

Robustness Evaluation of Different Controllers in the Presence of Flow Rate Variations

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This paper proposes a direct approach to evaluate achievable robustness to flow rate variations of different controllers (PID and advanced algorithms), for typical process dynamics (First Order Plus Time Delay with different time delay/lag ratio), able to represent very common heat exchange equipment.

Starting from comparable nominal performance, the effect of flow rate variations on process parameters and consequently on achievable performance is analyzed in simulation up to the onset of marginal stability conditions. Flow rate variations act as “structured” uncertainty on parameters and the proposed procedure is able to indicate maximum allowable variations in a more realistic and efficient way, avoiding the conservatism implicit in most of analytical design techniques available in the literature.

The proposed technique evaluates in a straightforward way the Maximum Allowed Changes (MAC) in flow rate. As inverse proportionality between flow rate and process parameters is present, also the efficacy of adopting equal percentage (EP) valves, which allow a local compensation of process gain nonlinearity, is investigated.

1. Introduction

Robustness of controllers is certainly a topic of great relevance in the design of industrial control systems, owing to the presence of uncertainty (errors) in the model used and to variations in operating conditions and process variables which make the process model inadequate, independently of large efforts spent in its identification and development. This is well known since the very beginning of process control and attempts to include some measures of robustness have been made starting from Gain and Phase Margin (GM, PM) incorporated into classical tuning rules of standard PID controllers (Ziegler and Nichols, 1942) or used as a design/tuning specification to be respected (Ho et al. 1995).

An evolution took place during the '80s, with the definition of Robust Stability (RS) and Robust Performance (RP) conditions and related methodologies to design control systems able to guarantee assigned specifications in the face of an estimated range of variation of process parameters: the compendium of the research activity in this field can be considered the book by Morari and Zafiriou (1989). In the light of possible follow-up of these results to industrial problems, we must say that they are seen as too complicated to seem worth being applied to the design or tuning of PID controllers. In fact, they require an evaluation of bounds on parameter uncertainty and for the complete solution of the problem (i.e., to guarantee RP) also the definition of suitable weighting functions to represent desired performance. Limiting the objective to the RS issue, variations in N parameters of a model between a minimum and a maximum value would require the construction of regions of uncertainty of 2^N vertices in the Nyquist plane, regions having different shapes according to the relative values of uncertainties on each specific parameter.

More than this aspect (complexity), still a difficulty arises in obtaining a representation not too conservative of the real situation: uncertainties on different parameters are very often correlated, in the sense that they are originated by variations of a main operating variable. Therefore, they are affected in a “structured” way, making the extent of “unstructured” uncertainty regions not realistic and then not reliable for a tight design. These considerations led researchers to the formulation of the Structured Singular Value Theory (Doyle et al., 1982),

very promising and able to eliminate conservatism issues, but even more cumbersome for a straight industrial implementation of its results.

Within the vast literature on robust control, recent practical design methods for traditional and two-degree-of-freedom PID controllers of Nobuyama and Kami (2018) and PID auto-tuning of Zhao et al. (2018) can be mentioned. Interesting industrial-oriented applications of robust PID control regard Plug Flow Reactors (Mikhalevich et al., 2015), Continuous Stirred-Tank Reactors (Prokop et al., 2019), and spray dryers (Bacci di Capaci and Scali, 2017), where also cascade and feed-forward control schemes have been evaluated.

After this recall, the point we want to focus on is the effect of flow rate variations on controller robustness, a phenomenon which must be considered unavoidable, since the flow rate is the most common operating process variable; very often it is the manipulated variable and then it must vary to counteract process disturbances or to track set-point changes. Referring to flow rate variations, two points become evident: i) this occurrence makes also the most accurate model inadequate to maintain the assumed specifications, and ii) variations induced on process parameters are certainly correlated.

From this introduction, the structure of the paper follows. Section 2 presents some motivating examples of process control schemes, where it is evident that flow rate variations affect all the process parameters; Section 3 illustrates a procedure to compare controllers designed according to different techniques, a procedure which can be used also to guarantee robustness specifications. Section 4 presents the results for different process dynamics (delay/lag dominant) and different valve types. Finally, Section 5 reports conclusions and future works on this topic.

2. The role of flow rate variations

2.1 Examples of process control schemes

The problem we want to tackle can be exemplified by referring to the very common case of temperature control in heat exchange equipment. In Figure 1a the simplest case, actually reported in almost every process control textbook, represents two fluid flow rates interacting in a jacketed vessel (same considerations apply for a vessel with coil). The corresponding block diagram is reported in Figure 1b.

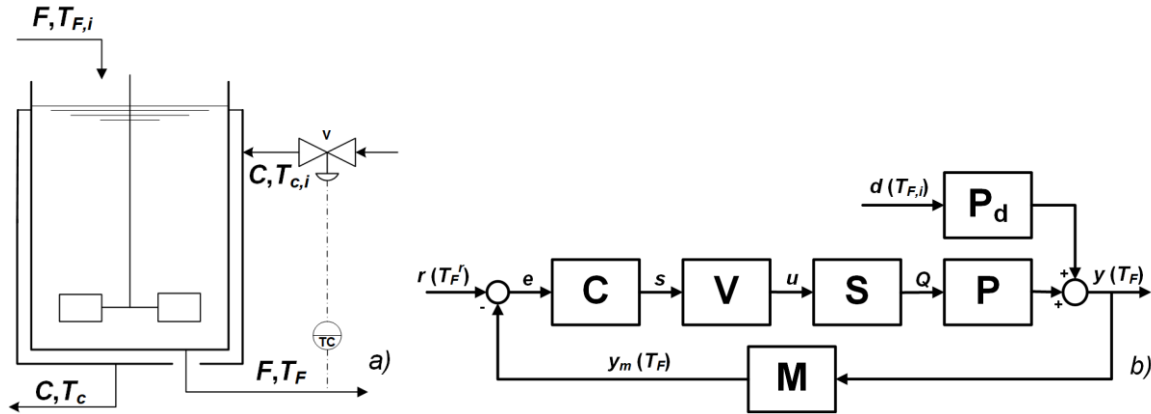


Figure 1: The classical heated tank: a) process scheme; b) block diagram.

Under the common hypothesis of perfect mixing in the vessel, no chemical reactions, and perfect dynamics of valve (V), heat exchange surface (S), and temperature sensor (M), an energy balance in the time domain for the cold fluid in dynamic conditions, in terms of deviation variables (x'), gives:

$$\tau \frac{dT_F'}{dt} + T_F' = T_{F,i}' + \frac{Q'}{FCp_F} \quad (1)$$

where:

- $F, T_{F,i}, T_F$ and $C, T_{c,i}, T_c$, indicate mass flow rate, inlet and outlet temperature of cold and hot fluid, respectively;
- Q is the heat power exchanged between the two fluids;
- $\tau = V \cdot \rho / F$ is the process time constant, being V the vessel volume and ρ, Cp_F the cold fluid density and heat capacity, respectively; all of them considered as constant.

Under the assumed hypotheses, the process dynamics (P) and disturbance dynamics (P_d) are both linear and can be represented by two Transfer Functions (TF) in the Laplace domain:

$$Q'(s) \neq 0; T'_{F,i}(s) = 0; \rightarrow P(s) = \frac{T'_F(s)}{Q'(s)} = \frac{1}{FCp_F} \cdot \frac{1}{1 + \tau s} = \frac{K}{1 + \tau s} \quad (2)$$

$$T'_{F,i}(s) \neq 0; Q'(s) = 0; \rightarrow P_d(s) = \frac{T'_F(s)}{T'_{F,i}(s)} = \frac{1}{1 + \tau s} \quad (3)$$

where $K = 1/(F \cdot C p_F)$ is the static gain of process P. Taking into account unavoidable delays Θ caused by the presence of volumes and lengths neglected in the above simplified approach, global dynamics represented by a First Order Plus Time Delay (FOPTD) model can be considered: $G = K \cdot e^{(-\Theta s)} / (\tau s + 1)$. This overall TF ($G = V \cdot S \cdot P$) relates control action (s) to temperature variations (T_F) in the forward path, through the valve, the heat exchanger, and the process.

2.2 Effect on process parameters

It is evident that the three parameters: gain, time constant, and delay (K, τ, Θ) are inversely proportional to the flow rate F . Also, in closed-loop operation, the hot fluid flow rate (C) is proportional to the cold fluid flow rate (F) and its variations affect in the same way (i.e., inversely) overall process parameters included in G (see Figure 2a). The situation is common to different possible control schemes in industrial exchanger networks. In Figure 2b, a simplified representation of a slightly different temperature control scheme is reproduced: in this case, flow rate F becomes also the manipulated variable, being used to counteract changes in C (depending on upstream equipment), in order to maintain the controlled variable T_F to its desired set point value (T_F^s).

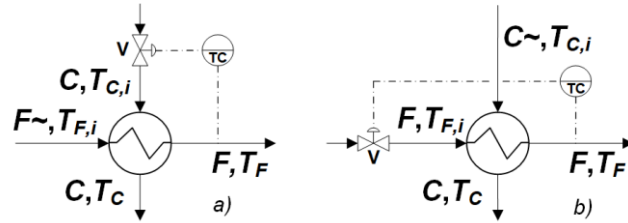


Figure 2: Heat exchange schemes: a) equivalent to Figure 1; b) with cold flow rate F as manipulated variable.

So, it is clear that changes in the heat exchange for whatever reason (disturbance on inlet flow rate and temperature, set-point changes in the controlled variable), require variations in the manipulated variable and affect process parameters (inverse dependence). Thus, representing changes of operative variables in terms of uncertainty on model parameters, a “structured” type uncertainty occurs, being that an increase/decrease of flow rate causes respectively a decrease/increase of all the three process parameters (in the case of FOPTD dynamics). For example, by indicating with (K, τ, Θ) , and $(K^\circ, \tau^\circ, \Theta^\circ)$, perturbed and nominal values, respectively, it follows: $K = K^\circ/R$; $\tau = \tau^\circ/R$; $\Theta = \Theta^\circ/R$; with $R = F/F^\circ$ and being F° the nominal flowrate. Note that, for the sake of simplicity, linear dynamics is here assumed time-invariant, that is, parameters are considered constants over the transient dynamics; time-variant systems will be then investigated in future works.

3. The proposed methodology

3.1 The basic idea

The controller design or tuning is generally based on the nominal linear model, so it is worth analyzing the effect of flow rate variations on loop performance. The worst case is that of a decrease in the flow rate, as all the parameters will increase, leading towards larger amplitude of oscillations and possibly to instability conditions. To keep the approach as simple as possible, we propose to simulate the effect of decreasing values of flow rate changes up to the onset of marginal stability conditions. Therefore, the procedure is the following:

- given the process model and a controller designed/tuned according to a predefined technique;
- analyze closed-loop response for increasing changes in process flow rate (with constant controller parameters);
- determine the maximum value of changes corresponding to marginal stability conditions.

Remarks:

1. This procedure can be implemented by few lines in a MATLAB code (or whatever simulation software) and does not require any time-consuming construction of uncertainty regions, as mentioned in the introduction.
2. This value is the maximum allowable flow rate variations and then indicates the Maximum Allowed Change (MAC) caused by inlet disturbance or set-point changes.

3. As illustrated, it refers to Robust Stability conditions; a slightly more complicated procedure would find the MAC to maintain Robust Performance specifications, stated in terms of time-domain constraints on maximum deviations from the desired value of the controlled variable, or settling time of the response.
4. Also, the procedure can be used for comparison of the performance of different controllers or to design a controller able to guarantee desired time-domain specifications. The first case is the more direct and also the more significant, as very often it is interesting to know which controller has large tolerance to flow rate variations, i.e., able to manage larger heat exchange variations for the equipment.

For the case of performance comparison, some more details are given below, in order to make the study as fair as possible.

3.2 Performance/Robustness comparison of different controllers

The comparison is straightforward in the case of controllers having the same algorithm and designed or tuned according to specific techniques. Here the investigated case refers to a comparison between an advanced IMC-algorithm controller and a traditional PI(D) controller tuned according to the Curve of Response (CR) Technique (Brambilla et al., 1990). Simulations deal with the case of change in the required heat exchange, and then on the cold flow rate F , to face set-point temperature variations (T_F).

In an IMC structure, for a step set-point change and a FOPTD model, the controller is given by q and becomes equivalent to a classical feedback controller C , as:

$$q = \frac{\tau s + 1}{K(\lambda s + 1)}; \quad C = \frac{q}{1 - Pq} \quad (4)$$

The traditional controller has a PI algorithm given by:

$$C = K_c \frac{(\tau_i s + 1)}{\tau_i s} \quad (5)$$

where parameters are computed by the CR tuning rule as: $K_c = (\tau + \Theta/2)/(K \cdot (c+1) \cdot \Theta)$; $\tau_i = \tau + \Theta/2$, with $c = 0.8$.

While for the PI controller K_c and τ_i are given, for the IMC controller the tuning parameter λ must be chosen for a desired robustness to parameter uncertainty or a desired speed of the closed-loop response: obviously, smaller values of λ determine smaller tolerance (to parameters variations) and faster speed (of response). A fair comparison between techniques can be made by choosing a value of λ such that the speed of response is comparable in the nominal case; that is, when process and model dynamics coincide. Starting from this condition, the effect of increasing errors on the model parameters, due to flow rate changes are examined: the onset of Marginal Stability conditions indicates the maximum value of flow rate changes (that is, MAC).

4. Results for different process dynamics

To show the application of the proposed method, FOPTD processes derived for the scheme of Figure 2b will be examined for different values of τ and Θ parameters. The first example is for a FOPTD process, with values $K=1$, $\tau_i=10$, $\Theta=10$, assumed constant along simulation horizon. Adopting the CR technique, controller parameters for this process are $K_c=0.83$, $\tau_i=15$. The corresponding value of the tuning parameter for the advanced IMC algorithm, having the same speed of response in the nominal case, is $\lambda_0=10$ (see Figure 3).

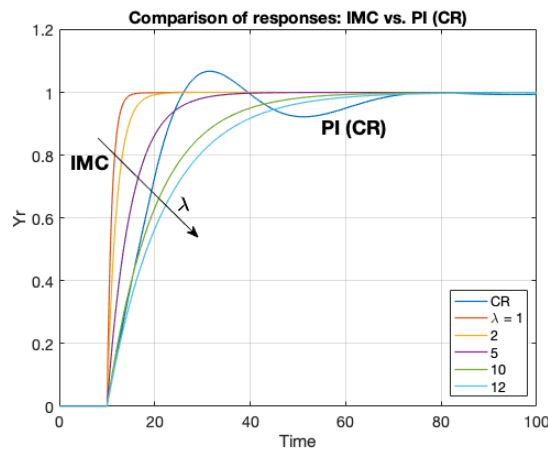


Figure 3: Set-point change responses: effect of the tuning parameter λ on the speed ($\lambda = 1, 2, 5, 10, 12$).

It can be seen that when the value of λ is increased, the response becomes slower; also, IMC allows in the nominal case to obtain smooth and fast responses, without the oscillation shown by the PI controller (having suboptimal algorithm).

Time responses to set-point variations for decreasing values of the cold flow rate F are reported in Figures 4a and 4b; corresponding variations in the flow rate are expressed as relative decrease, i.e., $\Delta F = (F^\circ - F)/F^\circ$; note that $\Delta F = 1 - R$.

Some comments are in order.

- As expected, responses become more and more oscillating, reaching Marginal Stability conditions.
- For the PI controller, this occurs for a flow variation ratio $\Delta F = (F^\circ - F)/F^\circ = 0.42$; the IMC controller shows larger robustness being able to maintain stability for larger flow rate variations (up to $\Delta F = 0.49$).
- In this case, MAC (Maximum Allowable Changes) to heat exchange are given by the IMC controller, showing a larger range of variation for the flow rate.
- Results cannot be considered general unless verified for different dynamics, for instance, delay dominant ($\Theta/\tau \gg 1$) and lag dominant cases ($\Theta/\tau \ll 1$). A synthesis of results is reported in Table 1.

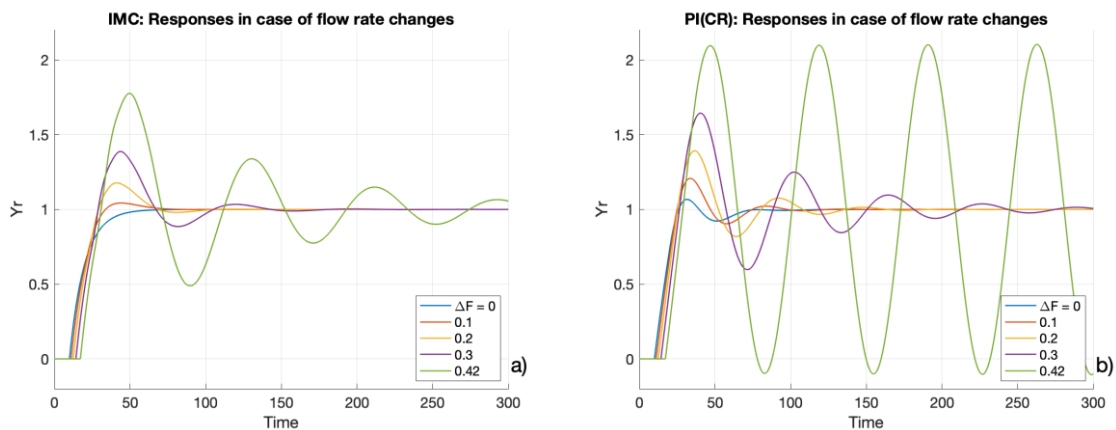


Figure 4: Effect of flow rate variations $\Delta F = (F^\circ - F)/F^\circ$ on set-point change responses: a) IMC; b) PI controller.

Table 1: MAC for IMC e PI controllers for different process dynamics; in square brackets values for EP valves.

$\Theta; \tau$	IMC: λ_0	PI: $K_c; \tau_i$	IMC: MAC [EP]	PI: MAC [EP]
10; 1	4.5	0.33; 6	0.40 [0.60]	0.40 [0.67]
10; 10	10	0.83; 15	0.49 [0.70]	0.42 [0.66]
1; 10	2	5.83; 10.5	0.75 [0.92]	0.60 [0.90]

From results reported in Table 1, some general conclusions seem to be possible from the examined cases:

- tolerance to flow rate variations increases from delay dominant to lag dominant processes.
- the tolerance is almost equivalent for the two controller types for delay dominant cases, while the IMC controller proves to become more tolerant for lag dominant processes.

These results may not be confirmed for different process dynamics, but an assessment of relative merits of controllers is beyond the scope of this study, its objective being to focus on the issue of flow rate variations and to present a methodology that permits an evaluation of Maximum Allowable Changes (MAC).

Another important point to recall is the beneficial effect of adopting Equal Percentage (EP) valves: owing to the special shape of their plug, assuming that the pressure drop across the control valve is nearly constant, they are able to generate an exponential relationship between the output flow rate and the input stem position such that the valve gain is proportional to the flow rate $K_v = k \cdot F$. In this way, a local compensation of process gain nonlinearity is obtained: that is, the overall (valve and process) loop gain $K_{OL} = K_v \cdot K$ becomes (approximately) independent of the flow rate, F .

The effect of this local compensation on nonlinearity has been investigated and results are reported in Figure 5, for the same situation simulated in Figure 4, that is, FOPTD process with $\Theta/\tau = 10/10$, IMC vs. PI controllers and same variations of flow rate. In these simulations, the process gain is considered constant, i.e., $K = K^\circ$. It can be seen that responses are now significantly less affected by flow rate changes.

MAC values by adopting equal percentage valves are reported among square brackets; values are indeed larger to indicate larger tolerance; also, there is less difference between the advanced and PI controller algorithm. Even though the compensation of detrimental effect of flow rate changes on performance is not complete (as it

regards only the overall loop gain K_{OL} and not also the other two parameters Θ and τ), EP valves allow operating in a wider range of flow rate values.

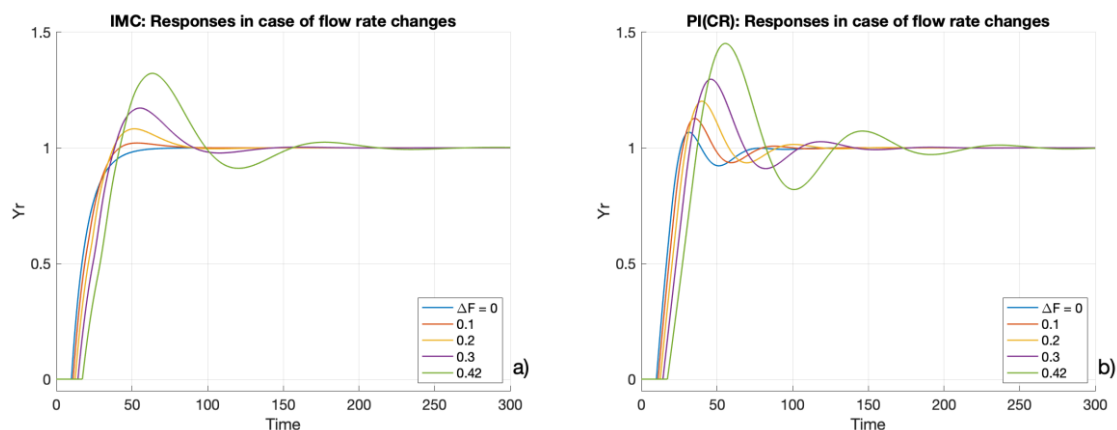


Figure 5: Effect of flow rate variations on set-point responses when EP valve is adopted: a) IMC; b) PI controller.

5. Conclusions

The proposed methodology permits a direct approach to the evaluation of the Maximum Allowed Changes (MAC) in process flow rate, by using different controllers. This approach gives an indication of robustness which is relevant for many processes, being the flow rate the most common manipulated variable, then necessarily subject to large variations. The obtained results for FOPTD dynamics show that tolerance is larger for lag dominant with respect to delay dominant processes. Also, advantages in extending the range of safe operation by adopting Equal Percentage valves are clearly put into evidence.

The implementation of the technique is simple and does not originate conservative results, which is the main drawback of most available methodologies having the same scope. The approach is based on linear simulation (i.e., Transfer Function representation of process dynamics) and does not take into account the presence of constraints on flow rate, the effect of variable pressure drops across the control valve, and also the fact that during the transient the time variations of flow rate cause consequent variations of all process parameters. These aspects will be the object of future works.

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