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A scaling methodology for high-power magnetically shielded Hall thrusters

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Abstract

The past two decades have seen an increasing interest in Hall thrusters in space propulsion, thanks to their favorable performance characteristics with respect to a wide variety of missions of current and future interest and to the significant extension in operational life potential achievable with magnetic shielding. Nevertheless, the physics underlying their behavior is complex and not yet fully understood, limiting the practical applications of models based on first principles due to their inability to self-consistently predict the device performance. Fortunately, modern Hall thrusters were developed through a lengthy process of gradual refinement, and thus they represent convenient reference devices to design new thrusters using appropriately defined scaling criteria. The objective of this work is to propose a new scaling methodology, especially intended for magnetically shielded high-power Hall thrusters. To this purpose, a novel phenomenological model for shielded thrusters is presented and discussed. This model includes free coefficients, whose values are chosen based on the agreement with the empirical data collected in a specially created high-power Hall thruster database. The proposed methodology features a new reference thruster and aims at keeping unchanged its main plasma intensive parameters in a scaling transformation. The possibility of creating performance maps at constant discharge power, which show how the scaling results vary with the channel dimensions, is also proposed as a preliminary design tool.

Keywords: Database, Hall thrusters, High-power, Magnetic shielding, Scaling methodology

Introduction

In recent years Hall thrusters (HTs) have found growing application opportunities in space due to their peculiar advantages, such as their combination of high thrust efficiency and thrust density, and easy scalability over a wide range of power levels. This last property well matches the increasing demand of electric thrusters for both microsatellites and very large spacecraft, on opposite sides of the power gamut. The adoption of low-power HTs for the SpaceX's Starlink constellation and the development of the NASA 12.5 kW Hall Effect Rocket with Magnetic Shielding (HERMeS) [1] may epitomize this general trend.

Currently, the preliminary design of these devices still represents an open challenge on several fronts. In fact, the physical processes involved in the operation of HTs are quite complex and some of them, such as the anomalous transport and high-frequency instabilities, are not totally understood yet [2], making the design of a thruster from scratch, on the basis of first principles, an impractical path to follow. In addition, an experimental optimization process can be very long and expensive, especially for high-power devices. Ultimately, the most practical and logical approach to follow is to scale the desired thruster starting from an already optimized device, using a suitable scaling methodology. To this purpose a set of thruster parameters and their interrelationships, the values of such parameters for a reference thruster and the way in which these parameters vary as a result of the change of scale must be identified. In particular, the scaling relations can be obtained assuming the validity of similarity criteria or through phenomenological or heuristic physical models describing the thruster operation.

Over the years a variety of scaling methodologies have been proposed (see, for instance, [3–7]), each one based on different and sometimes even contradictory approaches. One of the main problems common to most of these methodologies is that they were developed with the purpose of designing only optimal thrusters, ignoring the fact that other constraints make this impossible in certain situations (e.g., in nested Hall thrusters). In addition, the mean diameter and the height of the channel are often not treated separately, thus mixing scale and shape effects [8]. Furthermore, to the authors' best knowledge, none of these methodologies takes into consideration the possibility of scaling magnetically shielded (MS) thrusters. This is not optimal, considering that magnetic shielding has become a fundamental feature for HTs since its discovery [9], being acknowledged as the most effective strategy to reduce wall erosion and therefore to increase the operative lifetime of a thruster. However, a shielded thruster behaves in a different way with respect to an unshielded (US) device. As a matter of fact, a good degree of magnetic shielding involves major changes in the magnetic topology and a downstream shift of the region of the plasma characterized by higher electron temperatures, which usually leads to lower wall and anode losses but higher divergence of the plume [10, 11].

A scaling methodology for the preliminary design of magnetically shielded HTs is presented in this paper. This was created in the effort to overcome the critical issues of past methodologies and to implement the missing features discussed above. It was developed and validated specifically to scale medium–high power HTs working on xenon, but it might be easily adjusted to design devices operating at lower power or with different propellants.

The paper is organized as follows: in [Phenomenological scaling model](#) section the physical model on which the scaling methodology is based is presented and analyzed; in [Scaling methodology](#) section the scaling methodology is described in detail; in [Comparison with experimental data](#) section the scaling methodology is validated using the experimental data collected in a purposely created HTs database; [Discussion](#) section describes how the scaling procedure can be used to design new thrusters, directly or through performance maps, and shows its limits of applicability; [Conclusions](#) section presents a summary of the findings.

Phenomenological scaling model

The definition of a scaling model, i.e., a physical model that describes how the parameters of a thruster vary with respect to a reference device as a result of a change in size, is the most complex and delicate part of the development of a scaling methodology. In fact, the model must be simple enough to be used in the context of a preliminary design, but, at the same time, it must be sufficiently sophisticated to be able to grasp all the main aspects of the physics of a HT, in a range of operative conditions and geometric dimensions as wide as possible. The proposed model aims at being the best trade-off between these two opposite necessities.

In the model, the thruster will be considered as axisymmetric and operating in stationary conditions. All the spatially varying properties will be characterized through local or average values. In addition, their profiles along the radial and axial direction will be assumed to remain unchanged in terms of shape during a scaling transformation and varying proportionally to the chosen reference value.

Geometric model

The thruster is assumed to be geometrically described in terms of its discharge channel, represented in its cross section in Fig. 1. The geometric parameters chosen to characterize the channel are:

- The mean channel diameter d .
- The channel length L .
- The channel width (or height) b .

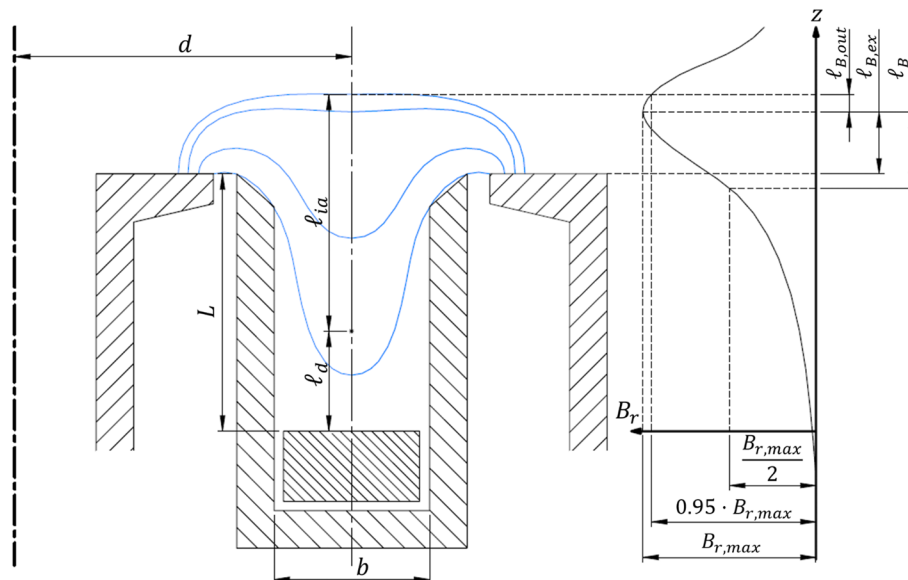


Fig. 1 Geometric and magnetic topology model of a MS thruster. On the right it is shown the profile of the radial component of the magnetic induction field on the centerline of the channel, together with the three characteristic magnetic lengths defined in [Magnetic field model](#) section

For simplicity, the anode is assumed to have the same frontal size as the discharge channel. As for what concerns the channel walls, their thickness has not been taken into consideration in the model, since the erosion of the ceramic should be negligible in a MS thruster [9]. In first approximation, this thickness can be taken equal to that of the reference thruster walls.

Magnetic field model

The magnetic field topology is assumed symmetrical with respect to the median line of the channel. The radial magnetic field profile along this line is assumed to be bell-shaped [2], i.e., monotonically increasing towards the channel exit and with a maximum value $B_{r,max}$ in its proximity, as shown in Fig. 1. The magnetic field profile reaches its peak value downstream the exit of the channel, as it is the case for a MS thruster [12]. The axial component of the magnetic induction field on the center-line of the channel is assumed negligible with respect to the radial one.

To characterize the magnetic field topology in the channel and in the near plume, three characteristic lengths of the radial magnetic field profile, indicated in Fig. 1, are defined:

- *Internal halving length* ℓ_B , the distance along the median line of the channel between the point where the radial magnetic induction field reaches the 50% of its maximum value in the portion of the magnetic field profile characterized by a positive axial gradient moving downstream, and the point where it reaches the maximum value itself.
- *External reduction length* $\ell_{B,out}$, the distance along the median line of the channel between the point where the radial magnetic induction reaches its maximum value and the point where it reaches the 95% of the maximum value, in the portion of the magnetic field profile characterized by a negative axial gradient moving downstream.
- *Exit-peak length* $\ell_{B,ex}$, the distance along the median line of the channel between the exit plane of the channel and the point where the radial magnetic induction reaches its peak value.

Plasma model

In general, the plasma produced in a HT presents a hot region between the magnetic lines near the radial magnetic field peak [2]. In an US thruster this region is located near the exit of the channel, and it extends between the walls. In a MS thruster, due to the difference in the magnetic field topology, the hot region is shifted downstream, and its extremities no longer touch the channel walls [13]. Consequently, the values of electron temperature on the walls tends to stay close to that of the anode, as it is expected based on the operating principle of magnetic shielding [14]. Furthermore, in the MS configuration, the maximum electron temperature is typically slightly higher, and this is generally attributed to the better confinement of the electrons [15]. Simulations conducted by Mikellides et al. [16, 17] for the H6MS thruster suggest that the increase of the maximum electron temperature with voltage (at a fixed discharge power) is not matched by

a proportional increase in the electron temperature near the walls. This idea is corroborated by measurements obtained by Shastry et al. [14] for the HERMeS thruster.

Based on these findings, it has been decided to take into consideration in the model two separate characteristic electron temperatures:

- The maximum electron temperature in the channel T_e .
- The average electron temperature at the walls and at the anode $T_{e,b}$.

These characteristics electron temperatures are clearly influenced by the geometry, by the magnetic topology and by the operative parameters of the thruster. Unfortunately, no meaningful relations in this sense have been found in the literature. However, numerous authors report the existence of a dependence of the maximum electron temperature in the channel on the discharge voltage. For example, Raitses et al. [18] measured a linear increase of the maximum electron temperature with the voltage at fixed power; Azziz [19], using a simplified model to correlate plume divergence measurements and electron temperature, obtained that the maximum electron temperature is roughly proportional to the square root of the discharge voltage; finally, Goebel and Katz [20] suggested, as a rule of thumb, to consider the electron temperature in eV equal to about 10% of the discharge voltage. Therefore, it has been decided to include in the model two parameterized semi-empirical scaling relationships between the characteristic temperatures and the voltage:

$$T_e \propto V_D^{\chi_1} \quad (1)$$

$$T_{e,b} \propto V_D^{\chi_2} \quad (2)$$

where χ_1 , χ_2 are non-dimensional parameters. In [Comparison with experimental data](#) section the effects of the variation of these two parameters on the model will be discussed, also showing the values that best fit the available experimental data.

The plasma density and the electron density distributions in the thruster are assumed to be in first approximation proportional to the average number density of the neutrals at the anode $n_{n,a}$. Applying the conservation of mass to the volume enclosed in the channel, a simple scaling relation can be obtained for $n_{n,a}$:

$$n_{n,a} \propto \frac{\dot{m}_a}{u_{a,z} db} \quad (3)$$

where $u_{a,z}$ is the mean axial velocity of the neutrals evaluated over the channel cross section at the anode exit and $\dot{m}_a$ is the anodic mass flow rate.

Following an approach widely used in the literature [20–24], the plasma region inside the channel and in the near plume can be divided into three further regions, according to the main phenomena taking place in them. These regions are typically referred as:

- *Diffusion region*, located directly in front of the anode sheath and characterized by a low-velocity flow of ions directed towards the anode, a low ion production, a low plasma temperature and an electron diffusion mainly driven by a quasi-linear pressure gradient.

- *Ionization region*, where most of the gas flow is ionized, which is located mostly upstream of the acceleration region. In this region the ion density, the ionization rate, and the electron temperature reach their maximum values [24], the ion velocity changes its sign [22], the electric field is low, and the neutral density drops moving in the downstream direction.
- *Acceleration region*, where the ions acquire most of their kinetic energy, which is located near the peak of radial magnetic field. Most of the potential drop established between the anode and the cathode is concentrated in this region, and consequently the electric field reaches its maximum, whereas the plasma density decreases due to the ion acceleration.

These trends along the channel for the plasma parameters have been confirmed by several experimental observations and 2D plasma simulations for both US and MS HTs [2, 13]. The main difference between the two configurations is that in MS HTs the ionization and acceleration regions are shifted downstream, beyond the exit section of the channel. Their displacement can be attributed to the shift of the radial magnetic induction field profile between the two configurations. In fact, the width of the radial magnetic induction field distribution determines to a great extent the longitudinal size of the acceleration and ionization regions and the position of the magnetic field maximum specifies their position in the channel [25]. In a real thruster, there is not a sharp separation between these regions, which can partially overlap each other. A reasonable simplifying hypothesis is that there is no overlap between the diffusion and the ionization regions. However, the same simplifying hypothesis cannot be applied for the ionization and the acceleration regions without appreciably reducing the accuracy of the model, since the extent of the overlap of these two regions can be much more significant [2]. Since defining a parameter of the model that takes into account the entity of the overlapping would have been an excessive complication, it has been chosen to follow the approach taken by Kim et al. [25–28] and Shashkov et al. [29], which consists in treating the ionization and acceleration regions as a single macro-region, which will be called *ionization-acceleration region* (IAR).

According to Andrenucci et al. [8], starting from a work of Ahedo and Martinez-Cerero [22], the characteristic length of the diffusion region can be expressed as

$$\ell_d \propto \sqrt{\frac{T_{e,b}^{3/2}}{n_{n,a} B_{r,max}}} \quad (4)$$

A relation for the length of the IAR can as well be found starting from the results of Kim et al. [26], based on which we have that

$$\ell_{ia} \propto r_{L,e} \sqrt{\frac{\nu_e}{\nu_i}} \quad (5)$$

where $r_{L,e}$ is the electron Larmor radius, ν_e is the effective electron collision frequency and ν_i is the ionization frequency. The ionization frequency can be approximated as the product of the ionization rate factor averaged over the electron velocity distribution function $\langle \sigma_i u_e \rangle$ and the average number density of neutrals in the region n_n [30], or

$$v_i = \langle \sigma_i u_e \rangle n_n \quad (6)$$

Remembering that in the model the neutral density in each point of the channel is assumed to vary proportionally to the average neutral density at the anode, and using the relation between the ionization rate factor and the maximum electron temperature in the channel found by Andrenucci et al. ($\langle \sigma_i u_e \rangle \propto T_e^{3/2}$) [30], Eq. (6) can be written in form of a scaling relation as

$$v_i \propto n_{n,a} T_e^{3/2} \quad (7)$$

Then, the electron Larmor radius $r_{L,e}$ can be expressed as

$$r_{L,e} = \frac{v_{\perp e}}{\omega_e} \cong \sqrt{\frac{8k_B T_e}{\pi M_e}} \frac{M_e}{eB_{r,max}} \propto \frac{V_D^{\chi_1/2}}{B_{r,max}} \quad (8)$$

where $v_{\perp e}$ is the electron speed perpendicular to the magnetic field, M_e is the electron mass, e is the elementary charge, k_B is the Boltzmann constant, and ω_e is the electron cyclotron frequency. As regards the effective electron-neutrals collision frequency, it can be written as the sum of multiple contributions, each one related to a different phenomenon. The main contributions are those related to the electron-neutral collision v_{en} , to the electron-ions collision v_{ei} and to the anomalous transport v_{an} , thus

$$v_e = v_{en} + v_{ei} + v_{an} \quad (9)$$

However, as Ahedo and Martinez-Cerezo [22] suggest, for typical thruster conditions the term related to Coulomb collisions can be neglected ($v_{ei} \cong 0$). Moreover, it is pointed out that the anomalous transport is typically the dominant electron transport mechanism in the IAR. Hence, using a Bohm diffusion model for v_{an} [31], it can be assumed as a first approximation that

$$v_e \cong v_{an} = \frac{1}{16} \frac{eB_{r,max}}{M_e} \propto B_{r,max} \quad (10)$$

Finally, by inserting Eqs. (7), (8) and (10) in Eq. (5), the length of the IAR can be expressed as

$$\ell_{ia} \propto \ell_B^{\chi_3} \sqrt{\frac{V_D^{\chi_1}}{T_e^{3/2} n_{n,a} B_{r,max}}} \quad (11)$$

Here, the additional term ℓ_B has been added in form of a power law to also consider the dependence of the IAR length on the axial gradient of the radial magnetic field as motivated by the fact that there are strong indications in this sense [28]. In a similar way to what has been done with Eqs. (1) and (2), this additional term is raised to a non-dimensional parameter χ_3 , subsequently evaluated through empirical data.

At this point, it is convenient to add two further simplifying assumptions to the model:

- Upstream boundary of the diffusion region coinciding with the surface of the anode (i.e., the thickness of the plasma sheath at the anode is assumed to be negligible).
- Downstream boundary of the IAR coinciding with the point where the radial magnetic field on the median line of the channel is decreasing and is equal to the 95% of its maximum value [27] (which is the right boundary of the region identified by the previously defined external reduction length $\ell_{B,out}$).

In this way, with reference to Fig. 1, the channel length can be related to the characteristic lengths of the magnetic field and of the plasma regions by the relation:

$$L + \ell_{B,ex} + \ell_{B,out} = \ell_{ia} + \ell_d \quad (12)$$

Performance model

The fundamental parameters that characterize the anodic performance of a HT are the thrust T , the anodic specific impulse $I_{sp,a}$, the anodic effective exhaust velocity $v_{e,a}$, and the anodic thrust efficiency $\eta_{T,a}$. These parameters are linked to each other and to the other parameters of the model by their definition

$$v_{e,a} = g_0 I_{sp,a} = \frac{T}{\dot{m}_a} \quad (13)$$

$$\eta_{T,a} = \frac{\dot{m}_a v_{e,a}^2}{2P_D} = \frac{g_0^2 \dot{m}_a I_{sp}^2}{2P_D} = \frac{T^2}{2P_D \dot{m}_a} \quad (14)$$

where P_D is the discharge power and g_0 is the standard acceleration due to gravity.

Using the method of the decomposition of the anodic thrust efficiency, the latter can be factorized into a series of specific efficiencies, each of which considers a particular loss mechanism. In this work the five-component method developed by Hofer et al. [32] is adopted. The effects of multiple ionization are also included in the model. According to this approach, the anodic thrust efficiency can be broken down into the following efficiencies:

- The *voltage utilization efficiency* η_v , which quantifies how much of the voltage provided by the discharge supply is used to accelerate the ions. It is given by

$$\eta_v = \frac{V_a}{V_D} = 1 - \frac{V_l}{V_D} \quad (15)$$

where V_a is the *effective acceleration voltage*, i.e., the potential difference that the ions should cross to reach the average exhaust speed outside the channel, and V_l is the *loss voltage*, a term which considers all the effects that lower the ions accelerating potential. With this definition, and assuming for simplicity a linear profile of the plasma potential in the IAR, it is also possible to write the peak of the axial component of the electric field in first approximation as

$$E_{z,max} = \frac{V_a}{\ell_{ia}} \quad (16)$$

- The *current utilization efficiency* η_j , which quantifies how much of the discharge current is carried by the ion beam. It is given by where I_j is the *beam current* (also called *jet current* or *ion current*), which is the current associated to the flux of ions ejected from the thruster, and I_e is the *electron current*, which is the current associated to the flux of electrons that flow to the anode.

$$\eta_j = \frac{I_j}{I_D} = 1 - \frac{I_e}{I_D} \quad (17)$$

- The *charge utilization efficiency*, which takes into account the efficiency loss due to the presence of multiple ionization. It is given by where N is the number of ion species, and Z_i and φ_i are respectively the charge state and the current fraction ($\varphi_i = I_{j,i}/I_j$) of the i -th ion species.

$$\eta_q = \frac{\left(\sum_{i=1}^N \frac{\varphi_i}{\sqrt{Z_i}} \right)^2}{\sum_{i=1}^N \frac{\varphi_i}{Z_i}} \quad (18)$$

- The *mass utilization efficiency* η_m , which quantifies how much of the mass flow exiting the thruster channel is in the form of ions. It is given by where M_{ion} is the mass of the ion, and $\dot{m}_j$ is the *beam mass flow rate* (also called *jet mass flow rate* or *ion mass flow rate*), i.e., the mass flow rate corresponding to the flux of accelerated ions.

$$\eta_m = \frac{\dot{m}_j}{\dot{m}_a} = \eta_j \frac{M_{ion} I_D}{\dot{m}_a e} \sum_{i=1}^N \frac{\varphi_i}{Z_i} \quad (19)$$

- The *plume divergence utilization efficiency* (or simply *divergence efficiency*) η_d , which quantifies how much of the kinetic energy imparted to the ions is used to produce the thrust. Given the plume divergence half-angle θ (defined as in [33]), it is expressed by

$$\eta_d = (\cos\theta)^2 \quad (20)$$

Therefore, the anodic thrust efficiency can be simply expressed as the product of these five efficiencies

$$\eta_{T,a} = \eta_v \eta_j \eta_q \eta_m \eta_d \quad (21)$$

Following this factorization, the thrust and the anodic specific impulse can be written as

$$T = \eta_j I_D \sqrt{\eta_v \eta_d \frac{2M_{ion} V_D}{e}} \sum_{i=1}^N \frac{\varphi_i}{\sqrt{Z_i}} = \eta_m \dot{m}_a \sqrt{\eta_v \eta_d \frac{2e V_D}{e M_{ion}}} \frac{\sum_{i=1}^N \frac{\varphi_i}{\sqrt{Z_i}}}{\sum_{i=1}^N \frac{\varphi_i}{Z_i}} \quad (22)$$

$$I_{sp,a} = \frac{\eta_m}{g_0} \sqrt{\frac{2eV_D}{\eta_v \eta_d} \frac{\sum_{i=1}^N \frac{\varphi_i}{Z_i}}{M_{ion} \sum_{i=1}^N \frac{\varphi_i}{Z_i}}} \quad (23)$$

Even the discharge power of the thruster can be rewritten in terms of the above defined efficiencies [20]:

$$P_D = V_D I_D = (V_a + V_l)(I_j + I_e) = \frac{V_a I_j}{\eta_v \eta_j} = \frac{P_K}{\eta_v \eta_j} \quad (24)$$

where P_K is the *kinetic power* i.e., the power used to accelerate the propellant. Hence, it is possible to think the discharge power as the sum of the kinetic power and the *dissipated power* P_L a term that considers the losses

$$P_D = P_K + P_L = V_a I_j + P_L \quad (25)$$

It is noteworthy that the kinetic power accounts for power used for the propellant acceleration in the thrust direction (the *jet power*, also called *directed kinetic power* or *beam power*), but also for the small fraction that is wasted in radial acceleration.

The dissipated power can in turn be broken down into several terms, each one related to a different dissipative phenomenon

$$P_L = P_D(1 - \eta_v \eta_j) = P_w + P_a + P_i \quad (26)$$

where P_w , P_a are respectively the power lost to the walls and to the anode due to the collection of ions and electrons, and P_i is the power spent to ionize the propellant. The power radiated away from the plasma has been neglected since it typically represents a very small part of the discharge power [34]. Similarly, the power deposited by counter-streaming ions on MS thruster pole pieces, and in particular on the inner one, can be considered negligible, even if this phenomenon can be relevant in terms of erosion [35].

Non-dimensionalizing the power losses with the discharge power, we obtain the *power loss fractions*:

$$\varepsilon_L = \frac{P_L}{P_D}; \quad \varepsilon_w = \frac{P_w}{P_D}; \quad \varepsilon_a = \frac{P_a}{P_D}; \quad \varepsilon_i = \frac{P_i}{P_D} \quad (27)$$

from which, using Eq. (26), it follows that

$$\varepsilon_L = \varepsilon_w + \varepsilon_a + \varepsilon_i = 1 - \eta_v \eta_j \quad (28)$$

According to Andrenucci et al. [30], for typical HT operations, the wall loss fraction is mainly due to the electron collisions with the channel and can be expressed as

$$\varepsilon_w = \frac{P_w}{P_D} \propto \frac{n_{n,a} T_{e,b}^{3/2} dL}{P_D} \propto \frac{\eta_j}{\eta_m} \frac{n_{n,a} T_{e,b}^{3/2} dL}{V_D \dot{m}_a} \sum_{i=1}^N \frac{\varphi_i}{Z_i} \quad (29)$$

In an analogous way, assuming a negative anode fall and neglecting the ionic current and the secondary emission, using the estimation for the power deposited to the anode made by Goebel and Katz [20], the corresponding loss fraction can be expressed as

$$\varepsilon_a = \frac{P_a}{P_D} = \frac{2T_{e,b}(eV)I_D}{P_D} \propto \frac{T_{e,b}}{V_D} \quad (30)$$

In addition, for what concerns the power spent for ionization, Battista et al. [36] proposed the following relation

$$\varepsilon_i = \frac{P_i}{P_D} = \frac{E_{ion}\dot{m}_a}{M_{ion}P_D} \propto \frac{\dot{m}_a}{P_D} \quad (31)$$

where E_{ion} is the energy required for ionization, which can be expressed as the product of the first ionization energy $E_{ion,1}$ of the propellant by a multiplicative coefficient c_{ion} , which takes in consideration multiple ionization and, in first approximation, can be assumed constant ($E_{ion} = c_{ion}E_{ion,1}$).

Although the above relations for the three main loss mechanisms are valid for both US and MS thrusters, the power loss fractions obtained with a scaling procedure can differ between the two configurations since the values of said parameters for the reference thrusters may be significantly different. In particular, MS thrusters usually require more power for ionization due to a greater presence of multiple ionizations [16, 37]. This effect, which translates into a greater value of c_{ion} , may be caused by the higher electron temperature in the bulk of the plasma. On the other hand, MS considerably reduces the power deposition on the walls in the acceleration zone of the discharge chamber [38]. As a result, for high-power devices, the term P_L is generally slightly lower for MS thrusters with respect to US thrusters. However, MS thrusters are usually characterized by a higher divergence of the beam [16, 37] and therefore a smaller fraction of the kinetic power is used efficiently. Consequently, from the perspective of overall thrust efficiency, all these opposite effects tend to counterbalance each other, with high-power MS thrusters often having comparable or slightly worse performance with respect to US thrusters.

Scaling methodology

The scaling methodology here proposed is based on the scaling theory developed in Pisa, which has been progressively refined over the past two decades [8, 30, 36, 39–42]. In this section, after a brief introduction to this theory, we construct the characteristic matrices of the methodology. Then, we address the problem of the reference thruster selection, motivating the choice made for this role. Finally, we propose a possible scaling criterion for the methodology.

Scaling theory

Consider a reference thruster which works at steady state in a certain operating point. The device can be characterized in terms of a series of parameters, which can represent its size, operating conditions, or performances. We define as a *scaling transformation* any variation of one or more of these parameters according to a set of scaling laws, which can also include the invariance of some of the involved variables.

The scaling theory developed in Pisa aims at giving a simple but effective mathematical formalism to the scaling problem, also allowing an easy code implementation of the scaling procedure. The basic details of this theory can be found in [Appendix](#) and in the

references provided at the beginning of this section. However, a summary of the procedure necessary to define a scale methodology and to perform a scale transformation is here provided in its main steps:

- A scaling model is defined through a set of n relevant *fundamental parameters*, forming the vector $\mathbf{y}$, and $m < n$ *scaling laws*, or simple power-law relations between these parameters. An additional number of *derived parameters*, forming the vector $\mathbf{y}_d$, and an equal number of equations are added to the model to describe those phenomena that cannot be expressed by scaling laws.
- $n - m$ fundamental parameters are chosen as independent variables of the model.
- The scaling laws are firstly rewritten as equations in terms of the scaling factors of each parameter φ_i , which are defined as the ratio of the value of the parameter y_i to the value of a the corresponding parameter for a reference thruster y_{i0} . Then, the equations are rewritten in terms of the natural logarithm of the scaling factors $\psi_i = \ln \varphi_i$. The coefficients of the homogeneous linear system of the scaling relations written in this way ($\mathbf{K} \cdot \boldsymbol{\psi} = 0$) are collected in the so-called *scaling matrix* $\mathbf{K} \in \mathbb{R}^{m \times n}$.
- A set of *scaling modes* are defined in terms of their *scaling exponent vector* $\boldsymbol{\alpha}_j$. The latter are vectors that can be used to specify any way to change the value of the system parameters in a scaling transformation. Each scaling exponent vector can be constructed solving the linear system $\mathbf{K} \cdot \boldsymbol{\alpha}_j = 0$, after assigning the value of some of his components in a number equal to the degrees of freedom of the model ($n - m$). The scaling exponents vectors are then collected in the so-called *scaling exponent matrix* $\mathbf{A}$ to form its columns.
- $s \leq n - m$ parameters are selected as *target parameters* $\bar{y}_h$, i.e., parameters which value is imposed in a scaling transformation, and s independent scaling modes are chosen. Among the target parameters, some can be chosen to vary arbitrarily, whereas the others can be maintained equal to a certain value for all the scaling transformations with the methodology. We will call *scaling set* the selected set of target parameters and scaling modes.

Ultimately, using this approach, a scaling methodology is univocally identified by the set of parameters chosen for the scaling model, by its characteristic matrices (scaling matrix and scaling exponent matrix), by the values of the reference thruster parameters and by the chosen scaling set. Once a scaling methodology has been defined, a scaling transformation can be performed by choosing a certain value for the selected target parameters. Then, through a simple procedure that involves the solution of linear systems, it is possible to find all the unknown parameters of the model. Finally, the derived parameters are obtained using the additional equations.

Characteristic matrices of the methodology

The scaling model described in [Phenomenological scaling model](#) section presents 15 degrees of freedom. Hence, additional simplifications to the model are necessary to avoid making the application of the scaling procedure excessively complex. Consequently, it has been assumed that the following parameters remain constant in a scaling

Table 1 Scaling exponent matrix of the scaling methodology

System Parameter	Scaling Modes							
	R	L	L _B	V	M	B	H _m	H _j
d	1	0	0	0	0	0	0	0
b	0	1	0	0	0	0	0	0
ℓ_B	0	0	1	0	0	0	0	0
V_D	0	0	0	1	0	0	0	0
$\dot{m}_a$	0	0	0	0	1	0	0	0
$B_{r,max}$	0	0	0	0	0	1	0	0
η_j	0	0	0	0	0	0	1	0
η_m	0	0	0	0	0	0	0	1
$n_{n,a}$	-1	-1	0	0	1	0	0	0
$I_j, \dot{m}_j$	0	0	0	0	1	0	1	0
I_D	0	0	0	0	1	0	1	-1
P_D	0	0	0	1	1	0	1	-1
ε_a	0	0	0	$\chi_2 - 1$	0	0	0	0
ε_i	0	0	0	-1	0	0	-1	1
ℓ_{ia}	$\frac{1}{2}$	$\frac{1}{2}$	χ_3	$\frac{1}{2} - \frac{3}{4}\chi_1$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
ℓ_d	$\frac{1}{2}$	$\frac{1}{2}$	0	$\frac{3}{4}\chi_2$	$-\frac{1}{2}$	$-\frac{1}{2}$	0	0
T_e	0	0	0	χ_1	0	0	0	0
$T_{e,b}$	0	0	0	χ_2	0	0	0	0
$E_{z,max}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\chi_3$	$\frac{1}{2} + \frac{3}{4}\chi_1$	$\frac{1}{2}$	$\frac{1}{2}$	0	0
$r_{L,e}$	0	0	0	$-\frac{\chi_1}{2}$	0	-1	0	0

transformation: the exit-peak length $\ell_{B,ex}$, the external reduction length $\ell_{B,out}$, the average axial velocity of the neutrals at the anode $u_{a,z}$, the plume divergence utilization efficiency η_d and the current fractions φ_i . The impact of these assumptions on the design freedom and the prediction accuracy of the methodology will be discussed in more detail in [Discussion](#) section. Ultimately, following these simplifications, the number of degrees of freedom of the model is reduced to eight. Therefore, an equal number of parameters must be chosen as independent. The ones selected are:

- The mean channel diameter d .
- The channel width b .
- The magnetic internal halving length ℓ_B .
- The maximum of the radial component of the magnetic field on the centerline of the channel $B_{r,max}$.
- The discharge voltage V_D .
- The anodic mass flow rate $\dot{m}_a$.
- The current utilization efficiency η_j .
- The mass utilization efficiency η_m .

Once the independent parameter set is chosen, the scaling matrix of the simplified model can be constructed using the procedure described previously. Then, also the various scaling mode vectors can be built starting from the scaling matrix. The chosen matrix contains only scaling exponent vectors corresponding to *fundamental*

scaling modes, i.e., the scaling modes that affect the independent parameters only one by one, keeping the others unchanged. This approach allows a greater control on the assignment of the target parameters, making the scaling procedure easier. The adopted scaling exponent matrix is shown in Table 1. Specifically, each mode of the selected matrix, if applied individually, has the following effect: mode R varies only d , mode L_B varies only b , mode LB varies only ℓ_B , mode B varies only $B_{r,max}$, mode V varies only V_D , mode M varies only $\dot{m}_a$, mode H_j varies only η_j and mode H_m varies only η_m . The scaling matrix of the model is not shown because, as demonstrated in Appendix, it corresponds to the transpose of the lower part (under the marked line in Table 1) of the scaling exponent matrix.

Reference thruster

The choice of the reference thruster is typically a long and laborious process since it can significantly affect the scaling methodology validity range. In addition, there are only few thrusters that are suitable for this role. In fact, a reference thruster must be characterized by a series of quite stringent requirements. Firstly, the nominal values of all the fundamental parameters of the scaling model for the selected thruster must be known. Nevertheless, relevant information about thrusters is often kept confidential or not measured. Moreover, to increase the accuracy of the methodology, it would be preferable to select a thruster with power level and performances similar to those of the thruster to be scaled. However, this requirement is particularly problematic in the case of scaling towards very high powers, given the low number of HTs above 20 kW. Finally, thrusters that have been amply tested or even flight models are preferable.

For all these reasons, the Fakel's SPT-100 [43] is usually chosen as reference thruster in scaling methodologies. In fact, this thruster is by far the most studied and tested HT, with tens of related publications and studies. It has also been used onboard of tens of satellites and has the advantage of working nominally at 1350 W, which can be nowadays considered as an intermediate level of power. Therefore, it is a particularly suitable candidate for the purpose of being taken as a reference for scaling toward both low and high-power levels. However, the SPT-100 is not magnetically shielded, and, in recent years, it has been overtaken in performance by many thrusters. Therefore, it is not an optimal choice for the methodology that we aim to define.

Thus, the choice of the reference thruster has fallen on the H6MS, which is the magnetically shielded version of the H6, a 6 kW HT that is the result of a joint venture of Plasmadynamics & Electric Propulsion Laboratory of the University of Michigan (PEPL), Air Force Research Laboratory (AFRL) at Edwards Air Force Base (AFB) and NASA Jet Propulsion Laboratory (JPL). Although the H6, both in its US and MS configurations, has never been used onboard of a spacecraft, it has been extensively tested, and the results are widely described in the literature. The H6 has proved to perform well in a broad range of operative conditions, reaching total efficiencies of 64% at 300 V and even 70% at 800 V, with total specific impulses varying between slightly less than 2000s at low voltages, to 3170 s at high voltages [37].

It has been possible to obtain directly or extrapolate the values for all the required parameters of the H6MS operating in its nominal condition from the literature. Its nominal operative and performance parameters (discharge power, voltage, current, anodic

mass flow rate, efficiencies, and current fractions) can be found in [37]. The geometric dimensions of the acceleration channel and the nominal radial magnetic field profile and peak value are reported in [16, 44, 45]. The total power loss P_L was obtained using Eq. (25), from the values of the nominal current and voltage utilization efficiencies. A conservative estimation of the power lost on the boundaries of the channel has been also found in [45]. It is worth noting that the value of the anode loss P_a obtained is very close to the one that could be obtained using Eq. (29), assuming an electron temperature at the anode equal to 2.5 eV (a value in line with simulations for the thruster [16]). The power lost to ionize the plasma P_i was estimated by difference from the calculated value of P_L , using Eq. (25). The value is consistent with the one that can be calculated using the relation provided by Goebel and Katz in [20] for P_i . The estimation of the diffusion and ionization-acceleration lengths required a way to define the separation between their respective regions that could be easily obtained from available data. Having assumed no overlap between the two regions, this problem is equivalent to that of the definition of the upstream boundary of the ionization region. Ideally, this boundary could be identified by the point in which a given arbitrary percentage of the maximum ionization rate is achieved. Nevertheless, the ionization rate profile of the H6MS has not been found in the literature by the authors. Consequently, it has been decided to assume the plasma density and electron temperature profiles along the channel as representative of the evolution of the ionization in the channel. In particular, the plasma region lengths have been extrapolated from the profiles reported in [15], arbitrarily assuming that the limit between the two regions is located in the point in which the plasma density is the 80% of its maximum value moving towards the channel exit (which approximately coincides with the point in which the electron temperature starts to steeply increase). For some parameters, as the neutral number density near the anode or the maximum electron temperature in the channel, a nominal value could not be found. Despite this, thanks to the way the scaling procedure is conceived, their relative variation with respect to the unknown reference value can still be obtained. In particular, in the procedure a unitary value has been assigned to these unknown reference parameters.

Scaling criterion

A possible criterion that can be used in scaling is to keep the main intensive parameter of the reference thruster unchanged. The goal of this approach is to maintain unaltered the basic phenomena behind the functioning of the thruster. In this way, we aim to preserve the favorable conditions that led to the excellent performance of the reference thruster while avoiding operating in conditions in which the thruster could not work properly (for example, because of breathing mode or other forms of plasma instabilities).

Ultimately, having decided to follow this criterion, the methodology is used with the modes R , L , L_B , V , M , B and d , b , P_D , $n_{n,a}$, $E_{z,max}$, $B_{r,max}$ as target parameters, maintaining the values of the main plasma intensive parameters ($n_{n,a}$, $E_{z,max}$, $B_{r,max}$) equal to that of the reference thruster in a scaling transformation.

Comparison with experimental data

The scaling methodology introduced in the last section can easily be modified to be used with other scaling criteria, i.e., with other scaling modes and target parameters. For example, using the same reference thruster and scaling model, it is possible to match the value of as many target parameters as possible to those of a known thruster; then, we can apply the scaling procedure and compare the predicted and real performance of the thruster. Such a variant of the methodology can be used to assess the prediction goodness of the selected scaling model. A good model should provide a match between the predicted data and the experimental ones over a wide range of thruster dimensions and operating conditions around those of the chosen reference thruster.

In this section, we first provide an estimation for the free coefficients of the model χ defined in [Phenomenological scaling model](#) section using this method. The experimental data used in this process had been collected in a specially developed high power thruster database, which will be presented in detail. Subsequently, the proposed scaling methodology is applied, with the obtained χ values, to scale thrusters with the same channel dimensions and power of those of each operative point of the database, and then the predicted and experimental performances are once again compared. Finally, the results of comparisons are discussed.

High-power Hall thruster database

The database contains the general data of 39 high-power HTs and the test campaigns they have undergone, with experimental data in more than 2100 operative points. This database represents an important advancement compared to that previously created at the University of Pisa [41], from which it stands out both for the number of thrusters included and for the amount of data collected, although it relates only to a specific HTs subset. In fact, only SPT-type thrusters with nominal discharge power greater than 1 kW and operating with xenon have been included. Less conventional configurations were disregarded, except for nested HTs operating in single-channel mode and double-stage HTs operating in single-stage mode. Table 2 shows a list of the thrusters contained in the database. Most of the high-power MS HTs whose tests information has been published to date have been included, each one with experimental data in a considerable number of operative points.

Figure 2 shows a simplified entity relationship diagram for the database, summarizing all the data collected. The data collection has been conducted by analyzing a large number of publications, conference proceedings, patents, Ph.D. thesis and commercial brochures. During this process, all the data have been critically reviewed to eliminate those considered not reliable or subject to manifest errors.

The knowledge of at least the mean channel diameter and width, the discharge power, the discharge voltage, the discharge current, the thrust, the anodic specific impulse, the anodic mass flow rate, and the anodic efficiency were considered as a necessary condition to include a particular operative point into the database. Missing anodic performance data, whenever possible, were obtained from available performance data by Eqs. (13) and (14). For some thrusters, the width and/or the average diameter of the discharge channel were extrapolated from images, based on the knowledge of at least one reference dimension. Particular care has been taken in choosing high resolution images,

Table 2 Thrusters included in the database and related main information

#	Thruster Name	Version(s)	Developer(s)	Magnetic Shielding	Tested Power Range [kW]	Ref
1	SPT-1	-	EDB Fakel	No	0.62 to 3.1	[46]
2	SPT-100	-	EDB Fakel	No	0.60 to 1.5	[43]
3	SPT-115	-	EDB Fakel	No	0.45 to 3.7	[47]
4	SPT-140	-	EDB Fakel	No	0.90 to 5.0	[47–49]
5	SPT-200	-	EDB Fakel	No	2.0 to 13	[50]
6	SPT-290	-	EDB Fakel	No	1.9 to 16	[51]
7	T-140	-	Space Power Inc	No	1.3 to 4.0	[52]
8	T-160	-	NIITP	No	1.9 to 5.2	[53]
9	T-220	-	Space Power Inc	No	4.5 to 10	[54]
10	P5	1	AFRL, PEPL	No	1.4 to 9.2	[55, 56]
		2	NASA GRC, PEPL	No	1.4 to 9.2	[57]
11	NASA-77 M	-	NASA GRC, Aerojet-Rocketdyne	No	0.20 to 2.8	[58]
12	NASA-103 M	XL	NASA GRC, Aerojet-Rocketdyne	No	0.32 to 3.5	[59]
13	HIVHAc	EDU-1, EDU-2	NASA GRC, Aerojet-Rocketdyne	No	0.29 to 4.0	[60–62]
14	NASA-120 M	v2	NASA GRC	No	0.40 to 1.5	[63]
15	NASA-173 M	v1, v2	NASA GRC, University of Michigan	No	1.2 to 9.7	[64]
16	NASA-300 M	-	NASA GRC	No	4.9 to 20	[65]
		MS		Yes	1.5 to 20	[66]
17	NASA-400 M	-	NASA GRC	No	4.0 to 47	[67]
18	NASA-457 M	v1, v2	NASA GRC	No	5.0 to 72	[68, 69]
19	NASA X2	Inner channel	AFRL, PEPL	No	0.47 to 6.0	[70]
		Outer channel		No	1.0 to 9.9	
20	NASA X3	Inner channel	AFRL, PEPL, NASA GRC, JPL, Electro-Dynamic Applications, Inc	No	3.9 to 13	[71–73]
		Middle channel		No	9.5 to 31	
		Outer channel		No	16 to 55	
21	NASA N30	Inner channel	NASA GRC, PEPL, US AFRL	Yes	4.5 to 9.0	[74]
22	H6	US	JPL, University of Michigan, US AFRL	No	1.0 to 10	[75, 76]
		MS		Yes	6.0 to 9.0	[13, 16, 77]
23	H9	-	JPL, University of Michigan, AFRL	Yes	4.5 to 12	[78, 79]
		C			4.5 to 12	[80]
24	HERMeS	TDU-1	NASA GRC, JPL	Yes	0.58 to 12.5	[81, 82]
25	XR-5	BPT-4000	Aerojet Rocketdyne, Busek	No	0.60 to 4.5	[83]
26	XR-12	-	Aerojet Rocketdyne, Busek	Yes	4.5 to 12	[84, 85]
27	BHT-1000	Single stage	Busek	No	0.19 to 2.4	[86]
28	BHT-1500	-	Busek	No	0.59 to 3.0	[87]
29	BHT-5000	EM1	Busek, Maxar	No	3.0 to 4.5	[88]
30	BHT-6000	EQM003	Busek, Maxar	No	3.0 to 6.0	[89]
31	BHT-8000	-	Busek	No	2.0 to 10	[90, 91]
32	BHT-20 K	-	Busek	No	5.0 to 20	[92]
33	HT-5 k	TDU-1	Sitael	No	1.6 to 4.6	[93, 94]
		TDU-3 (LL)		Yes	3.0 to 5.7	[95]
34	HT-20 k	DM2 M	Sitael	Yes	6.1 to 20	[96]
35	PPS-1350	G EM	Snecma Moteurs	No	0.62 to 3.4	[97, 98]
		MLM	Snecma Moteurs, Université d'Orléans	No	0.48 to 4.5	[99]
36	THT VI	-	Osaka Institute of Technology	No	0.80 to 4.5	[100, 101]
37	LHT-140	-	Lanzhou Institute of Physics	No	0.62 to 3.0	[102]
38	HEP-1350PM	-	Harbin Institute of Technology	No	0.87 to 2.2	[103]
39	HEP-140MF	-	Beijing Institute of Control Engineering	No	3.0 to 6.0	[104]

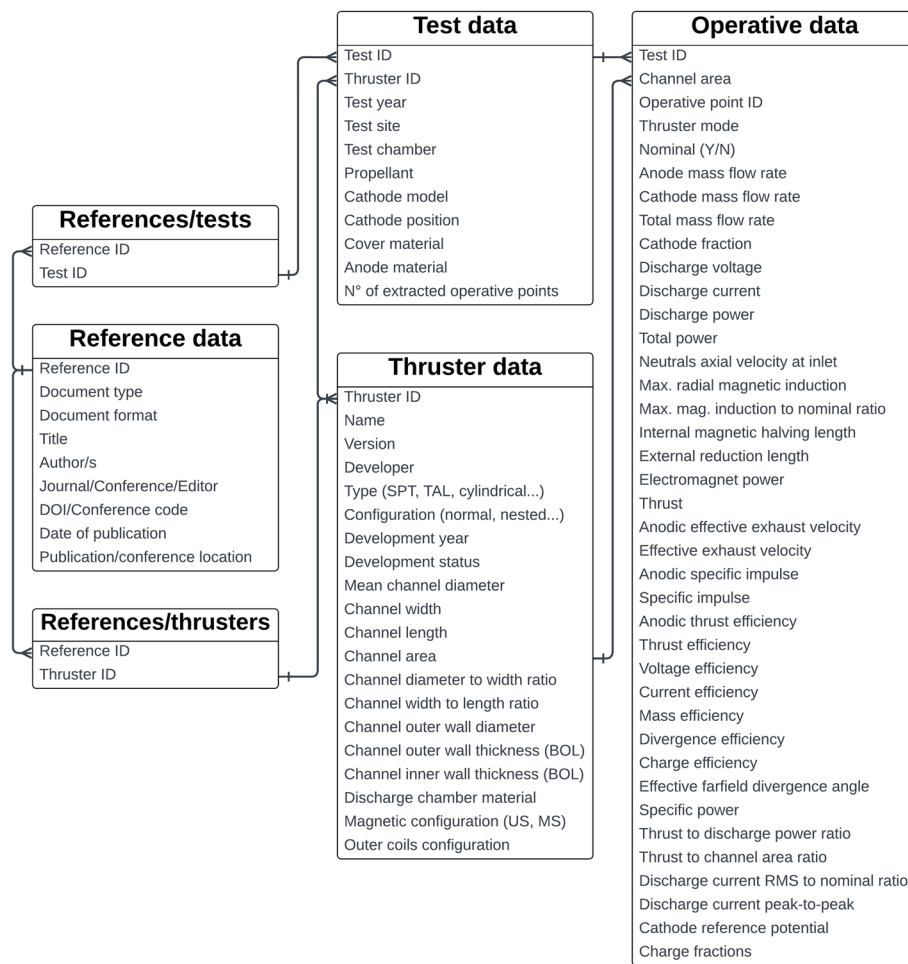


Fig. 2 Entity-relationship diagram for the high-power HT database

and in using reference dimensions only when an accuracy of at least 1 mm was clearly implied or reported in the source, so the authors could consider the measurement error to be negligible in the context of this work.

Procedure of comparison with the experimental data

The data contained in the database are compared with the performance data predicted by the scaling methodology in terms of thrust, anodic specific impulse, and anodic thrust efficiency. In fact, for each of the operative points of the database, a thruster can be scaled from the H6MS maintaining some of the original parameters of the corresponding database thruster and changing others. Specifically, two scaling datasets, containing the values of all the parameters of the model for the scaled thrusters, have been created:

- Dataset-1 is generated by scaling a thruster for each operative point of the database while maintaining the same channel geometry and overall operating conditions (P_D , I_D , $\dot{m}_a$, ℓ_B , $B_{r,max}$) of the corresponding database thruster. In this way, the main intensive parameters ($n_{n,a}$, $E_{z,max}$, $B_{r,max}$) of the scaled thrusters are generally dif-

Table 3 Target parameters for the two scaling datasets. The target parameters whose value is kept equal to that of the reference thruster present the subscript “0”

Target Parameters								
Dataset-1	d	b	P_D	$\dot{m}_a$	ℓ_B	$B_{r,max}$	I_D	$\eta_{m,0}$
Dataset-2 (Methodology)	d	b	P_D	$n_{n,a,0}$	$E_{r,max,0}$	$B_{r,max,0}$	$\eta_{j,0}$	$\eta_{m,0}$

ferent from the ones of the adopted reference thruster (the H6MS). This dataset has been also used to find the values of the χ s that best fit the experimental data.

- Dataset-2 is generated by scaling a thruster for each operative point of the database, maintaining the same channel geometry and discharge power of the corresponding database thruster, while keeping the main plasma intensive parameters ($n_{n,a}$, $E_{z,max}$, $B_{r,max}$) equal to those of the reference thruster (the H6MS). Thus, this dataset represents the results of the application of the scaling methodology proposed in this work.

Table 3 summarizes the target parameters of the two scaling datasets introduced, specifying which values are maintained equal to those of the reference thruster. In both cases the scaling modes adopted are the fundamental ones (R, L, L_B , V, M, B, H_m , H_j).

The data comparisons have been conducted using 2D or 3D plots of the scaling model parameters. These plots can be quickly generated using a specially developed MATLAB® App, which incorporates in its code the scaling procedure.

Dataset-1 and determination of the χ values

As stated before, the comparison between Dataset-1 and experimental data has been used to find the unknowns values of the χ s. As the scaling model was developed for MS thrusters, only the subset of Dataset-1 created starting from the MS thrusters of the database was considered in this procedure. The US thruster data are only used subsequently, in the graphical comparison of scaled and experimental data, to analyze the performance predicting capability of the model also for this configuration.

Fitting goodness between the Dataset-1 and the MS thruster experimental data has been evaluated both qualitatively, through visual inspection of the plots, and quantitatively, by means of a parameter f . The latter is defined as the square root of the sum of the squares of the normalized differences between the experimental and predicted values of anodic thrust efficiency, anodic specific impulse, and thrust for each operative point considered.

The set of χ values that minimizes f has been identified using the “ga” genetic optimization algorithm implemented in the MATLAB® optimization toolbox. The inputs of the algorithm were a specially developed function, which calculates the value of the objective function f for a specific set of χ s, and the interval in which the χ s were chosen to vary, which is between 0 and 2, considered to be the most plausible. A population of 500 for each generation and the “mutationadaptfeasible” mutation function were selected, whereas the “FunctionTolerance” stopping criteria was used, with a tolerance of 10^{-6} for the average relative change in the best fitness function between 10 consecutive generations. The algorithm converged after 86 generations.

After visual inspection of the plots, to verify that the trends in data were also respected, the final set of values for the χ s have been chosen as:

$$\chi_1 = 1 \quad \chi_2 = 0.25 \quad \chi_3 = 1 \quad (32)$$

The predicted data show both anomalous trends and poor fitting of experimental data for values of χ_2 and χ_3 slightly different from the ones selected, whereas the model seems to be less sensitive to the variation of χ_1 . Obviously, the approach used bases its effectiveness on the goodness of the scaling model adopted. However, the values found are in line with those expected for the behavior of a MS thruster. In particular, the value of χ_2 that provides a better fitting of the data is low, and this agrees with the assumption made in [Phenomenological scaling model](#) section for the electron temperature in a MS HT. In addition, also the values of χ_1 and χ_3 are in line with those that can be found in the literature, discussed also in [Phenomenological scaling model](#) section. Ultimately, these results suggest that the proposed scaling laws containing the χ s, i.e., Eqs. (1), (2) and (11), can be representative of the first order scaling behavior of the thruster, at least in the context of a simplified model such as the one defined.

The plots showing predicted and experimental data for the Dataset-1 obtained with the selected χ values are presented in Fig. 3. Specifically, it shows a group of 2D plots with the discharge power on the x-axis and the main performance parameters on the ordinate, comparing database and scaling data from Dataset-1, for all the database thrusters and then only for the MS ones.

Figure 4 shows the percentage difference in terms of thrust, anodic specific impulse, and anodic thrust efficiency between the Dataset-1 and the experimental data averaged over all the operative points for different discharge power ranges. Figure 5 shows instead the same percentage differences for each database thruster. The mean data percentage deviation averaged on all the operative points is about 3% for thrust and anodic specific impulse, and about 6% for the anodic thrust efficiency.

In the graphical comparison we have decided to show the data related to both MS and US thrusters. The comparison with US thruster is not necessarily a good judgement criterion of the accuracy of the model, being it been developed specifically for shielded thrusters. But it can be observed that the results appear to fit well and represent accurately the trends also for the US thrusters of the database, although higher deviation with respect to those of MS HTs can be observed in some cases (see Fig. 5). The relatively good fitting of US thruster data is probably due to the fact that differences in the loss mechanisms between the two configurations tend to counterbalance each other (as explained in [Phenomenological scaling model](#) section) leading to similar performances for the two configurations.

Through the analysis of the performance plots and the data deviations, the following additional observations can be made:

- Towards low powers (<5 kW) the expected anodic thrust efficiency appears to decrease less steeply with respect to the experimental measurements. In addition, the thrust efficiency data appear to be increasingly scattered around the expected value going towards higher powers (>5 kW), with a fan-shaped trend. These deviation in data going towards the extremes of the tested power range, can be attributed

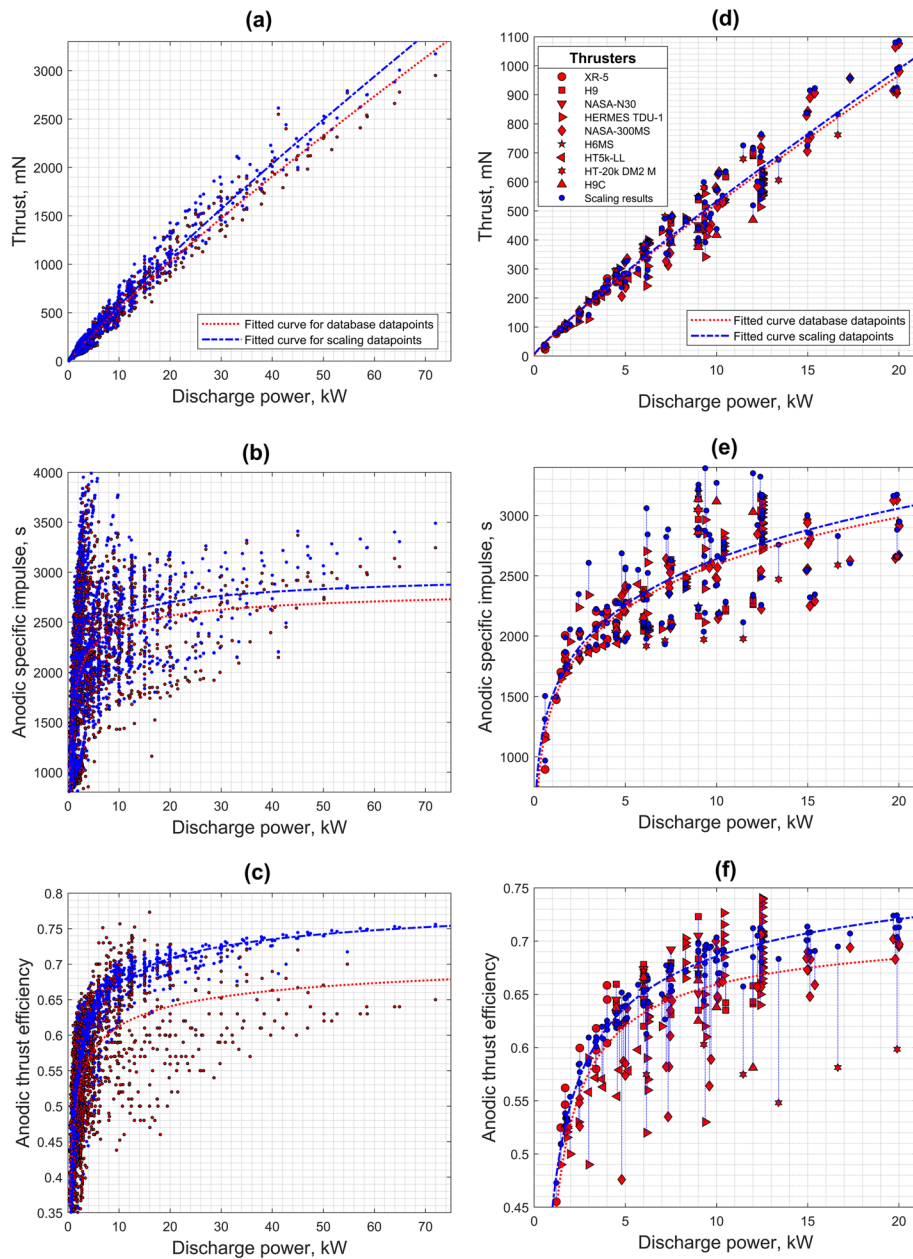


Fig. 3 **a, b, c** Plots of thrust, anodic specific impulse, and anodic thrust efficiency as a function of the discharge power, showing the Dataset-1 data points (in blue) and the database data points (in red) for all the database thrusters. The χ values used are those specified in Eq. (32). The corresponding value of f is 1.6. **d, e, f** Same plots showing only magnetically shielded thrusters. Light-blue lines connect each database data point to the respective Dataset-1 data point. In each plot the linear regression curves for both scaling datapoints (in blue) and database datapoints (in red) are reported. The curves are obtained using the MATLAB® "fit" function with a two-term power series model

mainly to the lack of modeling of plume divergence utilization efficiency η_d , mass utilization efficiency η_m and charge utilization efficiency η_q . On the other hand, as regards the scattering of data at high powers, it may be caused by the choice of keep-

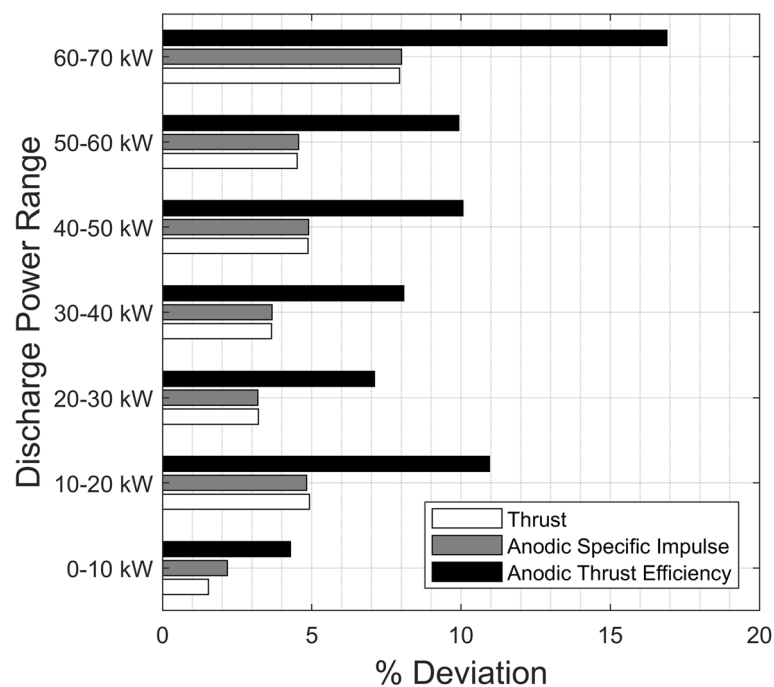


Fig. 4 Percentage difference between Dataset-1 and database performance data, averaged over all database thruster operative points, in different discharge power ranges

ing the axial inlet speed of the neutrals equal to that of the reference thruster, which is motivated by the almost total absence of experimental data in this regard.

- In general, the thrust efficiency is slightly overestimated in most of the operative points. Thus, the scaling data points appear to be part of the upper envelope of the experimental performance volume. This is probably a consequence of having selected one of the highest performing HTs as reference thruster.

Ultimately, considering the simplicity and the limits of the scaling model, the predicted data fit the experimental data very well and in a wide range of operating conditions and channel sizes. Therefore, the scaling model, can be considered reasonably validated, at least with the adoption of the H6MS as reference thruster, and can be used within the scaling methodology to design thrusters with an operational domain not too far from that of the tested thrusters. However, further studies will be for sure needed to establish the full validity of the method, especially using reference thrusters having worse performance and different scale of the H6MS.

Results of the application of the methodology (Dataset-2)

Figure 6 shows again a group of 2D plots with the discharge power on the x-axis and the main performance parameters on the y-axis, comparing database and scaling data from the Dataset-2.

It can be observed that the trends in the data are in reasonably good accordance with experimental results, even if the scaled thrusters generally exhibit a greater thrust and a lower specific impulse. Obviously in this case, the exact coincidence of experimental and

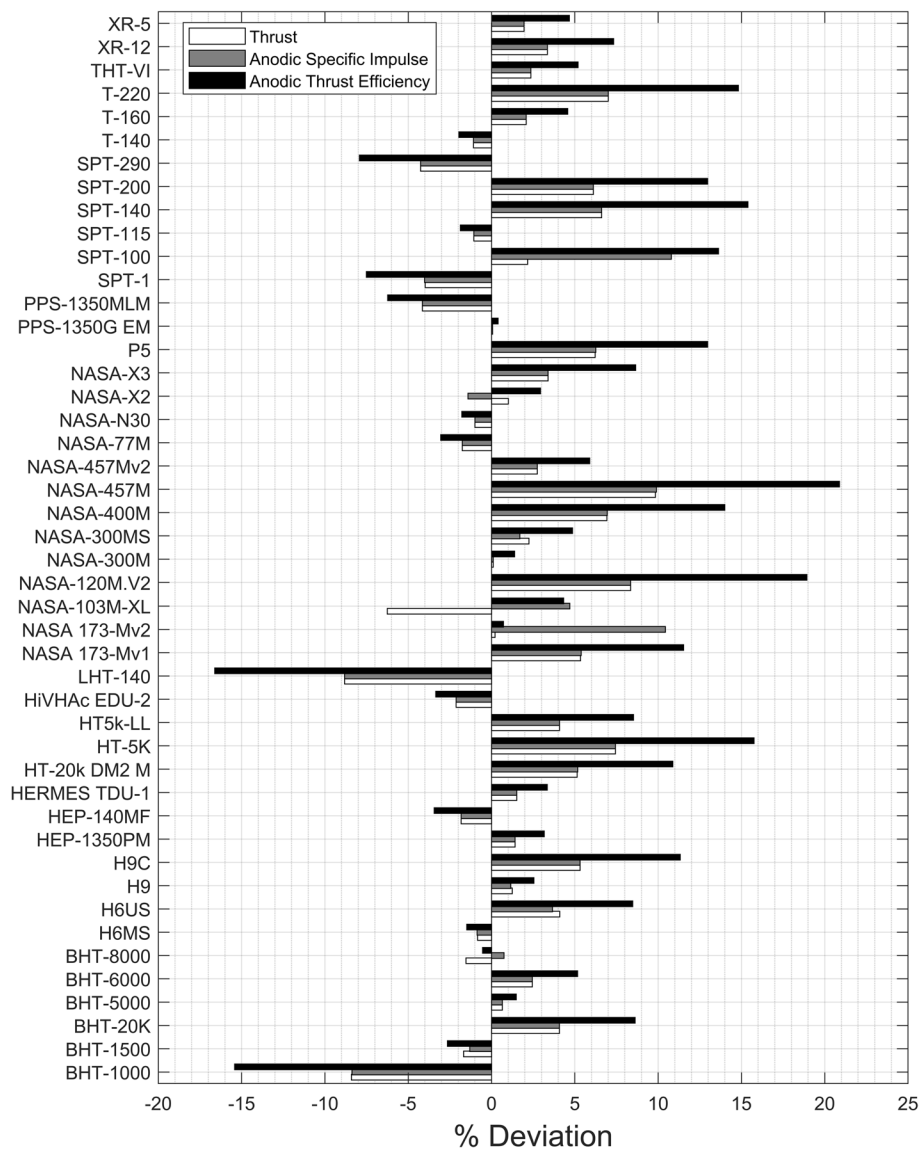


Fig. 5 Percentage difference between Dataset-1 and database performance data averaged over all the datapoints for each database thruster

predicted data is no longer a requirement to corroborate the validity of the methodology. In fact, the thrusters scaled with the methodology, while maintaining the same dimensions of the channels and discharge power of the thrusters in the corresponding database operating point, also preserve the intensive parameters of the reference thruster, which in general may be different from those of the tested thrusters. However, this outcome could suggest that the thrusters in the database operate with similar plasma intensive parameters as the reference thruster over all their range of operation, and thus that they have been designed scaling from one another (consciously or not) with criteria analogous to the one used in the proposed methodology.

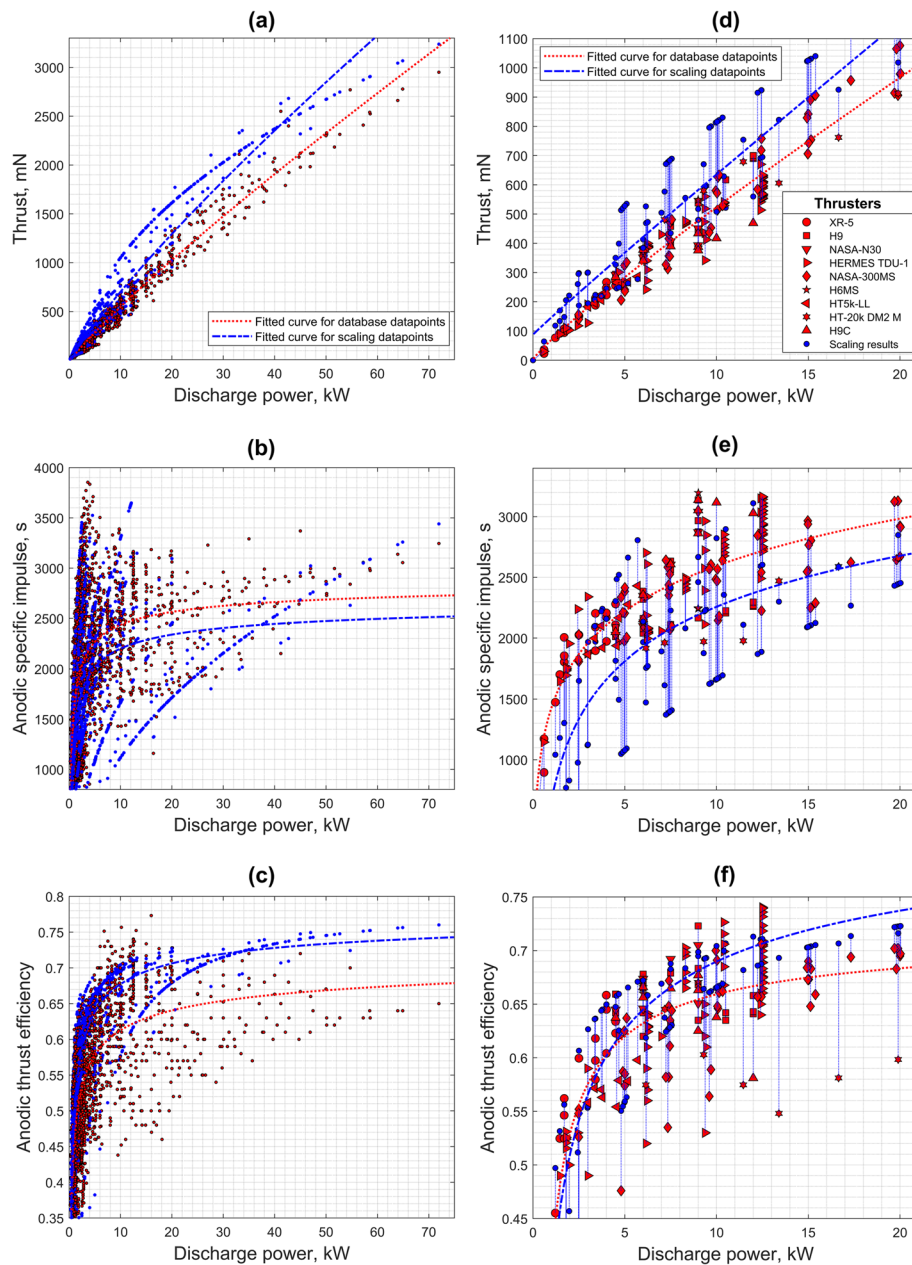


Fig. 6 **a, b, c** Plots of thrust, anodic specific impulse, and anodic thrust efficiency as a function of the discharge power, showing the scaling Dataset-2 data points (in blue) and database data points (in red) for all the database thrusters. **d, e, f** Same plots showing only magnetically shielded thrusters. Light-blue lines connect each database data point to the respective Dataset-2 data point. In each plot the linear regression curves for both scaling datapoints (in blue) and database datapoints (in red) are reported. The curves are obtained using the MATLAB® "fit" function with a two-term power series model

Discussion

In this section we describe the possible ways in which the proposed methodology can be used to design new thrusters. Subsequently, we discuss the main limits of the approach used in this work and the viable solutions to overcome these criticalities.

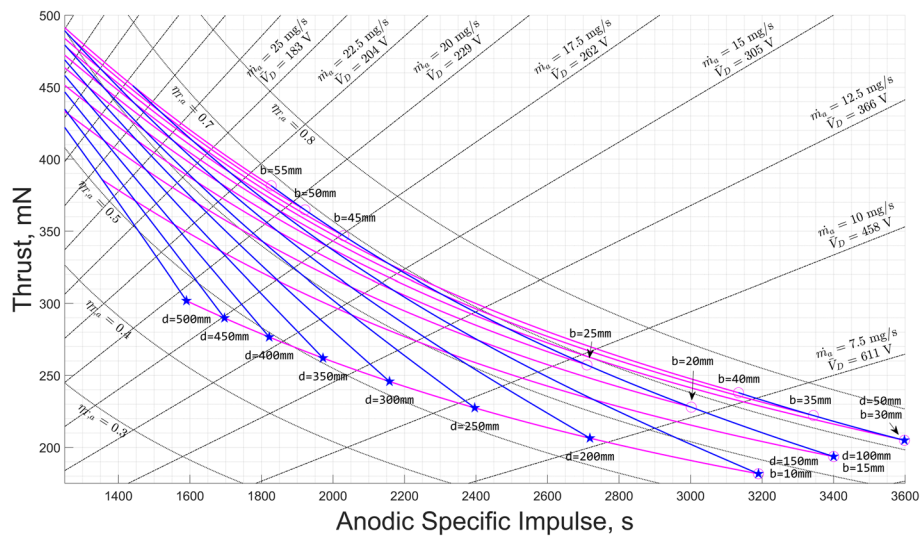


Fig. 7 Performance map with scaling data (blue and pink grid) at 5 kW

Use of the methodology for thruster preliminary sizing

The proposed scaling methodology can be used for the preliminary design of a new thruster in a straightforward manner. In fact, the scaling procedure requires only three inputs: the values of the discharge power, the channel width, and the mean channel diameter. The values of the other parameters are automatically obtained by maintaining the intensive parameters equal to those of the reference thruster. The scaling procedure can be applied using a specially developed MATLAB® App to instantaneously obtain all the data for the scaled thruster.

The direct utilization of the methodology through the App is especially useful to design thrusters with specific channel dimensions and discharge power. Nevertheless, the designer is usually provided with a requirement on the nominal discharge power only, whereas the width and the diameter can be chosen in relatively wide ranges. In this sense, an additional standalone design tool can be represented by performance maps at constant discharge power. These maps report the data obtained through scaling with the methodology for different channel diameters and widths, using a grid of diameter and width isolines. They also feature isolines for thrust efficiency, anode mass flow rate, and discharge voltage. The thrust efficiency isolines are easily generated because in a map at constant discharge power the thrust efficiency only depends on the thrust and on the anodic specific impulse. Moreover, the discharge voltage and current isolines correspond to the anodic mass flow rate isolines, being the mass and current efficiencies held equal to those of the reference thruster in a scaling transformation. These maps can be used in several ways, for instance to determine the channel width and mean diameter that maximize the thrust or the thrust efficiency at a given power. Ultimately, these maps allow to quickly assess the design possibilities for a thruster at a certain power level. Once selected a certain performance point, all the parameters not shown in the maps, such as the channel length, can be obtained assigning the selected discharge power, channel diameter and channel width in the MATLAB® App.

Two examples of constant discharge power performance maps, for 5 kW and 20 kW, are reported respectively in Figs. 7 and 8. To avoid operating conditions too extreme,

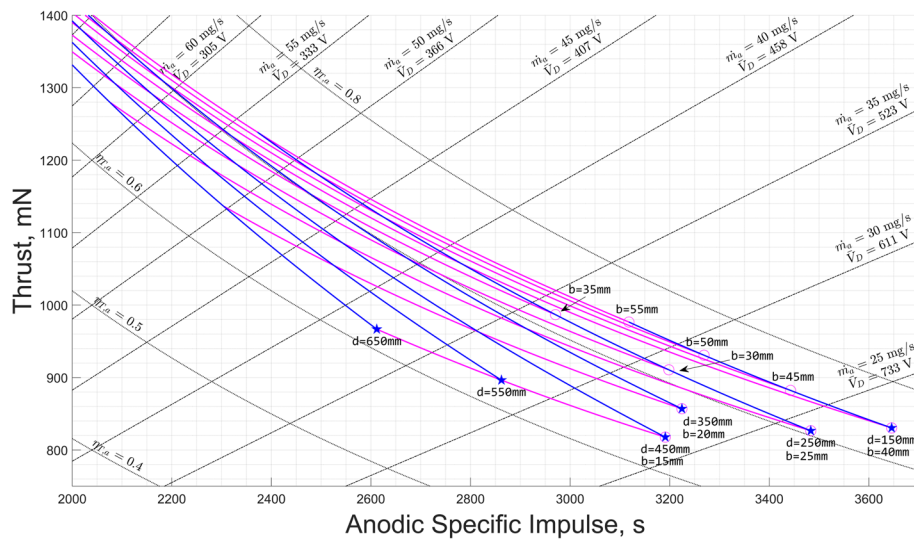


Fig. 8 Performance map with scaling data (blue and pink grid) at 20 kW

only results in the range of 100 to 900 V and with an electron Larmor radius to channel length ratio less than ten times that of the reference thruster have been included.

Limits of the methodology

Although the methodology shows promising results, its biggest limit lies in the difficulty in the validation of the model predictions for operating conditions far from the operating points of existing thrusters. In fact, by applying the methodology, in most cases, we generate thrusters operating in conditions not tested yet. For instance, even choosing to scale a thruster with the diameter and height of the channel and discharge power equal to a known thruster, in general their performance will be different; in fact, it is unlikely that the experimental thruster has operated with the same intensive parameters of the H6MS, which are kept constant in the scaling transformation. Obviously, this is a limit common to all scaling methodologies: the goodness of the predictions gradually becomes less satisfactory the more one moves away from tested operating conditions. Because of this, for example, the performance maps presented in Figs. 7 and 8 can be used with relative safety only in a reduced range of diameters and widths of the channel. In fact, as can be seen in Fig. 9, almost all the thrusters included in the database have a similar ratio between channel diameter and channel width, except for the outer channels of nested thrusters. An encouraging factor for using the methodology with exotic d/b is that, for nested thrusters, the predicted performances with Dataset-1 are no more distant from Database's data than those of thrusters with a typical d/b value, as can be seen in Fig. 5.

Another limit of the methodology derives from the choice of the scaling criterion. In fact, we focus only on sizing thrusters with intensive parameters equal to those of the H6MS. However, recent research works, such as that of Su and Jorns [105], have demonstrated the possibility of operating thrusters at much higher current densities (thus higher $n_{n,a}$) than those usually used (up to 2.7x). Scaling with variable intensive parameters allows a greater design freedom that could lead to the design of highly

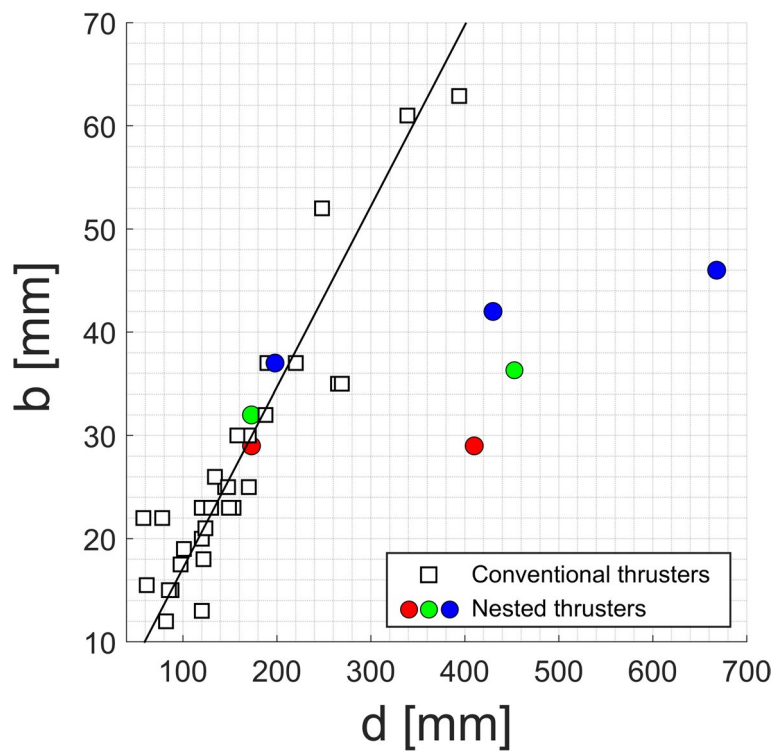


Fig. 9 Plot of the channel diameter and of the channel width for all the database thrusters. The circular markers are related to the three nested HTs included in the database (each one distinguished by a distinct color). It can be observed that for their outer channels the d/b ratio differs significantly from that of the majority of the other thrusters, which is $d/b \cong 5.7$. The black line represents the linear regression for all the points, with exception to those related to the outer channels of nested HTs

performing and/or compact thrusters. Fortunately, the methodology is structured in a way that also allows to vary the values of these intensive parameters in a scaling transformation, but as explained in [Scaling methodology](#) section it was a precise choice that of leaving them unchanged.

Ultimately, all these criticalities can be overcome as new experimental data are available. For this reason, it is in the authors' intention to keep the database updated and to improve the scaling model if needed.

A last constraint to the applicability of the methodology consists in the fact that it does not consider the magnetic structure design. In fact, in the ambit of a given mission, the thruster parameters that give the best performance may not coincide with the best thruster configuration, since aspects such as the total mass or the power used by the electromagnets are not taken into account. A more complete analysis in this sense would require an iterative approach to the thruster design. An algorithm enabling this process, conducted using the proposed methodology, has been already developed by the authors in the context of the TANDEM project [106, 107] and will be presented in a future article.

Conclusions

The objective of this work is to propose a scaling methodology for high-power MS HTs, starting from the previous works conducted on the topic at University of Pisa. To this purpose, a phenomenological model for high-power MS HTs is firstly presented. This model, based mainly on the decomposition of the anodic thrust efficiency, takes into consideration the main physical dimensions of the discharge channel, the characteristic lengths of the magnetic topology and the operative parameters of the thruster. All these quantities are linked together through a series of relations, which are used to estimate the values of the thruster performance parameters. In this sense, a key role is played by the modeling of the electron temperature and of the characteristic lengths of the main regions of the plasma discharge. This is also made possible by the introduction of a series of coefficients (χ_1 , χ_2 and χ_3), determined later through the analysis of experimental data.

Subsequently, after having provided the main concepts about the scaling theory developed in Pisa, the phenomenological model is used to create the corresponding fundamental scaling exponent matrix.

The choice of a scaling set and of a reference thruster is then discussed. In particular, the adopted reference thruster is the H6MS, a 6 kW MS HT with excellent measured performances. For what concerns the choice of the scaling set, a first one is used to determine the value of the χ s previously defined. This is done by the comparison of the results of the scaling methodology with the data collected in a specially created database, containing general, geometric, and operative information of several existing high-power HTs. Then, a second scaling set is suggested to design new thrusters. This set allows to keep the plasma intensive parameters of the reference thruster unchanged, with the objective of maintaining unaltered the fundamental physics governing the thruster operation. The agreement of the predicted data of the two scaling sets with the experimental data of the database is discussed, and in both cases a reasonably good accordance can be observed.

Then, to better illustrate the applicability of the scaling methodology, the possibility of creating performance maps at constant discharge power is briefly presented and discussed.

Finally, the main issues and limits of applicability of the methodology are discussed. Most of these limits, which are common to all the scaling methods, could be overcome with a larger amount of experimental data. In this sense, data from thrusters with a d/b ratio different from the one usually adopted would be extremely valuable.

In conclusion, the proposed method may constitute a valid tool for the preliminary design of HTs. In this sense, it has already been used in the framework of the development of a dual-channel nested HT, where the necessity of integrating coaxially two channels required a scaling method in which the mean channel diameter and width were independent [106, 107]. The details of the procedure adopted to optimize the size of the two discharge channels to obtain maximum performance will be covered in a future paper.

Appendix

This appendix is devoted to the description of the mathematical formalism behind the scaling methodology adopted in this paper. This approach is the result of a series of studies on the topic conducted in Pisa with the goal of addressing the scaling problem in a coherent and consistent way (see, for instance, [30]).

Scaling model

A *scaling model* can be defined as a physical model that describes the way in which each of the n system parameters characterizing a thruster and its operation would vary, with respect to a set of reference parameters, as a result of a change in the value of one or more of them.

Being $\mathbf{y}$ the vector of the *system parameters* and $\mathbf{y}_0$ the vector of the reference parameters, we can define a *scaling factor vector* $\boldsymbol{\varphi}$, each component of which is given by

$$\varphi_i = \frac{y_i}{y_{i0}} \quad i = 1, \dots, n \quad (33)$$

A scaling model includes $m < n$ independent scaling relations i.e., proportionality relations between certain powers of the model parameters, each one describing a specific physical phenomenon. This system of equations can be expressed in terms of scaling factors [30] as

$$\prod_i \varphi_i^{k_{ri}} = 1 \quad r = 1, \dots, m \quad (34)$$

where k_{ri} is the power to which is raised the i -th scaling factor in the r -th equation. The above system can be converted into a homogenous one by applying the natural logarithm on both sides of each equation

$$\sum_i^n k_{ri} \ln \varphi_i = \sum_i^n k_{ri} \psi_i = 0 \quad (35)$$

where we have defined ψ_i as

$$\psi_i = \ln \varphi_i \quad (36)$$

Hence, the scaling model is mathematically equivalent to a system of m homogenous linear equations in the natural logarithm of the n scaling factors

$$\mathbf{K} \cdot \boldsymbol{\psi} = \begin{bmatrix} k_{11} & \cdots & k_{1n} \\ \vdots & \ddots & \vdots \\ k_{m1} & \cdots & k_{mn} \end{bmatrix} \cdot \begin{bmatrix} \psi_1 \\ \vdots \\ \psi_n \end{bmatrix} = 0 \quad (37)$$

where the matrix $\mathbf{K} \in \mathbb{R}^{m \times n}$ is called *scaling matrix* of the model. It is worth noting that the model, as described above, has $n - m$ degrees of freedom, and therefore as many system parameters must be assigned externally for the system to have a unique solution. We will call *independent fundamental parameters* the ones whose value will be arbitrary assigned in the scaling procedure, and *dependent fundamental parameters* those obtainable as a function of the independent ones.

A scaling model can also include t additional parameters, linked to the fundamental ones by relationships that are not of simple proportionality. They take the name of *derived parameters* and are collected in the vector $\mathbf{y}_d$. Despite of their name, these parameters are not necessary of secondary importance for the characterization of the thruster, as demonstrated by the fact that the thrust efficiency, for instance, is often a derived parameter. The introduction of derived parameters does not change the degree of freedom of the model since a new equation is introduced for each new variable.

Scaling modes and transformations

A scaling mode is defined as any specific way to change the values of the reference thruster parameters. In this sense, each scaling mode is associated to a scaling factor vector $\boldsymbol{\varphi}$. For a given scaling mode, the i -th scaling factor can always be expressed as

$$\varphi_i = \frac{y_i}{y_{i0}} = \varsigma^{\alpha_i} \quad (38)$$

where ς is a positive real number called the *scaling coefficient*, and α_i is the i -th component of the *scaling exponent vector* $\boldsymbol{\alpha}$. Therefore, each scaling mode can also be described in terms of a *scaling vector*

$$\mathbf{a}_\varsigma = \begin{bmatrix} a_{\varsigma,i} \\ \vdots \\ a_{\varsigma,n} \end{bmatrix} = \begin{bmatrix} \varsigma^{\alpha_1} \\ \vdots \\ \varsigma^{\alpha_n} \end{bmatrix} \quad (39)$$

By inserting Eq. (38) in Eq. (35), we can rewrite the r -th scaling relation as a function of the components of the scaling vector of the selected scaling mode as

$$\sum_{i=1}^n k_{ri} \ln \varphi_i = \sum_{i=1}^n \ln \varsigma^{\alpha_i k_{ri}} = 0 \quad (40)$$

And being ς a positive real number different from zero

$$\sum_{i=1}^n k_{ri} \alpha_i = 0 \quad (41)$$

So, we have that the n variables α_i related to a given scaling mode must satisfy the following system of m homogenous linear equations

$$\mathbf{K} \cdot \boldsymbol{\alpha} = \begin{bmatrix} k_{11} & \cdots & k_{1n} \\ \vdots & \ddots & \vdots \\ k_{m1} & \cdots & k_{mn} \end{bmatrix} \cdot \begin{bmatrix} \alpha_1 \\ \vdots \\ \alpha_n \end{bmatrix} = 0 \quad (42)$$

From the result just obtained, a scaling mode is determined by defining only $n - m$ components of its scaling vector $\mathbf{a}_\varsigma$, a number equal to the number of degrees of freedom of the system. Once the $n - m$ components have been defined, the others can be obtained by solving the system (42).

The application of a scaling mode on a reference thruster is called *scaling transformation*, that is, therefore, a linear map between the vector of the reference parameters $\mathbf{y}_0$ and that of the scaled parameters $\mathbf{y}$.

Scaling mode algebra

To evaluate the cumulative effect of multiple consecutive scaling transformations, it is useful to define a scaling mode algebra.

Suppose to apply in series $\ell \geq 2$ scaling transformations to a reference thruster, it is always possible to define an equivalent scaling transformation [30]. The scaling vector $\mathbf{g}_\xi$, associated with the equivalent scaling mode, can be expressed as

$$\mathbf{g}_\xi = \begin{bmatrix} g_{\xi,i} \\ \vdots \\ g_{\xi,n} \end{bmatrix} = \begin{bmatrix} \xi^{\gamma_1} \\ \vdots \\ \xi^{\gamma_n} \end{bmatrix} \quad (43)$$

where the *equivalent scaling coefficient* ξ and the equivalent scaling exponent vector $\boldsymbol{\gamma}$ are given by

$$\xi = \prod_{j=1}^{\ell} \varsigma_j \quad (44)$$

$$\gamma_i = \sum_{j=1}^{\ell} p_j \alpha_{ij} \quad (45)$$

$$p_j = \frac{\ln \varsigma_j}{\sum_{j=1}^{\ell} \ln \varsigma_j} \quad (46)$$

The system given by all the Eqs. (45), which relates the scaling exponents for the overall transformation to those of the ℓ transformations and to their weights, can also be expressed in the form of a non-homogeneous linear system as

$$\boldsymbol{\gamma} = \begin{bmatrix} \gamma_{1,j} \\ \vdots \\ \gamma_{n,j} \end{bmatrix} = \mathbf{A} \cdot \mathbf{p} = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1\ell} \\ \vdots & \ddots & \vdots \\ \alpha_{n1} & \cdots & \alpha_{n\ell} \end{bmatrix} \cdot \begin{bmatrix} p_1 \\ \vdots \\ p_\ell \end{bmatrix} \quad (47)$$

where $\mathbf{p}$ is the *weight vector* whose components are the p_j defined in Eq. (46), and $\mathbf{A} \in \mathbb{R}^{n \times \ell}$ is the so-called *scaling exponent matrix*, which has for columns the scaling exponent vector of each transformation.

The scaling factors related to the equivalent transformation can be expressed as a function of the scaling coefficients and scaling exponents of the individual transformations. In fact, for the i -th scaling factor

$$\varphi_i = \frac{y_{i\ell}}{y_{i0}} = \frac{y_{i\ell}}{y_{i(\ell-1)}} \frac{y_{i(\ell-1)}}{y_{i(\ell-2)}} \cdots \frac{y_{i2}}{y_{i1}} \frac{y_{i1}}{y_{i0}} = \prod_{j=1}^{\ell} \varphi_j = \prod_{j=1}^{\ell} \varsigma_j^{\alpha_{ij}} \quad (48)$$

Hence, applying the natural logarithm on both sides

$$\ln \varphi_i = \sum_{j=1}^{\ell} \alpha_{ij} \ln \varsigma_j \quad (49)$$

Therefore, the scaling coefficients of the ℓ transformations are linked to the n global scaling factors by a non-homogeneous linear system of equations, which can be expressed as

$$\mathbf{A} \cdot \boldsymbol{\sigma} = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1\ell} \\ \vdots & \ddots & \vdots \\ \alpha_{n1} & \cdots & \alpha_{n\ell} \end{bmatrix} \cdot \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_\ell \end{bmatrix} = \begin{bmatrix} \psi_1 \\ \vdots \\ \psi_n \end{bmatrix} = \boldsymbol{\psi} \quad (50)$$

where $\boldsymbol{\sigma} \in \mathbb{R}^\ell$ is the vector of the natural logarithms of the scaling coefficients, whose j -th component is such that $\sigma_j = \ln \varsigma_j$, and $\boldsymbol{\psi} \in \mathbb{R}^n$ is the vector of the natural logarithms of the scaling factors already defined in Eq. (36).

Fundamental scaling exponent vectors

In principle, it is possible to define an infinite number of scaling modes. For this reason, it is useful to find a set of scaling exponent vectors that constitute a basis of their space vector, in such a way that any possible scaling exponent vector can be expressed as one of their linear combinations. For a model with $n - m$ degrees of freedom, we call *fundamental scaling exponent vectors* a set of $n - m$ vectors with the above-mentioned characteristics and *fundamental scaling modes* the associated scaling modes.

The simplest way to obtain one of these bases, is to build $n - m$ fundamental scaling exponent vectors, each one with a unitary value for the scaling exponent relative to one of the independent fundamental parameters and a null value to the scaling exponents relative to the remaining independent parameters. Then, the other components for each fundamental scaling exponent vector are obtained using Eq. (42). Thus, the scaling exponent matrix $\mathbf{A}_f \in \mathbb{R}^{n \times (n-m)}$ related to the fundamental vectors can be expressed as

$$\mathbf{A}_f = \begin{bmatrix} \mathbf{I}_{n-m} \\ \mathbf{K}' \end{bmatrix} \quad (51)$$

where $\mathbf{I}_{n-m} \in \mathbb{R}^{(n-m) \times (n-m)}$ is the identity matrix with size equal to the degrees of freedom of the model, and $\mathbf{K}' \in \mathbb{R}^{m \times (n-m)}$ is the submatrix extracted from the scaling matrix $\mathbf{K}$ considering only the columns corresponding to the selected $n - m$ independent parameters of the model.

Scaling procedure

Once a scaling model has been chosen, and a set of ℓ scaling modes has been adopted, it is possible to execute the actual *scaling procedure*. To do this, it is necessary to choose $s \leq n - m$ target parameters $\bar{y}_h$ ($h = 1, \dots, s$) and ℓ scaling modes. According to Eq. (50), we have that

$$\bar{\psi} = \begin{bmatrix} \bar{\psi}_1 \\ \vdots \\ \bar{\psi}_s \end{bmatrix} = \begin{bmatrix} \ln \bar{\varphi}_1 \\ \vdots \\ \ln \bar{\varphi}_s \end{bmatrix} = \mathbf{A}' \cdot \boldsymbol{\sigma} = \begin{bmatrix} \alpha_{11} & \cdots & \alpha_{1\ell} \\ \vdots & \ddots & \vdots \\ \alpha_{s1} & \cdots & \alpha_{s\ell} \end{bmatrix} \cdot \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_\ell \end{bmatrix} \quad (52)$$

where $\bar{\psi} \in \mathbb{R}^s$ is the vector of the natural logarithms of the *target scaling factors* $\bar{\varphi}_h$, whose h -th component is defined as $\bar{\psi}_h = \ln \bar{\varphi}_h$, and $\mathbf{A}' \in \mathbb{R}^{s \times \ell}$ is a submatrix extracted from the scaling exponent vector matrix $\mathbf{A}$, built choosing the rows related to the chosen target parameters and the columns related to the selected scaling modes. System consistency needs the rank of $\mathbf{A}'$ to be equal to the one of the augmented matrix $[\mathbf{A}' | \bar{\psi}]$

$$\text{rk}(\mathbf{A}') = \text{rk}([\mathbf{A}' | \bar{\psi}]) \quad (53)$$

In addition, for the solution to be unique it is also necessary that

$$\text{rk}(\mathbf{A}') = \ell \quad (54)$$

If the adopted scaling modes are linearly independent and no redundant scaling factor has been assigned, this is equivalent to assess that $\ell = s$, with $\mathbf{A}'$ being a non-singular square matrix. So we must have that

$$\det(\mathbf{A}') \neq 0 \quad (55)$$

Therefore, it is possible to rewrite the system of Eq. (50) as

$$\boldsymbol{\sigma} = \begin{bmatrix} \sigma_1 \\ \vdots \\ \sigma_\ell \end{bmatrix} = \begin{bmatrix} \ln \varsigma_1 \\ \vdots \\ \ln \varsigma_\ell \end{bmatrix} = (\mathbf{A}')^{-1} \cdot \bar{\psi} \quad (56)$$

from which it is possible to determine the scaling coefficients ς_j relative to each scaling transformation from the knowledge of the target parameters and of the scaling exponent vectors.

In summary, the procedure to perform a scaling transformation is articulated in the following steps:

- The values of the target parameters are chosen.
- The submatrix $\mathbf{A}' \in \mathbb{R}^{s \times s}$ is extracted from the scaling exponent matrix $\mathbf{A}$, choosing the rows related to the chosen target parameters and the columns related to the selected scaling modes. The compatibility between the selected scaling modes and target parameters is verified checking the fulfillment of the condition $\det(\mathbf{A}') \neq 0$.
- The scaling factors of the target parameters are calculated based on their definition.
- The scaling coefficients ς_j relative to the chosen scaling modes are obtained solving the linear system of Eq. (52) with the target scaling factors.
- The scaling coefficient ξ of the equivalent transformation and the related scaling exponent vector $\boldsymbol{\gamma}$ are calculated respectively from Eq. (44) and solving the linear system (47).

- The values of the $n - s$ missing components of the scaling factor vector $\boldsymbol{\varphi}$ are obtained from Eq. (38). Then, the corresponding fundamental parameters are calculated with Eq. (33).
- The derived parameters $\mathbf{y}_d$ are calculated according to their constitutive relations.

Conflicts of interest

The authors declare no conflict of interest.

Code availability

Not applicable

Authors' contributions

Conceptualization, F.M. and G.G.; methodology, F.M., G.G., M.A.; software, F.M.; validation, F.M. and G.G.; formal analysis, F.M. and G.G.; investigation, F.M. and G.G.; data curation, F.M. and G.G.; writing—original draft preparation, F.M. and G.G.; writing—review and editing, F.M., G.G., G.B., M.M.S., M.A., F.P.; visualization, F.M. and G.G.; supervision, F.P. All authors have read and agreed to the published version of the manuscript.

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