To abate, or not to abate? A strategic approach on green production in Cournot and Bertrand duopolies

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Abstract This research analyses firms' strategic choice of adopting an abatement technology in an environment with pollution externalities when the government levies an emission tax to incentivise firms to undertake emission-reducing actions. A set of different Nash equilibria – ranging from dirty to green production – arises in both quantity-setting and price-setting duopolies. Results show that if societal awareness toward a clean environment is relatively low (resp. high) and the index measuring the relative cost of abatement is relatively high (resp. low), the strategic interaction between two independent, competing, and selfish firms playing *the abatement game* leads them not to abate (resp. to abate) as the Pareto efficient outcome: no conflict exists between self-interest and mutual benefit to not undertake (resp. to undertake) emissions-reducing actions. Multiple Nash equilibria or a "green" prisoner's dilemma can also emerge in pure strategies. When the choice of adopting a green technology is an anti-prisoner's dilemma (deadlock), the society is better off, as social welfare under abatement is always larger than under no abatement. These findings suggest that living in a sustainable environment encourages the improvement of public education systems for the achievement of an eco-responsible attitude and the development of clean technologies through ad hoc R&D.

Keywords "Green" production; Abatement; Emissions tax; Cournot and Bertrand duopolies

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Declarations of interest None

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1. Introduction

"The future's uncertain and the end is always near."
Jim Morrison

In recent years, several companies have increasingly adopted measures oriented toward environmental protection, such as the reduction of carbon emissions and energy intensity (Wurlod and Noailly, 2018). In this regard, KPMG (2015) revealed that the worldwide rate of carbon reporting within the G250 Fortune Index is 82%, with rates in the 14 surveyed industries ranging from 67% (personal and household goods) to 100% (food and beverages). A solid majority of those companies have disclosed targets for cutting further carbon emissions; while in 2015 the percentage was 58%, in 2017, the figure climbed to 67%. However, only a minority (25%) of those firms directed their own behaviours to targets related to climate goals set by national governments, regional authorities, or the United Nations, such as The Paris Agreement (KPMG, 2017). This suggests that there is room for intervention to realign firms' targets with environmental targets, as planned by public authorities. At this point, a question arises: is it beneficial for a firm operating in a strategic oligopolistic (duopolistic) market to invest in a green or clean technology to abate emissions when an environmentally concerned government sets an optimal emissions tax to incentivise firms to undertake emissions-reducing actions? The present work concentrates on this issue, which is surprisingly absent in the environmental economics literature framed in strategic competitive markets. Given the growing concerns among the public all around the world due to the urgent request by climate scientists to reduce emissions of greenhouse gases because of their potential impact on global and local political and economic systems (see "The climate issue", The Economist, 2019), a timely investigation on this subject is critical and needs to be addressed, especially for policy purposes.

There is a vast body of literature on environmental issues in economic models, ranging from growth and development (see., e.g., Antoci et al., 2019, 2020) to strategic competitive markets. This work concentrates on the latter. The need to adopt cleaner technologies is especially relevant in many developing countries, as the process of industrialisation based on heavy industry is often crucial for these countries' long-term development. Therefore, the use of end-of-pipe technologies for reducing emissions is increasingly necessary for environmental protection. This article contributes to the literature by clarifying the non-trivial effects of environment-related regulatory policies.

Starting from the simple framework in which every firm produces a single homogenous good at a single production plant (see, for example, Simpson 1995; Katsoulacos and Xepapadeas 1995; Carlsson 2000), the economics literature has extended the analysis to different market configurations (e.g., multi-countries, multi-products, unionised, managerial, vertical, differentiated for types of products and competition). The first branch of this literature analyses the strategic environmental policy in an international context, examining the strategic behaviour of countries when they set environmental taxes unilaterally or cooperatively. A pioneering contribution in this direction is Ulph (1996), who analysed the strategic environmental policy in countries engaging in international trade by considering an imperfect competition context. Recently, Hambel et al. (2018) proposed a novel theoretical framework that analysed the problem of optimal carbon abatement in a dynamic non-cooperative game with multiple countries assumed to be open economies. Several empirical contributions also exist, analysing the effects of coordination of environmental taxation on competition in markets and their related effects on social welfare. We referred to works such as Conrad (1993), Kennedy (1994), Conrad (1996a), Conrad (1996b), Bárcena-Ruiz and Campo (2012), and Bárcena-Ruiz and Garzón (2014). Conrad (1993) considered firms that sell to a third-country market and showed that higher environmental taxes are set when countries do not cooperate than when they do. Alternatively, Kennedy (1994) and Conrad (1996a) assumed home- and foreign-segmented markets with cross-border pollution. Furthermore, Conrad (1996b) and Bárcena-Ruiz and

Campo (2012) extended the latter analysis to the Bertrand competition market and cross-ownership of firms, respectively. Finally, Bárcena-Ruiz and Garzón (2014) analysed the coordination of environmental taxes with a supranational organisation considering multiproduct firms.

A second branch of this literature examined the strategic interaction between environmental policy and the endogenous location of polluting firms (to name a few, Rauscher 1995; Markusen et al. 1993; Markusen 1997; Bárcena-Ruiz and Garzón 2003). For instance, Markusen et al. (1993) discussed environmental taxation in a two-market, two-firm model in which firms locate their plants endogenously, showing that the social cost can be very high if environmental taxation ignores market endogeneity.

A third branch of the literature extended the basic framework of oligopolistic rivalry by assuming on the one hand separation between ownership and control by also introducing managerial delegation and on the other hand the existence of labour market imperfections (unions). Regarding managerial delegation, Bárcena-Ruiz (2002) studied the effects of delegating to managers the sales and pollution abatement choices in the presence of environmental tax and damage in a context of a competitive labour market and homogeneous product. Pal (2012) extended the framework of Bárcena-Ruiz (2002) to product differentiation and alternative modes of product market competition. Regarding unionised oligopolies, Bárcena-Ruiz (2011) and Bárcena-Ruiz and Garzón (2003, 2009) analysed the role played by different wage-setting structures on the environmental taxes and standards, the preferences of governments, and the location of polluting firms.

Finally, a fourth branch of literature illustrated the relation between environmental policies and market structures. In this regard, the pioneering works of Lee (1975) and Smith (1976) showed that market structures have an important effect on the efficiency of environmental taxation. Since then, other studies have revisited this issue. Oates and Strassmann (1984) examined the efficiency of environmental taxation in a mixed (private and public firms) market, and Conrad and Wang (1993) compared pollution taxes and abatement subsidies under three market structures: perfect competition, oligopolistic competition, and a dominant firm with a competitive fringe. Focusing on market structures, Lee (1999) revisited environmental taxation under an endogenous oligopolistic market structure, Althammer and Bucholz (1999) investigated the effects of the market structures on the second-best choice of the environmental tax, and Katsoulacos and Xepapadeas (1996) found that the optimal emissions tax could exceed marginal environmental damage under an endogenous market structure. Finally, Cato (2010) proposed a three-part environmental tax policy in an endogenous market structure, and Lambertini et al. (2017) examined the relationship between competition and innovation in an industry in which production is polluting and R&D has been developed to reduce emissions with both exogenous and endogenous emissions taxes, whereas Fukuda and Ouchida (2020) considered the environmental concerns in markets with firms practicing corporate social responsibility.

However, despite the different aspects and contexts investigated, none of the contributions mentioned above has embedded into the oligopoly market theory the analysis of *the firms' endogenous strategic choice* of adopting an abatement technology in an environment with pollution externalities when the government levies an emission tax to incentivise firms undertaking emission-reducing actions. This article aims at filling this gap by considering Cournot and Bertrand rivalry settings. In doing so, it augments the existing related literature, such as Poyago-Theotoky and Teerasuwannajak (2002), Moner-Colonques and Rubio (2015), and Ouchida and Goto (2016), who examined a multi-stage game in which the government has an emissions tax rate precommitment ability by analysing only the subgame in which firms do abate, by studying a multi-stage game comparing the incentives to abate with those that do not abate.

Definitively, selfish profit-maximising firms are engaged in a three-stage non-cooperative *abatement game* in which they should choose whether to abate pollution through a cleaning technology in a context where the government levies a social-welfare-maximising emission tax (through a linear tax system). At stage zero (*pre-stage*), firms choose whether to adopt the green technology (that is, to abate or not to abate pollutants from industrial production). At stage one (*the* *regulator stage*), the government chooses the optimal emissions tax to maximise social welfare (defined as the algebraic sum of consumers' surplus, producers' surplus, total tax revenue, and environmental damage). At stage two (*the market stage*), firms either simultaneously choose the amount of abated pollutant (if they do abate) and the quantity in the output market or choose their own optimal amount of production (if they do not abate). Alternatively, they set the price in a Bertrand setting.[1]

The article begins with the basic Cournot setting with homogenous products. Though there exists an optimal level of abated pollutants for each single player (firm) in a subgame in which pollution abatement is not subject to any strategic choice and both firms abate, it is critical that researchers analyse whether each firm finds optimal the unilateral choice of non-abatement in the standard context of the simultaneous and independent non-cooperative choice between the strategic profiles "to abate" (A) and "not to abate" (NA). In other words, the decision of whether to adopt an abatement technology should emerge as the endogenous Nash equilibrium outcome of the abatement game. Indeed, after the investment in pollution abatement, a reduction in production cost equal to the amount of taxation computed on less emissions than the corresponding case of no abatement is achieved. Under no abatement, in fact, each firm incurs no costs for abatement but pays more taxes, as the pollutant amount is larger in that case. This implies that the existence of a possible trade-off between A and NA so that the outcome of the game is not unambiguous.

The article shows that the abatement game produces a set of different Nash equilibria, ranging from dirty to clean or green production, depending on the societal awareness towards environmental quality (or against the damage) and the relative importance of the technological progress in abatement that affect the firms' marginal benefits and the marginal costs of undertaking emissions-reducing actions.

On the one hand, when the awareness of the society against the damage generated by industrial production is relatively low and the index measuring the relative cost of abatement is relatively high, the strategic interaction between two non-cooperative profit-maximising firms leads firms not to abate (NA) as the Pareto efficient outcome of the game, and there is no conflict between self-interest and mutual benefit to not undertake emission-reducing actions. This is because the amount of abated pollutants is low due to the high abatement costs so that the reduction in production costs – equal to the amount of taxation computed on less emissions – is too low to overcome the costs to invest in pollution abatement.

On the other hand, when the awareness of the society against the damage generated by industrial production is relatively high and the index measuring the relative cost of abatement is relatively low, the strategic interaction between two non-cooperative profit-maximising firms leads to abate (A) as the Pareto efficient outcome of the game, and there is no conflict between self-interest and mutual benefit to undertake emission-reducing actions. This is because the amount of abated pollutant is high due to the low abatement costs so that the reduction in production costs – equal to the amount of taxation computed on less emissions – is high enough to overcome the costs to invest in pollution abatement.

Pollution and environmental damage in the abatement scenario are always lower than under no abatement, so that social welfare under A is larger than under NA. Therefore, if the degree of ecological awareness of the society is large and the cost-reducing R&D technological progress is high, adopting a green technology becomes a Pareto-efficient strategy for both firms and society, which are therefore willing to accept positive (optimal) taxation to improve the environmental conditions. These findings suggest that living in a sustainable environment challenges the

[1] This game structure could represent, e.g., an environment with end-of-pipe technologies requiring both variable and fixed costs. Fixed costs can enable firms to hesitate to invest in reducing abatement. In our model, the variable costs of pollution reduction are explicitly formulated, whereas the fixed costs are not. In a developing or emerging country lying in a trajectory of industrialisation, firms might find it difficult to pay fixed costs for the abatement device of end-of-pipe type, so a foreign developed country could assist them in this direction. If firms accept this support for abatement equipment, they can choose to abate incurring variable costs for pollution abatement. We warmly thank an anonymous reviewer for suggesting this kind of application that can also represent a stimulus for future research.

development of clean technologies through ad hoc R&D and the improvement of public education to achieve an eco-responsible attitude.

The work also compares the outcomes of the abatement game in quantity-setting and price-setting duopolies by considering the horizontal product differentiation à la Singh and Vives (1984) to facilitate capturing of the heterogeneity of consumers' tastes (see Nevo, 2000, for an empirical analysis of the market demand under product differentiation). With this kind of product differentiation, goods are different but and some consumers will prefer buying one of them and some will buy other products at the same price depending on their preferences. The main results of the article are confirmed in both cases of product substitutability and complementarity. However, as product differentiation increases the profitability of firms through the increase in their market power, product substitutability and product complementarity work out in the anti-ecological direction of letting non-abatement be the dominant and Pareto-efficient strategy of the game for a large range of values of the key parameters of the model.

The remainder of the article is organised as follows. Section 2 introduces the basic Cournot model with homogenous products. Section 3 discusses the key results by offering the economic intuition of the different scenarios the abatement game can generate (ranging from an anti-ecological deadlock to a green deadlock passing from a green prisoner's dilemma). Section 4 speculates about a possible ethical-historical-political projection exercise with the aim of distinguishing between the logical or philosophical time of the (timeless or static) one-shot, non-cooperative abatement game and the historical timing of the events (past, present, and future) that contributes to generate a chronological structure of sequences and describes possible policy recipes. Section 5 discusses some extensions of the basic model for robustness check by considering the horizontal product differentiation à la Singh and Vives (1984) and progressive taxation. Section 6 concludes the article, providing an outline for future research.

2. The abatement game with homogeneous products

Consider a Cournot duopoly industry in which firm 1 and firm 2 produce homogeneous products q_1 and q_2, respectively. The existing technology allows firm i ($i = 1,2$) to produce q_i units of output, causing e_i units of emissions (pollution), where $e_i = q_i - k_i$ (Ulph, 1996), with $0 \leq k_i < q_i$ representing the abatement level for environmental protection coming from a cleaning technology available to each firm, implying that emissions cannot be entirely eliminated.[2] Available technology is such that firm i ($i = 1,2$) produces with constant (marginal) returns to labour production function, that is $q_i = L_i$, where L_i represents the labour force employed by the firm and faces the same (constant) average and marginal cost $w \geq 0$ (representing the wage per unit of labour) for every unit of output produced. Therefore, firm i's cost function is linear and given by $C_i(q_i) = w\, L_i = w\, q_i$. In addition, the pollution abatement cost function of firm i is $CA_i(k_i) = \frac{z}{2} k_i^2$, where $z > 0$ is a parameter that scales up/down the total abatement cost and represents an exogenous index of technological progress measuring, for example, the appearance of a new, cost-effective cleaning technology. Indeed, it measures the degree at which the available technology for pollution abatement impacts the environment. A reduction in z can be interpreted as an improvement in the technological progress in abatement so that abating becomes cheaper. The adoption of a clean technology requires sustaining costs with decreasing returns to investment. This implies that when firms choose to abate emissions, they always face some costs. One can think of a cleaning technology not directly linked to output, for example "the number of the filters in a refinery's pipe for CO2 reduction or 'scrubbers' to remove SO2 from a fuel gas coal fired electric plant" (Asproudis and Gil-Moltó, 2015, p. 169). As

[2] See Appendix A for analytical details also supporting the assumption of an abatement technology allowing to eliminate pollution entirely, e.g., $0 \leq k_i \leq q_i$. Our assumption is in line with Ulph (1996) and differs from Asproudis and Gil-Moltó (2015), who assumed emissions (pollution) were represented by $e_i = q_i k_i$, so that $k_i \in [0,1)$ is a fraction of total production in that case.

k_i represents the pollutant abated per q_i units of output, a larger (resp. smaller) value of k_i corresponds ceteris paribus to a more (resp. less) efficient abatement.

Industrial production causes an environmental damage measured by the index $ED = \frac{g}{2}(e_1 + e_2)^2$, where $g > 0$ is the burden the government attaches to the environmental damage, representing the awareness of the overall society towards the environment and thus against the damage (computed as aggregate emission squared) generated by industrial production.[3] An increase in g implies *ceteris paribus* an increase in the extent of the relative weight of the environmental damage as measured by the overall society (government). This type of damage function is commonly used in the related literature and assumes that (i) the environmental damage is a convex function of total pollution and (ii) the damage is exogenous for consumers (see, for example, van der Ploeg and de Zeeuw, 1992; Ulph, 1996).[4]

We assume a linear (inverse) market demand given by $p = \alpha - \beta Q$, where α is a positive parameter representing the market size ($\alpha > w \geq 0$), $\beta > 0$ measures the slope of the market demand being part of its elasticity, and $Q = q_1 + q_2$ is total supply.[5] This kind of demand structure comes from the usual specification of quadratic utility for consumers' preferences, that is $U(q_1, q_2) = \alpha(q_1 + q_2) - \frac{\beta}{2}(q_1^2 + q_2^2 + 2q_1 q_2)$, as proposed by Dixit (1979) and subsequently used, amongst others, by Singh and Vives (1984), Häckner (2000), and Correa-López and Naylor (2004). For reasons of analytical tractability (and without loss of generality), we assume $\alpha = 1$, $\beta = 1$, and $w = 0$ henceforth.[6]

The government levies an emission tax $t \in (0,1]$ per each unit of polluting output to incentivise firms undertaking emissions-reducing actions with the aim of maximising social welfare. Consequently, the tax base of firm i without abatement is q_i and the corresponding tax revenue of the government is $t\, q_i$. Differently, if firm i chooses to abate emissions, the tax base becomes $q_i - k_i$ (i.e., the remaining pollution), and the corresponding tax revenue is $t(q_i - k_i)$.[7]

Definitively, firms are engaged in a three-stage non-cooperative *abatement game* in which they should choose whether to abate pollution through a cleaning technology in a context in which the government levies a social-welfare-maximising emission tax. At stage zero (pre-stage), firms choose whether to adopt the green technology. At stage one (the regulator stage), the government chooses the optimal emission tax to maximise social welfare. At stage two (the market stage), firms either simultaneously choose to adopt a clean (or green) technology to abate pollution and the quantity in the output market (if they do abate), or they choose their own optimal amount of production (if they do not abate).[8] As usual, the game is solved by adopting the backward induction logic.

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[3] The main results of this article are confirmed by assuming a linear environmental damage function $ED = g(e_1 + e_2)$ where $g > 0$ becomes the marginal damage from emissions (see Kennedy, 1999; Kennedy and Laplante, 1999; Requate, 2005; Asproudis and Gil-Moltó, 2015).

[4] Though g is assumed to be exogenous, it may be endogenized as it represents a parameter capturing the preferences of the overall society towards environmental quality. It may depend, for example, on the preferences of voters by capturing the awareness of the majority to environmental issues. However, this is beyond the aim of the present article and is left for future research.

[5] These inequalities should hold, as the (highest) price consumers are willing to pay for the first unit of goods must always be larger than the (lowest) marginal cost firms incur to produce the first unit of goods. Otherwise, the market would not exist.

[6] Indeed, the market size, the slope of the market demand, and the average and marginal costs do not affect all the relevant feasibility thresholds, profit differentials, environmental damage, and social welfare functions in the cases of homogeneous and heterogeneous products in quantity-setting and price-setting duopolies, as will be shown later in this article.

[7] As we assumed a normalised market demand, the emission tax $t \in (0,1]$. In the non-normalised case, we would have $t \in (0, \alpha]$ (see Eq. (3)).

[8] An additional issue concerns the choice of the timing of the game in the setting of the emission tax by the regulator. In this regard, we note that, with a linear tax scheme as the one used in this article, it is not possible to test the results in case the government would choose the emissions tax to incentivise firms undertaking emissions-reducing actions at the second stage of the game, and firms decide on output and (if they abate) on the level of abatement simultaneously at the first stage. This is because an optimal emission tax rate does not exist in that case. To test for an ex-post versus ex-ante

2.1 Firms do not to abate emissions (NA)

Let us first analyse the symmetric case in which both firms choose not to abate emissions, that is $k_i = 0$ ($i = 1,2$). The profit function of firm i is therefore the following:

$$\pi_1^{*NA} = (1 - q_i - q_j - t)q_i, i = 1,2; \quad i \neq j, \tag{1}$$

where the upper script NA stands for no abatement. In the second stage of the game, each firm chooses its optimal output. Maximisation of (1) with respect to quantities leads to the following system of reaction functions:

$$q_i = \frac{1-q_j-t}{2}, i = 1,2; \quad i \neq j, \tag{2}$$

whose solution gives the following equilibrium output chosen by firm i at the second stage of the game:

$$q_i = \frac{1-t}{3}. \tag{3}$$

Making use of (3), one may directly obtain producer surplus (PS), consumer surplus (CS), tax revenues (TR), and the environmental damage caused by industrial production (ED) under NA:

$$PS^{NA} = \pi_1^{NA} + \pi_2^{NA} = \frac{2}{9}(1-t)^2, \tag{4}$$

$$CS^{NA} = \frac{(q_1+q_2)^2}{2} = \frac{2}{9}(1-t)^2, \tag{5}$$

$$TS^{NA} = t(q_1 + q_2)^2 = \frac{2}{3}t(1-t), \tag{6}$$

$$ED^{NA} = \frac{g}{2}(q_1 + q_2)^2 = \frac{2}{9}g(1-t)^2. \tag{7}$$

The social welfare function is given by the following index:

$$SW^{NA} = PS^{NA} + CS^{NA} + TR^{NA} - ED^{NA} = \frac{2(1-t)[2+t-g(1-t)]}{9}. \tag{8}$$

In the first stage of the game, the government chooses the emissions tax to maximise social welfare, that is:

$$\frac{\partial SW^{NA}}{\partial t} = 0 \Rightarrow t^{*NA} = \frac{2g-1}{2(1+g)}. \tag{9}$$

Eq. (9) implies that a positive optimal emission tax under NA exists (i.e., $t^{*NA} > 0$) if and only if the overall social evaluation of the environmental damage (alternatively, the degree of societal awareness against the damage) is sufficiently large, that is $g > \frac{1}{2} := g^{NA}$. Moreover, further substitutions reveal that the non-negativity constraint on output is always fulfilled for $g > g^{NA}$, i.e., $q_i^{NA} > 0$ and $e_i^{NA} = q_i^{NA} > 0$ holds. Substituting back the optimal tax in (9), the first-stage equilibrium profits of firm 1 and firm 2 under NA are given by:

$$\pi_1^{*NA} = \pi_2^{*NA} = \frac{1}{4(1+g)^2}. \tag{10}$$

2.2 Firms adopt the abatement technology (A)

Consider now the symmetric case in which both firms adopt the abatement technology, with $k_i \in [0, q_i)$. The profit functions take the following form:

dilemma, one should introduce a non-linear environmental tax scheme in the abatement game or avoid considering a government that levies emission taxes by assuming a social planner that chooses the amount of abated pollutant, as in e.g., Asproudis et al. (2019) and the references cited therein. However, by assuming that the firm's choices about production and abatement are sequential, it is possible to build on a game in which the timing is such that, at stage 1, each firm chooses the emissions abatement effort to maximise profit; at stage 2, the regulator (government) chooses the emission tax to maximise social welfare; at stage 3, each firm chooses the output in the product market to maximise profit, and then there may exist situations such that (A,A) is the Pareto-efficient SPNE (deadlock) or multiple asymmetric Nash equilibria (A,NA) and (NA,A), in which only one firm abates pollution.

$$\pi_i^A = (1 - q_i - q_j)q_i - t(q_i - k_i) - \frac{z{k_i}^2}{2},\ i = 1,2; \quad i \neq j, \tag{11}$$

where the upper script A stands for abatement. In the second stage of the game, firms choose both the output and the abatement level. After the investment in pollution abatement, a reduction in production cost equal to the amount of taxation computed on less emissions is achieved beyond the corresponding case of no abatement.

Maximisation of (11) with respect to q_i and k_i leads to the following set of first-order conditions:

$$\text{A) } q_i = \frac{1 - q_j - t}{2};\ \text{B) } k_i = \frac{t}{z},\ i = 1,2; \quad i \neq j. \tag{12}$$

The solution of the system of output reaction functions A) in (12) implies that, in equilibrium, output is as in (3). Using (3) and condition B) in (12), one can get the following expressions for the producer surplus, consumer surplus, the tax revenue, and the environmental damage under A:

$$PS^A = \pi_1^{NA} + \pi_2^{NA} = \frac{2}{9}(1 - t)^2 + \frac{t^2}{z}, \tag{13}$$

$$CS^{NA} = \frac{(q_1 + q_2)^2}{2} = \frac{2}{9}(1 - t)^2, \tag{14}$$

$$TR^A = t[(q_1 - k_1) + (q_2 - k_2)] = \frac{2}{3}t\left[1 - \frac{t(3+z)}{z}\right], \tag{15}$$

$$ED^A = \frac{g}{2}[(q_1 - k_1) + (q_2 - k_2)]^2 = \frac{2}{9}g\left[1 - \frac{t(3+z)}{z}\right]^2. \tag{16}$$

Therefore, the expression of social welfare is as follows:

$$SW^A = PA^A + CS^A + TR^A - ED^A$$
$$= \frac{2z^2(1-t)[2-g+(1+g)t]+3zt[4g(1-t)-3t]-18gt^2}{9z^2}. \tag{17}$$

In the first stage of the game, the government chooses the emissions tax such that the expression in (17) is maximised, that is:

$$\frac{\partial SW^A}{\partial t} = 0 \Rightarrow t^{*A} = \frac{z[(2g-1)z+6g]}{2z^2(1+g)+3z(3+4g)+18g}. \tag{18}$$

Eq. (18) implies that a positive optimal emission tax under A exists (i.e., $t^{*A} > 0$) if and only if the overall societal evaluation of the environmental damage is sufficiently large, that is $g > \frac{z}{2(3+z)} := g^A(z) < g^{NA}$. Making use of the optimal tax in (18), one can easily check that if $g > g^A(z)$ then $q_i^A > 0$, $k_i^A > 0$, and the condition $e_i^A = q_i^A - k_i^A > 0$ is always fulfilled. Finally, substituting back the optimal tax in (18), the first-stage equilibrium profits of firm 1 and firm 2 under A are as follows:

$$\pi_1^{*A} = \pi_2^{*A} = \frac{2z^4+z^3(4g^2+4g+13)+2z^2(16g^2+18g+\ \ )+12zg(7g+6)+72g^2}{2[2z^2(1+g)+3(4gz+6g+3z)]^2}. \tag{19}$$

2.3 The asymmetric case: one firm abates, the rival does not (A/NA)

To analyse the firms' endogenous incentive to abate emissions, one must evaluate the outcomes of the asymmetric behaviour in which one firm, say firm 1, adopts the abatement technology and the rival (firm 2) does not. The two firms' optimisation problems lead to first-order conditions as in (12) for firm 1 and as in (2) for firm 2. Again, the firms' equilibrium output is given by (3) and, together with condition B) in (12), standard substitutions lead to the next expressions for the producer surplus, consumer surplus, tax revenue, and environmental damage in the asymmetric case:

$$PS^{A/NA} = \pi_1^{A/NA} + \pi_2^{A/NA} = \frac{1}{18}[4z(1 + t^2) + t(9t - 8z)], \tag{20}$$

$$CS^{NA} = \frac{(q_1 + q_2)^2}{2} = \frac{2}{9}(1 - t)^2, \tag{21}$$

$$TR^{A/NA} = t[(q_1 - k_1) + q_2] = \frac{t}{3z}[2z(1 - t) - 3t)], \tag{22}$$

$$ED^{A/NA} = \frac{g}{2}[(q_1 - k_1) + q_2\ ]^2 = \frac{g}{18z^2}[2z(1 - t) - 3t]. \tag{23}$$

The expression of the social welfare under A/NA is as follows:

$$SW^{A/NA} = PA^{A/NA} + CS^{A/NA} + TR^{A/NA} - ED^{A/NA}$$
$$= \frac{4z^2(1-t)[2-g+(1+g)t]+3zt[4g-(4g+3)t]-9gt^2}{18\ \ ^2}\qquad . \tag{24}$$

Therefore, in the first stage of the game, the government chooses the emissions tax such that the social welfare is maximised, that is:

$$\frac{\partial SW^{A/NA}}{\partial t} = 0 \Rightarrow t^{*A/NA} = \frac{2z[(2g-1)z+3g]}{4z^2(1+g)+3z(4g+3)+9g}. \quad (25)$$

Eq. (25) implies that a positive optimal emission tax in the asymmetric case exists (i.e., $t^{*A/NA} > 0$) if and only if the overall societal evaluation of the environmental damage is sufficiently large, that is $g > \frac{z}{3+2z} := g^{A/NA}(z) < g^{NA}$. Making use of the optimal tax in (25) and accounting for the constraint $g > g^{A/NA}(z)$, it can easily be shown that $q_1^{A/NA} > 0$, $k_1^{A/NA} > 0$, $q_2^{A/NA} > 0$ and $k_2^{A/NA} > 0$. In addition, the inequality $e_1^{A/NA} = q_1^{A/NA} - k_1^{A/NA} > 0$ is satisfied for any $g^{A/NA}(z) < g < \frac{z(2z+5)}{3+2z} := g^T(z)$. The second inequality in the last constraint is a threshold that should be fulfilled, as the perceived societal awareness towards the environment should not be too high; otherwise, the corresponding optimal tax should be set at too high a level and prevent the firm from implementing a feasible abatement technology. Finally, inserting back the optimal tax in (25), the first-stage equilibrium profits of firm 1 and firm 2 in the case of asymmetric behaviour are as follows:

$$\pi_1^{*A/NA} = \frac{4z^4+2z^3(4g^2+7)+z^2(28g^2+12g+9)+6zg(5g+3)+9g^2}{\{4z[z(1+g)+3g]+9(g+z)\}^2}, \pi_2^{*A/NA} = \frac{(2z+3)^2(z+g)^2}{\{4z[z(1+g)+3g]+9(g+z)\}^2}. \quad (26)$$

It is important to note that the optimal tax rates (9), (18), and (25) in the strategic profiles NA, A, and A/NA, respectively, positively depend on g ($\partial t^*/\partial g > 0$). The economic intuition is clear. As g represents the societal awareness towards environment quality (i.e., against the environmental damage generated by industrial production), an increase in g implies that the society is willing to pay more taxes to allocate more resources toward environmental protection by favouring the adoption of clean technologies. This is one important element to disentangle the effects allowing to incentivise the (endogenous) adoption of environmentally friendly technologies by firms.

The tax rate contributes to the increase in the marginal cost of industrial production and incentivise abatement. Therefore, the higher the optimal emission tax rate, the lower the quantity produced by each firm in the market and the higher the amount of pollutant will be abated in equilibrium.

Regarding the dependency of t^* on z, under the strategic profile NA, the optimal tax rate is independent of the parameter weighting technological progress in abatement. Differently, under A and A/NA, there exists a non-monotonic shape (first increasing and then decreasing but the increasing branch of the curve still holds for sufficiently high values of z) depending on the size of g. However, when g becomes larger, the optimal emission tax rate starts becoming a monotonic increasing function of g (this holds for values of z around 1 in both cases A and A/NA).

Therefore, technological progress favouring emissions abatement (through reduction in z) generally works in the direction of reducing the tax burden. This is another element that will be useful for the intuition of the results that will be presented later in this article.

3. The abatement game: Analysis and results

3.1 Nash equilibria

This section examines the pre-stage of the game, in which firms should choose whether to adopt or not to adopt the green technology. Making use of the firms' profits in (10), (19), and (26), it is possible to build on the payoff matrix summarised in Table 1 regarding the Cournot abatement game with homogenous products.

To satisfy all of the technical restrictions and have well-defined equilibria in pure strategies for every strategic profile, the analysis is restricted to the following feasibility constraints, $\frac{1}{2} := g^{NA} < g < g^T(z)$, that hold for any $z > 0.323$ (which is assumed to be always satisfied henceforth). Then, to derive all of the possible equilibria of the game, one must study the sign of the following profit differentials for $i = 1,2$ (see Appendix B for details):

$$\Delta\pi_1 := \pi_i^{A/NA} - \pi_i^{NA},\ \Delta\pi_2 := \pi_i^{NA/A} - \pi_i^{A} \text{ and } \Delta\pi_3 := \pi_i^{NA} - \pi_i^{A}. \quad (27)$$

Table 1. The abatement game (payoff matrix). Cournot competition with homogenous products.

Firm 2 → Firm 1 ↓	A	NA
A	π_1^{*A}, π_2^{*A}	$\pi_1^{*A/NA}, \pi_2^{*A/NA}$
NA	$\pi_1^{*NA/A}, \pi_2^{*NA/A}$	π_1^{*NA}, π_2^{*NA}

Let $g_{\Delta_1}(z)$, $g_{\Delta_2}(z)$, and $g_{\Delta_3}(z)$ be three threshold values of g as a function of z such that the profit differentials $\Delta\pi_1 = 0$, $\Delta\pi_2 = 0$, and $\Delta\pi_3 = 0$, respectively. The shape of $g_{\Delta_1}(z)$ (solid line), $g_{\Delta_2}(z)$ (dashed line), and $g_{\Delta_3}(z)$ (dotted line) is depicted in Figure 1 in the parameter space (g, z). The solid red increasing curve $g^T(z)$ is the boundary of the (red) area, representing the unfeasible parameter space (otherwise the optimal tax rate should be set at too high a level, thus preventing the firm from implementing a feasible abatement technology).

An analytical inspection of each profit differential in (27) reveals that, for $g < g_{\Delta_1}(z)$ (resp. $g > g_{\Delta_1}(z)$), we have that $\Delta\pi_1 < 0$ (resp. $\Delta\pi_1 > 0$), for $g < g_{\Delta_2}(z)$ (resp. $g > g_{\Delta_2}(z)$) we have that $\Delta\pi_2 > 0$ (resp. $\Delta\pi_2 < 0$), and for $g < g_{\Delta_3}(z)$ (resp. $g > g_{\Delta_3}(z)$) we have that $\Delta\pi_3 > 0$ (resp. $\Delta\pi_3 < 0$). As is clear by looking at the figure and the shapes of the thresholds, the larger the parameter measuring the relative cost of technological progress in abatement (z) and the smaller the societal awareness against the environmental damage (g), the higher the firms' incentive not to adopt an abatement technology, as the benefits of abatement (the amount of the emission tax levied by the regulator on the differences between production and abatement) are smaller than the costs (as measured by the effectiveness of the abatement technology). Differently, the firms' incentive to abate emissions increases if the abatement technology becomes more efficient (z reduces) and the society's degree of environmental awareness becomes higher (g increases). The relative shapes of the three thresholds contribute to determine the outcomes of the abatement game. For this purpose, Result 1 shows the spectrum of Nash equilibria of this three-stage non-cooperative game for different parameter values.

Result 1. If the regulator levies an emissions tax to incentivise firms undertaking emissions-reducing actions, the abatement game produces the following set of Nash equilibrium outcomes in pure strategies.

[1] Let $z > 3.702$ hold. (1) If $g^{NA} < g < g_{\Delta_2}(z)$ then (NA,NA) is the unique Pareto-efficient SPNE of the game (deadlock), and NA is the dominant strategy (there is no conflict between self-interest and mutual benefit to do not undertake emission-reducing actions). (2) If $g_{\Delta_2}(z) < g < g_{\Delta_1}(z)$, then there exist two pure-strategy Nash equilibria given by (NA,NA) and (A,A), and the NA payoff dominates A (coordination game). (3) If $g_{\Delta_1}(z) < g < g_{\Delta_3}(z)$ then (A,A) is the unique Pareto-inefficient SPNE of the game (prisoner's dilemma), and A is the dominant strategy (there is a conflict between self-interest and the mutual benefit of undertaking emission-reducing actions). (4) If $g_{\Delta_3}(z) < g < g^T(z)$, then (A,A) is the unique Pareto-efficient SPNE of the game (deadlock), and A is the dominant strategy (there is no conflict between self-interest and the mutual benefit of undertaking emissions-reducing actions).

[2] Let $2.107 < z < 3.702$ hold. (1) If $g^{NA} < g < g_{\Delta_2}(z)$, then (NA,NA) is the unique Pareto-efficient SPNE of the game (deadlock), and NA is the dominant strategy (there is no conflict between self-interest and mutual benefit to do not undertake emissions-reducing actions). (2) If $g_{\Delta_2}(z) < g <$

$g_{\Delta_3}(z)$, then there exist two pure-strategy Nash equilibria given by (NA,NA) and (A,A), and NA payoff dominates A (coordination game). (3) If $g_{\Delta_3}(z) < g < g_{\Delta_1}(z)$, then there exist two pure-strategy Nash equilibria given by (NA,NA) and (A,A), and the A payoff dominates NA (coordination game). (4) If $g_{\Delta_1}(z) < g < g^T(z)$, then (A,A) is the unique Pareto efficient SPNE of the game (deadlock), and A is the dominant strategy (there is no conflict between self-interest and the mutual benefit of undertaking emissions-reducing actions).

[3] Let $0.593 < z < 2.107$ hold. (1) If $g^{NA} < g < g_{\Delta_3}(z)$, then there exist two pure-strategy Nash equilibria given by (NA,NA) and (A,A), and the NA payoff dominates A (coordination game). (2) If $g_{\Delta_3}(z) < g < g_{\Delta_1}(z)$ then there exist two pure-strategy Nash equilibria given by (NA,NA) and (A,A), and A payoff dominates NA (coordination game). (3) If $g_{\Delta_1}(z) < g < g^T(z)$, then (A,A) is the unique Pareto-efficient SPNE of the game (deadlock), and A is the dominant strategy (there is no conflict between self-interest and mutual benefit of undertaking emission-reducing actions).

[4] Let $0.323 < z < 0.593$ hold. (1) If $g^{NA} < g < g_{\Delta_1}(z)$, then there exist two pure-strategy Nash equilibria given by (NA,NA) and (A,A), and NA payoff dominates A (coordination game). (2) If $g_{\Delta_1}(z) < g < g_{\Delta_3}(z)$ then (A,A) is the unique Pareto-inefficient SPNE of the game (prisoner's dilemma), and A is the dominant strategy (there is a conflict between self-interest and mutual benefit of undertaking emission-reducing actions). (3) If $g_{\Delta_3}(z) < g < g^T(z)$, then (A,A) is the unique Pareto efficient SPNE of the game (deadlock) and A is the dominant strategy (there is no conflict between self-interest and mutual benefit of undertaking emission-reducing actions).

3.2 Discussion and economic intuition

The analysis of the abatement game in the case of quantity competition with homogenous products shows that the comparison between the behaviours of selfish firms in a strategic context may have different outcomes resembling several paradigms of the game theory. To this purpose, Result 1 (whose geometric pattern is represented in Figure 1) clearly implies that the emergence of these outcomes depends on a combination of two parameters representing, respectively, the relative impact on profits of the costs that should be sustained by the firms to adopt a technology to reduce pollution (z) and the relative weight of the (optimal) emission tax in the different strategic profiles (g) to incentivise firms to undertake emissions-reducing actions. The higher (resp. lower) the parameter z, the higher (resp. lower) the cost each firm should incur to abate pollution by choosing the optimal amount of pollutant to be abated. This may happen, for example, because the technological progress for abatement has not yet adequately facilitated development of (resp. has already allowed) an efficient cleaning technology to reduce abatement costs and improve environmental quality. The lower (resp. higher) g, the lower (resp. higher) the societal awareness towards environmental quality and the lower (resp. higher) the amount of emission tax firms are paying to favour abatement. An obvious scenario emerging from the study of the game is that adopting a cleaning technology is a matter of comparing marginal benefits and marginal costs of abatement when the regulator levies an emissions tax whose burden changes depending on the amount of abated pollutant. In fact, on the one hand, the emissions tax increases the marginal cost of production. On the other hand, the abatement technology allows firm to reduce marginal costs, pollution, and eventually the tax base. However, it requires an investment with diminishing returns (Asproudis and Gil-Moltó, 2015) that indeed reduces profits.

The larger the quantity of pollutant abated (which, at the optimum, positively depends on the emission tax rate and thus on g, and negatively depends on z), the higher the costs for abatement, but the lower the tax base (i.e., the difference between industrial production and abated pollutant) and the amount of emission tax that should be paid in equilibrium. The smaller the quantity of pollutant abated, the lower the costs for abatement but the higher the tax base and the amount of emission tax should be paid in equilibrium. In the ultimate case of no abatement, each firm does not incur abatement costs, but the tax base and the emissions tax are the highest. Therefore, there are conflicting effects, and the outcome of the abatement game depends on whether the benefits arising from adopting an abatement technology overcome the corresponding costs or not.

The economic intuition of the different results of the abatement game can be carried out following the narrative of and the cases detailed in Result 1 (whose ethical-historical-political reading is presented in Section 3), starting from a combination of parameters in (g, z) plane belonging to the north-west area of Figure 1 (i.e., high abatement costs and low awareness towards environmental quality) and moving to the right by increasing the degree of awareness of the society towards the environment. In this case, the comparison between profits of two profit-maximising firms that must choose to abate or not to abate pollutants from industrial production supports the strategic profile NA. Indeed, on the one hand, a firm that chooses not to abate pollution pays a relatively low amount of emissions tax, though it is computed on the whole quantity produced. This is because the awareness towards environmental quality is low, and it does not incur any abatement cost so that the tax rate at the optimum is low enough in that case. On the other hand, choosing to abate pollution implies that firms incur high abatement costs (as z is high) that will thereby imply a small amount of pollutant abated at the optimum but the rate of environmental taxation applied by the government in that case will be relatively high because of the high value of z (though g is low). In addition, a lower amount of goods will be produced in the product market (which will be sold at a higher price) than in the case of non-abatement. The combination of high costs and low benefits of abatement implies that adopting an abatement technology is expensive, and profits under A will be lower than profits under NA, which is therefore a dominant strategy. This outcome is Pareto efficient for firms, and there is no conflict between self-interest and mutual benefit not to undertake emissions-reducing actions.

By increasing slightly environmental awareness when abatement costs are high implies entering a parametric area in which there are multiple pure-strategy Nash equilibria, in which NA payoff dominates A. The increase in g causes an increase in the emissions tax rate that should be paid at the optimum in the different strategic profiles of the game. This implies a reduction in the amount of industrial production and an increase in the price consumers are willing to pay in the market. In addition, the amount of pollutant abated increases as g is increased.

The combination of these effects implies a drop in profits in all (symmetric and asymmetric) strategic profiles, as the negative effects on profits of the reduction in the amount of industrial production overcomes the positive effects due the increase in the market price. However, the fall in profits is larger under NA than under A, as pollution abatement (implying a higher amount of abated pollutant than when g was lower) also works in the direction of reducing the overall burden of the emission tax and provides an incentive for firms to deviate towards abatement if the rival also chooses to abate. No dominant strategies exist in this case. Definitively, the outcome of the game is *a priori* uncertain. In fact, on the one hand, each player is interested in NA if the rival chooses not to abate pollution, because everyone would like to play NA. On the other hand, no one is willing to adopt an abatement technology if the rival does not abate, as no one wants to be the sole player to incur abatement costs. Again, no one would like to make decisions that he might regret, but no one can foresee the rival's decisions (this is because firms are playing a simultaneous non-cooperative game).

It could happen, however, that the two players make non-consistent choices: if someone chooses not to abate to avoid losing the opportunity to get higher profits and the rival chooses to abate to reduce the tax burden, players could end up in a situation in which only one of them adopts the abatement technology, representing for both an unsatisfactory outcome, as everyone could be better off with a different choice. Differently, if decisions were compatible, both players would choose not to abate pollution if they are not willing to be the unique player to adopt the abatement technology (which is relatively expensive), thus achieving a Pareto-efficient outcome. Players could also make compatible decisions ensuring a sub-optimal result, and thus they can both choose to adopt the abatement technology if they are willing to reduce the tax burden in a context in which social awareness against pollution is not sufficiently high. In this context, however, players are interested in agreeing not to abate (NA). Furthermore, by choosing NA, everyone becomes interested in complying with the non-binding agreement, as no one would have the incentive to deviate towards A with a Pareto-dominant outcome. Though the game is non-cooperative, both players have an incentive to coordinate towards NA by producing with dirty technology.[9]

A further increase in the societal awareness of environmental quality (g) allows entering a small portion of the plane (area c) in which the game becomes a prisoner's dilemma, where there is a conflict between self-interest and the mutual benefit of undertaking emissions-reducing actions. The increase in g causes a significant increase in the emissions tax rate under NA and a slight increase in the tax rate under A. The quantity of goods produced in all strategic profiles reduces, and the market price increases. The increase in the amount of abated pollutant also reduces the tax burden of emissions in comparison with the case of no abatement. The combination of these effects implies that the reduction in profits under NA is larger than the reduction in profits under A. Payoffs in this case imply that each player would be fully satisfied to adopt the cleaning technology with the lowest tax burden, but each player would prefer to cooperate with the rival not to abate (thus producing the highest possible amount of goods), thereby achieving higher profits than when both firms choose to adopt the abatement technology. Each player, however, would prefer to abate and reduce the tax burden rather than being the only one choosing not to abate, thus paying the highest possible amount of emissions taxes and getting the lowest possible amount of profits. In this context, each player has a dominant strategy (A) allowing the best results, regardless of the rivals' choice. In fact, no one is interested in playing NA if the rival plays A, as everyone prefers to be the only one to abate with the lowest tax burden (and highest profit), rather than being the only one not to abate with the highest tax burden (and lowest profit). Also, no one is interested in not abating pollution, even when the rival chooses not to abate, as everyone prefers to adopt the cleaning technology to reduce the tax burden rather than producing with a dirty technology by paying more taxes. Regardless of the rival's decision, no one will decide to play NA, and both players will agree to reduce their profits by adopting an abatement technology. If both players had decided to cooperate by playing NA, however, they would have jointly achieved a better outcome. This result is interesting, as it highlights the existence of a conflict between self-interest and mutual benefits. By making decisions that guarantee each selfish player the best result unilaterally, both achieve a Pareto dominated outcome, which they would not have obtained if both had chosen not to abate.[10]

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[9] A result with multiple equilibria in pure strategies like the one outlined here can be obtained in area b of the parametric space (g, z) in Figure 1, where A payoff dominates NA. By increasing environmental awareness when abatement costs are lower than before causes an increase in the amount of pollutant abated (as g is increased) that may make it convenient to adopt a cleaning technology through the reduction in the marginal costs of production. No dominant strategies do exist, and the outcome of the game is *a priori* uncertain also in this case. If decisions were compatible, both players would choose to abate pollution if they are not willing to be a unique player who does not adopt the abatement technology (which is cheaper), thus achieving a Pareto efficient outcome. Players may also make compatible decisions ensuring a sub-optimal result, and thus they both choose not to abate if each player is not willing the be unique in reducing the tax burden in a context in which social awareness against pollution is larger than before and abatement costs are lower than before. In this context, however, players are interested in agreeing to coordinate towards A by producing a clean technology, though the game is non-cooperative. By choosing A, everyone would be interested in complying with the agreement, as no one would have the incentive to deviate towards NA with a Pareto-dominated outcome.

[10] The same kind of prisoner's dilemma can be obtained with low values of z and g (area a in Figure 1). In this case, in fact, technological progress for abatement has worked out by sharply reducing the costs firms should incur to adopt a green technology so that the environment is already clean, and a small value of g is enough to go through the adoption of a cleaning technology as a dominant strategy.

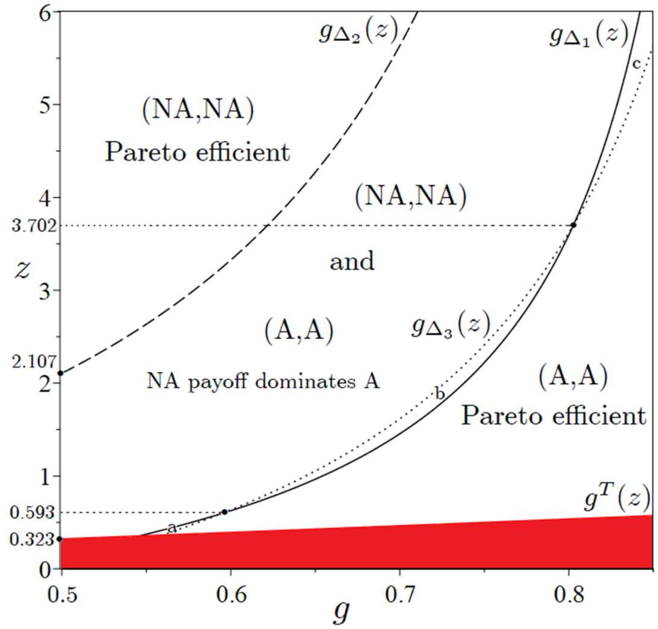


Figure 1. The abatement game. Nash equilibrium outcomes in (g, z) plane. The solid (resp. dashed) [resp. dotted] black line represents the threshold value $g_{\Delta_1}(z)$ (resp. $g_{\Delta_2}(z)$) [resp. $g_{\Delta_3}(z)$] such that $\Delta\pi_1 = 0$ (resp. $\Delta\pi_2 = 0$) [resp. $\Delta\pi_3 = 0$]. The solid red increasing curve $g^T(z)$ is the upper boundary of the red area representing the unfeasible parameter space (alternatively, the technical non-feasibility condition of pollution abatement). The SPNE in the areas denoted by a and c is (A,A). This outcome is Pareto inefficient (prisoner's dilemma). Area b denotes a region with multiple Nash equilibria in pure strategy: (NA,NA) and (A,A), where A payoff dominates NA.

Though the strategic profile A was a dominant strategy in area c of Figure 1, there was a conflict between self-interest and mutual benefit of undertaking emission-reducing actions. However, the model reveals that raising public awareness regarding environmental quality solves the dilemma, and the game becomes a deadlock. Indeed, an increase in g increases taxation in all the strategic profiles (causing a reduction in the industrial production of goods and an increase in the market price) but also contributes to increase the amount of abated pollutant in equilibrium. The combined effect of these changes works in the direction of reducing profits for both NA and A. The reduction in profits under NA, however, is much larger than those observed under A. This is because the increase in the amount of emissions abated reduces the tax burden for the firm adopting the cleaning technology to the extent that profits under A become larger than profits under NA. Therefore, there is no longer any conflict between self-interest and mutual benefit of undertaking emission-reducing actions, as the adoption of a clean technology is Pareto efficient for firms.[11]

3.3 Environmental damage and social welfare

Now, having identified the endogenous choice of the firms regarding abatement decisions, let us investigate the effects (at the equilibrium) on total pollution (P^*), environmental damage (ED^*), and social welfare (SW^*) by comparing the cases (NA,NA) and (A,A), as mixed Nash equilibria do not exist in this game. Making use of the expressions of the optimal tax rates in (9) and (18), one obtains the equilibrium values of the amount of pollution generated by industrial production, the

[11] The same kind of Pareto-efficient outcome can be achieved with low values of g by reducing z adequately. In fact, a sufficient reduction of z through R&D for cleaning purposes sharply reduces the costs a firm should incur for adopting a cleaning technology and makes it convenient to increase the amount of emissions abated so that the tax burden also decreases.

corresponding environmental damage, and social welfare under the strategic profiles NA and A, that is:

$$P^{*NA} = e_1^{*NA} + e_2^{*NA} = q_1^{*NA} + q_2^{*NA} = \frac{1}{1+g}, \quad (28)$$

$$P^{*A} = e_1^{*A} + e_2^{*A} = (q_1^{*A} - k_1^{*A}) + (q_2^{*A} - k_2^{*A}) = \frac{2z(4+z)}{2z^2(1+g)+3[3z+2g(3+z)]}, \quad (29)$$

$$ED^{*NA} = \frac{g}{2}(e_1^{*NA} + e_2^{*NA})^2 = \frac{g}{2(1+g)^2}, \quad (30)$$

$$ED^{*A} = \frac{g}{2}(e_1^{*A} + e_2^{*A})^2 = \frac{2z^2 g(4+z)^2}{\{2z^2(1+g)+3[3z+2g(3+z)]\}^2}, \quad (31)$$

$$SW^{*NA} = \frac{1}{2(1+g)}, \quad (32)$$

$$SW^{*A} = \frac{(4+z)(z+2g)}{2z^2(1+g)+3[3z+2g(3+z)]}. \quad (33)$$

Straightforward comparison of the expressions in (28)-(33) yields:

$$P^{*A} - P^{*NA} = \frac{-[z(1+4g)+18g]}{(1+g)\{2z^2(1+g)+3[3z+2g(3+z)]\}} < 0, \quad (34)$$

$$ED^{*A} - ED^{*NA} = \frac{-g(z+18g+4zg)(4z^2g+20zg+4z^2+18g+17z)}{2(1+g)^2\{2z^2(1+g)+3[3z+2g(3+z)]\}^2} < 0, \quad (35)$$

$$SW^{*A} - SW^{*NA} = \frac{z(4g^2-1)+2g(8g-1)}{2(1+g)\{2z^2(1+g)+3[3z+2g(3+z)]\}} > 0, \quad (36)$$

for any $\frac{1}{2} := g^{NA} < g < g^T(z)$ and $z > 0.323$. Therefore, the following proposition holds.

Proposition 1. The aggregate amount of pollution and the environmental damage generated by industrial production when firms abate are smaller than when firms do not abate. Social welfare when firms abate is higher than when firms do not abate for any $\frac{1}{2} := g^{NA} < g < g^T(z)$ and $z > 0.323$.

Proof. The proof easily follows from the sign of the expressions in (34)-(36).

The result of Proposition 1 has an interesting and simple intuition: the reduction in environmental damage coming from adopting a cleaning technology always compensates for the reduction in consumers' and producers' surpluses due to the reduction in the quantity of goods from industrial production that abatement implies. Combining Result 1 and Proposition 1, the next result immediately follows.

Result 2. The abatement game produces the following Pareto-efficient outcomes for the society depending on the relative size of z and g.

[1] Let $z > 3.702$ hold. If $g_{\Delta_3}(z) < g < g^T(z)$, then (A,A) is the unique Pareto-efficient SPNE of the game (deadlock) and $SW^{*A} > SW^{*NA}$.

[2] Let $2.107 < z < 3.702$ hold. If $g_{\Delta_1}(z) < g < g^T(z)$, then (A,A) is the unique Pareto-efficient SPNE of the game (deadlock) and $SW^{*A} > SW^{*NA}$.

[3] Let $0.593 < z < 2.107$ hold. If $g_{\Delta_1}(z) < g < g^T(z)$, then (A,A) is the unique Pareto-efficient SPNE of the game (deadlock) and $SW^{*A} > SW^{*NA}$.

[4] Let $0.323 < z < 0.593$ hold. If $g_{\Delta_3}(z) < g < g^T(z)$, then (A,A) is the unique Pareto-efficient SPNE of the game (deadlock) and $SW^{*A} > SW^{*NA}$.

The analyses of Results 1 and 2 and Proposition 1 show a clear policy statement that we believe is important to stress here. Moving from northwest to southeast in Figure 1 marks the start from a polluted environment in which firms produce a dirty technology and the society is not very aware of environmental issues (this outcome is Pareto efficient for firms, but it is Pareto dominated for the society as $SW^{*A} > SW^{*NA}$ always holds) to facilitate a sustainable or clean environment, in which the awareness towards environmental quality is high, and firms and the society are better off than under no abatement. This result confirms the rationale for working for a cleaner world by properly modifying individual incentives. Indeed, the improvement in R&D fostering cleaner technology (Jaffe et al., 2003) allows firms to reduce the costs to be incurred to install and adopt environmentally beneficial technologies, allowing for a sharp decrease in the number of emissions from industrial production. By stimulating innovations, profit-maximising firms can reduce marginal costs and gain from the adoption of new technologies spreading the beneficial effects of reducing emissions for society. In addition, the government should pursue the aim of increasing public awareness towards environmental quality through the improvement of the public education system (school, universities, etc.) as well as by ad hoc programmes to be shared by the media for a thoughtful understanding of environmental issues. Indeed, education is associated with growth trajectories with higher technological progress, human capital, and working income (Galor, 2011) and eventually increased environmental awareness (Gori et al., 2020) than trajectories in which education is lower or even absent. This could help individuals escape poverty and starvation to experience health gains and improve their own education decisions by creating a virtuous circle that may generate – amongst other things – demand for goods and services produced with cleaner technologies and less emissions, thereby protecting the environment from degradation.

We should pinpoint that, though improvements in R&D are a stylised fact in history so that z can unambiguously represent the degree of technological progress, which can also reasonably be the subject to ongoing improvements (through reductions in z), less obvious is the timeline of parameter g. Indeed, g can represent both the degree of ecological awareness of the society/government (in this case one should conjecture that g increases over time), or alternatively, it can be the actual degree of pollutant damage coming from industrial production. In this case, it is hard to have a timeline with increasing values of g, as a low g can be related to the production of goods coming from a non-polluting, low value-added technology and a high g instead to a polluting, high valued-added technology (the petrochemical sector or the steel sector). This would lead to a different, counterintuitive interpretation: social welfare could be larger with the production of polluting goods than with the production of non-polluting goods.

The next section focuses on the analysis of a possible ethical-historical-political scenario based on the conjecture that the timeline of z is decreasing and the timeline of g is increasing for the reasons discussed above.

4. An ethical-historical-political projection exercise

The aim of this section is to speculate about a possible ethical-historical-political trajectory that the analysis detailed in the previous section may generate. To do this, a methodological premise is useful, as we need to distinguish between the logical or philosophical time of the (timeless or static) one-shot, non-cooperative abatement game (out of historical time) played by two selfish (profit-maximising) firms and the historical timing of the events (past, present, and future) that contributes to generate a chronological structure of sequences.

The logical structure of sequences of the model presented in Section 2 represents the usual linkage of variables (having a causal relationship amongst them) that move towards the same logical (not necessarily chronological) direction. However, given the structure of the model, nothing can be said about the chronological linkage amongst the variables in terms of involvement in the analysis. Therefore, to write down a narrative of the story behind our results outlining it in space and historical time, it is important to abstract beyond the implicit logical narrative of the model (which was just detailed in Section 2 to give an economic intuition of Result 1) following the methodological premise made at the end of the previous section, based on the conjecture that the timeline of z is decreasing and the timeline of g is increasing (Figure 2). This implies that a reduction in z can be associated with technological improvements observed in history (assuming also that we will continue to observe them in the future), and an increase in g can be conjectured to be associated with improvements in the degree of ecological awareness of the society.

Our projection exercise aims to speculate about the possible trajectories an economy can follow depending on the relative size of g and z. To this purpose, Figure 2 depicts a timeline of the events in which the future with different environmental conditions can be reached following trajectories with improvements in either cost-reducing R&D for cleaning purposes (for a given g) or public awareness towards the environment (for a given z) or both (see the direction of the arrows in the figure). In this regard, we believe our model can represent a useful tool to examine the historical evolution of capitalist economies (starting at least from the industrial revolution) and the improvement in technological progress and related environmental issues. As economic development based on heavy industries in Western countries has been and is still increasingly impacting the environment, the environmental challenge certainly represents a bifurcation according to which the future may evolve in dramatically different scenarios. In this sense, our research has identified eco-friendly or eco-unfriendly patterns emerging endogenously from individual economic interactions and related green or anti-green policies.[12]

The story started from the north-west of Figure 2, resembling a moment in history in which industrial production took place with dirty technologies and the societal awareness towards the environmental damage was relatively low. This occurred since the beginning of the Industrial Revolution until almost the middle of the 20th century. Firms had an incentive to adopt a dirty technology as the costs of abatement were relatively high; alternatively, the benefits of the green technology were still low (indeed, cleaning R&D was almost absent or not adequately developed). This is because industrial production had not yet caused relevant (perceived) damages to the environment to generate individual incentives for sustaining abatement costs. Since this point in time, development trajectories can be different, generating a timeline along dystopian, utopian, or sustainable paths with dirty, clean, or sustainable environments depending on whether the economy will follow, respectively, a trajectory with improvements in the public awareness towards environmental quality (increases in g) for a given (high) abatement cost, a trajectory with improvements in cleaning R&D that contribute to reduce the costs to abate pollution (reductions in z) for a given (low) value of the degree of ecological awareness, and a trajectory with gradual improvements in cleaning R&D and ecological awareness.

The first case represents a dystopian path (according to the view of the epic science fiction film by Christopher Nolan, we define this as the "Interstellar scenario"), as the environment would be highly polluted and degraded due to the lack of developments in adequate cleaning R&D over the course of human history. The adoption of an abatement technology will be delayed and will happen only when g will be high enough to increase the emissions tax rate to incentivise firms to increase abatement and reduce the tax burden to compensate for the high degree of pollution. Environmental quality following this trajectory, however, is in danger of being compromised, and the future of human beings on Earth could include coexisting with environmental disasters or a catastrophic decline in society or, alternatively, it may not be on Earth, due to the severe environmental degradation coexisting with the development of technologies allowed to leave the planet.

In the second scenario, economies would follow a utopian development path, as the environment would be clean thanks to the dramatic improvements in cleaning R&D that make it convenient to adopt abatement, even with a very low degree of ecological awareness (individual-selfish behaviour matters). The combination of low z and g would enable firms to keep the emissions tax rate to incentivise emissions-reducing actions at a low level.

[12] As the historical evolution of capitalist economies has lied on an increasingly impacting environmental trajectory, the environmental challenge is crucial for the incoming future. Our research has then identified pros and cons of different paths emerging endogenously from individual interactions. For a recent analysis of the evolution of capitalism see Acemoglu and Robinson (2015).

The third case represents a scenario resembling the actual development paths of economies observed in history that gradually become aware of the need to increase environmental awareness and at the same time to develop and adopt cost-reducing clean technologies, leading them to rely on sustainable environmental trajectories. Indeed, since almost the middle of the 20th century, environmental sustainability started to become part of the political agendas in several Western governments due to the increasingly perceived environmental damages coming from industrial production (see Brock and Scott Taylor, 2010 for an empirical finding of the so-called Environmental Kuznets Curve, EKC, shown by augmenting the Solow growth model to incorporate technological progress in abatement).

As far as the societal awareness towards environmental quality increased and cost-reducing R&D to favour abatement reduced, firms started having an incentive to adopt green technologies through pollution abatement, though they may get entrapped in a situation in which choosing to abate or not to abate was *a priori* uncertain. This resembles an intermediate phase in history from the 1960s to the 1980s. As the EKC represents a condition to converge towards a sustainable path, further improvements in cost-reducing R&D and ecological awareness (from the 1980s onwards) are a necessary condition to live in a sustainable environment.

In addition, we believe some observations on the trade-off between taxes and R&D investments can be addressed here. In this regard, the work has assumed that there is no link between environmental taxation and resources to be used by each firm for investments in cleaning technologies. Paying taxes can reduce the resources available to firms for cleaning purposes and thus discourage related investments. This trade-off can certainly be addressed in future works. A green-oriented policy maker, however, can use the revenues collected to incentivise firms undertaking emission-reducing R&D by letting parameter z be dependent on the fiscal burden or, alternatively, fostering the public education system, making g become endogenous.

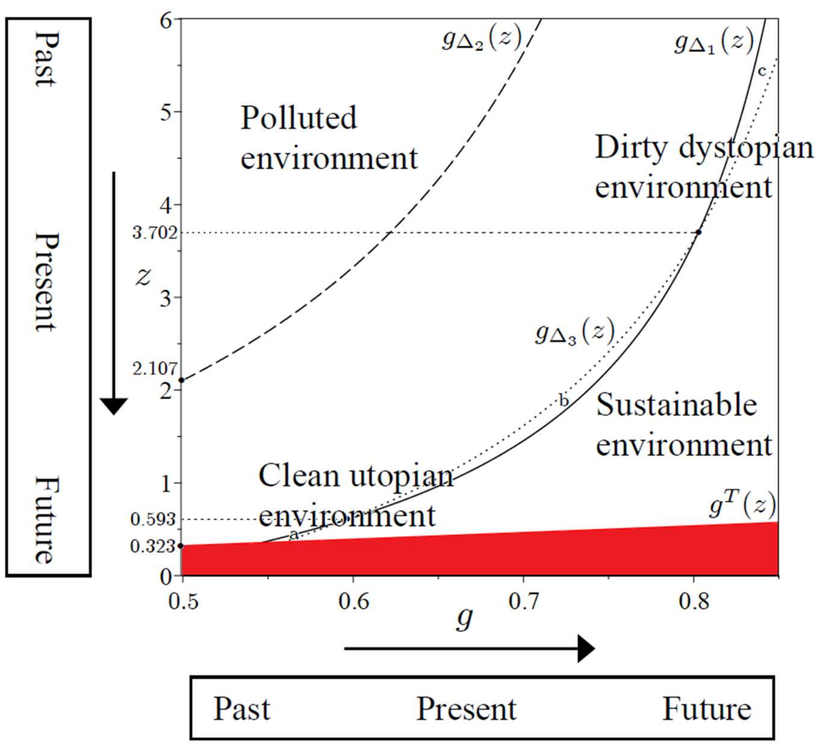


Figure 2. Timeline

5. Extensions: horizontal product differentiation and progressive taxation

The robustness of the key feature of the basic model detailed in Section 2 is now tested in the case of Cournot and Bertrand competition with differentiated products (Singh and Vives, 1984). If products are perceived as heterogeneous, the linear inverse market demand becomes:

$$p_i = 1 - q_i - dq_j, \quad i,j = 1,2; \quad i \neq j, \tag{37}$$

where p_i denotes the price of firm i, q_i and q_j are the firms' output levels, and $-1 < d < 1$ is the parameter that weights the degree of product differentiation. When $d < 0$ (resp. $d > 0$), products are complements (resp. substitutes); when $d = 0$, goods are completely differentiated, and each firm is a monopolist for its own product.

Therefore, under Cournot competition, profits of firm i in the cases of abatement (A) and no abatement (NA) become the following (the first superscript, *C*, stands for Cournot):

$$\pi_i^{C,A} = (1 - q_i - dq_j)q_i - t(q_i - k_i) - \frac{zk_i^2}{2}, \tag{38}$$

and

$$\pi_i^{C,NA} = (1 - q_i - dq_j)q_i. \tag{39}$$

Differently, the Bertrand rivalry setting implies the use of the direct version of the expression of the inverse market demand in (37), that is:

$$q_i = \frac{1-p_i-d(1-p_j)}{1-d^2}, \quad i,j = 1,2; \quad i \neq j, \tag{40}$$

and the corresponding firm i's profits in the cases of abatement (A) and no abatement (NA) are (the first superscript, B, stands for Bertrand):

$$\pi_i^{B,A} = p_i\left[\frac{1-p_i-d(1-p_j)}{1-d^2}\right] - t\left[\frac{1-p_i-d(1-p_j)}{1-d^2} - k_i\right] - \frac{zk_i^2}{2}, \tag{41}$$

and

$$\pi_i^{B,NA} = (p_i - t)\left[\frac{1-p_i-d(1-p_j)}{1-d^2}\right], \tag{42}$$

By using the standard optimisation techniques detailed in Section 2, it is possible to build on the payoff matrices of the abatement game in both Cournot and Bertrand rivalry settings with differentiated products. The corresponding equilibrium outcomes (profits) of the differentiated Cournot and Bertrand duopoly games are summarised in Table 2 and Table 3, respectively (see Appendix B for details about the equilibrium profit equations).

Table 2. The abatement game (payoff matrix). Cournot competition with differentiated products.

Firm 2 → Firm 1 ↓	A	NA
A	$\pi_1^{*C,A}, \pi_2^{*C,A}$	$\pi_1^{*C,A/NA}, \pi_2^{*C,A/NA}$
NA	$\pi_1^{*C,NA/A}, \pi_2^{*C,NA/A}$	$\pi_1^{*C,NA}, \pi_2^{*C,NA}$

Table 3. The abatement game (payoff matrix). Bertrand competition with differentiated products.

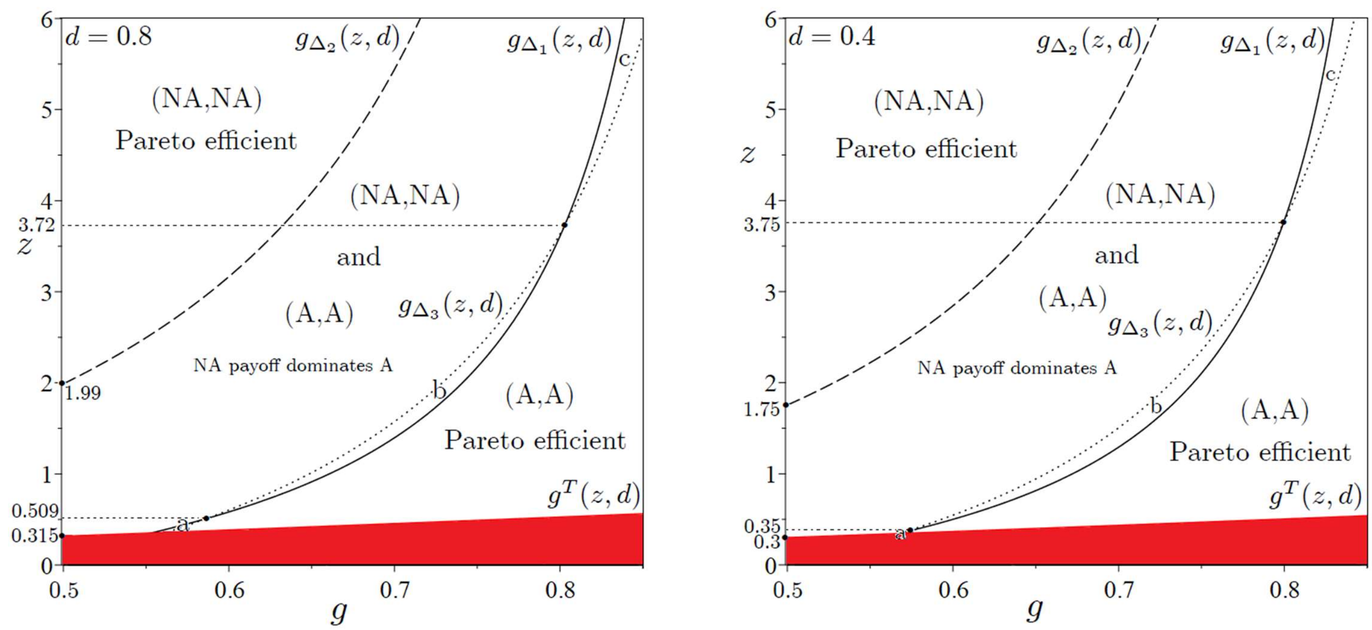
Firm 2 → Firm 1 ↓	A	NA
A	$\pi_1^{*B,A}, \pi_2^{*B,A}$	$\pi_1^{*B,A/NA}, \pi_2^{*B,A/NA}$
NA	$\pi_1^{*B,NA/A}, \pi_2^{*B,NA/A}$	$\pi_1^{*B,NA}, \pi_2^{*B,NA}$

From the payoffs in Table 2 and Table 3, it is possible to compute the profit differentials as in (27). The relevant parametric constraints and Nash equilibria are shown in Panels A-D of Figure 3 (Cournot) and Panels A-D of Figure 4 (Bertrand), plotted in (g, z) plane for four different values of the degree of product differentiation (Panel A: $d = 0.8$, Panel B: $d = 0.4$, Panel C: $d = -0.4$, Panel D: $d = -0.8$).

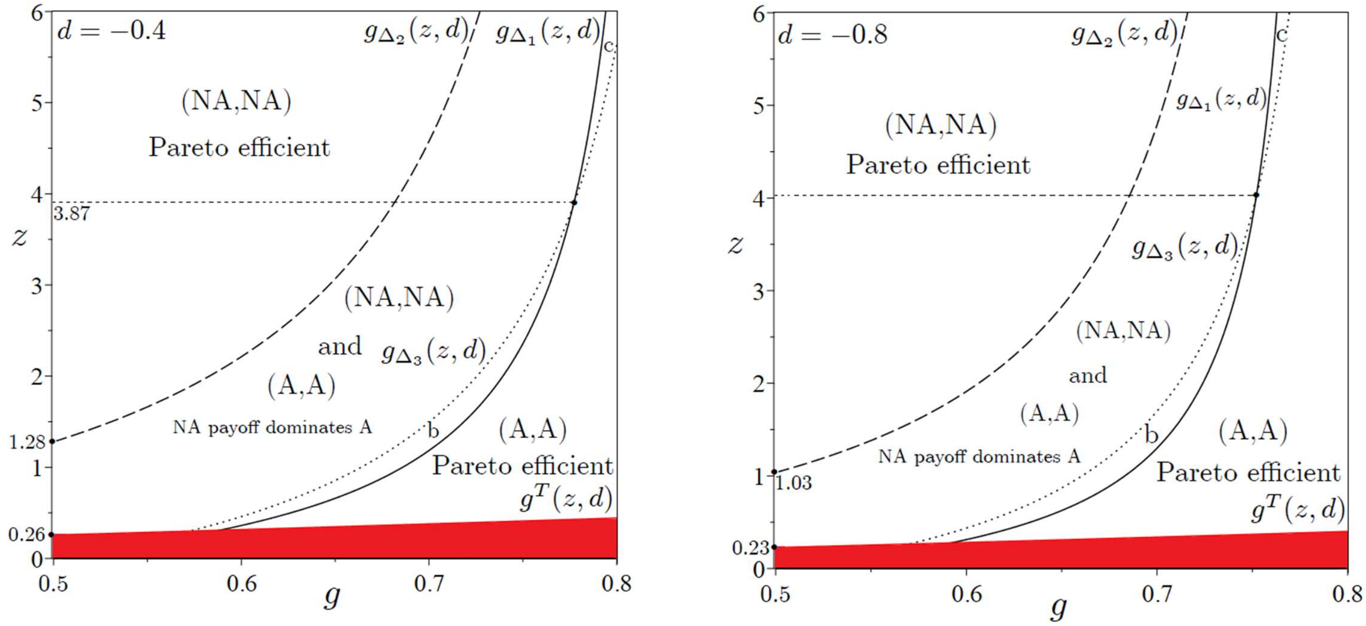
The results shown in the figures qualitatively hold for the whole range of values of d for the cases of both product substitutability and product complementarity (we do not report them in other figures to save space) and basically confirm the findings of the Cournot model with homogeneous products. Product differentiation, however, works in the direction of widening the area in which (NA,NA) is the Pareto efficient Nash equilibrium. Indeed, increasing the degree of product differentiation of consumers implies an increase in the market power of firms and this in turn causes an increase in profits. The incentives to abate therefore require extra efforts either in term of increasing the degree of ecological awareness of the society or in cost-reducing cleaning R&D (or both), thus raising the optimal emission tax rate levied by the government that should be used to incentivise firms undertaking emission-reducing actions. The same result holds for product complementarity.

Definitively, product differentiation – which is a measure of the degree of market (monopoly) power – in Cournot and Bertrand duopolies is anti-ecological. Our result therefore implies that not only the degree of market power is harmful for social welfare, as is known in the traditional market theory, but – by including the problem of the negative externality in production into the model – it also degrades environment quality.

Moreover, we have tested the robustness of the main results of the Cournot model with homogeneous products by considering a linear progressive tax scheme (superscript *pro*) according to which abating pollution up to a given threshold implies a lump-sum tax deduction (subsidy), whereas above the threshold firms pay emission taxes according to the standard linear tax scheme, that is $t^{pro} = -T + t(q_i - k_i)$. The results (available upon request from the authors) substantially remain unaltered compared to those of the model specification of Section 2.

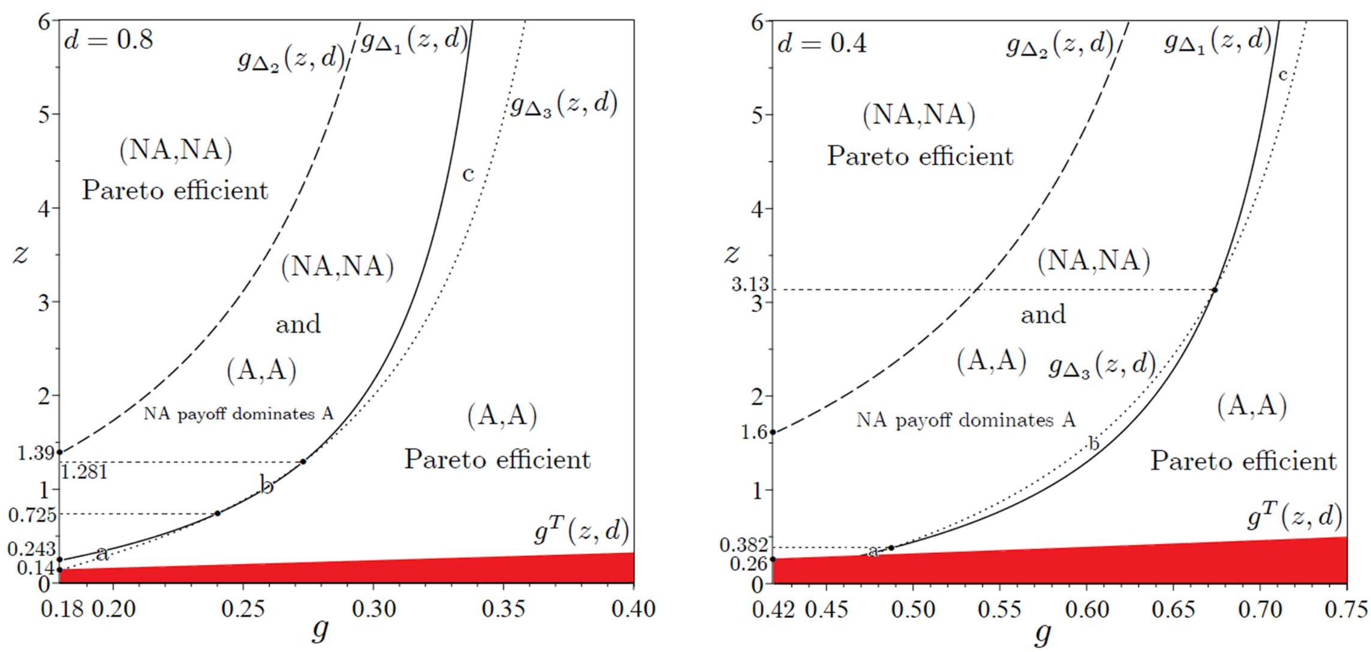


(A) (B)

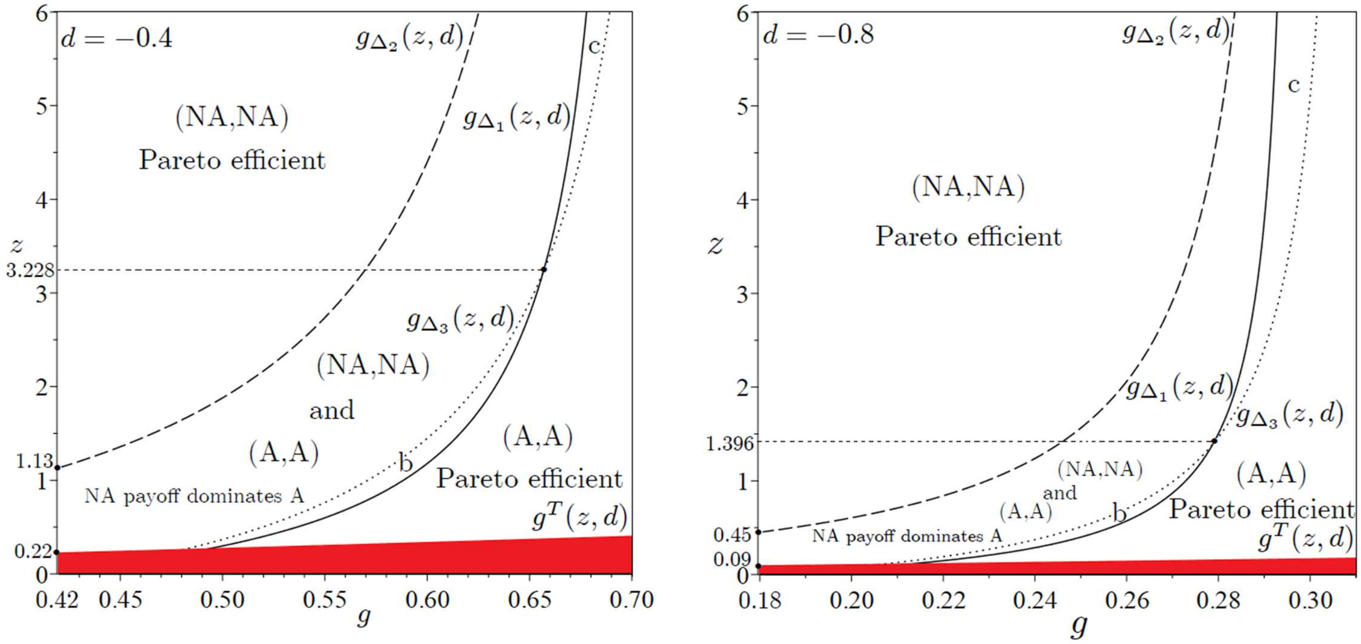


(C) (D)

Figure 3. The abatement game with differentiated products (Cournot competition). Nash equilibrium outcomes in (g, z) plane. The solid (resp. dashed) [resp. dotted] black line represents the threshold value $g_{\Delta_1}(z, d)$ (resp. $g_{\Delta_2}(z, d)$) [resp. $g_{\Delta_3}(z, d)$] such that $\Delta\pi_1 = 0$ (resp. $\Delta\pi_2 = 0$) [resp. $\Delta\pi_3 = 0$]. The solid red increasing curve $g^T(z, d)$ is the upper boundary of the red area representing the unfeasible parameter space (alternatively, the technical non-feasibility of pollution abatement). The SPNE in the areas denoted by a and c is (A,A). This outcome is Pareto inefficient (prisoner's dilemma). In the area denoted by b there are multiple Nash equilibria in pure strategy: (NA,NA) and (A,A), where A payoff dominates NA. Panel (A): $d = 0.8$. Panel (B): $d = 0.4$. Panel (C): $d = -0.4$. Panel (D): $d = -0.8$.



(A) (B)



(C) (D)

Figure 4. The abatement game with differentiated products (Bertrand competition). Nash equilibrium outcomes in (g, z) plane. The solid (resp. dashed) [resp. dotted] black line represents the threshold value $g_{\Delta_1}(z,d)$ (resp. $g_{\Delta_2}(z,d)$) [resp. $g_{\Delta_3}(z,d)$] such that $\Delta\pi_1 = 0$ (resp. $\Delta\pi_2 = 0$) [resp. $\Delta\pi_3 = 0$]. The solid red increasing curve $g^T(z,d)$ is the upper boundary of the red area representing the unfeasible parameter space (alternatively, the technical non-feasibility of pollution abatement). The SPNE in the areas denoted by a and c is (A,A). This outcome is Pareto inefficient (prisoner's dilemma). In the area denoted by b there are multiple Nash equilibria in pure strategy: (NA,NA) and (A,A), where A payoff dominates NA. Panel (A): $d = 0.8$. Panel (B): $d = 0.4$. Panel (C): $d = -0.4$. Panel (D): $d = -0.8$.

6. Conclusions

This article developed a novel three-stage non-cooperative game to study the firms' strategic adoption of an abatement technology for cleaning purposes, which was so far surprisingly overcome in the existing literature, by considering a textbook Cournot duopoly with homogeneous products. For doing this, it was assumed that 1) industrial production generates pollution externalities causing environmental damages, and 2) a government choosing an optimal (welfare-maximising) emission tax to incentivise firms undertaking emission-reducing actions. After the investment in pollution abatement a reduction in production cost equal to the amount of taxation computed on less emissions than the corresponding case of no abatement is achieved. Therefore, there exists a trade-off between abating or not abating.

Choosing to abate (or not to abate) depends on the relative size of the ecological awareness of the society and the degree of improvement in technological progress in abatement. The firms' strategic interactions in this context generates a set of rich Nash equilibrium outcomes ranging from the (Pareto efficient) choice of dirty technology to clean production. Our results confirm the rationale that working for a cleaner world should pass through individual incentives: 1) improvements in R&D fostering cleaner production to reduce the costs should be incurred to install and adopt environmentally beneficial technologies, and 2) increasing the public awareness towards environmental quality through the improvement in public education system for a thoughtful understanding of environmental issues.

The model analysed in this article made simplicity its major point by considering a set of precise assumptions that immediately call for substantial extensions. In this direction, one may consider the separation between ownership and control and analyse how and who would take abatement decisions in managerial firms. In addition, the interplay between managers and unions in the adoption of clean technologies would also be of interest.

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Compliance with Ethical Standards

Disclosure of potential conflict of interest The authors declare that they have no conflict of interest.

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Informed consent Informed consent was obtained from all individual participants included in the study.

Declarations of interest None.

Appendix A

Assuming that emissions cannot be eliminated entirely (e.g., $0 \leq k_i < q_i$) is supported by the stylised fact that an abatement technology allowing to completely abate pollutant from industrial production does not exists in actual world. Results of our models, however, do not change assuming an abating technology such that $0 \leq k_i \leq q_i$. In this regard, the profit maximisation problem is firm i ($i = 1,2; \quad i \neq j$) at the second stage of the game can be written as follows:

$$\max_{\{q_i,k_i\}} \pi_i(q_i, q_j, k_i) = (1 - q_i - q_j)q_i - t(q_i - k_i) - z\frac{k_i^2}{2}.$$
$$s.t. \quad q_i - k_i \geq 0$$

The Lagrangian function for this problem is:

$$\mathrm{L} = (1 - q_i - q_j)q_i - t(q_i - k_i) - z\frac{k_i^2}{2} + \lambda(q_i - k_i).$$

According to the Kuhn-Tucker conditions, the optimal level of output and pollution abatement must satisfy the following constraints:

$$\frac{\partial \mathrm{L}}{\partial q_i} = \frac{\partial \pi_i}{\partial q_i} + \lambda \leq 0, \quad q_i \geq 0 \quad \text{and} \quad q_i \frac{\partial \mathrm{L}}{\partial q_i} = 0$$
$$\frac{\partial \mathrm{L}}{\partial k_i} = \frac{\partial \pi_i}{\partial k_i} + \lambda \leq 0, \quad k_i \geq 0 \quad \text{and} \quad k_i \frac{\partial \mathrm{L}}{\partial k_i} = 0 \quad .$$
$$\frac{\partial \mathrm{L}}{\partial \lambda} = q_i - k_i \geq 0, \quad \lambda \geq 0, \quad \text{and} \quad \lambda \frac{\partial \mathrm{L}}{\partial \lambda} = 0$$

If $\lambda > 0$, then $\frac{\partial \mathrm{L}}{\partial \lambda} = 0$ so that $q_i - k_i = 0 \Rightarrow q_i = k_i$, and both $\frac{\partial \pi_i}{\partial q_i} < 0$ and $\frac{\partial \pi_i}{\partial k_i} < 0$. If $\lambda = 0$, then $\frac{\partial \mathrm{L}}{\partial \lambda} > 0$ so that $q_i - k_i > 0 \Rightarrow q_i > k_i$, and both $\frac{\partial \pi_i}{\partial q_i} = 0$ and $\frac{\partial \pi_i}{\partial k_i} = 0$. Therefore, given the Kuhn–Tucker conditions, the firm's profit maximisation with respect to k_i allows us to derive the following optimal emission intensity:

$$k_i = \frac{t}{z} > 0 \quad \text{if} \quad t > 0 \quad \text{or} \quad k_i = 0 \quad \text{if} \quad t = 0.$$

The former represents the interior solution of the problem, whereas the latter is a corner solution. Moreover, it can be easily shown that the second-order conditions for a maximum $\frac{\partial^2 \pi_i}{\partial q_i^2} < 0$ and $\frac{\partial^2 \pi_i}{\partial k_i^2} < 0$ are fulfilled.

Appendix B

This appendix reports the profit differentials defined by Eq. (27) in the main text (referred to the Cournot game with homogenous products) and the profit equations summarised in Table 2 and Table 3 about differentiated Cournot and Bertrand rivalry settings.

The profit differentials are the following:

$$\Delta\pi_1 = \frac{1}{4(1+g)^2[9(z+g)+4z^2(1+g)+12gz]^2} \times$$
$$(32g^4z^3 + 122g^4z^2 + 64g^3z^3 + 120g^4z + 272g^3z^2 - 8g^2z^3 + 36g^4 + 312g^3z + 28g^2z^2 - 56gz^3$$
$$+72g^3 + 48g^2z - 168gz^2 - 16z^3 - 45g^2 - 90gz - 45z^2)$$

,

$$\Delta\pi_2 = \frac{-z}{2[9(z+g)+4z^2(1+g)+12zg]^2[9(z+2g)+2z^2(1+g)+12\quad]^2} \times$$
$$(64g^2z^6 + 864g^4z^5 + 128g^3z^6 + 4800g^4z^4 + 1632g^3z^5 - 16g^2z^6 + 14040g^4z^3$$
$$+7920g^3z^4 + 120g^2z^5 - 112gz^6 + 22788g^4z^2 + 1836g^3z^3 - 372g^2z^4 - 1032gz^5 - 32z^6,$$
$$+19440g^4z + 20412g^3z^2 - 702g^2z^3 - 3024gz^4 - 198z^5 + 6804g^4$$
$$+8748g^3z - 567g^2z^2 - 2754gz^3 - 243z^4)$$

and

$$\Delta\pi_3 = \frac{-1}{4(1+g)^2[9(z+2g)+2z^2(1+g)+12gz]^2} \times$$
$$(8g^4z^3 + 64g^4z^2 + 24g^3z^3 + 168g^4z + 200g^3z^2 + 2g^2z^3 + 144g^4 + 480g^3z + 28g^2z^2 - 24gz^3$$
$$+288g^3 + 24g^2z - 144gz^2 - 10z^3 - 180g^2 - 180gz - 45z^2)$$

.

The profit equations referred to the differentiated Cournot duopoly (see the payoff matrix, Table 2, in the main text) are the following:

$$\pi_i^{*C,A} = \frac{1}{8\left\{\left[\frac{1}{2}(1+d)+g\right]z^2+\frac{z(2+d)(2+d+4g)}{2}+g(2+d)^2\right\}^2} \times \{12z^4$$
$$+z^3(4g^2 + 4d + 4g + 9) + z^2[8g^2(3 + d) + 12g(1 + d) + 2(2 + d)^2]'$$
$$+4(2 + d)[g(6 + d) + 2d + 4]gz + 8g^2(2 + d)^2\}$$

$$\pi_i^{*C,NA} = \frac{1}{(1+d+2g)^2},$$

$$\pi_1^{*C,A/NA} = \frac{1}{[2z^2(d+2g+1)+z(2+d)(d+4g+2)+g(2+d)^2]^2} \times \{4z^4 + 2z^3(4g^2+2d+5) + z^2[4g^2(5+2d) + 4g(2+d) + (2+d)^2] + 2gz(2+d)[(4+d)g+2+d] + g^2(2+d)^2\},$$

and

$$\pi_2^{*C,A/NA} = \frac{(g+z)^2(d+2z+2)^2}{[2z^2(d+2g+1)+z(2+d)(d+4g+2)+g(2+d)^2]^2}.$$

The profit equations referred to the differentiated Bertrand duopoly (see the payoff matrix, Table 3, in the main text) are the following:

$$\pi_1^{*B,A} = \frac{1}{8\left\{\left[\frac{1}{2}(1+d)+g\right]z^2 + \frac{z(1+d)(2-d)(2+d+4g-d^2)}{2} + g(1+d)^2(2-d)^2\right\}^2} \times \{2z^4(1-d^2) + z^3[5d^4 - 4d^3 - 2d^2(7+2g) + 4(d+g^2+g) + 9] - 2z^2(1+d)[d^5 - 3d^4 - d^3(1+6g) + d^2(7+12g) + 2dg(3+g) - 12g(1+g) - 4] + 8zg(2-d)(1+d)^2\left[d^3 - 2d^2 - d\left(1+\frac{5g}{2}\right) + 3g + 2\right] + 8g(1-d)(1+d)^3(2-d)^2\},$$

$$\pi_i^{*B,NA} = \frac{1-d^2}{(1+d+2g)^2},$$

$$\pi_1^{*B,A/NA} = \frac{1}{\{d^3(g+z)(d-2) - d^2[3g+z(3+4g)] + 2d[z^2+2z(1+g)+2g] + 2z(2+z)(1+2g) + 4g\}^2} \times \{4z^4(1-d^2) + 2z^3(3d^4 - 2d^3 - 8d^2 + 4g^2 + d + 5) - z^2(1+d)[d^5 - 3d^4 - d^3(1+4g) + d^2(8g+7) + 4dg(1+3g) - 4(5g^2+2g+1)] + 2gz(2-d)(1+d)^2[d^3 - 2d^2 - d(1+3g) + 4g + 2] + g^2(1-d)(2-d)^2(1+d)^3\},$$

$$\pi_2^{*B,A/NA} = \frac{(1-d^2)(g+z)^2[2(1+z)+d(1-d)]^2}{\{d^3(g+z)(d-2) - d^2[3g+z(3+4g)] + 2d[z^2+2z(1+g)+2g] + 2z(2+z)(1+2g) + 4g\}^2}.$$

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