analysis may not allow bringing some results out that can be at odds with those emerging through a global study. This is even more important in models aiming to explain economic development, which is a long-term phenomenon whose evolution cannot be regarded solely in a neighbourhood of the steady state (transitional dynamics matter). This article wants to fill this gap and represents the first attempt of inquiring about long-term demo-economic behaviour by explicitly considering individual lifetime related to specific health investments in the OLG literature with endogenous labour supply. In this setting, economic development can be explained based on two distinct reasons: history (path dependence) and global indeterminacy (expectations driven). In the former case, for a given level of public health expenditure, convergence towards either a low-development regime or a high-development regime is determined by the initial condition

[5] Other works dealing with labour supply and health expenditure are Hokayem and Ziliak (2014) and French (2005). See also the survey of Currie and Madrian (1999), where it clearly emerges a strong relationship between health and labour market outcomes including the supply of labour.

[6] Two exceptions are the works of Gori and Sodini (2014) and Antoci et al. (2016) that analysed global (in)determinacy in OLG models with endogenous preferences (habits) and environmental issues (free-access natural resources), respectively. More recently, Antoci et al. (2019) tackled out this issue in a growth model with production-induced environmental degradation and maladaptation.

of the economy (i.e. the stock of capital installed at the very beginning). However, as history alone cannot always be taken as a reasonable matter of a long-term scenario an economy may approach with, the model provides an alternative explanation for which setting the initial condition on the state variable (the capital stock) is no more sufficient to characterise the long-term dynamics of the system. This holds for (initial) values of the stock of capital close to (local indeterminacy) or far away from (global indeterminacy) the stationary state value of K. In fact, agents can coordinate themselves on different values of the control variable (labour supply) leading an economy towards very different development regimes. These outcomes depend on both an individual's own decision and decisions of the others. When there are multiple equilibria and indeterminacy there may be problems of coordination failures that can be solved with an element of self-fulfilling prophecy. For a given value of public health spending, if everyone believes that the economy will approach a low-growth regime where labour supply, capital stock, private health spending, longevity and welfare are little, then this outcome will occur despite the existence of a high-growth regime (that Pareto dominates the other one).

Although the approach used here is mainly theoretical, it is also related to the political debate regarding whether transforming or not the public-based European welfare system (the public health expenditure is one of the pillars of the welfare state in Europe) to a private-based one. The welfare state in several European countries is experiencing some concerns. This is because of the reduction in per capita GDP and the demographic shift leading to observing a steadily reducing number of young workers and a steadily increasing number of old and healthy pensioners in these countries. The article contributes to explain how the choices on how much spending on health determines different dynamic paths in income, longevity and labour supply in a macroeconomic setting and stresses the importance of public health investments as a source of economic development.

The rest of the article is organised as follows. Sections 2 outlines the model and provides the main analytical results. Section 3 describes the global properties of the map and gives a detailed description of three different development scenarios the model can generate. Scenario 1 explains economic development based on initial conditions (history). Scenarios 2 and 3 instead focus on local and global (in)determinacy as possible theoretical justifications for observing different development trajectories of economies whose initial values of the state variable are quite similar (expectations). These last two scenarios are able to provide a reason why an originally less developed country such as South Korea (experiencing low values of capital accumulation and longevity) has entered a tremendous growth trajectory leading towards high values of capital accumulation and longevity, whereas an originally (sufficiently) rich country such as the Philippines (experiencing relatively high values of capital accumulation and longevity) has moved to a lower level of economic development with low values of capital accumulation and longevity. Section 4 discusses the main results and Section 5 outlines the conclusions. Appendix A reports some analytical details of the model. Appendix B augments the model developed in the main text with young-age consumption and briefly sketches some outcomes.

2 The model

The model is developed along the lines of the work of BQ. Consider an OLG closed economy inhabited by a continuum of rational and identical individuals of measure one per generation. In every period, three generations are alive, two of which are economically active (Diamond, 1965): the young and the old. Each (economically active) generation overlaps for one period with the previous generation and then overlaps for one period with the next generation. Time is discrete and indexed by $t = 0, 1, 2, ...$ Economic life of the typical agent born at time t is divided between youth and old age and the length of each period consists of 30 years.[7] In the first period of active life (youth), the individual of generation t is endowed with $\bar{\ell}$ units of time and supplies the share $\ell_t \in [0, \bar{\ell}]$ to firms in exchange for wage w_t per unit of labour. The remaining share $\bar{\ell} - \ell_t$ is used for leisure activities. The individual also chooses the amount of resources that should be allocated between (private) health investments and saving. Material consumption when young is not a choice variable. This assumption follows a well-established literature that includes, amongst others, the contributions of Woodford (1984), Reichlin (1986), Galor and Weil (1996), Grandmont et al. (1998), Bhattacharya and Qiao (2007), Gori and Sodini (2019). In the second period of life (old age), the individual retires and consumes based on his saving (capitalised at the expected interest rate that will prevail from time t to time $t + 1$). The narrative behind this story is the following. The representative individual in the standard OLG model à la Diamond (1965), whose (lifetime) preferences are defined over consumption bundles of material goods in two subsequent periods, faces a trade-off between consumption when young and consumption when old, so that increasing material consumption during youth (time t) reduces (ceteris paribus) resources for saving and material consumption in the next period (time $t+1$). Therefore, competition for resources when young (available working income) is between consumption and saving. In the present model, choosing consumption when young is not an economic problem (i.e., the time endowment spent for labour force participation is high enough to avoid struggle for existence) and competition for resources during youth (available working income) is between private health expenditure and saving. In addition, the individual allocates his time endowment $\bar{\ell}$ between working and leisure activities. This choice affects his total labour income (determined by the wage per unit of labour multiplied by the amount of time spent working) as well as the amount of private health investments and saving that will in turn represent the basis for determining the consumption bundle when old. Therefore, the individual trades leisure off for health investment and saving when young and consumption when old. Ceteris paribus, enjoying leisure today reduces labour income, health investment and future consumption. Appendix B shows details of the model (individual optimisation problem and dynamic system) augmented with material consumption when young.

[7] As a child, an individual does not make economic decisions. He spends time in the parent's household by consuming resources directly from him and enters young age with certainty. The length of this period is conventionally set at 30 years as well.

An individual survives at the onset of old age with certainty, and he is alive only for a fraction $\theta \in (0,1]$ of the second period of his lifetime. Then, $1+\theta$ represents a measure of individual longevity. The probability of surviving when old is endogenous and determined by the individual state of health. An agent can lengthen his lifetime horizon when old by incurring private investments in health when young. These investments are accompanied by tax-financed health expenditure. This (toy) structure can adequately describe health systems in several actual European economies, where both public and private components coexist and the former component represents a relevant portion of total expenditure on health over per capita GDP (World Health Statistics, 2015).[8] Therefore, population is endogenous because the length of life of the typical agent when old varies as long as adult mortality varies due to changes in health spending when young. Therefore, the lifetime of an individual when old depends upon his health status when young. This health status is augmented by incurring private investments in health and tax-financed public investments in health. The former is represented by private effort to better health and longevity directly financed by individuals, which can reasonably be represented by "annual diagnostic health screening, opportunity cost of regular exercise, taking vitamins, nutrients, and other supplements, eating organically grown food, health benefits from quitting unhealthy habits such as smoking, etc." (Bhattacharya and Qiao, 2007, p. 2520). The latter ones can be summarised, following Chakraborty (2004, p. 121), by the provision of "clinical facilities, sanitation, inoculation and disease control programs", or being represented by policies to promote healthy environments. Therefore, adult mortality can be reduced through the rise in health spending when young. In this regard, we assume that the survival probability when old of the typical agent of generation t is determined by $\theta_t = \theta(x_t, \eta_t)$, where θ is the so-called longevity production function, with x_t being the private input (private investments in health) and η_t the public input (public investments in health). Unlike BQ, we assume that these two inputs can generally be viewed as complements irrespective of the size of the private effort. This is because an increase in public investments in health always acts as an incentive to increase private investments. In other words, the public expenditure on health increases the marginal productivity of the private one. This assumption enables to capture the interrelationship between private and public inputs for developed (rather than developing) countries, where there is no crowding out effects of the public health spending on the private health spending. It represents a difference with respect to the longevity production function used by BQ, which is reported here for clarity: $\theta_t = b\eta_t x_t^{b\eta_t}$, where $b>0$. In that case the elasticity of longevity with respect to private investments in health depends on public investments in health, and the expression $\theta''_{x_t,\eta_t} = b^2\eta_t x_t^{b\eta_t}\left(\frac{2+b\eta_t\ln(x_t)}{x_t}\right)$ is negative for small values of x_t. This in turn implies

[8] In the European Union the ratio of domestic general government health expenditure to total health expenditure was almost 79.5 percent in 2016 (in Italy, it reached a peak of 78 per cent in 2010 and then decreased to 74 per cent in 2016). In the US this ratio was almost 48 per cent until 2013 and sharply raised to slightly above 80 percent in 2016. In Sub Saharan Africa, it fluctuated around 30-38 per cent from 2000 to 2016 (https://data.worldbank.org/indicator).

that an increase in public health spending reduces the productivity of the private input to higher longevity when the private health spending is sufficiently low, so that x_t and η_t are substitutes in such a case. The formulation of the longevity production function used by BQ may represent a realistic scenario for least-developed and developing countries, where individuals earn low wages and cannot allocate adequate private resources for health care, but it does not capture the effects of health spending on longevity in developed countries. Complementarity between public and private inputs in the longevity production function means that a marginal increase in private effort to better health is more efficient as a means of higher longevity when public investments in health are high (Dow et al., 1999). The present work considers a technology where both the private and public inputs are complements for every x_t. Therefore, an increase in public health investments always generates an increase in the marginal productivity of private health investments, that is $\theta''_{x_t,\eta_t}(x_t, \eta_t) > 0$. This is captured by the following Cobb-Douglas longevity production function:

$$\theta(x_t, \eta_t) := \frac{x_t^{\rho} \eta_t^{1-\rho}}{Z}, \tag{1}$$

where $\rho \in (0, 1)$ is the constant elasticity of longevity with respect to private investments in health and Z is a positive parameter that will appropriately be fixed to get well-defined economic dynamics ($\theta(x_t, \eta_t) \in (0, 1]$).[9] We note that Z is a scaling constant that has the same unit of measurement of x_t and η_t. The expression in (1) implies that public and private inputs tend to be complementary in forming the society's health capital. As an example, one may think about the beneficial effects that some health programmes financed via public health investments can play in reducing air or water pollution as well as how they can indeed complement (private) health support via, for instance, better nutrition or medical screening.

The public input in the longevity production function is determined by public investments in health per young person at time t, p_t, that is $\eta_t = \eta(p_t)$. These services are provided by the government at a balanced budget by levying labour income taxes at the constant rate $0 \leq \tau < 1$ (Chakraborty, 2004; Bhattacharya and Qiao, 2007; Fanti and Gori, 2014). Therefore,

$$p_t = \tau w_t \ell_t. \tag{2}$$

We assume that $\eta(p_t)$ takes the form:

$$\eta(p_t) := \underline{\eta} + p_t^{\delta}, \tag{3}$$

where $0 < \delta \leq 1$ is a parameter that weights the intensity of the effects of the public health expenditure as a means of higher longevity (i.e., η is always an increasing concave function).

[9]To obtain endogenous fluctuations, BQ assumed that the elasticity of longevity with respect to private investments depends on public health investments. This elasticity measures how private health spending reacts to a change in the public component. More importantly, BQ adjusted parameter b to get $\theta < 1$ at the steady state. However, in their model nothing guarantees that longevity remains below unity if the economy lies on a trajectory that generates either monotonic or complex dynamics.

The expression in (3) implies that if $\tau = 0$, $\eta(0) = \underline{\eta} > 0$ (i.e., a minimum level of public health support exists even in the absence of public intervention) and if $\tau > 0$, $\eta(p_t) > \underline{\eta}$.

The budget constraint of a young individual of generation t is $s_t + x_t = w_t \ell_t (1 - \tau)$. This constraint implies that available labour income is divided between saving, s_t, and private health expenditure, x_t. From (1), (2), (3) and the budget constraint when young, the government takes resources away from the private sector through a proportional (linear) income taxation uses the revenue so collected to finance health services in a non-linear way. Consumption when old, C_{t+1}, is constrained by the amount of resources saved when young plus expected interest accrued from t time to time $t+1$, $C_{t+1} = R^e_{t+1} s_t$, where R^e_{t+1} is the expected interest factor, which will be realised at time $t+1$. The return to savings when young is used starting from the end of youth or, alternatively, the beginning of old age. Therefore, the lifetime budget constraint of an individual of generation t can be written as follows:

$$C_{t+1} = R^e_{t+1}[w_t \ell_t (1 - \tau) - x_t]. \tag{4}$$

By normalising the utility from death to zero, the individual representative of generation t has preferences (over the lifetime) towards leisure, private health spending and consumption described by the following Constant Inter-temporal Elasticity of Substitution (CIES) expected utility function:

$$U(\bar{\ell} - \ell_t, x_t, C_{t+1}) = \frac{(\bar{\ell} - \ell_t)^{1-\gamma}}{1-\gamma} + \theta(x_t, \eta_t) \frac{C_{t+1}^{1-\mu}}{1-\mu}, \tag{5}$$

where $\theta(x_t, \eta_t)$ is defined in (1), $\gamma > 0$ ($\gamma \neq 1$) measures the constant elasticity of marginal utility of leisure and $\mu > 0$ measures the constant elasticity of marginal utility of material consumption. We additionally require that $\mu < 1$ to avoid paradoxical effects of longevity on utility (Rosen, 1988; Hall and Jones, 2007; Gori et al., 2019a). The condition $\mu < 1$ also implies gross substitutability between leisure and consumption, that is $-CU''_C / U'_C < 1$ (Grandmont et al., 1998). The general isoelastic specification for the expected utility function in (5) is concave if and only if $\rho < \mu$, which is assumed to hold throughout the article. Decisions about leisure and private health spending when young, and decisions about consumption when old determine saving behaviour of the individual, who takes η_t, τ, and w_t and R^e_{t+1} as given. By using (4) and (5), the optimisation programme can be written as follows:

$$\max_{x_t, \ell_t} \left\{ \frac{(\bar{\ell} - \ell_t)^{1-\gamma}}{1-\gamma} + \frac{x_t^{\rho} \eta_t^{1-\rho}}{Z} \frac{\left\{ R^e_{t+1}[w_t \ell_t (1-\tau) - x_t] \right\}^{1-\mu}}{1-\mu} \right\}, \tag{6}$$

with $0 \leq x_t \leq w_t \ell_t (1 - \tau)$, and $0 \leq \ell_t \leq \bar{\ell}$. Maximising (6) with respect to ℓ_t and x_t gives the first order conditions:

$$\frac{x_t^{\rho} \eta_t^{1-\rho} \{R^e_{t+1}[w_t \ell_t (1-\tau) - x_t]\}^{1-\mu} w_t (1-\tau)}{Z[w_t \ell_t (1-\tau) - x_t]} = (\bar{\ell} - \ell_t)^{-\gamma}, \tag{7}$$

$$\frac{x_t}{\ell_t} = \frac{\rho w_t (1-\tau)}{1 + \rho - \mu}. \tag{8}$$

Eq. (8) implies that the ratio between x_t and ℓ_t is independent of the interest factor, R^e_{t+1}, but depends on the constant elasticity of the health technology with respect to the private input, ρ. By using the implicit function theorem, it follows that (see Appendix A for details): 1) an increase in the wage per unit of labour always increases the labour supply, $\frac{\partial \ell_t}{\partial w_t} > 0$ (i.e., the substitution effect dominates the income effect); an increase in the interest factor positively affects the return to labour in the current period so that leisure reduces, $\frac{\partial \ell_t}{\partial R^e_{t+1}} > 0$; an increase in public health investments improves the individual health status and the labour supply, $\frac{\partial \ell_t}{\partial \eta_t} > 0$; 2) an increase in the wage per unit of labour makes the individual richer and increases the resources for private health investments $\frac{\partial x_t}{\partial w_t} > 0$; an increase in the interest factor positively affects the return to private health investments in the current period, $\frac{\partial x_t}{\partial R^e_{t+1}} > 0$; private health investments complement the public ones, that is $\frac{\partial x_t}{\partial \eta_t} > 0$. By using these results and Eq. (8), it follows that the ratio of private health spending to *labour* and the ratio of private health spending to *leisure* are both increasing functions of the wage rate. In fact, we have that $\partial(\frac{x_t}{\ell_t})/\partial w_t = \frac{\rho(1-\tau)}{1+\rho-\mu} > 0$ and $\partial(\frac{x_t}{\bar{\ell}-\ell_t})/\partial w_t = \frac{\partial x_t}{\partial w_t}/(\bar{\ell}-\ell_t) + x_t \frac{\partial \ell_t}{\partial w_t}/(\bar{\ell}-\ell_t)^2 > 0$. Therefore, an increase in the wage rate makes private health investment more important for the welfare of individuals. In addition, given the individual budget constraint when young $s_t = w_t \ell_t(1-\tau) - x_t$ and the expression in (8) we get $\frac{\partial s_t}{\partial w_t} = \frac{(1-\tau)(1-\mu)}{1+\rho-\mu}\left[\ell_t(w_t) + \frac{\partial \ell_t}{\partial w_t} w_t\right] > 0$. Therefore, an increase in w_t generates an increase in the available working income $(w_t \ell_t(1-\tau))$ that more than compensates the increase in private health spending so that saving and consumption increase. Finally, $\frac{\partial s_t}{\partial R^e_{t+1}} = \frac{w_t(1-\tau)(1-\mu)}{1+\rho-\mu}\frac{\partial \ell_t}{\partial R^e_{t+1}} > 0$. This implies that an increase in the interest factor increases saving and consumption.

At time t identical and competitive firms produce a homogeneous good (Y_t) by combining capital (K_t) and labour (L_t) through a Cobb-Douglas production function with constant returns to scale, that is:

$$Y_t = AF(K_t, L_t) = AK_t^\alpha L_t^{1-\alpha}, \tag{9}$$

where $A > 0$ is a parameter that measures technological progress and $0 < \alpha < 1$ is the capital share. Profits are given by $AK_t^\alpha L_t^{1-\alpha} - w_t L_t - R_t K_t$. Then, by assuming that the capital stock fully depreciates at the end of every period and output is sold at unit price, profit maximisation by the typical firm that takes factor prices as given implies that the marginal productivity of labour and the marginal productivity of capital are respectively equal to the wage rate and the interest factor. Knowing that the temporary equilibrium condition in the labour market is $L_t = \ell_t$, we have that:

$$w_t = (1-\alpha)AK_t^\alpha \ell_t^{-\alpha}, \tag{10}$$

$$R_t = \alpha AK_t^{\alpha-1}\ell_t^{1-\alpha}. \tag{11}$$

The market-clearing condition in the capital market is given by the condition $K_{t+1} = s_t = w_t \ell_t(1-\tau) - x_t$. Then, by using (2), (3), (7), (8), (10), (11) and knowing that individuals have perfect foresight, the two-dimensional map characterising the equilibrium dynamics of the economy is the following:

$$M : \begin{cases} K_{t+1} = V_1(K_t, \ell_t) := A(1-\alpha)\left[\frac{(1-\tau)(1-\mu)}{1+\rho-\mu}\right] K_t^{\alpha} \ell_t^{1-\alpha} \\ \ell_{t+1} = V_2(K_t, \ell_t) := P K_t^{-\frac{\alpha[(1-\mu)\alpha+\rho]}{(1-\alpha)(1-\mu)}} \frac{\ell_t^{\frac{(1-\mu)\alpha^2+(1-\rho)(1-\alpha)+\mu\alpha}{(1-\alpha)(1-\mu)}}}{(\bar{\ell}-\ell_t)^{\frac{\gamma}{(1-\alpha)(1-\mu)}}} \{\underline{\eta} + [\tau A(1-\alpha) K_t^{\alpha} \ell_t^{1-\alpha}]^{\delta}\}^{\frac{\rho-1}{(1-\alpha)(1-\mu)}} \end{cases}, \tag{12}$$

where

$$P := \left\{ \frac{Z(1+\rho-\mu)^{(1-\mu)\alpha-1+\rho}(1-\mu)^{1-(1-\mu)\alpha}}{\rho^{\rho} A^{(1-\mu)\alpha+1+\rho-\mu} \alpha^{1-\mu} [(1-\tau)(1-\alpha)]^{(1-\mu)\alpha+\rho}} \right\}^{\frac{1}{(1-\alpha)(1-\mu)}}. \tag{13}$$

Compared to the equation of capital accumulation in a standard model with endogenous labour supply, the extra term in brackets in the first equation of (12) is due to the coexistence of private and public expenditures in health. In fact, if $\tau = \rho = 0$, function V_1 boils down to $A(1-\alpha)K_t^{\alpha}\ell_t^{1-\alpha}$.

Given the couple (K_t, ℓ_t), it is possible to compute its subsequent iterate if and only if we start from a point in set:

$$W_1 = \{(K_t, \ell_t) \in R^2 : K_t > 0, 0 < \ell_t < \bar{\ell}\}. \tag{14}$$

Nevertheless, feasible trajectories lie in a set smaller than W_1. This is because by starting from an initial condition in W_1 it is possible to have an iterate from which the existence of the subsequent one is not guaranteed. Then, we introduce the set of feasible trajectories, which is defined as follows:

$$W_2 = \{(K_t, \ell_t) \in R^2 : M^n(K_t, \ell_t) \in W_1, \forall n > 0\}, \tag{15}$$

where $M^n(K_t, \ell_t)$ is the nth iterate of the map applied to point (K_t, ℓ_t).

As one of the main objectives of this article is the global analysis of map M, we should guarantee that $\theta \in (0, 1]$ for every iterate by defining an appropriate threshold value of Z. From a mathematical point of view, this procedure would not apply due to the existence of basins of attraction for which K may take values without bounds. However, from the first equation in (12) we have that:

$$K_{t+1} = A(1-\alpha)\left[\frac{(1-\tau)(1-\mu)}{1+\rho-\mu}\right] K_t^{\alpha}\ell_t^{1-\alpha} \leq d(K_t) := A(1-\alpha)\left[\frac{(1-\tau)(1-\mu)}{1+\rho-\mu}\right] K_t^{\alpha}\bar{\ell}^{1-\alpha}, \tag{16}$$

where the right-hand side of the inequality in (16) gives the accumulation of capital if individuals entirely allocate their time endowment to labour. Let $K_{\max}$ be the stationary-state value of map d. Then, every feasible trajectory (i.e., belonging to set W_2) generated by map M is economically meaningful if $K_0 < K_{\max}$. Starting from $K_0 < K_{\max}$ then $K_t < K_{\max}$ for any t. By substituting $K_{\max}$ and $\bar{\ell}$ for K_t and ℓ_t, respectively, in (1) we can define the threshold value

$$\widetilde{Z} := (\bar{\ell}\rho)^{\rho}\left[\frac{A(1-\alpha)(1-\tau)(1-\mu)^{\alpha}}{1+\rho-\mu}\right]^{\frac{\rho}{1-\alpha}} \left\{\underline{\eta} + (\bar{\ell}\tau)^{\delta}\left[A(1-\alpha)\left(\frac{(1-\tau)(1-\mu)}{1+\rho-\mu}\right)^{\alpha}\right]^{\frac{\delta}{1-\alpha}}\right\}^{1-\rho}, \tag{17}$$

such that $\theta(x_t, \eta_t) < 1$ holds for every t by letting $Z > \widetilde{Z}$. In what follows, we will consider feasible trajectories such that $K_0 < K_{\max}$.

Regarding the existence of fixed points of map M, we have the following result.

Proposition 1 *(a) Map M always admits at least a fixed point in W_2. (b) The number of fixed points is generically odd and it is at most three.*

Proof. Fixed points of map M are solutions of the system:

$$\begin{cases} K = V_1(K, \ell) \\ \ell = V_2(K, \ell) \end{cases} . \tag{18}$$

From the first equation in (18), we have that

$$K = q(\ell) := \left[\frac{A(1-\alpha)(1-\tau)(1-\mu)}{1+\rho-\mu}\right]^{\frac{1}{1-\alpha}} \ell. \tag{19}$$

By substituting (19) in the second equation of system (18) for K, the stationary-state coordinate values of ℓ are solutions of the following equation:

$$g(\ell) = 1, \tag{20}$$

where

$$g(\ell) := \frac{V_2(q(\ell), \ell)}{\ell} = N \ell^{\frac{\mu-\rho}{(1-\alpha)(1-\mu)}} (\bar{\ell} - \ell)^{\frac{-\gamma}{(1-\alpha)(1-\mu)}} \left\{ \underline{\eta} + (\ell\tau)^{\delta} \left[A(1-\alpha) \left(\frac{(1-\tau)(1-\mu)}{1+\rho-\mu} \right)^{\alpha} \right]^{\frac{\delta}{1-\alpha}} \right\}^{\frac{\rho-1}{(1-\alpha)(1-\mu)}}, \tag{21}$$

and

$$N := P \left[\frac{1+\rho-\mu}{A(1-\alpha)(1-\tau)(1-\mu)} \right]^{\frac{(1-\mu)\alpha+\rho}{(1-\alpha)^2(1-\mu)}} . \tag{22}$$

Then, $\lim_{\ell \to 0^+} g(\ell) = 0$, $\lim_{\ell \to \bar{\ell}^-} g(\ell) = +\infty$ as $\rho < \mu$ holds. From the continuity of g it follows that at least a fixed point always exists. In order to determine the number of fixed points of map M, we characterise the monotonic intervals of g. For $0 < \ell_t < \bar{\ell}$ we have that $sgn\{g'(\ell)\} = sgn\{H(\ell)\}$, where

$$\begin{aligned} H(\ell) := {} & (\tau\ell)^{\delta} (1+\rho-\mu)^{\frac{\alpha\delta}{\alpha-1}} \{[\mu - (1-\delta)\rho - \delta](\bar{\ell} - \ell) + \gamma\ell\} + \\ & + \underline{\eta} \left[A(1-\alpha)(1-\tau)^{\alpha}(1-\mu)^{\alpha}\right]^{\frac{\delta}{\alpha-1}} \left[(\mu-\rho)(\bar{\ell}-\ell) + \gamma\ell\right]. \end{aligned} \tag{23}$$

By computing $H''(\ell)$, there exists at most a point $\hat{\ell} \in (0, \bar{\ell})$ such that $H''(\hat{\ell}) = 0$. Then, H'' changes sign at most once in the interval $(0, \bar{\ell})$ so that H' changes sign at most twice. From $\lim_{\ell \to 0^+} H(\ell) > 0$ and $\lim_{\ell \to \bar{\ell}^-} H(\ell) > 0$, it follows that H changes sign at most twice in the interval $(0, \bar{\ell})$ $(+, -, +)$. Therefore, g can have at most two changes in its monotonicity and the maximum number of fixed points of map M is three. ■

The study of the expression in H shows that a necessary condition for the existence of multiple equilibria is

$$\rho < \mu < (1-\delta)\rho + \delta, \tag{24}$$

whereas a sufficient condition for the existence of a unique equilibrium is

$$(1-\delta)\rho + \delta < \mu < 1. \tag{25}$$

We recall that $\mu < 1$ implies gross substitutability between material consumption and leisure. Therefore, multiple equilibria can exist only when the degree of substitutability between consumption and leisure is sufficiently low (i.e., the degree of complementarity between consumption and labour is sufficiently high). This result is confirmed by 1) the assumption of exogenous labour supply (i.e. individuals inelastically supply their whole time endowment to firms)[10] and 2) the existence of either young-age material consumption or young-age and old-age material consumption. In these cases, in fact, the model exhibits one and only one globally asymptotically stable steady-state equilibrium (starting from $K_0 > 0$). Therefore, the more anelastic the labour supply, the less an increase in the marginal health tax rate of a proportional tax scheme distorts individual choices about labour and the less the excess burden or deadweight loss of taxation (if the labour supply were perfectly anelastic there would be no distortion and no deadweight loss when the marginal tax rate changes). Therefore, the number of fixed points (i.e., the number of the long-term outcomes) of the economy changes depending on whether labour is supplied elastically or inelastically to firms. In the former case, labour is a control variable and an increase in the marginal health tax rate changes the individual incentives to substitute out leisure with material consumption and private health spending. An increase in the marginal tax rate reduces the opportunity cost of leisure time as it reduces the net wage per unit of labour and thus increases the demand for leisure by reducing the supply of labour (if the substitution effect dominates). This contributes to change the magnitude of the marginal rates of substitution between labour and consumption and between labour and private health expenditure. The reduction in the net wage also implies a reduction in the disposable income. Therefore, if the individual wants to maintain consumption and private health expenditure at the level before the increase in the marginal tax rate, he must increase the labour supply. A change in the marginal tax rate also affects the amount of resources allocated to public expenditure in health. This is because an increase in τ causes a direct increase in the tax revenue but also causes an indirect reduction in it through the reduction in the labour supply and the tax base. If the tax rate is not too high the direct effect dominates and the tax revenue increases following an increase in τ. This in turn contributes to increase (ceteris paribus) the weight of

[10]The proof is available on request.

the public component in the longevity production function, which positively affects the health status of the representative individual by also lengthening his quantity of life. This contributes to raise both the labour supply and savings. Definitively, a change in τ produces counterbalancing effects on longevity, savings, the dynamics of the labour supply and the dynamics of capital accumulation (map M). Therefore, also the shape of g, which determines the mechanism of creation and disruption of fixed points, changes when τ varies (Figure 2). By the analysis of g, it is possible to verify that for values of the marginal tax rate close or equal to 0 or close to 1 there exists one and only one stationary-state equilibrium (irrespective of the other parameters of the model). The intuition is simple as for $\tau \to 0$ the distortions on the labour supply are low and the weight of the public component in the longevity production function is also low. When $\tau \to 1$ the government tends to confiscate the income produced by the individuals so that the labour supply, savings and the accumulation of capital tend to 0. From this discussion, we can conclude that the conditions for the existence of multiple equilibria are a low degree of substitutability between consumption and leisure and an intermediate value of the health tax rate by avoiding to distort (reduce) too much the individual choices about labour through the effects on the tax base and thus the relative weight of the public component in the longevity production function. We stress that these two conditions are only necessary as the final outcome depends on the magnitude of the other parameters of the model. In contrast, in the latter (limit) case the whole time endowment is inelastically supplied to firms and an increase in the marginal tax rate does not change individual incentives to substitute out leisure with material consumption and private health spending. The reduction in the disposable income following the increase in τ reduces capital accumulation (as a direct effect) but also increases the government total revenue, the public health expenditure, the weight of the public component in the longevity production function and eventually the quantity of life and the incentive to save (as an indirect effect) as individuals live longer. In this case, there are no effects on the tax base as the labour supply is constant. Unlike Chakraborty (2004), the assumption of exogenous labour supply in the present context implies that the negative direct effect always dominates and capital accumulation monotonically reduces.

In the case of multiple fixed points, we introduce the following notation to define the stationary-state values of K and ℓ: (K^*, ℓ^*), (K^{**}, ℓ^{**}) and (K^{***}, ℓ^{***}), where $K^* < K^{**} < K^{***}$ and (see Eq. (19)) $\ell^* < \ell^{**} < \ell^{***}$. It is now interesting to analyse the individual wellbeing corresponding to each of them for comparison purposes. In this regard, we have the following result.

Proposition 2 *In the case of three fixed points we have that* $U\mid_{K=K^*,\ell=\ell^*} < U\mid_{K=K^{**},\ell=\ell^{**}} < U\mid_{K=K^{***},\ell=\ell^{***}}$.

Proof. The result follows by using the expressions in (1), (2), (3), (4), (8), (10), (11), (19), and by noting that the utility function (5) evaluated at a generic stationary state (K^{ss}, ℓ^{ss})

$$U(\ell^{ss}) := \frac{(\bar{\ell}-\ell^{ss})^{1-\gamma}}{1-\gamma} + \frac{1}{Z}\rho^{\rho}\alpha^{1-\mu}A^{\frac{1+\rho-\mu}{1-\alpha}}\left[\frac{(1-\tau)(1-\alpha)}{1+\rho-\mu}\right]^{\frac{(1-\mu)\alpha+\rho}{1-\alpha}}(1-\mu)^{\frac{1-(2+\rho-\mu)\alpha}{\alpha-1}}(\ell^{ss})^{1+\rho-\mu}\times \\ \times\left\{\underline{\eta}+(\tau\ell^{ss})^{\delta}\left[A(1-\alpha)\left(\frac{(1-\tau)(1-\mu)}{1+\rho-\mu}\right)^{\alpha}\right]^{\frac{\delta}{1-\alpha}}\right\}^{1-\rho}, \quad (26)$$

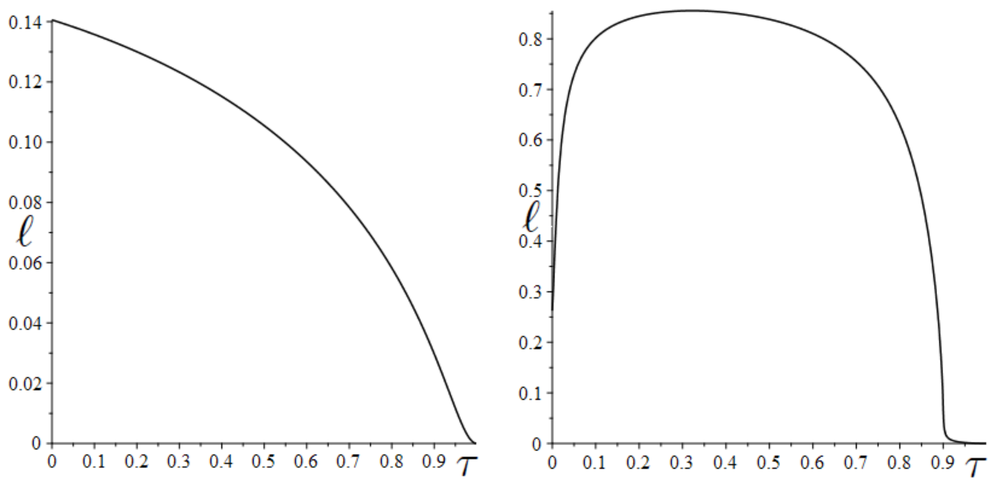
is increasing in ℓ^{ss}. ■

We now discuss the effects of a change in τ on the long-term values of both the labour supply and capital accumulation (comparative statics) if there exists only one fixed point, (K^*, ℓ^*). Following the narrative of Chakraborty (2004), an increase in τ generates a twofold effect on ℓ (and K) at the stationary state:[11] a direct depressing effect through the reduction in the disposable income of the individual and an indirect positive feedback effect that passes through the increase in the public health expenditure and the improvement in the individual health status that causes an increase in longevity. This in turn generates an increase in saving that makes resources available to increase private health investments, thus implying an additional positive feedback effect on longevity. If the former (resp. the latter) effect dominates, an increase in τ reduces (resp. increases) the labour supply (and capital accumulation). This is illustrated in Figure 1, Panels (a) and (b). Panel (a) shows a monotonic decreasing relationship between the stationary-state value of ℓ and the health tax rate τ. Differently, Panel (b) illustrates an inverted U-shaped relationship between the stationary-state value of ℓ and the health tax rate τ. In this case, the indirect positive feedback of an increase in τ on the stationary-state value of ℓ dominates for values of τ smaller than almost 0.3. When τ increases further, the negative effect due to the reduction in the disposable income overcomes the indirect positive effect and the labour supply reduces. The narrative behind this story may also be explained, according to standard economic theory, in terms of substitution effect and income effect. An increase in the marginal tax rate causes a reduction in the net wage per unit of time spent working (disposable income), but also implies a reduction in the price that the representative agent must pay to buy one additional unit of leisure. As leisure is now cheaper, the representative agent substitutes the demand of material consumption and private health spending out with the demand for leisure so that the labour supply reduces through this channel. An increase in the marginal tax rate also causes a reduction in the purchasing power of the representative agent. As leisure is a normal good (see Eq. (43), Appendix A), this leads to a reduction in demand for leisure and an increase in labour supply through this channel. Panel (a) shows that the substitution effects always dominates, whereas in Panel (b) the substitution effects dominates only when τ is sufficiently high. However, this is only part of the story as Figure 1 illustrates the behaviour of the labour supply in the long term, i.e. once the general equilibrium effects of a change in τ have been considered. In fact, an increase in the tax rate also implies better health and higher

[11] We recall that there exists a monotonic increasing relationship between K and ℓ at the stationary state (see Eq. (19)). Therefore, an increase in ℓ always implies an increase in K.

longevity. As an individual life is longer, he also tends to increase savings to finance future consumption. For doing this, the labour supply must increase. When the elasticity of marginal utility of leisure (γ) is sufficiently high (alternatively, when the proxy for measuring the (constant) elasticity of substitution in leisure ($1/\gamma$) is sufficiently low) (Panel (a)), the substitution effect dominates and labour supply reduces as the disposable income reduces. This is because the long-term demand for leisure does not react sufficiently to let the income and general equilibrium effects dominate over the substitution effect. When the elasticity of marginal utility of leisure (γ) is sufficiently low (alternatively, when the proxy for measuring the (constant) elasticity of substitution in leisure ($1/\gamma$) is sufficiently high) (Panel (b)), the income effect and the general equilibrium effects dominate and labour supply increases as the disposable income reduces.

In addition, as was stressed in the discussion above, τ can also play an important role on the existence of one fixed point or three fixed points. This is shown in Figure 2 that illustrates the graph of g and its intersection points with the horizontal straight line at 1 (according to Eq. (20)) when τ varies. The red (resp. blue) [resp. black] line refers to $\tau = 0.2$ (resp. $\tau = 0.4$) [resp. $\tau = 0.55$]. When the health tax rate is sufficiently small (red line) there exists only one fixed point (K^*, ℓ^*). For an intermediate value of τ (blue line) there exist three fixed points, (K^*, ℓ^*), (K^{**}, ℓ^{**}) and (K^{***}, ℓ^{***}). With this parameter set (see the caption of Figure 2), an increase in the health tax rate generates an increase in the public health expenditure and longevity that imply an increase in the supply of labour and savings that in turn change the shape of g (see the discussion above) and cause an increase in the accumulation of capital (and private spending on health) to give birth to two additional fixed points, (K^{**}, ℓ^{**}) and (K^{***}, ℓ^{***}), that Pareto dominate (K^*, ℓ^*) thus following a development trajectory. If τ increases further (black line), the direct negative effect on the individual disposable labour income dominates and both the labour supply and capital accumulation reduce so that the economy comes back one fixed point with smaller stationary-state values of K and ℓ than those observed at $\tau = 0.2$ (red line).



(a) (b)

Figure 1. Comparative statics with respect to τ if there exists one fixed point (K^*, ℓ^*). (a) Parameter set: $A = 3$, $Z = 2$, $\bar{\ell} = 1$, $\alpha = 0.33$, $\delta = 1$, $\gamma = 9.9$, $\mu = 0.9$, $\rho = 0.5$ and $\underline{\eta} = 0.53$. (b) Parameter set: $A = 8$, $Z = 2$, $\bar{\ell} = 1$, $\alpha = 0.33$, $\delta = 1$, $\gamma = 0.9$, $\mu = 0.9$, $\rho = 0.5$ and $\underline{\eta} = 0.013$.

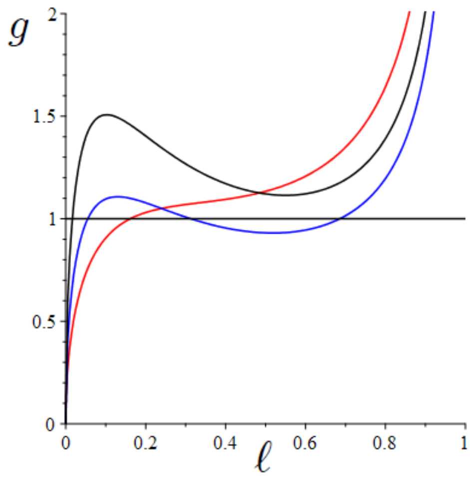


Figure 2. Mechanism of creation and disruption of fixed points when τ varies. Parameter set: $A = 5$, $Z = 2.1$, $\bar{\ell} = 1$, $\alpha = 0.33$, $\delta = 1$, $\gamma = 0.29$, $\mu = 0.19$, $\rho = 0.5$ and $\underline{\eta} = 0.5$. Red line ($\tau = 0.2$); blue line ($\tau = 0.4$); black line ($\tau = 0.55$).

We now study the local stability properties of the stationary states and summarize the results in Proposition 3. Before doing this, however, we recall that a fixed point is locally indeterminate (resp. locally determinate)[12] if for every arbitrarily small neighbourhood of it, and for a given value of the state variable close enough to its coordinate value at the stationary state, there exists a continuum of values (resp. a unique value) of the control variable for which an equilibrium trajectory converges (resp. approaches) towards the fixed point. From a mathematical point of view, the fixed point is a sink (resp. a saddle).

Proposition 3 *(a) The fixed point (K^*, ℓ^*) is a saddle (locally determinate). (b) If there exist three distinct fixed points, (K^{***}, ℓ^{***}) is a saddle (locally determinate) and the intermediate one (K^{**}, ℓ^{**}) is either unstable (source) or a sink (locally indeterminate).*

Proof. In order to evaluate the stability properties of the system at a generic fixed point (K^{ss}, ℓ^{ss}) let us consider the following Jacobian matrix:

$$J(K^{ss}, \ell^{ss}) := \begin{pmatrix} J_{1,1} & J_{1,2} \\ J_{2,1} & J_{2,2} \end{pmatrix}, \tag{27}$$

[12] A locally determinate fixed point is usually defined as a saddle-path stable equilibrium.

where

$$J_{1,1} := \alpha, \tag{28}$$

$$J_{1,2} := (1-\alpha)\left[\frac{A(1-\alpha)(1-\tau)(1-\mu)}{1+\rho-\mu}\right]^{\frac{1}{1-\alpha}}, \tag{29}$$

$$J_{2,1} := \frac{\alpha[(\mu-1)\alpha-\rho]}{(1-\alpha)(1-\mu)}\frac{\ell^{ss}}{K^{ss}} + \frac{\alpha\delta(\rho-1)}{(1-\alpha)(1-\mu)}\frac{\ell^{ss}p^{ss}}{K^{ss}\eta^{ss}}, \tag{30}$$

$$J_{2,2} := \frac{(1-\mu)\alpha^2+(\mu+\rho-1)\alpha+1-\rho}{(1-\alpha)(1-\mu)} + \frac{\gamma}{(1-\alpha)(1-\mu)}\frac{\ell^{ss}}{(\bar{\ell}-\ell^{ss})} + \frac{(\rho-1)(1-\alpha)\delta}{(1-\alpha)(1-\mu)}\frac{p^{ss}}{\eta^{ss}}, \tag{31}$$

η^{ss} and p^{ss} are the value of η and p evaluated at a generic fixed point (K^{ss}, ℓ^{ss}). From (27) we have that:

$$\begin{array}{c} Det(J(K^{ss},\ell^{ss})) := \frac{(\bar{\ell}-\ell^{ss})^{\frac{-\gamma}{(1-\alpha)(1-\mu)}}(\bar{\ell}-\ell^{ss}+\gamma\ell^{ss})[\underline{\eta}+(\tau A(1-\alpha)(K^{ss})^{\alpha}(\ell^{ss})^{1-\alpha})^{\delta}]^{\frac{\rho-1}{(1-\alpha)(1-\mu)}}}{(1+\rho-\mu)K^{ss}\ell^{ss}(\bar{\ell}-\ell^{ss})}\times \\ \times P\alpha A(1-\tau)(K^{ss})^{\alpha}(\ell^{ss})^{1-\alpha}(K^{ss})^{\frac{\alpha[(\mu-1)\alpha-\rho]}{(1-\alpha)(1-\mu)}}(\ell^{ss})^{\frac{(1-\mu)\alpha^2+(\mu+\rho-1)\alpha+1-\rho}{(1-\alpha)(1-\mu)}} > 0 \end{array}. \tag{32}$$

Focusing on the dependence of ℓ_{t+1} on ℓ_t, such an expression is the ratio of two positive and increasing functions of ℓ_t, $I_1(\ell_t) := PK_t^{-\frac{\alpha[(1-\mu)\alpha+\rho]}{(1-\alpha)(1-\mu)}}\frac{\ell_t^{\frac{(1-\mu)\alpha^2+(1-\rho)(1-\alpha)+\mu\alpha}{(1-\alpha)(1-\mu)}}}{(\bar{\ell}-\ell_t)^{\frac{\gamma}{(1-\alpha)(1-\mu)}}}$ and $I_2(\ell_t) := [\underline{\eta} + I_3(\ell_t)]^{\frac{1-\rho}{(1-\alpha)(1-\mu)}}$, with $I_3(\ell_t) := (\tau A(1-\alpha)K_t^{\alpha}\ell_t^{1-\alpha})^{\delta}$. By evaluating $\partial\ell_{t+1}/\partial\ell_t$ when $\underline{\eta} = 0$ (implying $I_2(\ell_t) = I_3(\ell_t)^{\frac{1-\rho}{(1-\alpha)(1-\mu)}}$), we get:

$$\begin{array}{c} \frac{\partial\ell_{t+1}}{\partial\ell_t}\Big|_{\underline{\eta}=0} = \frac{PK_t^{-\frac{\alpha[(1-\mu)\alpha+\rho]}{(1-\alpha)(1-\mu)}}[A(1-\alpha)K_t^{\alpha\delta}]^{\frac{\rho-1}{(1-\alpha)(1-\mu)}}}{(1-\alpha)(1-\mu)}\times \\ \times\left\{\gamma\ell_t^{\frac{\alpha[(1-\mu)\alpha+\mu]}{(1-\alpha)(1-\mu)}}(\bar{\ell}-\ell_t)^{\frac{(1-\mu)\alpha+\mu-1-\gamma}{(1-\alpha)(1-\mu)}} + \alpha[\alpha+(1-\alpha)\mu]\ell_t^{\frac{(1-\mu)\alpha^2+\alpha-1+\mu}{(1-\alpha)(1-\mu)}}(\bar{\ell}-\ell_t)^{\frac{\gamma}{(1-\alpha)(1-\mu)}}\right\} > 0 \end{array}. \tag{33}$$

By direct computation, it follows that

$$\frac{\partial\left(\frac{I_1(\ell_t)}{I_3(\ell_t)^{\frac{1-\rho}{(1-\alpha)(1-\mu)}}}\right)}{\partial\ell_t} > 0 \Rightarrow \frac{\partial\left(\frac{I_1(\ell_t)}{I_2(\ell_t)}\right)}{\partial\ell_t} > 0, \tag{34}$$

and then $J_{2,2} > 0$. It follows that $Tr(J(K^{ss},\ell^{ss})) > 0$. From the results on trace and determinant of $J(K^{ss},\ell^{ss})$, a fixed point is a saddle (locally determinate) if $1 - Tr(J(K^{ss},\ell^{ss})) + Det(J(K^{ss},\ell^{ss})) < 0$, whereas it is a sink (locally indeterminate) or unstable if $1-Tr(J(K^{ss},\ell^{ss}))+Det(J(K^{ss},\ell^{ss})) > 0$. It is possible to study the sign of $1 - Tr(J(K^{ss},\ell^{ss})) + Det(J(K^{ss},\ell^{ss}))$ by considering how the graph of $g(\ell) = \frac{V_2(q(\ell),\ell)}{\ell}$ intersects the horizontal line $\ell = 1$. In fact, we have that

$$g'(\ell) = \left[\frac{\partial V_2(q(\ell),\ell)}{\partial K_t}\frac{\frac{\partial V_1(q(\ell),\ell)}{\partial\ell_t}}{1-\frac{\partial V_1(q(\ell),\ell)}{\partial K_t}} + \frac{\partial V_2(q(\ell),\ell)}{\partial\ell_t}\right]\frac{1}{\ell} - \frac{V_2(q(\ell),\ell)}{\ell^2}, \tag{35}$$

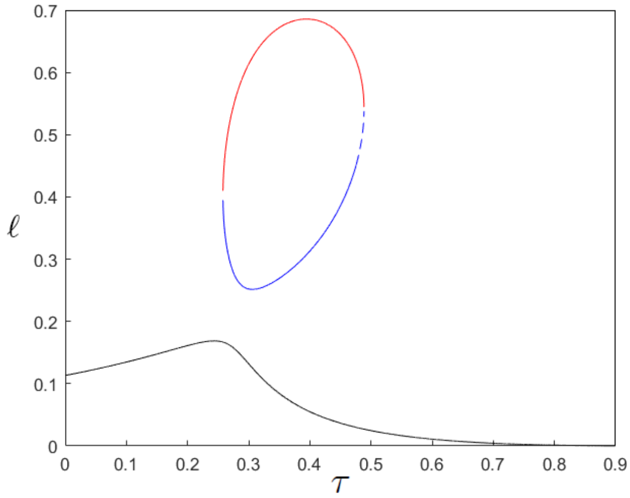
and

$$\begin{array}{c} g'(\ell^{ss}) = \frac{-1+(J_{1,1}+J_{2,2})-(J_{1,1}J_{2,2}-J_{2,1}J_{1,2})}{(1-\alpha)\ell^{ss}} = \\ = -\frac{1-Tr(J(K^{ss},\ell^{ss}))+Det(J(K^{ss},\ell^{ss}))}{(1-\alpha)\ell^{ss}} \end{array}. \tag{36}$$

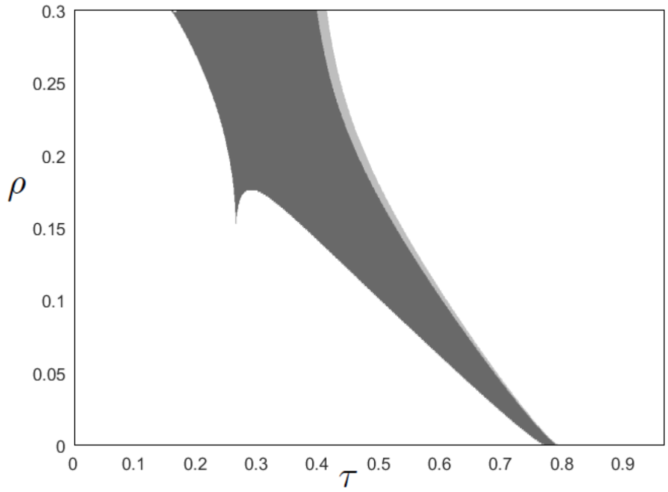
The expression in (36) is positive at the first fixed point (K^*, ℓ^*) and possibly at the third one (K^{***}, ℓ^{***}) if it exists, and it is negative at the intermediate fixed point (K^{**}, ℓ^{**}) if it exists. ■

A possible scenario – referred to the case in which the intermediate equilibrium (K^{**}, ℓ^{**}) is indeterminate – about the existence and stability of the fixed points is depicted in Figure 3, Panel (a). The figure shows the behaviour of the stationary-state values of ℓ of the system and their stability when τ varies. For low values of τ there exists only one stationary state, (K^*, ℓ^*), which is a saddle point (black line). Then, the map undergoes a saddle-node bifurcation at $\tau \cong 0.258$ through which both a locally asymptotically stable equilibrium (K^{**}, ℓ^{**}) (blue-solid line) and another saddle path stable fixed point (K^{***}, ℓ^{***}) (red line) also arise. For $\tau \cong 0.476$, the interior fixed point (K^{**}, ℓ^{**}) becomes unstable (source, which is represented by the blue-dashed line) through a (sub-critical) Neimark-Sacker bifurcation. For $\tau \cong 0.488$, the fixed points (K^{**}, ℓ^{**}) and (K^{***}, ℓ^{***}) disappear as a second saddle-node bifurcation occurs. For larger values of τ, the system admits only one fixed point, (K^*, ℓ^*), which is saddle path stable. However, it is possible to have scenarios in which the fixed point (K^{**}, ℓ^{**}) born through the occurrence of a saddle-node bifurcation is a source instead of an indeterminate equilibrium. Also, this last event can give rise to interesting phenomena that will analysed in the next section by using global analysis techniques.

Figure 3, Panel B shows a two-dimensional bifurcation diagram in the plane (τ, ρ). The figure is important as it outlines several possible long-term scenarios depending on whether the longevity production function depends on (i) private investment in health only ($\tau = 0$), (ii) public investment in health only ($\rho = 0$) or (iii) both public and private investments in health. The white regions in the figure identify the existence of a unique saddle point (K^*, ℓ^*). The dark grey region shows the existence of an indeterminate equilibrium (K^{**}, ℓ^{**}) that coexists with two saddles (K^*, ℓ^*) and (K^{***}, ℓ^{***}). The light grey region identifies the parameter space in which the interior equilibrium (K^{**}, ℓ^{**}) is a source coexisting with the saddles (K^*, ℓ^*) and (K^{***}, ℓ^{***}). The transition from a scenario in which there are three stationary states to the one in which there is only one stationary state can take place through the disappearance of (K^*, ℓ^*) and (K^{**}, ℓ^{**}) or the disappearance of (K^{**}, ℓ^{**}) and (K^{***}, ℓ^{***}) (in this case, given the notation introduced above, the fixed point (K^{***}, ℓ^{***}) becomes (K^*, ℓ^*)). We pinpoint that for $\tau = 0$ or $\rho = 0$ the fixed point is unique and it is a saddle (determinate). From the continuity of g with respect to τ it follows that multiple equilibria are possible only for values of τ sufficiently large (for a given positive value of ρ). This implies that the existence of a public expenditure in health that complements the private one is responsible for the existence of distinct development regimes. From a theoretical point of view, we stress that multiple equilibria exist by satisfying the gross substitutability condition between consumption and leisure ($\mu < 1$).



(a)



(b)

Figure 3. Parameter set: $A = 5$, $Z = 2.1$, $\bar{\ell} = 1$, $\alpha = 0.33$, $\delta = 1$, $\gamma = 0.29$, $\mu = 0.39$ and $\underline{\eta} = 0.5$. (a) One-dimensional bifurcation diagram for τ ($\rho = 0.19$). (b) Two-dimensional bifurcation diagram in the (τ, ρ) plane ($\rho < \mu$).

3 Global analysis

Since Cazzavillan et al. (1998) and Grandmont et al. (1998), several works belonging to either the classical OLG literature with finite lived agents or the discrete-time set up with infinite horizon optimising agents focused on the study of the local dynamics around the normalised equilibrium or models producing multiple equilibria, but leaving the issue of the global analysis unexplored. This is the case, for instance, of the relevant works of Nourry (2001) and Lloyd-Braga et al. (2007), who respectively studied problems of existence and local stability of steady-state equilibria in a general equilibrium economy à la Diamond (1965) with endogenous labour supply and problems of indeterminacy in a model with production externalities. The continuous-time framework, instead, generally accounts for models with a unique steady state in which the global study of the system does not provide further insights than those shown by the local analysis (Benhabib and Farmer, 1994; Benhabib and Perli, 1994).

None of the articles belonging to this literature, however, deals with health expenditure and endogenous longevity, and few of them concentrate on the study of the dynamics of a general equilibrium economy with productive public spending (Cazzavillan, 1996) or unproductive public spending (Guo and Lansing, 1998; Nourry et al., 2013; Nishimura et al., 2015) focusing just on the local analysis.

Instead, especially when multiple stationary equilibria exist and there are both state and control variables, the global analysis allows to clarify what happens far away from a stationary equilibrium and then get deeper results both from a theoretical point of view and policy perspective. Notably, this is relevant in growth models dealing with economic development, which is a long-term phenomenon. *By global analysis*, we consider the study of the basins of attraction of the stationary equilibria and the manifolds of the saddle points. As for the study of manifolds, we resort to several numerical exercises and techniques: from the simple backward iteration of the local approximation of the stable manifold of a saddle point to more sophisticated techniques (see Panchuck, 2015). Related to the global analysis of a dynamic system there exists the concept of *global indeterminacy*. In this regard, we recall that a system is globally indeterminate when there exist values of the state variable such that different choices on the control variable lead to different invariant sets. In this case, the initial condition of the stock of capital is not sufficient to define the long-term dynamics of the economy.

As a first step in the global analysis, the following Lemma shows that map M is invertible on the non-negative orthant. This property is shared with Agliari and Vachadze (2011), but not with Grandmont et al. (1998), where the map is invertible only in a neighbourhood of the fixed point.

Lemma 4 *Map M is invertible on W_2.*

Proof. The inverse map of M, i.e. M^{-1}, is solution of the following system:

$$M^{-1}: \begin{cases} \frac{\ell_t}{(\bar{\ell}-\ell_t)^{\gamma}} = \frac{\rho^{\rho}\alpha^{1-\mu}(1-\mu)^{(1-\mu)\alpha-1}\left[\frac{1+\rho-\mu}{A(1-\mu)}\right]^{(1-\mu)\alpha+\rho}}{ZA^{(1-\mu)(1+\alpha)-\rho}(1+\rho-\mu)^{(1-\mu)\alpha+\rho-1}} \ell_{t+1}^{(1-\alpha)(1-\mu)} K_{t+1}^{(1-\mu)\alpha+\rho} \left\{\underline{\eta} + \left[\frac{\tau(1+\rho-\mu)}{(1-\tau)(1-\mu)}\right]^{\delta} K_{t+1}^{\delta}\right\}^{1-\rho} \\ K_t = K_{t+1}^{\frac{1}{\alpha}} \left[\frac{1+\rho-\mu}{A(1-\alpha)(1-\tau)(1-\mu)}\right]^{\frac{1}{\alpha}} \ell_t^{\frac{\alpha-1}{\alpha}} \end{cases}. \quad (37)$$

■

The invertibility of a map is an important result for the study of the global properties of a dynamic system. For instance, it implies that the basins of attraction of any attracting set of a map are connected sets. Furthermore, as was previously recalled, the inverse map may allow to obtain the boundary of the basins of attraction and, more generally, the stable manifolds of the saddles. The importance of studying invariant manifolds in dynamic models with control variables relies on the fact that economic agents may coordinate themselves on these manifolds through their choices. In other words, these curves represent sets on which an economy develops.

Before performing the global analysis of map M, we recall the definitions of both the stable manifold

$$G^s(a) = \{x : M^n(x) \to a \ as \ n \to +\infty\}$$

and unstable manifold

$$G^u(a) = \{x : M^n(x) \to a \ as \ n \to -\infty\}$$

of a fixed point a. If the periodic point $a \in R^2$ is a saddle, then the stable (resp. unstable) manifold is a smooth curve through a, tangent at a to the eigenvector of the Jacobian matrix evaluated at a corresponding to the eigenvalue λ with $|\lambda| < 1$ (resp. $|\lambda| > 1$), see, e.g., Guckenheimer and Holmes (1983). Outside the neighbourhood of a, the stable and unstable manifolds may even intersect each other with dramatic consequences on the global dynamics of the model (Guckenheimer and Holmes, 1983, p. 22).[13] The global analysis of the map allows us to identify some scenarios (summarised below) related to different explanations (history driven or expectations driven) of economic development.[14]

Scenario 1. *History matters (path dependence).* Depending on the initial condition of the state variable (K), it is possible to converge towards different fixed points (path dependence). This phenomenon cannot be observed in BQ, where the labour supply is exogenous. This is because the long-term behaviour of their model is characterised by the convergence towards a unique regular or chaotic attractor and these two scenarios are mutually alternative. The authors interpret the second one as a possible explanation for the existence of different development paths the economy may follow. However, although it is well known that in a chaotic system small changes in the initial conditions may let an economy follow very different trajectories, the statistical properties of the resulting different time series are the same. This means that the frequencies with which the various levels of the state variable (capital stock) arise over time are the same for different initial conditions.[15] Therefore, *the model of BQ (with exogenous labour supply and private and public expenditures on health) cannot explain the existence of*

[13] Non-trivial intersection points of stable and unstable manifolds of a unique saddle cycle, or non-trivial intersection points between the stable manifold of one cycle and the unstable manifold of the other one are really important from a dynamic point of view because they sharply change the topological structure of the phase plane (global bifurcations). About this phenomenon, which is not deepen in this article, we refer to Agliari and Vachadze (2011).

[14] All the figures that follow have been obtained by numerical techniques and the arrows are added manually for reasons of clarity.

[15] Consider the following sentence reported in Bhattacharya and Qiao (2007, p. 2531-2532): "...imagine two

different development regimes. In other words, even starting from different initial conditions all economies will experience the same development scenario though this may happen at different dates. Things change dramatically by considering endogenous labour supply. To show how the model can generate different development trajectories, we use the following parameter set: $A = 10$, $Z = 3.35$, $\bar{\ell} = 1$, $\alpha = 0.34$ (Gollin, 2002), $\delta = 0.96$, $\gamma = 0.014$, $\mu = 0.8$, $\rho = 0.3$, $\tau = 0.041$ and $\underline{\eta} = 0.04$. Figure 4 shows the existence of two (determinate) saddle points (K^*, ℓ^*) and (K^{***}, ℓ^{***}), represented by the black dot and red dot respectively, towards which the economy approaches depending on the initial condition of the state variable K. In fact, these two saddles are separated by the unstable equilibrium (K^{**}, ℓ^{**}) (the white dot in the figure). For every $K^* < K^{**}$, there exists one and only one initial value of the control variable ℓ leading the economy on the stable manifold approaching[16] (K^*, ℓ^*) (the black stable manifold). For every $K^* > K^{**}$, there exists one and only one initial value of the control variable ℓ leading the economy on the stable manifold approaching (K^{***}, ℓ^{***}) (the black stable manifold). The stationary equilibrium (K^*, ℓ^*) represents a low-development regime (poverty trap), corresponding to which individuals offer a small fraction of their time endowment to firms and capital accumulation is small. Consequently, longevity will be low as the small capital stock installed in the economy does not allow adequate private investments in health, a subsequent improvement in the health status and an increase in the individual life span (there exists a vicious circle due to poorly treated diseases that keeps the productivity of labour and the individual willingness to invest in health at small levels). The economy is not able to capture the possible benefits that the complementarity between the public and private inputs in the longevity of the production technology may cause, so that a takeoff towards economic development is prevented. In contrast, the stationary equilibrium (K^{***}, ℓ^{***}) represents a high-development regime in which individuals choose to offer a larger fraction of their time endowment to firms and capital accumulation is high. Both the productivity of labour and capital accumulation increase, and this contributes to make private investments in health adequate to capture the benefits of the complementarity between the private and public

economies that differ only slightly in their initial conditions. One may approach k^* monotonically and eventually achieve a high per capita income and high level of life expectancy for its residents. The other may get "infected" by cyclical fluctuations in all its endogenous variables, sometimes coming 'very close' to the steady state k^*, but eventually being repelled away. Such an economy will not be able to sustain a permanently high level of real activity or a high level of life expectancy for its citizens. In other words, the model can generate dramatic reversals in life expectancy. This is in line with such reversals experienced by many countries in the last three decades". This sentence is not correct. This is because a unimodal map admits the existence of a unique attractor. Therefore, unless one decides to vary some key parameters of the model (*not the initial conditions*) all trajectories converge towards the same ω-limit set. Definitely, the model of BQ is about nonlinear (long-term) dynamics in an economic growth model. It is not a work about economic development as it cannot explain the coexistence of different long-term equilibria.

[16] This represents the unique possible choice of ℓ such that the economy approaches the long-term equilibrium (given an initial condition K_0). Other choices for ℓ define unfeasible trajectories (i.e., trajectories that do not belong to W_2).

inputs in the longevity production function. This in turn causes an increase in the health status and longevity. Saving therefore will be larger as individuals live longer producing an additional positive effect at the macroeconomic level. In this case, there exists a virtuous circle due to well treated diseases that makes economic development possible. Converging towards the low regime of development or the high regime of development is a matter of the initial condition (history matters) of the state variable (Chakraborty, 2004; Blackburn and Cipriani, 2002). Although the results detailed in this scenario can help understanding some reasons (related to health investments and mortality) why some countries prosper and others are entrapped in stagnation or poverty, the model is not yet able to explain the reasons why two economies starting from similar states (or the same state) converge towards different long-term regimes. We refer to the example of Korea and Philippines described by Lucas (1993) and reported in BQ. In other words, " "History" alone cannot in general determinates where the economy will end up." (Matsuyama, 1991, p. 617). To overcome this concern, the cases studied in Scenario 2 and Scenario 3 will allow to get plausible explanations of economic development based on reasons different from path dependence.

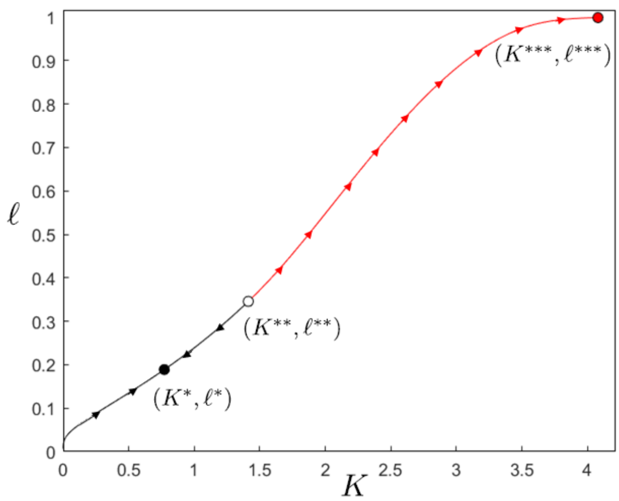
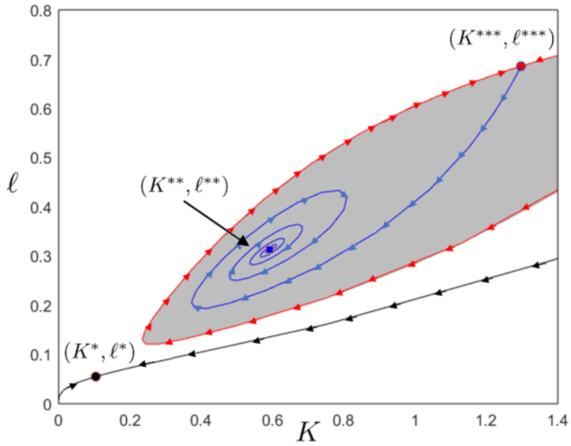


Figure 4. History matters (path dependence). Parameter set: $A = 10$, $Z = 3.35$, $\bar{\ell} = 1$, $\alpha = 0.34$, $\delta = 0.96$, $\gamma = 0.014$, $\mu = 0.8$, $\rho = 0.3$, $\tau = 0.041$ and $\underline{\eta} = 0.04$. The black (resp. red) curve is the stable manifold of the (black) saddle (K^*, ℓ^*) [resp. (red) saddle (K^{***}, ℓ^{***})]. (K^{***}, ℓ^{***}) Pareto dominates (K^*, ℓ^*). The steady-state coordinate values of K and ℓ are: $(K^*, \ell^*) = (0.7713, 0.1888)$, $(K^{**}, \ell^{**}) = (1.4124, 0.3457)$ and $(K^{***}, \ell^{***}) = (4.0806, 0.9987)$.

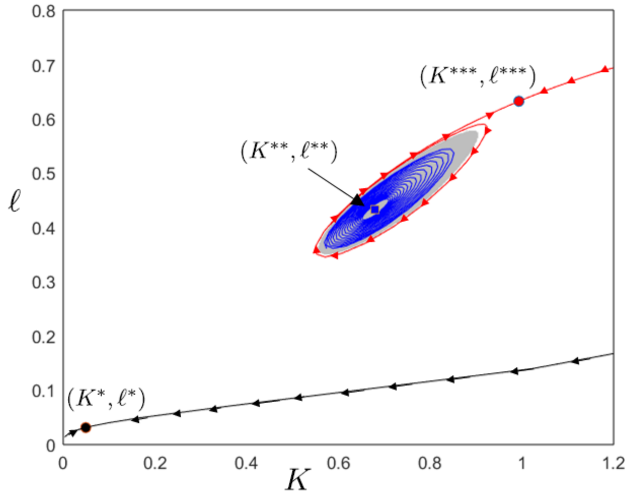
Scenarios 2.A, 2.B and 3.A below illustrate the behaviour of the system in the cases of local and global indeterminacy by using the parameter set of Figure 3 for three different values of the health tax rate τ (Scenario 2.A: $\tau = 0.4$; Scenario 2.B: $\tau = 0.47$; Scenario 3.A: $\tau = 0.477$). These scenarios generally show that the initial conditions of the state variable are no longer able to determine where the economy will end up in the long term. Therefore, underdevelopment becomes a matter of coordination failure.

Scenario 2.A. *Local and global indeterminacy (expectations driven).* This scenario shows the possibility of different long-term regimes capturing the existence of least developed, developing and developed countries in the same setting. Figure 5, Panel (a), depicts a phase plane for $\tau = 0.4$ showing the two (locally determinate) saddle points (K^*, ℓ^*) and (K^{***}, ℓ^{***}) as well as the interior (locally indeterminate) attractor (K^{**}, ℓ^{**}) with its basin of attraction (grey region). The first steady-state equilibrium (K^*, ℓ^*) resembles the case of a poverty trap. The economy in this case produces a low level of real activity and adult mortality is high. In this scenario, labour income is low and private health expenditure is not adequate to complement public health services. This implies that the labour productivity is low because of the resulting poor state of health. The second equilibrium represents the paradigm of developing countries, where the transition from a pre-industrial environment to an industrialised one has not yet completed. There exist damped fluctuations and the economy is not able to sustain a permanently high level of income or a high level of life expectancy. The third equilibrium describes the case of a developed economy producing a high level of real activity and adult mortality is low. In this scenario, labour income is high and private health expenditure is adequate to complement the public one thus getting larger values of labour productivity because of the resulting good state of health. We note that any initial condition of K belonging to the grey region allows individuals to coordinate on different choices of ℓ_0 leading towards different development regimes. Specifically, (i) there exists a sufficiently small value of ℓ_0 such that the economy approaches the poverty trap (the black stable manifold), (ii) there exist two other values of ℓ_0 such that the economy approaches the high saddle point (the red stable manifold), (iii) there exist infinite values of ℓ_0 that generate trajectories converging towards the intermediate equilibrium.

Scenario 2.B. *Local and global indeterminacy (expectations driven).* Figure 5, Panel (b), shows a phase plane for $\tau = 0.47$. The basin of attraction of the indeterminate equilibrium (K^{**}, ℓ^{**}) is bounded by the closed invariant curve generated by a sub-critical Neimark-Sacker bifurcation. In this case, the (red) stable manifold of (K^{***}, ℓ^{***}) twists around the invariant curve. Therefore, even if individuals coordinate themselves on the stable manifold of the high saddle point, the economy can be characterised by several fluctuations of economic and demographic variables.



(a)

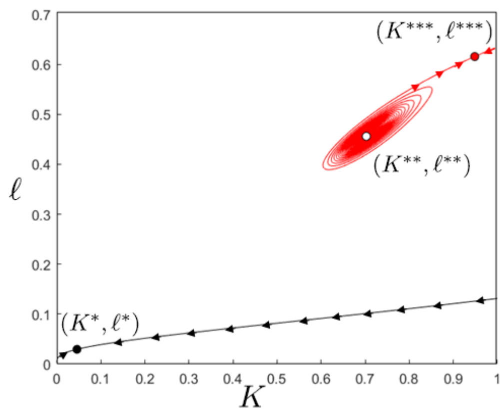


(b)

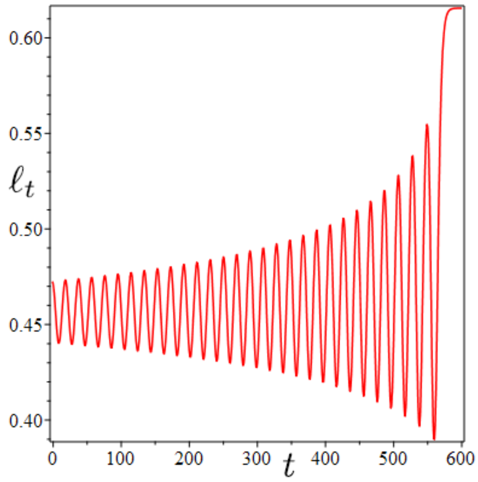
Figure 5. Local and global indeterminacy (expectations driven). Parameter set: $A = 5$, $Z = 2.1$, $\bar{\ell} = 1$, $\alpha = 0.33$, $\delta = 1$, $\gamma = 0.29$, $\mu = 0.39$, $\rho = 0.19$ and $\underline{\eta} = 0.5$. (a) Phase plane for $\tau = 0.4$. The steady-state coordinate values of K and ℓ are: $(K^*, \ell^*) = (0.1040, 0.0550)$, $(K^{**}, \ell^{**}) = (0.5920, 0.3130)$ and $(K^{***}, \ell^{***}) = (1.2965, 0.6855)$. The blue trajectory in Panel (a) is a heteroclinic connection between (K^{***}, ℓ^{***}) and (K^{**}, ℓ^{**}). (b) Phase plane for $\tau = 0.47$. The steady-state coordinate values of K and ℓ are: $(K^*, \ell^*) = (0.0491, 0.0312)$, $(K^{**}, \ell^{**}) = (0.6803, 0.4328)$ and $(K^{***}, \ell^{***}) = (0.9935, 0.6321)$. The blue trajectory in Panel (b) is generated by $K_0 = 0.574$ in the case individuals coordinate themselves on $\ell_0 = 0.3609$. The black (resp. red) curve in both panels is the stable manifold of the (black) saddle (K^*, ℓ^*) [resp. (red) saddle (K^{***}, ℓ^{***})]. (K^{***}, ℓ^{***}) Pareto dominates (K^{**}, ℓ^{**}) and (K^{**}, ℓ^{**}) Pareto dominates (K^*, ℓ^*).

Scenario 3.A. *Local determinacy and global indeterminacy (expectations driven).* In this scenario obtained for $\tau = 0.477$ there are no attractors of the system. In fact, there exist the (locally determinate) saddle points (K^*, ℓ^*) and (K^{***}, ℓ^{***}) and the unstable equilibrium

(K^{**}, ℓ^{**}). Although the two fixed points the economy can reach in the long term are locally determinate, the numerical approximation of the stable manifolds allows to show that the system is globally indeterminate. In fact, there exist values of the state variable such that different choices on the control variable lead to different development trajectories (Figure 6, Panel (a)). If the initial condition of K is sufficiently small, the stationary state the economy will approach is (K^*, ℓ^*). This is because there exists one and only one (small) initial value of ℓ allowing the economy to be on the stable manifold of the low-development regime. The economy, therefore, is entrapped in poverty as capital accumulation do not allow an adequate private health investment and the subsequent triggering of the beneficial (multiplier) effects that endogenous lifetime introduces via the productivity of labour and the saving rate. A takeoff towards economic development is prevented and this leads individuals to live in a low-longevity scenario. For larger values of the initial condition of K, agents may coordinate themselves either on the stable manifold of the saddle leading towards the low regime of development (K^*, ℓ^*) (for small values of the labour supply) or on the stable manifold of the saddle leading towards the high regime of development (K^{***}, ℓ^{***}) (for larger values of the labour supply). Like in Scenario 2.A, also in this case even if agents coordinate on the stable manifold (red line) of the high saddle point, the growth trajectory of K and ℓ can be non-monotonic and characterised by several oscillations. This is illustrated in Figure 6, Panel (b) that shows the time series ℓ.



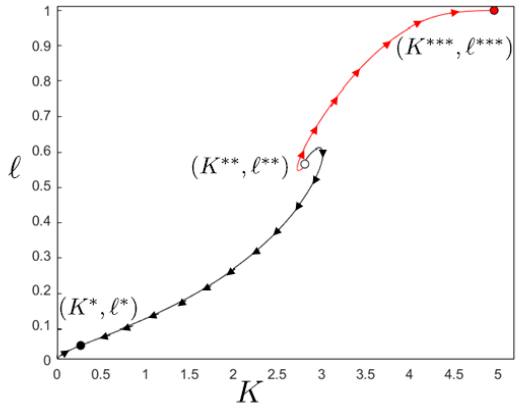
(a)



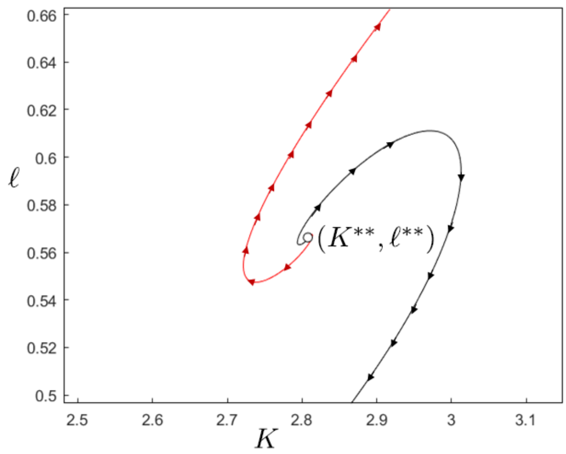
(b)

Figure 6. Local determinacy and global indeterminacy (expectations driven). Parameter set: $A = 5$, $Z = 2.1$, $\bar{\ell} = 1$, $\alpha = 0.33$, $\delta = 1$, $\gamma = 0.29$, $\mu = 0.39$, $\rho = 0.19$ and $\underline{\eta} = 0.5$. (a) Phase plane for $\tau = 0.477$. The black (resp. red) curve is the stable manifold of the (black) saddle (K^*, ℓ^*) [resp. (red) saddle (K^{***}, ℓ^{***})]. (K^{***}, ℓ^{***}) Pareto dominates (K^*, ℓ^*). The steady-state coordinate values of K and ℓ and the following: $(K^*, \ell^*) = (0.0455, 0.0295)$, $(K^{**}, \ell^{**}) = (0.7023, 0.4558)$ and $(K^{***}, \ell^{***}) = (0.9484, 0.61553)$. (b) Time series of ℓ.

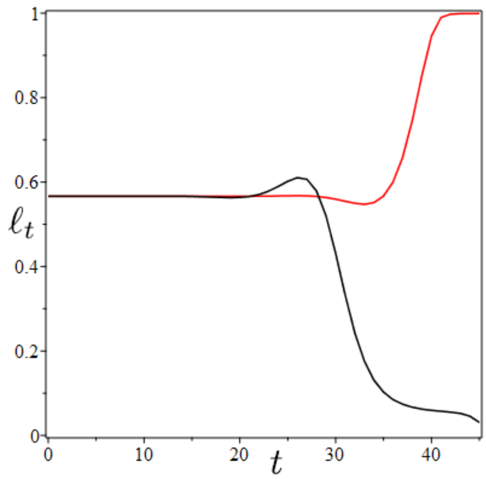
Scenario 3.B. *Local determinacy and global indeterminacy (expectations driven).* By using a different parameter set i.e., $A = 10$, $Z = 3.35$, $\bar{\ell} = 1$, $\alpha = 0.34$, $\delta = 0.96$, $\gamma = 0.014$, $\mu = 0.75$, $\rho = 0.3$, $\underline{\eta} = 0.04$ and $\tau = 0.041$, Figure 7, Panel (a) depicts a phase plane in which the saddles (K^*, ℓ^*) and (K^{***}, ℓ^{***}) (the black dot and red dot, respectively) belong to the boundary of the basin of attraction of the dynamics created by map M^{-1} (backward dynamics). The example shows that starting from an initial condition of K (K_0) close to K^{**} agents may coordinate on (slightly) different values of ℓ (see also Figure 7, Panel (b) that represents an enlargement of Panel (a) close to the unstable equilibrium (K^{**}, ℓ^{**})) leading to two opposite development regimes: the red stable manifold represents the growth trajectory towards the saddle point (K^{***}, ℓ^{***}) with high values of demo-economic variables; the black stable manifold, instead, represents the trajectory leading to the poverty trap with low values of demo-economic variables. To further clarify the evolution of the system and the importance of the choices on the control variable, Figure 7, Panel (c) shows a time series of ℓ followed by two economies starting (very close to the coordinate value of the unstable equilibrium K^{**}) from the same initial condition of K $(K_0 = 2.807817)$. The example shows that choosing two slightly different values of ℓ (that is, $\ell_0 = 0.566243$ and $\ell_0 = 0.566246$) leads towards trajectories that share at the beginning a similar development pathway ending up however in two completely different long-term equilibria.



(a)



(b)



(c)

Figure 7. Local determinacy and global indeterminacy (expectations driven). Parameter set: $A = 10$, $Z = 3.35$, $\bar{\ell} = 1$, $\alpha = 0.34$, $\delta = 0.96$, $\gamma = 0.014$, $\mu = 0.75$, $\rho = 0.3$ and $\underline{\eta} = 0.04$. (a) Phase plane for $\tau = 0.041$. The steady-state coordinate values of K and ℓ and the following: $(K^*, \ell^*) = (0.267, 0.0538)$, $(K^{**}, \ell^{**}) = (2.8078, 0.5662)$ and $(K^{***}, \ell^{***}) = (4.9546, 0.9991)$. (b)

Enlarged view of the phase plane of Panel (a). (c) Time series of ℓ showing underdevelopment (black) and development (red) trajectories.

Definitively, the model always shows simple dynamics if an economy starts close to the two saddles. The most complex situation occurs when the economy is sufficiently far from these two equilibria and there exists an attracting intermediate equilibrium (Figures 5 and 6). In this case, financing a public expenditure on health may remove the obstacles to economic development by eliminating the (intermediate) poverty trap thus favouring in the medium-long term (this is because the dynamics are oscillatory close to the equilibrium) an increase in labour supply (for the reasons discussed above) and therefore an increase in capital accumulation. Given the complex chain of interactions and transmission mechanisms amongst the variables, a small change in the marginal health tax rate can generate sharp differences in the long-term outcomes. The importance of health investment policies can also be observed if there is no attracting intermediate equilibrium. In fact, by considering a parameter set for which history matters (Figure 4) and taking an initial value of the capital stock lower than the value of the unstable equilibrium, then an appropriate change (i.e., neither too high nor too low, otherwise we would observe the existence of a unique equilibrium) of the health tax rate makes possible (i.e. if agents coordinate on the path leading to the Pareto dominant equilibrium), due to the property of global indeterminacy (i.e., from a mathematical point of view the intermediate equilibrium passes from node to focus), the convergence to the high equilibrium, as shown in Figure 7 (Krugman, 1991).

4 Discussion of the main results

Scenario 2.A outlined in Figure 5, Panel (a) provided an interesting result to interpret the development process of some countries, for example the case of South Korea and the Philippines outlined in Lucas (1993). "In 1960, The Philippines and South Korea had about the same standard of living, as measured by their per capita GDPs of about \$640 U.S. 1975. The two countries were similar in many other respects..." (Lucas, 1993, p. 251). Indeed, GDP per capita (current US\$) and life expectancy at birth (years) in South Korea were 158\$ and 53 years in 1960, whereas the corresponding values for the Philippines were 254\$ and 57 years. In the last decades, South Korea transformed dramatically its society experiencing a growth "miracle" and the Philippines did not.[17] Marked differences can in fact can be observed regarding the trend of both the total health expenditure over GDP and domestic general government health expenditure (as a percentage of current health expenditure) in the two countries. On one hand, South Korea observed a steady growth in total health expenditure compared to GDP

[17] GDP per capita in 2018 (current US\$) in South Korea and The Philippines were 31,362\$ and 3,102\$, respectively. This is even more impressive knowing that in the 1960s, GDP per capita was comparable with levels in the poorer countries of Africa and Asia.

from 4% in 2000 to 7.3% in 2016, whereas this ratio fluctuated around 3-4% over the same period adjusting downwards to 2.7% in 2002 in the Philippines. On the other hand, the general government health expenditure was almost 50% of total health expenditure in South Korea and this ratio rose to 60% in 2006, settling on those values until 2016. The Philippines experienced a monotonic reduction in this ratio from around 44% in 2000 to 26% in 2013 and then increased to 31% in 2016 (https://data.worldbank.org/indicator) under the Aquino administration that worked out to raise the budgets for education, health and cash transfers to the poor.

While much of the literature since Lucas (1993) has interpreted different development scenarios due to changes in labour productivity, technology, knowledge and human capital accumulation, little attention was devoted to how changes in life expectancy affected individual decisions about labour supply, savings and health expenditures as well as how these decisions in turn fed back on demography.[18] Our theory explained this gap by considering a model, where the public health expenditure complements the private one, in which individuals' expectations can affect labour supply and saving that in turn cause changes in private health investment and life expectancy. For instance, with the parameter values used in Scenario 2.A, the interior equilibrium (K^{**}, ℓ^{**}) and the saddle (K^{***}, ℓ^{***}) can be used to interpret the different development process experienced in those countries by concentrating on demography.[19] In this regard, the values of life expectancy at birth corresponding to the (long-term) stationary states in (K^{***}, ℓ^{***}) (South Korea) and (K^{**}, ℓ^{**}) (The Philippines) are 78 and 70, respectively (these values are close to the actual values of life expectancy observed in those countries in 2018 as detailed in the World Bank Indicator). As these two economies switched after 1960, history cannot explain their development process. Scenario 2.A can then explain the reason for the existence of distinct development paths that countries may follow starting from the same (or similar) initial conditions of the state variable. One of the two economies can coordinate on the saddle path allowing to follow a development trajectory and the achievement of the stationary equilibrium (K^{***}, ℓ^{***}). The other one can define a choice on ℓ allowing the achievement of (K^{**}, ℓ^{**}) (middle income scenario). The existence of different regular or non-regular paths of similarly endowed economies can therefore be explained by coordination

[18] An interesting recent contribution by Fiaschi et al. (2019) pinpointed - amongst other things - the role of religious beliefs on life expectancy. In this regard, South Korea is essentially characterised by no formal affiliation with a religion, though Protestantism and Buddhism represent the most relevant religious organisations in the country. Differently, The Philippines is the only Christian nation in Asia. This may determine different participation in the labour market (especially between male and female), income and life expectancy.

[19] We recall that in this model an individual becomes economically active at the beginning of youth. As we assumed the length of each generation consists of 30 years, an individual becomes economically active at 30 and enters old age at 60 (the very first period of life represents a paradigm of a non-economically active generation consuming resources in the parents' household). As we assumed the length of the interval between t and t+1 consists of 30 years, all individuals get healthy at the age of 60 with probability one. Then, one faces a probability smaller than one, measured by θ, describing the percentage of 30 years of old age an individual is staying alive the last part of his time horizon.

failures (global indeterminacy). In this case, the initial condition of the state variable is not sufficient to define the long-term behaviour of the system. Interestingly, if agents coordinate themselves on the stable manifold of the saddle leading towards (K^{***}, ℓ^{***}), the convergence behaviour can be monotonic or non-monotonic (endogenous damped fluctuations) depending on the initial condition of the state variable. Fluctuations are related to choices of ℓ associated with values of K far away from the steady state. Monotonic behaviour instead is related to choices of ℓ associated with values of K closer to the steady state. There exists a twofold effect of health taxation on labour at the macroeconomic level. The former effect is the standard negative (partial equilibrium) direct effect that contributes to reduce disposable income and thus saving, private health spending, capital accumulation and labour supply. The latter one is a positive (general equilibrium) indirect effect that passes through the increase in public health expenditure, the individual health status and the longevity rate causing, in turn, an increase in saving, capital accumulation and the labour supply. When the initial value of the stock of capital is sufficiently large, the choice of a (large) value of ℓ makes the positive indirect effect of taxation high enough to compensate the negative direct effect, thus favouring labour supply and capital accumulation. When the initial value of capital is smaller, fluctuations are possible. This is because the combination of not too large values of both capital and labour supply makes the beneficial effects of an improvement in the individual health status and labour productivity at relatively small levels. Then, an increase in the private health expenditure and a reduction in saving and capital accumulation are indeed required. This in turn causes a further reduction in labour supply. However, private health expenditure plays a positive role on the individual health status and thus on labour productivity. This tends to favour labour supply and capital accumulation. When this combined effect of private and public health expenditures becomes important enough, the indirect effect dominates and a monotonic behaviour is established leading the economy towards the saddle (K^{***}, ℓ^{***}).

Scenario 2.A pinpoints the importance of the role played by individual choices about labour supply and health investment and public health support regarding whether a country can eventually experience a phase of economic development with low mortality.

5 Conclusions

Economic development is a multi-dimensional phenomenon by which a nation is aiming at improving economic conditions, institutional and political environments, environmental quality as well as individual and social well-being. Economic development is also related to the magnitude and shape of some demographic variables, fertility and mortality in the first place (Galor, 2011). Economic growth is a phenomenon of market productivity, which is strictly dependent on GDP growth. Economic growth therefore represents only a part of the process of development of nations. Ill health can produce losses in individual utility or social welfare, or it can compromise other economic objectives such as the production of market goods. It is well recognised that some key channels through which disease or injury can affect macroeconomic outcomes include (public and private) health expenditures, the productivity of labour, and investments in human and physical capital. The aim of this research is to build on a theory to explain *economic development* based on the interaction between private expenditure and public expenditure on health by using an *economic growth* model with overlapping generations and endogenous labour supply. Specifically, it links the theoretical literature on endogenous labour supply and indeterminacy with the theoretical literature on endogenous lifetime and economic growth by revisiting the work of Bhattacharya and Qiao (2007), and the main findings are in sharp contrast with the existing ones. Due to the combined effect of the externality of the health technology on the macro-economy and the assumption of endogenous labour supply, multiple equilibria are possible and *history* or *global indeterminacy* become the main *alternative* driving forces of economic development of nations. These results hold under the assumption of gross substitutability between consumption and leisure and without production externalities. Often, history alone is not sufficient to determine which kind of trajectories an economy may follow. History can effectively be referred to in cases where the initial conditions of two or more economies are different enough. In contrast, when the initial conditions of countries are equal or like each other what determines the reason why an economy develops and others instead fail to develop, thus moving towards a lower development regime with higher mortality, can be a matter of coordination failures (self-fulfilling expectations). In this regard, the present work can effectively be invoked to explain the puzzle raised in Lucas (1993) for which although South Korea and the Philippines started with very similar initial conditions some years after the end of the Korean war they ended up in very different development environments some decades later.[20] Citizens in the former country spent a larger amount of time at work (given the rise of the Korean manufacturing industry) than citizens in the latter country. The increase in labour productivity has contributed to the rise of an efficient health system that has been a source of better health and steadily increasing longevity eventually producing a development trajectory in South Korea.[21] This happened though it was one of the poorest countries in East Asia until the early 1960s. In contrast, the Philippines, one of the richest countries in East Asia until the early 1960s, started experiencing under-performances that, amongst other things, cause a contraction in the supply of labour in the manufacturing industry. The fall in labour productivity just triggered a vicious circle that prevented improving private health

[20] By taking human capital as the main engine of (endogenous) growth, Lucas tried to explain the economic "miracle" of the steadily increasing South Korea output growth rate in comparison with the stagnating ones of the Philippines and other East Asian countries, especially during the late 1970s and the early 1980s, based on the observed marked differences in investment rates.

[21] For an analysis of growth trajectories in South Korea and the Philippines, labour productivity, the wage gap between high-income and lower-middle-income countries see ILO (2013). A recent thoughtful study of labour shares in several countries around the world including South Korea and the Philippines is Guerriero (2019). These values are in line with those used in the present paper.

care and health conditions of people at the economy-wide level, implying a steadily reducing longevity that eventually produced an underdevelopment trajectory in the Philippine economy. The present work explains these differences based on individual expectations and coordination.

The novel contribution of this work is to create a link between the theoretical literature on endogenous lifetime and economic growth and the theoretical literature on endogenous labour supply and indeterminacy. Existing articles belonging to the former group have often concerned about the study of the nature and causes of economic development in neoclassical growth models that bring forth to mechanisms leading towards multiple stationary state equilibria. The existence of least-developed, developing or developed countries is explained based on historical reasons (initial conditions) and sometimes there exist possible policy recipes to escape from the poverty trap (investing in education, strengthening the public healthcare system, family policies and so on). Articles belonging to the latter group have generally analysed the possibility of (in)determinacy of one of the stationary-state equilibria and tackled out this issue by means of the local analysis. This article proposes a model that can explain economic development in a neoclassical growth set up by concentrating on the global study of a two-dimensional map characterising the evolution of the capital stock (state variable) and the labour supply (control variable). The existence of distinct development trajectories is possible due to historical reasons or they may depend on individual expectations about some key variables of the model (labour supply). Individuals may coordinate on a (development) trajectory with high levels of labor supply allowing an adequate accumulation of capital. This in turn increases the amount of resources available to private investments in health allowing an increase in life expectancy. However, they may also coordinate on a Pareto dominated trajectory with low levels of labour supply, inadequate accumulation of capital and a lack of health investments leading to lower values of life expectancy (underdevelopment).

Though this article is mainly theoretical it may also contribute to the debate on the effects of the so-called austerity measures in Europe on the side of the health of people with dramatic potential effects on child survival rates and longevity rates reversals. To this purpose, Kentikelenis et al. (2014) has reported the Greek public health tragedy due to the cuts in health services in Greece (see also Kentikelenis et al., 2011), including expenditures to prevent and treat illicit drug and infectious diseases (such as HIV/AIDS).[22] This topic, therefore, deserves attention also in theoretical macroeconomic models, given the importance of mortality in the process of economic development.

Acknowledgements The authors are indebted to two anonymous reviewers of the journal for insightful comments allowing an improvement in the general quality of the article. They also gratefully acknowledge Costas Azariadis, Alessandro Cigno, Guido Cozzi, Carl-Johan Dalgaard, Oded Galor, Neri Salvadori, Thomas Seegmuller, Alain Venditti, Joseph Zeira and conference participants at NED 2013 held at University of Siena (Italy), IEA 2014 World Congress held in

[22] See the works of Chakraborty et al. (2010, 2016) and Gori et al. (2019a, 2019b).

Jordan, 13th Journées LAGV 2014 held at Aix Marseille Université (France), Summer School in Economic Growth 2014 held in Capri (Italy), 55th Italian Economic Association held at University of Trento (Italy), Rimini Centre for Economic Analysis (RCEA) 2015 Workshop held in Rimini (Italy), CRISIS 2016 held at University of Ancona (Italy), NED 2017 held at University of Pisa (Italy), 18th Journées LAGV 2019 held at Aix Marseille Université (France) and seminar participants at University of Pisa, University of Florence and University of Eastern Piedmont. Mauro Sodini also acknowledges financial support by the research project PRA 2017-2018 (University of Pisa) entitled "Globalization, population and sustainability". The usual disclaimer applies.

Conflict of Interest The authors declare that they have no conflict of interest.

Appendix A

We show here how a change in the wage rate, the interest factor and the public health investment affects the individual choices about the labour supply and the private health investment. Substituting out (8) in (7) for x_t we get:

$$\Omega(\ell_t, w_t, R^e_{t+1}, \eta_t) := (\bar{\ell}-\ell_t)^{-\gamma}+w_t(1-\tau)R^e_{t+1}\frac{\eta_t^{1-\rho}}{Z}\left[\frac{\rho w_t\ell_t(1-\tau)}{1+\rho-\mu}\right]^{\rho}\left[\frac{w_t\ell_t(1-\tau)(1-\mu)R^e_{t+1}}{1+\rho-\mu}\right]^{-\mu} = 0. \tag{38}$$

From (38), we obtain:

$$\frac{\partial\Omega(.)}{\partial\ell_t} = -\gamma(\bar{\ell}-\ell_t)^{-\gamma-1}-(\mu-\rho)w_t(1-\tau)R^e_{t+1}\frac{\eta_t^{1-\rho}}{Z\ell_t}\left[\frac{\rho w_t\ell_t(1-\tau)}{1+\rho-\mu}\right]^{\rho}\left[\frac{w_t\ell_t(1-\tau)(1-\mu)R^e_{t+1}}{1+\rho-\mu}\right]^{-\mu} < 0, \tag{39}$$

$$\frac{\partial\Omega(.)}{\partial w_t} = (1-\tau)(1+\rho-\mu)R^e_{t+1}\frac{\eta_t^{1-\rho}}{Z}\left[\frac{\rho w_t\ell_t(1-\tau)}{1+\rho-\mu}\right]^{\rho}\left[\frac{w_t\ell_t(1-\tau)(1-\mu)R^e_{t+1}}{1+\rho-\mu}\right]^{-\mu} > 0, \tag{40}$$

$$\frac{\partial\Omega(.)}{\partial R^e_{t+1}} = w_t(1-\tau)(1-\mu)\frac{\eta_t^{1-\rho}}{Z}\left[\frac{\rho w_t\ell_t(1-\tau)}{1+\rho-\mu}\right]^{\rho}\left[\frac{w_t\ell_t(1-\tau)(1-\mu)R^e_{t+1}}{1+\rho-\mu}\right]^{-\mu} > 0, \tag{41}$$

and

$$\frac{\partial\Omega(.)}{\partial\eta_t} = w_t(1-\tau)(1-\rho)R^e_{t+1}\frac{\eta_t^{-\rho}}{Z}\left[\frac{\rho w_t\ell_t(1-\tau)}{1+\rho-\mu}\right]^{\rho}\left[\frac{w_t\ell_t(1-\tau)(1-\mu)R^e_{t+1}}{1+\rho-\mu}\right]^{-\mu} > 0. \tag{42}$$

Then, by applying the implicit function theorem the following results hold:

$$\frac{\partial\ell_t}{\partial w_t} = -\frac{\frac{\partial\Omega(.)}{\partial w_t}}{\frac{\partial\Omega(.)}{\partial\ell_t}} > 0, \tag{43}$$

$$\frac{\partial \ell_t}{\partial R^e_{t+1}} = -\frac{\frac{\partial \Omega(.)}{\partial R^e_{t+1}}}{\frac{\partial \Omega(.)}{\partial \ell_t}} > 0, \tag{44}$$

$$\frac{\partial \ell_t}{\partial \eta_t} = -\frac{\frac{\partial \Omega(.)}{\partial \eta_t}}{\frac{\partial \Omega(.)}{\partial \ell_t}} > 0. \tag{45}$$

From (8) and (43)-(45) we also find that:

$$\frac{\partial x_t}{\partial w_t} = \frac{\rho(1-\tau)}{1+\rho-\mu}\left[\ell_t(w_t) + \frac{\partial \ell_t}{\partial w_t} w_t\right] > 0, \tag{46}$$

$$\frac{\partial x_t}{\partial R^e_{t+1}} = \frac{\rho(1-\tau)}{1+\rho-\mu}\frac{\partial \ell_t}{\partial R^e_{t+1}} w_t > 0, \tag{47}$$

and

$$\frac{\partial x_t}{\partial \eta_t} = \frac{\rho(1-\tau)}{1+\rho-\mu}\frac{\partial \ell_t}{\partial \eta_t} > 0. \tag{48}$$

Appendix B

This appendix shows the functioning of the model by including material consumption during youth (c_t). The lifetime utility function of the individual representative of generation t modifies to become the following:

$$U(c_t, \bar{\ell} - \ell_t, x_t, C_{t+1}) = \frac{c_t^{1-\mu}}{1-\mu} + \frac{(\bar{\ell} - \ell_t)^{1-\gamma}}{1-\gamma} + \theta(x_t, \eta_t)\frac{C_{t+1}^{1-\mu}}{1-\mu}, \tag{49}$$

whereas his budget constraint when young now implies that available labour income is divided between saving, s_t, private health expenditure, x_t, and young-age material consumption, that is $c_t + s_t + x_t = w_t \ell_t (1-\tau)$. Consumption when old is still given by the equation $C_{t+1} = R^e_{t+1} s_t$. Therefore, the inter-temporal budget constraint can be written as follows:

$$C_{t+1} = R^e_{t+1}[w_t \ell_t (1-\tau) - c_t - x_t]. \tag{50}$$

The conditions $\mu < 1$ (to avoid paradoxical effects of longevity on utility) and $\mu > \rho$ (to preserve concavity of the utility function) continue to hold. Decisions about consumption, leisure and private health spending when young, and decisions about consumption when old determine saving behaviour of the individual. By using (49) and (50), the optimisation programme can be written as follows:

$$\max_{c_t, x_t, \ell_t} \left\{ \frac{c_t^{1-\mu}}{1-\mu} + \frac{(\bar{\ell} - \ell_t)^{1-\gamma}}{1-\gamma} + \frac{x_t^{\rho}\eta_t^{1-\rho}}{Z}\frac{\left\{R^e_{t+1}[w_t \ell_t (1-\tau) - c_t - x_t]\right\}^{1-\mu}}{1-\mu} \right\}, \tag{51}$$

with $0 \leq x_t \leq w_t \ell_t (1-\tau) - c_t$, and $0 \leq \ell_t \leq \bar{\ell}$. The the first order conditions for this problem are given by:

$$\frac{x_t^{\rho}\eta_t^{1-\rho}\{R^e_{t+1}[w_t \ell_t (1-\tau) - c_t - x_t]\}^{1-\mu} w_t (1-\tau)}{Z[w_t \ell_t (1-\tau) - c_t - x_t]} = (\bar{\ell} - \ell_t)^{-\gamma}, \tag{52}$$

$$[w_t\ell_t(1-\tau)-c_t-x_t]\rho=(1-\mu+\rho)x_t, \tag{53}$$

$$\frac{x_t^{\rho}\eta_t^{1-\rho}\{R_{t+1}^e[w_t\ell_t(1-\tau)-c_t-x_t]\}^{1-\mu}}{Z[w_t\ell_t(1-\tau)-c_t-x_t]}=c_t^{-\mu}. \tag{54}$$

The market-clearing condition in the capital market is given by $K_{t+1}=s_t=w_t\ell_t(1-\tau)-c_t-x_t$. Then, by using (2), (3), (10), (11), (52), (53), (54) and knowing that individuals have perfect foresight, the two-dimensional map characterising the equilibrium dynamics of the economy is the following:

$$\tilde{M}:\begin{cases} K_{t+1}=V_3(K_t,\ell_t) \\ \ell_{t+1}=V_4(K_t,\ell_t)\end{cases}, \tag{55}$$

where

$$V_3(K_t,\ell_t):=\frac{1-\mu}{1-\mu+\rho}\left\{A(1-\alpha)(1-\tau)K_t^{\alpha}\ell_t^{1-\alpha}-\left[\frac{K_t^{-\alpha}\ell_t^{\alpha}}{A(1-\alpha)(1-\tau)}\right]^{\frac{-1}{\mu}}(\bar{\ell}-\ell_t)^{\frac{\gamma}{\mu}}\right\}, \tag{56}$$

$$\begin{aligned} V_4(K_t,\ell_t) \;:&=\tilde{P}\left\{A(1-\alpha)(1-\tau)K_t^{\alpha}\ell_t^{1-\alpha}-\left[\frac{K_t^{-\alpha}\ell_t^{\alpha}}{A(1-\alpha)(1-\tau)}\right]^{\frac{-1}{\mu}}(\bar{\ell}-\ell_t)^{\frac{\gamma}{\mu}}\right\}^{\frac{1-\rho+\alpha(\mu-1)}{(1-\alpha)(1-\mu)}} \\ &\times\left[K_t^{-\alpha}\ell_t^{\alpha}(\bar{\ell}-\ell_t)^{\frac{\gamma}{\mu}}\right]^{\frac{1}{(1-\alpha)(1-\mu)}}\left\{\underline{\eta}+\left[\tau A(1-\alpha)K_t^{\alpha}\ell_t^{1-\alpha}\right]^{\delta}\right\}^{\frac{\rho-1}{(1-\alpha)(1-\mu)}}, \end{aligned} \tag{57}$$

and

$$\tilde{P}:=\left\{\frac{Z(1+\rho-\mu)^{(1-\mu)\alpha-1+\rho}(1-\mu)^{1-(1-\mu)\alpha}}{\rho^{\rho}A^{2-\mu}\alpha^{1-\mu}(1-\tau)(1-\alpha)}\right\}^{\frac{1}{(1-\alpha)(1-\mu)}}. \tag{58}$$

As can be seen from map $\tilde{M}$, in this case it is difficult to obtain clear-cut results about existence of stationary states and their stability. However, it is possible to show that multiple equilibria can exist even with young-age consumption. To illustrate this result, let us take the following parameter set: $A=5$, $Z=2.1$, $\bar{\ell}=1$, $\alpha=0.33$, $\delta=1$, $\gamma=0.29$, $\mu=0.19$, $\rho=0.2$, $\underline{\eta}=0.5$ and $\tau=0.5$. Then, we get the stationary states $(K^*,\ell^*)=(0.0075,0.1363)$, $(K^{**},\ell^{**})=(0.0677,0.4105)$ and $(K^{***},\ell^{***})=(0.1988,0.6232)$. The first and third equilibria are saddles and the second one is a source. The parameters used to show the existence of three equilibria in this case are different than those used in the main text. This is because the existence of young-age consumption modifies the optimal allocation of resources of the representative individual by reducing resources available for saving and capital accumulation.

References

[1] Acemoglu, D., Johnson, S., 2007. Disease and development: the effect of life expectancy on economic growth. Journal of Political Economy 115, 925–985.

[2] Agénor, P.-R., 2015. Public capital, health persistence and poverty traps. Journal of Economics 115, 103–131.

[3] Agliari, A., G. Vachadze, G., 2011. Homoclinic and heteroclinic bifurcations in an overlapping generations model with credit market imperfection. Computational Economics 38, 241–260.

[4] Antoci, A., Gori, L., Sodini, M., 2016. Nonlinear dynamics and global indeterminacy in an overlapping generations model with environmental resources. Communications in Nonlinear Science and Numerical Simulation 38, 59–71.

[5] Antoci, A., Gori, L., Sodini, M., Ticci, E., 2019. Maladaptation and global indeterminacy. Environment and Development Economics, forthcoming, https://doi.org/10.1017/S1355770X19000251.

[6] Becker, G.S., 1960. An economic analysis of fertility. In: Demographic and economic change in developing countries, National Bureau Committee for Economic Research, Princeton University Press, Princeton (NJ).

[7] Benhabib, J., Farmer, R.E.A., 1994. Indeterminacy and increasing returns. Journal of Economic Theory 63, 19–41.

[8] Benhabib, J., Perli, R., 1994. Uniqueness and indeterminacy: on the dynamics of endogenous growth. Journal of Economic Theory 63, 113–142.

[9] Bhattacharya, J., Qiao, X., 2007. Public and private expenditures on health in a growth model. Journal of Economic Dynamics & Control 31, 2519–2535.

[10] Blackburn, K., Cipriani, G.P., 2002. A model of longevity, fertility and growth. Journal of Economic Dynamic & Control 26, 187–204.

[11] Cazzavillan, G., 1996. Public spending, endogenous growth, and endogenous fluctuations. Journal of Economic Theory 71, 394–415.

[12] Cazzavillan, G., 2001. Indeterminacy and endogenous fluctuations with arbitrarily small externalities. Journal of Economic Theory 101, 133–157.

[13] Cazzavillan, G., Pintus, P.A., 2006. Capital externalities in OLG economies. Journal of Economic Dynamics & Control 30, 1215–1231.

[14] Cazzavillan, G., Lloyd-Braga, T., Pintus, P.A., 1998. Multiple steady states and endogenous fluctuations with increasing returns to scale in production. Journal of Economic Theory 80, 60–107.

[15] Cervellati, M., Sunde, U., 2005. Human capital, life expectancy, and the process of development. American Economic Review 95, 1653–1672.

[16] Cervellati, M., Sunde, U., 2011. Life expectancy and economic growth: the role of the demographic transition. Journal of Economic Growth 16, 99–133.

[17] Cervellati, M., Sunde, U., 2015. The economic and demographic transition, mortality, and comparative development. American Economic Journal: Macroeconomics 7, 189–225.

[18] Cervellati, M., Sunde, U., Zimmermann, K.F., 2017. Demographic dynamics and long-run development: insights for the secular stagnation debate. Journal of Population Economics 30, 401–432.

[19] Chakraborty, S., 2004. Endogenous lifetime and economic growth. Journal of Economic Theory 116, 119–137.

[20] Chakraborty, S., Das, M., 2005. Mortality, human capital and persistent inequality. Journal of Economic Growth 10, 159–192.

[21] Chakraborty, S., Papageorgiou, C., Pérez Sebastián, F., 2010. Diseases, infection dynamics and development. Journal of Monetary Economics 57, 859–872.

[22] Chakraborty, S., Papageorgiou, C., Pérez Sebastián, F., 2016. Health cycles and health transitions. Macroeconomic Dynamics 20, 189–213.

[23] Currie, J., Madrian, B.C., 1999. Health, health insurance and the labor market. In: Ashenfelter, O., Card, D., eds., Handbook of Labor Economics, Vol. 3, Elsevier, Amsterdam, 3309–3415.

[24] de la Croix, D., Doepke, M., 2003. Inequality and growth: why differential fertility matters. American Economic Review 93, 1091–1113.

[25] de la Croix, D., Doepke, M., 2004. Public versus private education when differential fertility matters. Journal of Development Economics 73, 607–629.

[26] de Vilder, R., 1996. Complicated endogenous business cycles under gross substitutability. Journal of Economic Theory 71, 416–442.

[27] Diamond, P., 1965. National debt in a neoclassical growth model. American Economic Review 55, 1126–1150.

[28] Dow, W.H., Philipson, T.J., Sala-i-Martin, X., 1999. Longevity complementarities under competing risks. American Economic Review 89, 1358–1371.

[29] Ehrlich, I., Lui, F.T., 1991. Intergenerational trade, longevity, and economic growth. Journal of Political Economy 99, 1029–1060.

[30] Fanti, L., Gori, L., 2014. Endogenous fertility, endogenous lifetime and economic growth: the role of child policies. Journal of Population Economics 27, 529–564.

[31] Fiaschi, D., Fioroni, T., 2019. Transition to modern growth in Great Britain: the role of technological progress, adult mortality and factor accumulation. Structural Change and Economic Dynamics, forthcoming, https://doi.org/10.1016/j.strueco.2019.02.007.

[32] Fiaschi, D., Lavezzi, A.M., Parenti, A., 2019. Deep and proximate determinants of the world income distribution. Review of Income and Wealth, forthcoming, https://doi.org/10.1111/roiw.12418.

[33] Fioroni, T., 2010. Child mortality and fertility: public vs private education. Journal of Population Economics 23, 73–97.

[34] Fogel, R.W., 2004. The escape from hunger and premature death. Cambridge University Press, New York (NY).

[35] French, E., 2005. The effects of health, wealth, and wages on labour supply and retirement behaviour. Review of Economic Studies 72, 395–427.

[36] Galor, O., 2005. The Demographic transition and the emergence of sustained economic growth. Journal of the European Economic Association 3, 494–504.

[37] Galor, O., 2011. Unified growth theory. Princeton University Press, Princeton (NJ).

[38] Galor, O., Moav, O., 2002. Natural selection and the origin of economic growth. Quarterly Journal of Economics 117, 1133–1191.

[39] Galor, O., Weil, D.N., 1996. The gender gap, fertility, and growth. American Economic Review 86, 374–387.

[40] Galor, O., Weil, D.N., 1999. From Malthusian stagnation to modern growth. American Economic Review 89, 150–154.

[41] Galor, O., Weil, D.N., 2000. Population, technology, and growth: from Malthusian stagnation to the Demographic Transition and beyond. American Economic Review 90, 806–828.

[42] Garthwaite, C., Gross, T., Notowidigdo, M.J., 2013. Public health insurance, labor supply, and employment lock. NBER Working Paper No. 19220.

[43] Gollin, D., 2002. Getting income shares right. Journal of Political Economy 110, 458–474.

[44] Gori, L., Sodini, M., 2014. Local and global bifurcations in an economic growth model with endogenous labour supply and multiplicative external habits. Chaos: An Interdisciplinary Journal of Nonlinear Science 24, 013122.

[45] Gori, L., Sodini, M., 2019. A contribution to the theory of fertility and economic development. Macroeconomic Dynamics, forthcoming, https://doi.org/10.1017/S1365100519000415.

[46] Gori, L., Manfredi, P., Sodini, M., 2019a. A parsimonious model of longevity, fertility, HIV transmission and development. Macroeconomic Dynamics, forthcoming, https://doi.org/10.1017/S1365100519000609.

[47] Gori, L., Lupi, E., Manfredi, P., Sodini, M., 2019b. A contribution to the theory of economic development and the demographic transition: fertility reversal under the HIV epidemic. Journal of Demographic Economics, forthcoming, doi:10.1017/dem.2019.21.

[48] Grandmont, J.M., Pintus, P.A., de Vilder, R., 1998. Capital-labor substitution and competitive nonlinear endogenous business cycles. Journal of Economic Theory 80, 14–59.

[49] Guckenheimer, J., Holmes, P., 1983. Nonlinear Oscillations, Dynamical Systems, and Bifurcations of Vector Fields. Springer, Berlin.

[50] Guerriero, M., 2019. The labor share of income around the world: evidence from a panel dataset. Asian Development Bank Institute Working Paper No. 920.

[51] Guo, J.T, Lansing, K.J., 1998. Indeterminacy and stabilization policy. Journal of Economic Theory 82, 481–490.

[52] Hall, R.E., Jones, C.I., 2007. The value of life and the rise in health spending. Quarterly Journal of Economics 122, 39–72.

[53] Hazan, M., Zoabi, H., 2006. Does longevity cause growth? A theoretical critique. Journal of Economic Growth 11, 363–376.

[54] Hokayem, C., Ziliak, J.P., 2014. Health, human capital, and life cycle labor supply. American Economic Review 104, 127–131.

[55] Juhn, C., Kalemli-Ozcan, S., Turan, B., 2013. HIV and fertility in Africa: first evidence from population-based surveys. Journal of Population Economics 26, 835–853.

[56] Kalemli-Ozcan, S., 2002. Does the mortality decline promote economic growth? Journal of Economic Growth 7, 411–439.

[57] Kalemli-Ozcan, S., 2008. The uncertain lifetime and the timing of human capital investment. Journal of Population Economics 21, 557–572.

[58] Kalemli-Ozcan, S., 2012. AIDS, "reversal" of the demographic transition and economic development: evidence from Africa. Journal of Population Economics 25, 871–897.

[59] Kalemli-Ozcan, S., Turan, B., 2011. HIV and fertility revisited. Journal of Development Economics 96, 61–65.

[60] Kalemli-Ozcan, S., Ryder, H.E., Weil, D.N., 2000. Mortality decline, human capital investment, and economic growth. Journal of Development Economics 62, 1–23.

[61] Kentikelenis, A., Karanikolos, M., Papanicolas, I., Basu, S., McKee, M., Stuckler, D., 2011. Health effects of financial crisis: omens of a Greek tragedy. Lancet 378, 1457–1458.

[62] Kentikelenis, A., Karanikolos, M., Reeves, A., McKee, M., Stuckler, D., 2014. Greece's health crisis: from austerity to denialism. Lancet 383, 748–753.

[63] Krugman, P., 1991. History versus expectations. Quarterly Journal of Economics 106, 651–667.

[64] ILO, 2013. Global Wage Report 2012/13. Wages and equitable growth. International Labour Office, Geneva.

[65] Leibenstein, H.M., 1957. Economic Backwardness and Economic Growth. Wiley, New York (NY).

[66] Livi-Bacci, M. (2017) A concise history of world population, 6th edn. Wiley-Blackwell, Malden.

[67] Lloyd-Braga, T., Nourry, C., Venditti, A., 2007. Indeterminacy in dynamic models: when Diamond meets Ramsey. Journal of Economic Theory 134, 513–536.

[68] Lorentzen, P., McMillan, J., Wacziarg, R., 2008. Death and development. Journal of Economic Growth 13, 81–124.

[69] Lucas, R.E. Jr., 1993. Making a miracle. Econometrica 61, 251–272.

[70] Matsuyama, K., 1991. Increasing returns, industrialization, and indeterminacy of equilibrium. Quarterly Journal of Economics 106, 617–650.

[71] Nishimura, K., Seegmuller, T., Venditti, A., 2015. Fiscal policy, debt constraint and expectations-driven volatility. Journal of Mathematical Economics 61, 305–316.

[72] Nourry, C., 2001. Stability of equilibria in the overlapping generations model with endogenous labor supply. Journal of Economic Dynamics & Control 25, 1647–1663.

[73] Nourry, C., Venditti, A., 2006. Overlapping generations model with endogenous labor supply: general formulation. Journal of Optimization Theory and Applications 128, 355–377.

[74] Nourry, C., Seegmuller, T., Venditti, A., 2013. Aggregate instability under balanced-budget consumption taxes: a re-examination. Journal of Economic Theory 148, 1977–2006.

[75] Panchuk, A., 2015. Computing one-dimensional invariant manifolds for saddle points of discrete time dynamical systems. Gecomplexity Discussion Paper Series. Action IS1104 "The EU in the new complex geography of economic systems: models, tools and policy evaluation".

[76] Reichlin, P., 1986. Equilibrium cycles in an overlapping generations economy with production. Journal of Economic Theory 40, 89–102.

[77] Rosen, S., 1988. The value of changes in life expectancy. Journal of Risk and Uncertainty 1, 285–304.

[78] Sorger, G., 2018. Cycles and chaos in the one-sector growth model with elastic labor supply. Economic Theory 65, 55–77.

[79] Varvarigos, D., Zakaria, I.Z., 2013. Endogenous fertility in a growth model with public and private health expenditures. Journal of Population Economics 26, 67–85.

[80] Weil, D.N., 2007. Accounting for the effect of health on economic growth. Quarterly Journal of Economics 122, 1265–1306.

[81] Woodford, M., 1984. Indeterminacy of equilibrium in the overlapping generations model: a survey. Columbia University Working Paper, New York (NY).

[82] World Health Statistics, 2015. Part II. Global Health Indicators, France.