Frame Synchronization for FSO Links with Unknown Signal Amplitude and Noise Power

Michele Morelli, Marco Moretti, Antonio A. D'Amico, and Giulio Colavolpe, *Senior Member IEEE*

***Abstract*—In this work, we investigate the problem of frame synchronization in a free-space optical (FSO) communications link, where a known synch pattern is periodically embedded in the transmitted bitstream. The modulation format is on-off keying (OOK) and the electrical signal provided by the photo-detector is plagued by a mixture of thermal and shot noise with signal-dependent power. Due to atmospheric turbulence, channel attenuation can exhibit large randomic fluctuations, so that no prior knowledge of the signal level and noise variances is assumed. These parameters, together with the start-of-frame, are jointly estimated using a simplified maximum likelihood (ML) approach.**

Numerical simulations indicate that the proposed scheme is able to effectively exploit the presence of shot noise to improve its detection capability, and outperforms the standard frame synchronization method tailored for an AWGN channel with signal-independent noise power.

I. Introduction

In recent years, free-space optical (FSO) communication systems have received a great deal of attention because of their capability of providing high capacity point-to-point links over line-of-sight (LoS) channels [1]. Compared to radio-frequency (RF) systems, the FSO technology provides larger available bandwidth, longer operational range and increased security. Despite all these merits, optical propagation in free space has also some disadvantages, including the high sensitivity to pointing errors, adverse weather conditions and atmospheric turbulence.

In FSO packet-based transmissions, frame synchronization represents a crucial task for successful data reception. The common approach to reveal the frame boundaries is the detection of a known synchronization pattern, called Unique-Word (UW), which is periodically embedded in the bitstream. In his pioneering work, Barker demonstrated that the optimal metric for UW detection in the binary symmetric channel (BSC) is the correlation between the observed data sequence and the synch pattern [2]. Later, Massey showed that the optimum rule in an AWGN channel is the sum of the correlation plus a correction term that accounts for the presence of random data surrounding the UW [3]. The authors of [4] extended Massey's work to pulse position modulation (PPM) and OOK optical communications over a Poisson channel. A high SNR approximation of the optimum rule for PPM and OOK links can be found in [5], while a further extension of [3] to ultra-wideband (UWB) signals with square-law devices in the receiver is presented in [6]. Sequential frame synchronization for data packets of unknown length is investigated in [7] and [8], where suitable metrics are derived following a hypothesis testing approach. In these schemes, the presence of the UW is declared if the metric exceeds a proper threshold. In [9], the authors studied the maximum likelihood (ML) preamble detection for packet-based FSO communications. The modulation format is OOK and the link is modelled as a signal-dependent AWGN channel, where the noise variance is higher for a received 1 than for a received 0. The proposed scheme exhibits optimum performance, but requires knowledge of the current channel realization and signal-dependent noise variances, which must be estimated in some manner. An alternative method for frame synchronization in optical links is presented in [10] by resorting to the Hough transform.

In this letter, we investigate the problem of frame synchronization in FSO systems by detecting the position of a UW in the received bitstream. We assume OOK signaling in combination with an avalanche photo-diode (APD) employed as a photo-detector at the receiver side. Since one peculiarity of APD is the presence of a *shot noise* term for a received 1, we adopt the same signal-dependent noise model of [9]. The difference with respect to [9] is that in our study we do not assume any prior knowledge of the channel attenuation and noise variances. Rather, the joint estimation of all these unknown parameters is embedded in the UW detection process by resorting to the ML estimation principle. Our choice is motivated by the fact that, in many cases, the atmospheric turbulence caused by absorption and scattering phenomena may result into significant random fluctuations of the channel fading coefficient. As shown in [11], the coherence time of such fluctuations is in the order of a few milliseconds. Observing that the shot noise power depends on the intensity of the received optical signal, joint estimation of the channel attenuation and noise variances is required in order to design the optimum threshold level employed in the data detection process [12]. Periodic estimation of the channel and noise parameters is also needed in emerging LEO satellite FSO links, where adaptive modulation and coding is proposed to cope with the remarkable variations (more than 6 dB) of the channel coefficient as a function of the satellite elevation angle [13]. Since long blocks of data are usually transmitted by the LEO satellite to the ground station, the atmospheric

Michele Morelli, Marco Moretti and Antonio Alberto D'Amico ({michele.morelli,marco.moretti,antonio.damico}@unipi.it) are with the Dipartimento di Ingegneria dell'Informazione, University of Pisa, Italy. Their work is partially supported by the Italian Ministry of Education and Research (MIUR) in the framework of the CrossLab project (Departments of Excellence). Giulio Colavolpe (giulio.colavolpe@unipr.it) is with Department of Engineering and Architecture, University of Parma, Italy. The work by Giulio Colavolpe is funded by the European Space Agency, ESA-ESTEC, Noordwijk, The Netherlands. The view expressed herein can in no way be taken to reflect the official opinion of the European Space Agency.

Corresponding author: Marco Moretti.

turbulence should be estimated at the beginning of each new frame. Similarly, due to unavoidable timing instabilities, a confirmation of the position of the UW is required at every block.

In order to reduce the system complexity, in our derivations we neglect any information conveyed by data symbols surrounding the UW. This produces a practical, albeit suboptimal, synchronization scheme which can be implemented with affordable complexity and greatly outperforms the pure correlation rule widely adopted in engineering practice.

II. System model and problem formulation

We consider an FSO communication link for packet-based transmissions over an atmospheric turbulence channel. The modulation format is non-return-to-zero (NRZ) OOK and an APD is used at the receiver for direct detection of the transmitted data. The photocurrent signal is integrated over each bit period to produce a set of statistics suitable for the detection process. Assuming that symbol synchronization has already been acquired, the receiver is aligned with the transmitted data symbol epochs. Hence, the discrete-time statistics are given by

$$x(k) = Ab_k + n(k) \tag{1}$$

where $b_k \in \{0, 1\}$ denotes the kth OOK symbol, $n(k)$ represents the noise contribution and A is the channel state, which is typically unknown as a consequence of the random nature of the scintillation and absorption phenomena characterizing an FSO link. We assume the presence of both thermal and shot noise. While the former is present in the electrical signal at any time instant, shot noise only appears when a unitary symbol $b_k = 1$ is received. Accordingly, we model $\{n(k)\}$ as

$$n(k) = (1 - b_k)n_0(k) + b_k n_1(k) \tag{2}$$

where $n_0(k)$ and $n_1(k)$ are statistically independent Gaussian processes with zero mean and variance σ_0^2 and $\sigma_1^2 > \sigma_0^2$, respectively [14]. Data transmission is formatted in successive frames, each of which consists of N_F symbol periods. In order to achieve frame synchronization, a UW composed of L pilot symbols is periodically embedded in the bitstream with period N_F. We collect the pilots into an L-dimensional vector $\mathbf{a}_{UW} = [a_0, a_1, \ldots, a_{L-1}]$ and assume that the UW is located at the indices $k_0 \le k \le k_0 + L - 1$ in the received sample stream. Our goal is the joint estimation of the unknown quantities $\{A, \sigma_1^2, \sigma_0^2, k_0\}$, which are all necessary for reliable data detection. For this purpose, we adopt an ML-oriented approach and process the sequence $\{x(k)\}$ over a time interval spanning $N_F + L - 1$ symbol periods. The corresponding samples are collected into an observation vector $\mathbf{x} = [x(0), x(1), \ldots, x(N_F + L - 2)]^T$, whose length is chosen such that it contains a whole UW. Letting $I = \{0, 1, 2, \ldots, N_F + L - 2\}$ and $I_{UW} = \{k_0, k_0 + 1, \ldots, k_0 + L - 1\}$, we can rewrite the entries of $\mathbf{x}$ as

$$x(k) = \begin{cases} Ab_k + n(k) & k \in I \setminus I_{UW} \\ Aa_{k-k_0} + n(k) & k \in I_{UW}. \end{cases} \tag{3}$$

It is worth noting that several statistical models have been suggested in the literature to characterize the atmospheric turbulence in FSO systems, including lognormal, gamma-gamma and negative exponential fading [14], [15]. Since in this work we follow the *classical* approach to estimation where, in contrast to the Bayesan philosophy, A is treated as a deterministic unknown quantity rather than as a random variable, in the foregoing discussion there is no need to adopt a specific channel model. The main advantage of this procedure is that it dispenses from any prior information about the channel statistics, which should be acquired in some manner.

One last point is related to the statistical model of the shot-noise generated by the APD. As discussed in [12], the shot-noise power, say σ_{shot}^2, is strictly related to the channel state A and to the APD noise figure F. Hence, assuming that F is known and recalling that $\sigma_1^2 = \sigma_0^2 + \sigma_{shot}^2$, it turns out that σ_1^2 can be expressed as a function of the pair (A, σ_0^2). Although such a relationship can reduce the number of unknown parameters involved in the estimation procedure, it is not considered in the subsequent analysis. The reason is that the exact value of the APD noise figure is not available in practice as this parameter can exhibit large deviations from its nominal value due to long-term fluctuations of the operational conditions.

III. Estimation of the unknown parameters

Frame synchronization in FSO links with signal-dependent noise variance is investigated in [9]. It turns out that the optimum ML solution requires knowledge of the noise variances and signal level, which is not available in many FSO applications. For this reason, it is of interest to study the joint ML estimation of $\{A, \sigma_1^2, \sigma_0^2, k_0\}$ based on $\mathbf{x}$. From (3), we see that this task is complicated by the presence of the unknown symbols $\{b_k\}$ surrounding the UW. In [9], this problem is solved by averaging the probability density function of $\mathbf{x}$ with respect to the statistics of $\{b_k\}$. The resulting metric is composed of two parts, one of which is a kind of energy correction term required to account for the random data. This procedure, however, becomes impractical when $\{A, \sigma_1^2, \sigma_0^2\}$ are treated as unknown parameters, as it leads to a cumbersome expression of the log-likelihood function (LLF), whose maximization turns out to be mathematically intractable. To circumvent this obstacle, we suggest to exclude the data symbols $\{b_k\}$ from the estimation process. Although suboptimal, this approach has the advantage of producing a practical estimation algorithm whose metric is independent of $\{A, \sigma_1^2, \sigma_0^2\}$.

Under the above assumption, the LLF for the joint estimation of all the unknown quantities is given by

$$\Gamma(\tilde{A}, \tilde{\sigma}_1^2, \tilde{\sigma}_0^2, \tilde{k}_0) = -\frac{1}{2} \sum_{k=\tilde{k}_0}^{\tilde{k}_0+L-1} \left\{ \ln[\tilde{\sigma}^2(k - \tilde{k}_0)] + \frac{[x(k) - \tilde{A}a_{k-\tilde{k}_0}]^2}{\tilde{\sigma}^2(k - \tilde{k}_0)} \right\} \tag{4}$$

where $\{\tilde{A}, \tilde{\sigma}_1^2, \tilde{\sigma}_0^2, \tilde{k}_0\}$ are trial values of $\{A, \sigma_1^2, \sigma_0^2, k_0\}$ and $\tilde{\sigma}^2(k) = (1 - a_k)\tilde{\sigma}_0^2 + a_k\tilde{\sigma}_1^2$. To proceed further, we define the two sets $\mathcal{K}_1 = \{k \in \{0, 1, \ldots, L - 1\} : a_k = 1\}$ and $\mathcal{K}_0 = \{k \in \{0, 1, \ldots, L-1\} : a_k = 0\}$, collecting the indices

k for which the pilot symbol is either 1 or 0. Let L_1 and $L_0 = L - L_1$ be the cardinality of $\mathcal{K}_1$ and $\mathcal{K}_0$, respectively. Then, we can rewrite (4) as

$$\Gamma(\tilde{A}, \tilde{\sigma}_1^2, \tilde{\sigma}_0^2, \tilde{k}_0) = -\frac{L_0}{2}\ln(\tilde{\sigma}_0^2) - \frac{L_1}{2}\ln(\tilde{\sigma}_1^2) - \sum_{k\in\mathcal{K}_0}\frac{x^2(k+\tilde{k}_0)}{2\tilde{\sigma}_0^2} - \sum_{k\in\mathcal{K}_1}\frac{[x(k+\tilde{k}_0)-\tilde{A}]^2}{2\tilde{\sigma}_1^2}. \tag{5}$$

The ML estimate of $\{A, \sigma_1^2, \sigma_0^2, k_0\}$ is the location where $\Gamma(\tilde{A}, \tilde{\sigma}_1^2, \tilde{\sigma}_0^2, \tilde{k}_0)$ achieves its global maximum. Maximizing with respect to $\tilde{A}, \tilde{\sigma}_1^2$ and $\tilde{\sigma}_0^2$ yields

$$\hat{A}(\tilde{k}_0) = \frac{1}{L_1}\sum_{k\in\mathcal{K}_1} x(k+\tilde{k}_0) \tag{6}$$

$$\hat{\sigma}_1^2(\tilde{k}_0) = \frac{1}{L_1}\sum_{k\in\mathcal{K}_1} x^2(k+\tilde{k}_0) - \hat{A}^2(\tilde{k}_0) \tag{7}$$

and

$$\hat{\sigma}_0^2(\tilde{k}_0) = \frac{1}{L_0}\sum_{k\in\mathcal{K}_0} x^2(k+\tilde{k}_0). \tag{8}$$

Then, plugging the results (6)-(8) back into (5), produces the *concentrated* LLF in the form

$$\Gamma(\tilde{k}_0) = -L_0\ln[\hat{\sigma}_0^2(\tilde{k}_0)] - L_1\ln[\hat{\sigma}_1^2(\tilde{k}_0)], \tag{9}$$

which provides an estimate of k_0 as

$$\hat{k}_0 = \arg\min_{\tilde{k}_0\in J_0}\left\{L_0\ln\left[\sum_{k\in\mathcal{K}_0} x^2(k+\tilde{k}_0)\right] + L_1\ln\left[\sum_{k\in\mathcal{K}_1}[x(k+\tilde{k}_0)-\hat{A}(\tilde{k}_0)]^2\right]\right\} \tag{10}$$

with $J_0 = \{0, 1, 2, \ldots, N_F - 1\}$. Finally, replacing $\tilde{k}_0$ by $\hat{k}_0$ into (6)-(8) yields the estimated values of A, σ_1^2 and σ_0^2. We denote this scheme as the ML-oriented algorithm (MLA).

IV. Performance analysis

The accuracy of the estimates $\{\hat{A}, \hat{\sigma}_1^2, \hat{\sigma}_0^2\}$ provided by MLA is assessed under the assumption of ideal frame detection, i.e., $\hat{k}_0 = k_0$. The reason is that analytical evaluation of the probability of failure, defined as $P_f = \Pr\{\hat{k}_0 \neq k_0\}$, does not appear to be generally tractable when using the MLA metric shown in (10). Through standard computations, it is found that $\hat{A}$ is unbiased with variance

$$\text{var}\{\hat{A}\} = \frac{\sigma_1^2}{L_1}. \tag{11}$$

The estimator (7) and (8) can be reformulated as

$$\hat{\sigma}_1^2 = \frac{1}{L_1}\mathbf{n}_1^T\mathbf{A}\mathbf{n}_1 \tag{12}$$

$$\hat{\sigma}_0^2 = \frac{1}{L_0}\mathbf{n}_0^T\mathbf{n}_0 \tag{13}$$

where $\mathbf{n}_1 = \{n_1(k+k_0); k\in\mathcal{K}_1\}$ and $\mathbf{n}_0 = \{n_0(k+k_0); k\in\mathcal{K}_0\}$ are zero-mean Gaussian vectors with covariance matrices $\sigma_1^2\mathbf{I}_{L_1}$ and $\sigma_0^2\mathbf{I}_{L_0}$, respectively. Furthermore, $\mathbf{A}$ is the idempotent matrix defined by

$$\mathbf{A} = \mathbf{I} - \frac{1}{L_1}\mathbf{u}_1^T\mathbf{u}_1 \tag{14}$$

where $\mathbf{u}_1$ is an L_1-dimensional vector with all unitary entries. Then, using the identities $\mathbb{E}\{\mathbf{n}^T\mathbf{B}\mathbf{n}\} = \text{tr}\{\mathbf{B}\mathbf{C}_n\}$ and $\text{var}\{\mathbf{n}^T\mathbf{B}\mathbf{n}\} = 2\,\text{tr}\{\mathbf{B}\mathbf{C}_n\mathbf{B}\mathbf{C}_n\}$ which hold true for any symmetric matrix $\mathbf{B}$ and zero-mean Gaussian vector $\mathbf{n}$ with covariance matrix $\mathbf{C}_n$, it is found that $\hat{\sigma}_0^2$ is unbiased with variance

$$\text{var}\{\hat{\sigma}_0^2\} = \frac{2\sigma_0^4}{L_0}, \tag{15}$$

while

$$\mathbb{E}\{\hat{\sigma}_1^2\} = \frac{\sigma_1^2}{L_1}(L_1 - 1). \tag{16}$$

From (16), we see that $\hat{\sigma}_1^2$ is a biased estimator. On the other hand, an unbiased estimate of σ_1^2 is easily obtained as

$$\hat{\sigma}_{1,unb}^2 = \frac{L_1}{L_1 - 1}\hat{\sigma}_1^2 \tag{17}$$

and its variance is given by

$$\text{var}\{\hat{\sigma}_{1,unb}^2\} = \frac{2\sigma_1^4}{L_1 - 1}. \tag{18}$$

It is interesting to compare the accuracy of MLA with the Cramer Rao bound (CRB). Letting $\tilde{k}_0 = k_0$ in the LLF shown in (5), the Fisher information matrix for the estimation of $\{A, \sigma_1^2, \sigma_0^2\}$ is found to be

$$\mathbf{F} = \begin{bmatrix} L_1/\sigma_1^2 & 0 & 0 \\ 0 & L_1/(2\sigma_1^4) & 0 \\ 0 & 0 & L_0/(2\sigma_0^4) \end{bmatrix} \tag{19}$$

and the corresponding bounds are the diagonal entries of $\mathbf{F}^{-1}$

$$\text{CRB}(A) = \frac{\sigma_1^2}{L_1} \tag{20}$$

$$\text{CRB}(\sigma_0^2) = \frac{2\sigma_0^4}{L_0} \tag{21}$$

$$\text{CRB}(\sigma_1^2) = \frac{2\sigma_1^4}{L_1}. \tag{22}$$

These results indicate that the estimates $\hat{A}$ and $\hat{\sigma}_0^2$ provided by MLA attain the relevant CRBs, while $\hat{\sigma}_{1,unb}^2$ approaches the bound only asymptotically.

V. Simulation results

Computer simulations have been run to assess the performance of MLA. Comparisons are made with a standard frame detection algorithm, which is derived over the traditional AWGN channel with signal-independent noise variance. This scheme, denoted hereafter as the conventional algorithm (CA), provides an estimate of k_0 in the form

$$\hat{k}_{0,CA} = \arg\min_{\tilde{k}_0\in J_0}\left\{\sum_{k\in\mathcal{K}_0} x^2(k+\tilde{k}_0) + \sum_{k\in\mathcal{K}_1}[x(k+\tilde{k}_0)-\hat{A}(\tilde{k}_0)]^2\right\} \tag{23}$$

where $\hat{A}(\tilde{k}_0)$ is still given in (6). To the best of our knowledge, CA is the only available method in the literature to acquire frame synchronization in FSO systems without any knowledge of the signal level and signal-dependent noise variances. The UW is a maximum length sequence with $L_1 = 32$ and $L_0 = 31$, while the average SNR over the UW is defined as

$$\text{SNR} = \frac{\mathbb{E}\{a_k^2\}A^2}{(\sigma_0^2 + \sigma_1^2)/2} \tag{24}$$

with $\mathbb{E}\{a_k^2\} = L_1/L$. Letting $\alpha = \sigma_1^2/\sigma_0^2$, from (24) we have

$$\sigma_0^2 = \frac{2L_1A^2}{(1+\alpha)L}(\text{SNR})^{-1}. \tag{25}$$

Values of α as large as 30 can be encountered in practical applications [9]. Fig. 1 illustrates P_f for both MLA and CA.

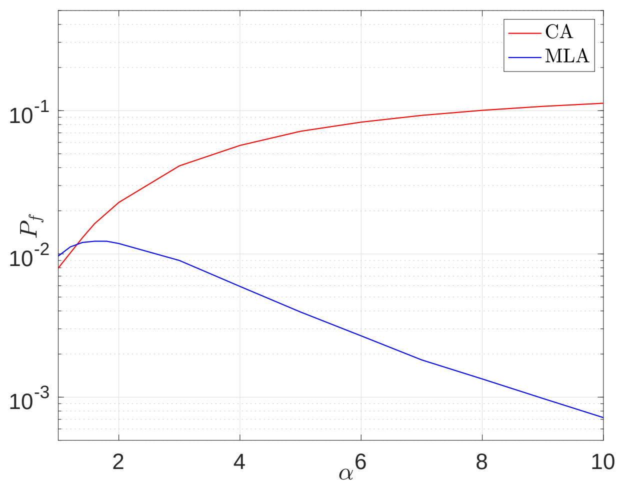

Fig. 1. P_f for both MLA and CA vs. α, SNR = 4 dB.

The SNR is kept fixed to 4 dB, while α varies from 1 to 10. When $\alpha = 1$, no shot noise is present in the received sample stream and CA performs slightly better than MLA. The reason is that, while CA is tailored to such a specific situation, MLA tries to estimate two different noise variances when in fact the noise power is the same over any symbol period. As α increases the detection capability of CA deteriorates due to its mismatched noise model, while MLA, after exhibiting a maximum of P_f around $\alpha = 1.6$, steadily improves its performance. Such an improvement is a consequence of the information conveyed by the shot noise on the received bit. In fact, shot noise is signal-dependent and, accordingly, it may prove useful to discriminate whether the received bit is 1 or 0. Since MLA effectively exploits this information, at large values of α it exhibits an improved capability of localizing the UW. Numerical simulations (not shown for space limitations) indicate that the maximum of P_f around $\alpha = 1.6$ disappears when the estimate $\hat{A}(\tilde{k}_0)$ in (10) is replaced by the true value of A. This means that the slight increase of P_f occurring with MLA in the range $\alpha \in [1, 1.6]$ is due to a lower accuracy in the estimation of A. At higher values of α, such a negative effect is overcome by the increased capability of detecting the UW offered by the presence of shot noise, as discussed previously.

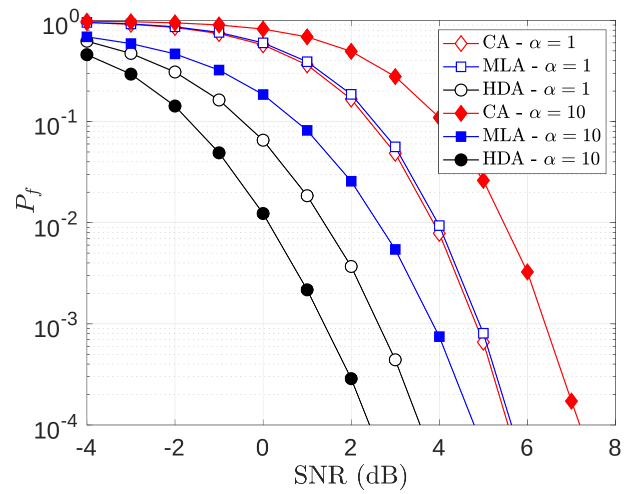

Fig. 2. P_f vs. SNR for CA, MLA and HDA with $\alpha = 1, 10$.

Fig. 2 shows P_f as a function of the average SNR obtained with $\alpha = 1$ and $\alpha = 10$. In addition to the results pertaining to MLA and CA, we also assess the performance of the Hammons and Davidson algorithm (HDA) presented in [9]. Since this scheme provides the ML estimate of k_0 under perfect knowledge of $\{A, \sigma_1^2, \sigma_0^2\}$, its accuracy can be regarded as a lower bound to P_f. For $\alpha = 1$, we see that MLA and CA perform similarly at all the considered SNR values. On the other hand, when $\alpha = 10$ a gain of approximately 2.5 dB is achieved by MLA with respect to CA. These results are in line with those reported in Fig. 1 and corroborate the superiority of MLA in the presence of shot noise. Compared to HDA, the loss of MLA is nearly 2.5 dB with both $\alpha = 1$ and $\alpha = 10$. The improvement exhibited by HDA in passing from $\alpha = 1$ to $\alpha = 10$ is a further evidence of how shot noise may reveal useful in localizing the UW.

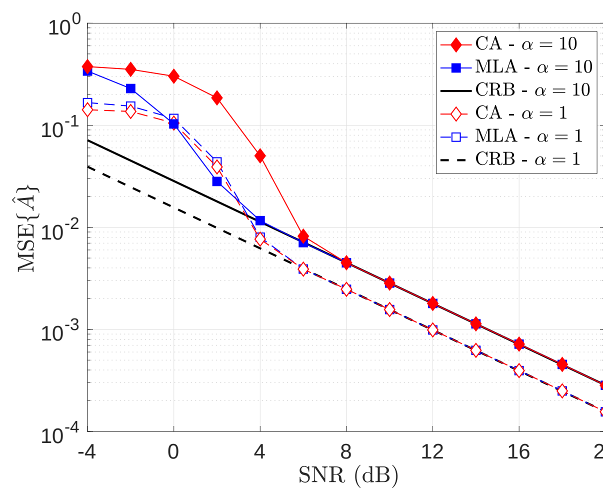

Fig. 3. MSE($\hat{A}$) vs. SNR for CA and MLA with $\alpha = 1, 10$.

Fig. 3 gives the mean square error (MSE) in the estimation of A vs. SNR in the same operating conditions of Fig. 2. As expected, at high SNR values both MLA and CA attain the relevant CRB with either $\alpha = 1$ or $\alpha = 10$. When the SNR reduces, however, the metrics employed for UW detection are so distorted by noise that their peak may occasionally occur far from k_0. In these circumstances, the UW position is incorrectly detected and the accuracy of the channel estimates departs from its theoretical value (11) since the latter was evaluated in Sect. IV under the optimistic assumption $\hat{k}_0 = k_0$. This fact justifies the abrupt increase of the MSE curves observed in Fig. 3 in the low SNR region (estimator threshold). In the absence of shot noise ($\alpha = 1$), the threshold is the same for both MLA and CA. When $\alpha = 10$, however, the use of MLA allows a reduction of the SNR threshold by nearly 3 dB compared to CA due to its improved capability of detecting the correct UW position.

The accuracy of the noise variance estimates $\hat{\sigma}_0^2$ and $\hat{\sigma}_{1,unb}^2$ provided by MLA is reported as a function of the SNR in Figs. 4 and 5, respectively. The experimental results obtained with $\alpha = 1$ and $\alpha = 10$ are compared with the theoretical analysis given in (15) and (18), obtained under the assumption of perfect frame detection. The curves are qualitatively similar to those pertaining to the channel estimates $\hat{A}$ shown in Fig. 3. Specifically, we see that the accuracy of MLA validates the theoretical results in the high SNR regime, while at low SNR values it deviates from the ideal curves due to the occurrence of failures in the detection of the UW.

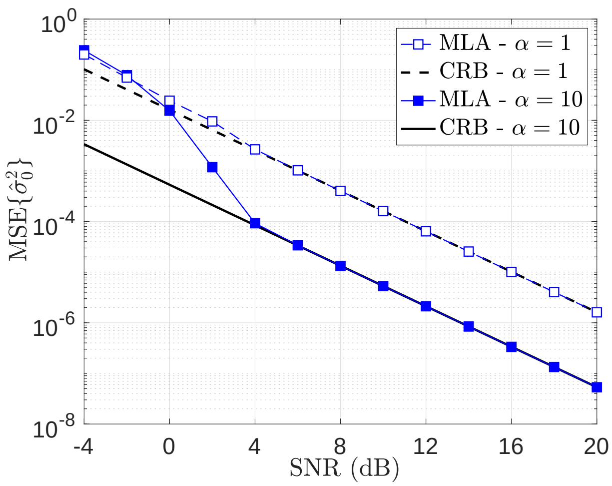

Fig. 4. MSE of $\hat{\sigma}_0^2$ as a function of the SNR for $\alpha = 1, 10$.

VI. Conclusions

We have addressed the problem of frame synchronization in a packet-based FSO link employing a NRZ-OOK modulation format. Since an APD is employed for data detection, the noise variance depends on the received bit value as a consequence of the shot noise. By applying the ML estimation principle, we have derived a scheme, named MLA, which can effectively exploit the presence of shot noise to improve the UW detection capability. In contrast to existing methods, the MLA lends itself to a practical implementation as it operates without any prior knowledge of the signal level and noise variances. Computer simulations indicate that MLA provides a substantial SNR gain with respect to the conventional UW detection method, which is derived under the traditional AWGN channel with signal-independent noise statistics. Such a gain is achieved without any increase of the computational complexity, which makes MLA a promising candidate for frame detection in FSO communications.

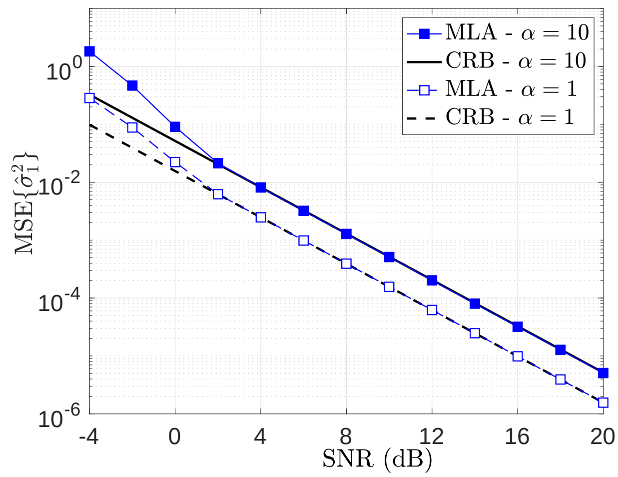

Fig. 5. MSE of $\hat{\sigma}_{1,unb}^2$ as a function of the SNR for $\alpha = 1, 10$.

References

[1] H. Kaushal and G. Kaddoum, “Optical Communication in Space: Challenges and Mitigation Techniques”, *IEEE Commun. Surveys & Tutorials*, vol. 19, no. 1, pp. 57-96, 2017.

[2] R. H. Barker, “Group synchronization of binary digital systems”, in *Commun. Theory*, W. Jackson (Ed.), Butterworths, New York 1953.

[3] J. L. Massey, “Optimum frame synchronization”, *IEEE Trans. on Commun.*, vol. 20, no. 2, pp. 115-119, Apr. 1972.

[4] C. N. Georghiades and D. L. Snyder, “Locating data frames in direct-detection optical communication systems”, *IEEE Trans. on Commun.*, vol. 32, no. 2, pp. 118-123, Feb. 1984.

[5] G. L. Lui and H. H. Tan, “Frame synchronization for direct-detection optical communication systems”, *IEEE Trans. on Commun.*, vol. 34, no. 3, pp. 227-237, Mar. 1986.

[6] A. A. D’Amico, U. Mengali, and L. Taponecco, “TOA estimation with the IEEE 802.15.4a standard”, *IEEE Trans. on Wireless Commun.*, vol. 9, no. 7, pp. 2238-2247, Jul. 2010.

[7] M. Chiani and M. G. Martini, “Practical frame synchronization for data with unknown distribution on AWGN channels”, *IEEE Commun. Letters*, vol. 9, no. 5, pp. 456-458, May 2005.

[8] M. Chiani and M. G. Martini, “On sequential frame synchronization in AWGN channels”, *IEEE Trans. on Commun.*, vol. 54, no. 2, Feb. 2006.

[9] A. R. Hammons and F. Davidson, “Near-optimal frame synchronization for free-space optical packet communications”, in *Proc. of Military Commun. Conf. (MILCOM) 2010*, pp. 797-801, 2010.

[10] H. Yin, S. Li, Z. Huang, J. Chen, “A novel data-aided frame synchronization method based on Hough transform for optical communications”, *Photonics*, Aug. 2020.

[11] Z. Ghassemlooy and W. O. Popoola, “Terrestrial free-space optical commmunications”, in *Mobile and wireless communications: Network layer and circuit level design, Chapter 17*, Edited by S. A. Fares and F. Adachi, 2010.

[12] K. Kiasaleh, “Receiver architecture for channel-aided, OOK, APD-based FSO communications through turbulent atmosphere”, *IEEE Trans. on Communications*, vol. 63, no. 1, pp. 186-194, Jan. 2015.

[13] D. Giggenbach, A. Shrestha, C. Fuchs, C. Schmidt, and F. Moll, “System aspect of optical LEO-to-ground links”, *Proc. of Int. Conf. on Space Optics*, Biarritz, France, 18-21 Oct. 2016.

[14] M. L. B. Riediger, R. Schober, and L. Lampe, “Fast multiple-symbol detection for free-space optical communications”, *IEEE Trans. on Commun.*, vol. 57, no. 4, pp. 1119-1128, Apr. 2009.

[15] M. A. Khalighi and M. Uysal, “Survey on free space optical communications: a communication theory perspective”, *IEEE Commun. Surveys & Tutorials*, vol. 16, no.4, pp. 2231-2257, Fourth quarter 2014.