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Research article

INTERFACIAL FRACTURE TOUGHNESS OF UNCONVENTIONAL SPECIMENS: SOME KEY ISSUES

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ABSTRACT

Laboratory specimens used to assess the interfacial fracture toughness of layered materials can be classified as either conventional or unconventional. We call conventional a specimen cut from a unidirectional composite laminate or an adhesive joint between two identical adherents. Assessing fracture toughness using conventional specimens is a common practice guided by international test standards. In contrast, we term unconventional a specimen resulting from, for instance, bimaterial joints, fiber metal laminates, or laminates with an elastically coupled behavior or residual stresses. This paper deals with unconventional specimens and highlights the key issues in determining their interfacial fracture toughness(es) based on fracture tests. Firstly, the mode decoupling and mode partitioning approaches are briefly discussed as tools to extract the pure-mode fracture toughnesses of an unconventional specimen that experiences mixed-mode fracture during testing. Next, we elaborate on the effects of bending-extension coupling and residual thermal stresses often appearing in unconventional specimens by reviewing major mechanical models that consider those effects. Lastly, the paper reviews two of our previous analytical models that surpass the state-of-the-art in that they consider the effects of bending-extension coupling and residual thermal stresses while they also offer mode partitioning.

KEYWORDS: interlaminar cracking, non-standard specimen, laminated material, bending-extension coupling, residual thermal stresses, analytical modeling

1. Introduction

Interfacial cracking is a common but also critical failure mode for many different types of layered materials. For example, a separation between two successive laminae of a laminated composite, commonly referred to as delamination or interlaminar crack, is today among the most prominent “enemies” of this material class. Similarly, interfacial cracking phenomena usually appear in the bondline of an adhesively bonded joint. An interfacial crack can be created either during manufacturing or in service. Then, it can propagate even under service loads, eventually leading to a large-scale failure of the structural component. For this reason, the accurate assessment of interfacial fracture toughness of layered materials is a crucial research task.

Evaluating the interfacial fracture toughness of *conventional* materials, such as composite laminates with unidirectional or symmetric layups and adhesive joints between two identical adherents, is a well-known procedure guided by international test standards (e.g., [1]). Moreover, short closed-form expressions derived from simple mechanical models based on Euler or Timoshenko beam theories are typically employed for experimental data reduction.

However, current research interests and industrial applications call for a broader range of multilayered materials and structures than those covered by the existing standard procedures. Examples include bimaterial adhesively bonded joints [2], fiber metal laminates [3], multidirectional composites with an unsymmetric layup that leads to an elastically coupled behavior [4], sandwich plates [5], thin laminates that need to be stiffened before testing (using backing beams) [6], and structures containing residual thermal stresses [7]. All those *unconventional* material types are attracting increasing interest over the last decades. To characterize the interfacial fracture of such materials or structures, beam-shaped laboratory specimens are cut from them that inherently feature a material asymmetry with respect to the crack plane. This asymmetry introduces mode mixity at the crack front even if the specimen is globally loaded in pure mode during the characterization test. In addition, elastic couplings, residual thermal stresses, and various other effects may appear in the mechanical behavior of the specimen, which complicates the data reduction. To account for a combination of new effects, refined mechanical models that inevitably are more complex than those for symmetric specimens should be developed.

The present paper briefly reviews the key issues in computing the interfacial fracture toughness of unconventional specimens. Section 2 defines two fundamental concepts: interfacial fracture within a layered material; and conventional versus unconventional specimens. Next, Section 3 discusses the two main approaches currently used in the literature to compute the pure-mode fracture toughness of an unconventional specimen: mode decoupling and mode partitioning. Then, Section 4 focuses on the effects of bending-extension coupling and residual thermal stresses, explaining the basic phenomena and reviewing major analytical models considering those effects. After that, Section 5 outlines two mechanical models previously proposed by two subsets of the present Authors independently, which can consider the previously discussed issues and go beyond state-of-the-art. The paper concludes with Section 6.

2. Basic concepts

2.1. Interfacial fracture in a layered material

In this paper, the term *layered* denotes a material that consists of two or more layers with different elastic properties. Within the general class of layered materials, we include, among others, dissimilar adhesively bonded joints, multidirectional composite laminates, fiber metal laminates, and sandwich plates.

Fig. 1a shows a beam-shaped layered material featuring a straight, through-the-width interfacial crack. Supposing this layered material is a (fiber-reinforced) composite laminate, the interfacial crack represents the separation between two successive laminae, a failure mode known as delamination or interlaminar fracture (Fig. 1b–i). Alternatively, the interfacial crack may represent a disbonding between the two adherents of a bimaterial adhesive joint (Fig. 1b–ii). Interfacial cracks are observed in several other layered materials, such as in sandwich structures as disbondings between core and skin (Fig. 1b–iii).

2.2. Conventional and unconventional specimens

A crack can develop in a material under a mix of three basic fracture modes: opening (or mode I), sliding (or mode II), and tearing (or mode III), each characterized by a different fracture toughness value. To assess the fracture toughness of material under both pure and mixed fracture modes, specific laboratory tests have been developed, and some of them are now standardized. For example, the double cantilever beam (DCB) test is the standard laboratory test to determine the pure-mode I fracture toughness according to a specified procedure (see, for instance, the ASTM

D5528-13 standard [1]). The test standards not only define testing procedures and conditions but also specify the material and geometric properties of the specimen.

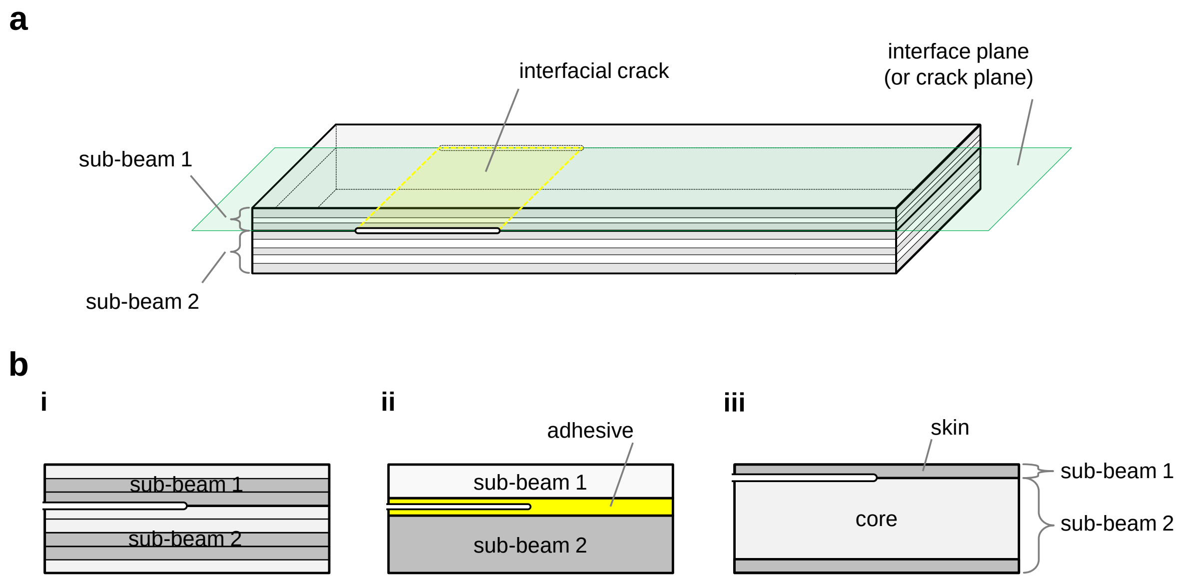

Figure 1. (a) A through-the-width interfacial crack in a beam-shaped layered material. **(b)** (i) An asymmetric delamination in a symmetric composite laminate; (ii) an interfacial disbonding in a bimaterial adhesive joint; and (iii) a crack in the core-to-skin interface of a sandwich beam

In this paper, we term *conventional* a test specimen specified by existing test standards. For example, a specimen cut from a unidirectional composite laminate (Fig. 2a–i) or a similar adhesive joint (Fig. 2a–ii) are both called conventional specimens. Studying fracture behavior using conventional specimens is today a common practice. In contrast, we term *unconventional* a specimen extracted from one of the layered materials discussed in Section 2.1. As a typical example, a specimen cut from a symmetric composite laminate is considered unconventional if the crack exists out of the laminate's middle plane (Fig. 2b–i). In such case, the two sub-beams into which the crack splits the specimen will be unsymmetric and may be elastically coupled, even though the total laminate is symmetric and elastically uncoupled. Another typical example of an unconventional specimen is the bimaterial adhesive joint (Fig. 2b–ii), where the two adherents have different material properties and thicknesses. Other examples of unconventional specimens that have been studied in the literature include sandwich composites with an interfacial crack between core and skin [5], specimens featuring stiffening beams [6, 7], and joints with a thick adhesive layer [8].

3. Mode decoupling and mode partitioning

The interfacial fracture toughness of an unconventional specimen cannot be determined using standard testing procedures and "classical" analytical models. As schematically shown in Fig. 2b, the two sub-beams constituting the specimen have different material properties — and different thicknesses in many cases. For this reason, during a pure-mode fracture test, such as the DCB or end-notched flexure test, the interfacial crack displays mixed-mode behavior.

To determine the pure-mode fracture toughnesses of such specimens, a usual approach is to properly design the specimen (i.e., select the material properties and thicknesses of sub-beams) to achieve fracture mode decoupling [9, 10]. For the bimaterial adhesive joint (Fig. 2b–ii), for example, a very simple design equation has been proposed in [9] so that when the specimen is globally loaded in mode I, only mode I fracture takes place (and not mixed-mode I/II). A more recent paper [10] proposes a design of fully uncoupled multidirectional stacking sequences to assess the pure-mode I interlaminar fracture of laminates with arbitrary fiber orientation angles in delamination interfaces.

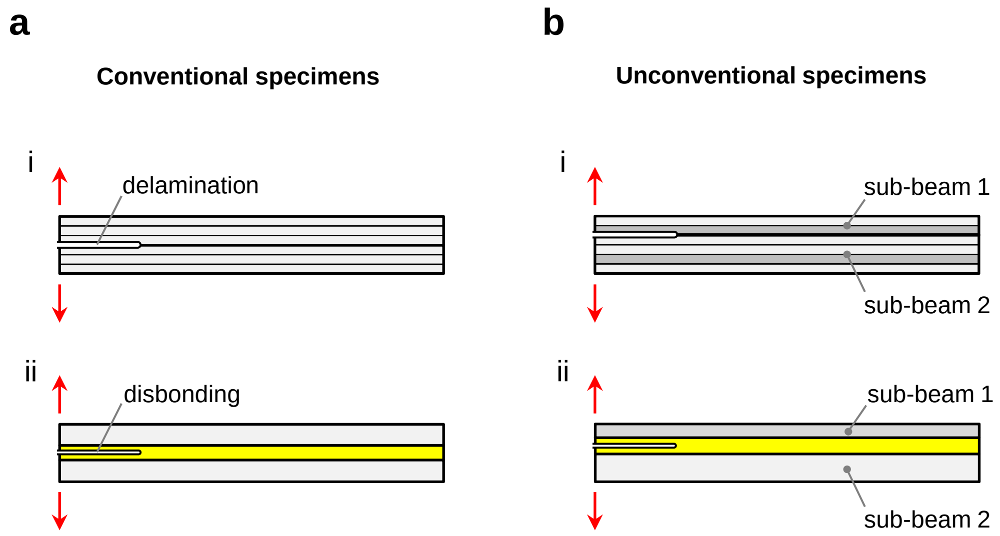

Figure 2. (a) Two characteristic examples of conventional specimens for assessing interfacial fracture toughness: (i) unidirectional composite laminate with a middle-plane delamination; and (ii) similar adhesive joint with an interfacial disbonding. **(b)** Two typical examples of unconventional specimens for assessing interfacial fracture toughness: (i) symmetric composite laminate with an asymmetric delamination; and (ii) bimaterial adhesive joint with an interfacial disbonding

It is often necessary to determine the interfacial fracture toughness of a specimen with pre-defined material properties and thicknesses that cannot be changed. Without tailoring specimen design, mode decoupling is impossible. In addition, in the presence of residual thermal stresses, common for layered materials manufactured at high temperatures, no mode decoupling conditions are available. In such cases, an analytical model capable of performing accurate mode partitioning should be used to compute the mode I and mode II (and mode III, if present) fracture toughnesses. Several mode partitioning methods have been proposed during the last 35 years, which are reviewed in a dedicated paper by a subset of the present Authors [11]. These partitioning methods are characterized by variable accuracies, while the correctness of some of them has been questioned. Thus, a certain challenge in determining the interfacial fracture toughness of an unconventional specimen is selecting an analytical model that can perform correct mode partitioning.

4. Effects of an elastically coupled behavior and residual thermal stresses

After discussing mode decoupling and mode partitioning, this section elaborates on two more specific issues challenging the fracture analysis of unconventional specimens: the effects of bending-extension coupling and residual thermal stresses.

4.1. Elastically coupled behavior

A beam or plate structure from a layered material can feature an elastically coupled behavior resulting from three types of elastic couplings: bending-extension, shear-extension, and bending-twisting couplings [12, pp. 127, 128]. If one of the two sub-beams of an unconventional specimen is elastically coupled (see, for instance, Fig. 2b–i), the displacement magnitudes of the specimen are modified, and mode mixity is induced.

Most analytical mechanical models for the study of interfacial cracks neglect the effect of elastic couplings by modeling the specimen as an assemblage of two *homogeneous* sub-beams. Among the few models considering the elastically coupled response of the specimen, those offering data-reduction expressions are even less. One example is Valvo's model [13], which concerns the delamination of shear-deformable laminated beams with bending-extension coupling. Assuming a clamped crack tip, this model offers closed-form expressions for the mode I and mode II contributions to the energy release rate. Another example is Tsokanas and Loutas's model [14], which can be seen as an extension of Valvo's work to include the effects of crack-tip displacements and residual hygrothermal stresses. Bennati et al. [15] presented an elastic-brittle interface joint model for delaminated shear-deformable beams with bending-extension coupling. Bennati et al.'s model is more accurate than the previous two models. Nonetheless, the final expressions for the energy release rate are not closed-form.

4.2. Residual thermal stresses

There are generally two cases of practical interest where a temperature difference is introduced into an unconventional specimen: (a) if the specimen has been produced at high temperature and cooled down to the service temperature, a typical case for both composites and adhesive joints; and (b) if the specimen has been manufactured at room temperature, but the service temperatures are significantly different (e.g., in cryogenic applications).[1] The two sub-beams constituting an unconventional specimen generally feature different thermo-mechanical properties — and different (effective) coefficients of thermal expansion, in particular. As a result of a temperature change, the two sub-beams will tend to volumetrically deform differently, which gives rise to distortional stresses in the specimen, known as residual thermal stresses [12, pp. 127–129].

The residual thermal stresses affect the stress and displacement fields of the entire specimen. For a bimaterial joint (see Fig. 2b–ii), for example, bending moments and deformations will be induced along the crack plane, which, of course, affect the interfacial fracture toughness by generating an energy release rate in the specimen without any applied mechanical load.

The effect of residual thermal stresses on fracture toughness is ignored by the "classical" theoretical models targeting conventional specimens. Nonetheless, some models aiming to evaluate the interfacial fracture toughness of layered materials subjected to temperature difference have been proposed so far. Nairn [16] proposed an analytical method for calculating the energy release rate in the presence of residual thermal stresses by modeling cracked beams consisting of heterogeneous sub-beams with different coefficients of thermal expansion. This method was later used by Yokozeki et al. [17] to compute the energy release rate in bimaterial specimens loaded using the DCB, end-notched flexure, and mixed-mode bending configurations. Yokozeki [18] proposed a different modeling approach that extends Wang and Qiao's semi-rigid joint model [19] to compute the modal contributions to the energy release rate in bimaterial specimens with residual thermal stresses. Tsokanas and Loutas's model [14], already mentioned in the previous section, extends Yokozeki's model to include the effect of bending-extension coupling on the energy release rate and associated mode mixity.

5. Analytical models for data reduction

After referring to the essential issues when determining the interfacial fracture toughness of an unconventional specimen, we now review two recent analytical models that consider those issues. Both models have been published by two subsets of the present Authors independently.

[1] Similarly to thermal stresses, residual hygroscopic stresses may develop in a structure when exposed to a high-humidity environment. In the simultaneous presence of both thermal and hygroscopic stresses, the term *hygrothermal stresses* is typically used. Nevertheless, the interest of the literature is mainly focused on thermal stresses and less on hygroscopic ones.

5.1. Problem statement

We consider a two-dimensional[2] cantilever beam made of a layered material, as shown in Fig. 3, having a width (not shown in the figure) equal to b and a thickness equal to h. The beam features an interfacial crack of length a at its free end, which ideally splits the beam into two sub-beams: the upper one ($i = 1$) and the lower one ($i = 2$), with thicknesses h_1 and h_2, respectively. We assume that both sub-beams exhibit bending-extension coupling. By way of illustration, we will discuss the simple case that the beam is loaded at its cracked end by two equal and opposite forces, P (DCB configuration). We also assume that the beam has been produced at a high temperature and cooled down to room temperature before applying the mechanical loading. From this temperature difference, ΔT, residual thermal stresses have been generated.

Because of the geometric and material asymmetry of the two sub-beams with respect to the interface plane, the specimen is subjected to mixed-mode I/II fracture. As already stated, only a few analytical models exist for determining the energy release rate and the induced mode mixity of specimens with bending-extension coupling and residual thermal stresses. Two of the most up-to-date models, already introduced in Section 4, are now outlined, which will be named here *clamped crack-tip model* [13] and *semi-rigid interface joint model* [14]. Below, we present the basic assumptions of those models and display the final expressions giving the modal contributions to the energy release rate.

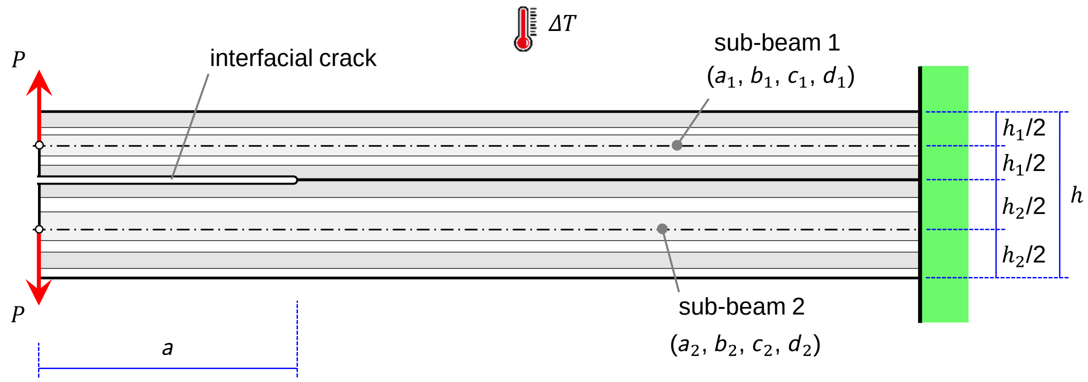

Figure 3. Layered cantilever beam with an asymmetric interfacial crack. Sub-beams 1 and 2 feature bending-extension coupling, while the specimen contains residual thermal stresses

5.2. Clamped crack-tip model

Valvo proposed an analytical method that aimed to "retrieve the spirit of Williams" [13, p. 117] and determine the energy release rate and mode mixity based solely on beam theory. This method may, indeed, be seen as an extension of Williams's method to a generally layered beam with an elastically coupled behavior that features an asymmetrically located crack and is loaded at its crack tip by arbitrary forces and moments. The Author used the first-order shear deformation theory for laminated beams [12, pp. 132–142] and introduced the so-called crack-tip displacement rates as the relative displacements at the crack tip per unit increase of the crack length. Using this definition, the Author provided analytical expressions for the energy release rate and its mode I and mode II partitions via an adaption of the virtual crack closure technique. In particular, mode partitioning was based on two conditions: (a) pure mode I fracture is obtained if the relative sliding displacement at the crack tip is zero; and (b) the crack-tip forces and moment should be partitioned into the sum of

[2] Our problem is two-dimensional, so mode III fracture and other out-of-plane effects (e.g., shear-extension and bending-twisting couplings) are excluded from our analysis.

two energetically orthogonal systems. The second condition ensures that no coupling term, $\mathcal{G}_{\mathrm{I/II}}$, arises.

The clamped crack-tip model does not consider the effect of residual thermal stresses, and we thus ignore this effect for now. According to this model, the mode I and mode II contributions to the energy release rate for the DCB test configuration (see Fig. 3) are given as

$$\mathcal{G}_{\mathrm{I}} = \frac{P^2}{2}\left\{c_1 + c_2 + \left[d_1 + d_2 - \frac{\left(b_1+b_2+d_1\frac{h_1}{2}-d_2\frac{h_2}{2}\right)^2}{a_1+a_2+b_1h_1-b_2h_2+d_1\frac{h_1^2}{4}+d_2\frac{h_2^2}{4}}\right]a^2\right\} \quad \text{and}$$

$$\mathcal{G}_{\mathrm{II}} = \frac{P^2}{2}\frac{\left(b_1+b_2+d_1\frac{h_1}{2}-d_2\frac{h_2}{2}\right)^2}{a_1+a_2+b_1h_1-b_2h_2+d_1\frac{h_1^2}{4}+d_2\frac{h_2^2}{4}}a^2, \tag{1}$$

respectively [13]. In (1), a_{i}, b_{i}, c_{i}, and d_{i} are the extensional compliance, bending-extension coupling compliance, shear compliance, and bending compliance of sub-beam i, respectively, which are conventionally computed using the first-order shear deformation theory for laminated beams [12]. The total energy release rate is provided by the sum of the two modal contributions:

$$\mathcal{G} = \mathcal{G}_{\mathrm{I}} + \mathcal{G}_{\mathrm{II}} = \frac{P^2}{2}[c_1 + c_2 + (d_1 + d_2)a^2]. \tag{2}$$

5.3. Semi-rigid interface joint model

Tsokanas and Loutas [14] proposed an analytical model that simultaneously considers the effects of bending-extension coupling and residual hygrothermal stresses on the energy release rate and mode mixity. This model is an extension of previous models [18, 19] by different groups, as specified in Section 4.2. In this model, the beam results by assembling two layered beams that feature a bending-extension coupled behavior and are modeled as Timoshenko beams. The interface between the two sublaminates follows the semi-rigid (or rotationally flexible) interface joint model [19]: the relative axial and transverse displacements are zero, while the relative rotation is non-zero.

According to the semi-rigid interface joint model, and assuming now that the beam contains residual thermal stresses (as explained in Section 5.1), the expressions for the mode I and mode II contributions to the energy release rate of the DCB configuration (Fig. 3) are

$$\mathcal{G}_{\mathrm{I}} = \frac{1}{2}(c_1 + c_2)\left\{P(1 + \lambda a) + \frac{2\lambda\xi\left[\alpha_{\mathrm{N2}}-\alpha_{\mathrm{N1}}+\frac{\eta}{\xi}(\alpha_{\mathrm{M2}}-\alpha_{\mathrm{M1}})+\frac{h_1+h_2}{2}\alpha_{\mathrm{M2}}\right]}{2(d_1+d_2)\eta+[2b_1+2b_2+(h_1+h_2)d_2]\xi}\right\}^2 \quad \text{and}$$

$$\mathcal{G}_{\mathrm{II}} = \frac{1}{h_1\xi+2\eta}\left(-\xi Pa - \alpha_{\mathrm{N1}} + \alpha_{\mathrm{N2}} + \frac{h_1}{2}\alpha_{\mathrm{M1}} + \frac{h_2}{2}\alpha_{\mathrm{M2}}\right)^2, \tag{3}$$

respectively [14]. In (3), the parameters λ, ξ, and η are defined as follows:

$$\lambda = \left\{\frac{\left[(d_1+d_2)\eta+\left(b_1+b_2+\frac{h_1+h_2}{2}d_2\right)\xi\right]}{(c_1+c_2)\left(\eta+\frac{h_1}{2}\xi\right)}\right\}^{1/2},$$

$$\xi = -b_1 - b_2 + \frac{h_1}{2}d_1 - \frac{h_2}{2}d_2, \text{ and} \tag{4}$$

$$\eta = a_1 + a_2 - \frac{h_1}{2} b_1 + \left(\frac{h_1}{2} + h_2\right) b_2 + \frac{h_2(h_1+h_2)}{4} d_2.$$

We observe that these parameters are functions of the compliances a_i, b_i, c_i, and d_i and the thicknesses h_1 and h_2. The parameters α_Ni and α_Mi in (3) respectively are the axial strain and curvature of sub-beam i due to residual thermal stresses. The analytical expressions giving them are too lengthy to be reported here but can be found in [14]. In the end, α_Ni and α_Mi are functions of the in-plane elastic moduli (i.e., E_{11}, E_{22}, G_{12}, G_{13}, G_{23}, v_{12}, and v_{21}), coefficients of thermal expansion, and thickness of each layer constituting both sub-beams, as well as of the temperature difference, ΔT. Again, the total energy release rate is the sum of the two modal contributions.

5.4. Commenting on the two models

The clamped crack-tip model is the simplest available analytical model for determining the interfacial fracture toughness of layered beams with material and thickness asymmetries and sub-beams featuring bending-extension coupling. The semi-rigid interface joint model is more refined but more complex than the clamped model as it does not constrain the displacement magnitudes at the crack tip and along the uncracked region of the specimen. It is also more refined since it considers the effect of the residual hygrothermal stresses that the clamped model does not consider.

6. Concluding remarks

This paper reviewed some critical issues in determining the interfacial fracture toughness of unconventional layered specimens not covered by current standard testing procedures. It started by providing fundamental concepts on the interfacial fracture of unconventional layered materials and structures. Next, it discussed the mode decoupling and mode partitioning approaches towards determining the pure-mode fracture toughnesses of a specimen experiencing mixed-mode fracture. After that, the issues of bending-extension coupling and residual thermal stresses were discussed by explaining the underlying phenomena and reviewing major analytical models considering those effects.

Next, the paper outlined two up-to-date mechanical models that consider, among others, the effects of bending-extension coupling and residual (hygro)thermal stresses. Both models, proposed by two subsets of the present Authors independently, provide closed-form expressions to determine the fracture toughness of unconventional specimens. In addition, both models can perform mode partitioning, in contrast to most other models in the literature.

In our opinion, an interesting future step is the comparison of the two analytical models reviewed in this paper against the analytical models used in the existing test standards for determining fracture toughness. Such a comparison can concern not only energy release rate computations by the various models but also calculations of the generalized displacements [20]. Even though preliminary work on this topic has already been published in [21], ongoing work is saved for future publications.

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