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Integrating NDVI and agronomic data to optimize the variable-rate nitrogen fertilization

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Abstract

The success of Variable Rate Application (VRA) techniques is closely linked to the algorithm used to calculate the different fertilizer rates. In this study, we proposed an algorithm based on the integration between some estimated agronomic inputs and crop radiometric data acquired by using a multispectral sensor. Generally, VRA algorithms are evaluated by comparing the yields, but they can often be affected by factors acting in the final phase of the crop cycle and not dependent on the fertilization treatments. Therefore, we decided to compare our algorithm (ALG) versus the traditional application of fertilizer (TRD) by evaluating the crop growth 1.5 months after the fertilization time. The algorithm was tested on a sorghum crop under organic farming, managed with or without manure. The saving of N obtained with ALG was equal to 14 and 5 kg ha^{-1} (-14 and −10% for the non-manure and fertilized treatments, respectively). The NDVI values acquired after fertilization showed a remarkable reduction of relative standard deviation for ALG system (from 22 to 9% and from 34 to 14% for manured and not manured, respectively), which was not found for TRD system (from 16 to 17% and from 29 to 18% for manured and not manured, respectively). The above ground biomass produced was statistically equivalent for the two systems in the manured plots and significant higher for ALG in not-manured plots (+0.74 t ha^{-1} of dm, equal to +23%). Finally, the indices calculated to evaluate the Nitrogen Use Efficiency (NUE) were consistently better in the ALG theses.

Keywords Sorghum · Nitrogen use efficiency · Precision agriculture · Remote sensing · NDVI map

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Introduction

In 1965, the world consumption of nitrogen fertilizers stood at 46.3 million tonnes. Nowadays, this amount has exceeded 190 million tonnes (International Fertilizer Association, 2024), of which 60% is devoted to the cereal cultivation even if only a third of this amount is actually adsorbed by crops (Raun et al., 2002). The low Nitrogen Use Efficiency (NUE), in addition to the economic damage for farmers, is the cause of some well-known environmental issues such as groundwater contamination, eutrophication of surface water bodies and nitrous oxide emissions (Perego et al., 2013; De Antoni, 2014).

Thanks to the huge technological development occurred in the last few years, many studies have been carried out to improve NUE and reduce nutrient releases from cultivated fields by using precision farming techniques (Cao et al., 2017; Goron et al., 2017; Maresma et al., 2016; Quemada et al., 2014; Raun et al., 2005). The use of proximal and remote sensing has made possible to acquire timely information about the nutritional status of crops on large areas (Campbell & Wynne, 2011; Comparetti & Marques da Silva, 2022), in order to spatially adjust the rate of the nitrogen fertilizer on the real crop requirements. The availability of spreaders able to distribute different fertilizer rates on different field areas (Variable Rate Application, VRA) allowed farmers to optimize the fertilizer use, by avoiding surpluses or deficiencies caused by its uniform application on the whole field. The processed large datasets generated by sensors provides a spatial representation of the nutritional status of the analyzed crop. This is the key to identify the various Management Zones (MZs) i.e. the field areas characterized by a different level of the yield-limiting factors and, therefore, by a different fertilizer rate to supply (Nawar et al., 2017). The MZs result from the aggregation of the elementary map units (pixels), that are the smallest areas for which sensors are able to record data.

Although the techniques for data acquisition and processing have been extensively developed so as to make them relatively inexpensive and no time-consuming, the rules to be used for the definition of N fertilizer rate are still to be improved and require further research activities (Guerrero et al., 2021; Halcro et al., 2013). As matter of fact, crop yield, and the consequently N requirement, are the result of the linear/non-linear interaction among many factors (Di Paola et al., 2016; Stockle & Debaeke, 1997), acting after the application of the top dressing fertilizer.

Initially, stress spectral indices correlated with the nitrogen demand (Normalized Difference Vegetation Index, NDVI; Green Normalized Difference Vegetation Index, GNDVI; Modified Chlorophyll Absorption in Reflectance Index, MARI; etc.), were used as co-variables to create the VRA fertilization map. In particular, NDVI (Tucker, 1979) has been the most widely used vegetation index to predict the crop nitrogen demand both for research activities and farming applications (Hatfield et al., 2008; Schwab et al., 2005). As it is known, NDVI is calculated as the ratio between the difference and the sum of the reflectance values of the red (RED) and near infrared (NIR) spectral bands and, therefore, it can be easily acquired by many sensors working in these spectral regions (Mirzakhaninafchi et al., 2022; Tagarakis & Ketterings, 2018).

However, the use of vegetation indices has not always led to remarkable results (Gobbo et al., 2022), since the response of the crop to nitrogen fertilizer is affected by many interacting factors changing over time (Colaço & Bramley, 2018; Raun et al., 2005), that cannot be represented through a single variable. A precious contribution in this regard can be supplied by the Crop Simulation Models (CSMs) used to simulate crop growth and yield under different environmental conditions (Morari et al., 2021). The CSMs are able to estimate the effects of limiting factors acting within the different MZs and, therefore, calculate the real N requirement for each of them.

Two distinct approaches are possible in using the CSMs in relation to the source of meteorological data considered (Gobbo et al., 2022): (i) historical data (Basso et al., 2012; Miao et al., 2006); ii) weather forecasts (Cantelaube & Terres, 2005; Pagani et al., 2017). Both techniques show some difficulties. For the former, the lack of sufficiently long time series of meteorological data reduces the consistency of the results, while for the latter it must be considered that the reliability of weather forecasts decreases rapidly when exceeding 10–15 days. Moreover, detailed information about the physical and chemical soil parameters and their spatial variability are required in both cases.

The uncertainties related to the quality of the input data, coupled with the knowledge required to use properly CSMs, significantly limit the viability of this solution, especially for small or non-high digital farms.

An acceptable compromise to produce a consistent VRA map starting only from a stress index map passes through the exploitation of the farmer's knowledge of their fields and the possibility to collect some information by monitoring the crop/soil parameters.

This study aimed at creating an algorithm to define the variable rate of nitrogen fertilizer to apply, by using a NDVI stress map and some agronomic information (potential crop yield, nitrogen content of crop biomass and crop growth at fertilization time), obtained by collecting crop samples and using data already available on farm. Finally, an experimental verification of the proposed method is provided, in order to evaluate its viability and to point out the limits and benefits of the whole procedure, compared to the traditional N application practice.

Materials and methods

Algorithm implementation

The N rate for each MZ (N_{MZ}) was calculated in reference to the broadcast N rate (N_F, fertilizer rate uniformly spread all over the field), in turn obtained as product of the Yield Goal (YG) at verification time and its nitrogen content in crop biomass (N_C), subtracted of the nitrogen rate eventually supplied with the fertilisation at sowing time (N_{ST}) (1).

The estimation of YG was based on the knowledge of: (i) the potential yield at harvest time under the soil and climatic conditions of the site (Y_{HT}); (ii) the effective growth reached by the crop at fertilization time (Y_{FT}); (iii) the growing cycle duration (D_{GC}). The N_C can be quantified either by starting from the current concentration of N in crop biomass or by using data already available (previous analyses carried out by farmers or provided by extension services).

N_{MZ} is driven by the ratio between YG and the yield crop goal per each management zone (YG_{MZ}) at verification time (2).

$$\mathrm{N_F} = (\mathrm{YG} * \mathrm{N_C}) - \mathrm{N_{ST}} \tag{1}$$

$$N_{MZ} = N_F * YG_{MZ} / YG \tag{2}$$

where: N_F = broadcast N fertilizer rate (g m^{-2}); YG=Yield Goal at verification time (g m^{-2}); N_C = nitrogen content per crop biomass unit (dimensionless); N_{ST} = rate of nitrogen eventually supplied with the fertilisation at sowing time (g m^{-2}); N_{MZ} = N fertilizer rate for each MZ (g m^{-2}); YG_{MZ} = yield goal per each Management Zone at verification time (g m^{-2}).

The NDVI map of the field was used to estimate the YG_{MZ} value. NDVI values are dimensionless and range from 0.0 to 1.0, with 0.0 being poor crop growth and 1.0 being very vigorous and healthy. Each pixel of the map can be considered as a MZ, whose area size depends on the spatial resolution of the sensor used. The average of NDVI values for the whole field ($NDVI_F$) was calculated by dividing the sum of all $NDVI_{MZ}$ values by the total number of MZ in the field (n_{MZ}) (3).

$$NDVI_F = \sum NDVI_{MZ} / n_{MZ} \tag{3}$$

Therefore, we estimated YG_{MZ} by using the ratio between $NDVI_{MZ}$ and $NDVI_F$.

$$YG_{MZ} = YG * NDVI_{MZ} / NDVI_F \tag{4}$$

Substantially, any deviation of $NDVI_{MZ}$ from the average value of the field ($NDVI_F$) determined an equal variation in YG_{MZ} estimation. By coupling the Eqs. (2) and (4), we can calculate the value of N_{MZ} (5).

$$N_{MZ} = N_F * NDVI_{MZ} / NDVI_F \tag{5}$$

At this point, there are two possible strategies to follow. The former, called low-input (LI), aims at reaching the crop yield goal by distributing on the field a smaller amount of fertilizer rather than for the traditional fertilization method ($N_{MZ} \le N_F$). The latter, called high-output (HO), aims at obtaining a higher crop yield rather than YG by distributing about the same total amount of fertilizer used for the traditional fertilization ($N_{MZ} \approx N_F$).

The choice between the two strategies depends on the main path followed by farmers to increase their profit: to minimize costs (LI) or to maximize gross margin (HO). In any case, the option of distributing a uniform N fertilizer rate on the whole field (traditional fertilization method) would be dominated "*Pareto sensu*" by the two VRA solutions because of the higher NUE. For both strategies, the variations of the N_{MZ} with respect to N_F were linearly driven by the $NDVI_{MZ}$ / $NDVI_F$ ratio unless the low or high cut-off thresholds were reached. Beyond the cut-off thresholds, any application of fertilizer is considered useless either because the crop is too stressed to valorize N fertilizer (low cut-off) or because further increases in fertilizer rate cannot produce a significant yield gain (high cut-off).

In the Table 1 the description of the two different VRA strategies is reported.

The rationale was that if $NDVI_{MZ} > NDVI_F$ ($\Delta NDVI_{MZ} > 0$) the crop thrives and, therefore, N_{MZ} can be reduced; whereas if $NDVI_{MZ} < NDVI_F$ ($\Delta NDVI_{MZ} < 0$) the crop is stressed and, therefore, N_{MZ} must be increased (HO strategy) or fixed equal to N_F (LI strategy).

Table 1 Description of the low-input (LI) and high-output (HO) strategies in response to NDVI values

Conditions	LI strategy	HO strategy
$NDVI_{MZ} / NDVI_F < 0.3$ (low cut-off)	$N_{MZ} = 0$	$N_{MZ} = 0$
$0.3 \leq NDVI_{MZ} / NDVI_F < 1.0$	$N_{MZ} = N_F$	$N_{MZ} > N_F$
$NDVI_{MZ} / NDVI_F = 1.0$	$N_{MZ} = N_F$	$N_{MZ} = N_F$
$1.0 < NDVI_{MZ} / NDVI_F \leq 1.5$	$N_{MZ} < N_F$	$N_{MZ} < N_F$
$NDVI_{MZ} / NDVI_F > 1.5$ (high cut-off)	$N_{MZ} = 0$	$N_{MZ} = 0$

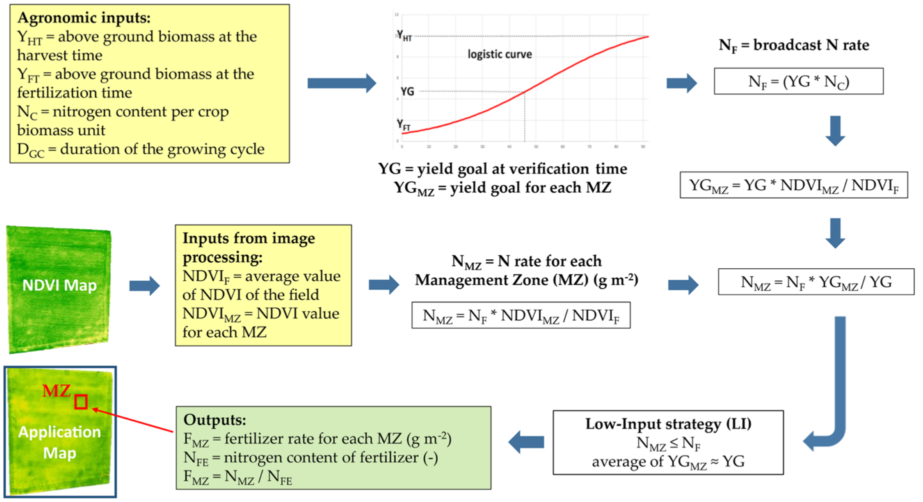


Fig. 1 Scheme of the proposed algorithm

In order to calculate the fertilizer rate to apply on each MZ (F_{MX}), the N_{MZ} was divided by the N content in the commercial product (N_{FE}), expressed as ratio between the weight of N contained in the fertilizer and the weight of the fertilizer (6):

$$F_{MZ} = N_{MX} / N_{FE} \tag{6}$$

where: F_{MZ} = fertilizer rate for each MZ (g m^{-2}); N_{FE} = nitrogen content into the fertilizer (dimensionless).

Finally, we can obtain the total fertilizer requirement for the whole field (F_F) by calculating the average value of F_{MZ}, multiplying it by the field area (F_{AR}, m^2) and dividing it by 1000 (to convert g in kg) to express the results in kg (7). The flow chart of the whole process is summarized in Fig. 1.

$$F_F = \left(\sum F_{MX} / n_{MZ}\right) * F_{AR} / 1000 \tag{7}$$

where: F_F = total fertilizer requirement for the whole field (kg); F_{AR} = field area (m^2).

Experimental site and trial set-up

In order to evaluate the viability and the functionality of the proposed algorithm, a field test was carried out during the year 2021 in an area of about 5 hectares located within the Natural Park Migliarino San Rossore Massaciuccoli (Tuscany, Central Italy) at the San Rossore estate (43° 74’ N, 10° 30’ E).

The soil of the experimental area was sandy-loam, sub-alkaline (pH of 7.6) and with a medium content of organic matter (about 2% w/w). The climate is classified as Mediterranean (Csa) according to the Köppen-Geiger classification (Kottek et al., 2006).

The experimental area was sowed with *Sorghum bicolor* (L.) Moench to forage destination (silage), managed according to organic agriculture principles.

The experimental area was divided in two main sub-areas: the cattle manured sub-area (M) and the not manured sub-area (NM). In each sub-area, we compared the VRA based on the proposed algorithm (ALG_M and ALG_{NM}) with the traditional fertilization (TRD_M and TRD_{NM}) based on the broadcast fertilizer application. Each sub-area consisted of two contiguous fields, each treated according to one of the two fertilization methods (ALG or TRD). The two fields of M sub-area each measured 0.8-1.0 ha, whereas those of NM sub-area 0.5–0.6 ha.

The main crop operations are reported in Table 2.

For both ALG theses (ALG_M and ALG_{NM}), the LI strategy was chosen to contain the N use in relation to the organic system farming adopted and to the host location (Natural Park).

The amount of N supplied with manure was estimated by considering the content of organic N (0.5%, w/w on the manure wet weight) and its mineralization rate (50% over the sorghum cycle span). Therefore, the amount of mineral N available for the crop resulted equal to 50 kg ha^{-1} (5 g m^{-2}).

Among the organic fertilizers, the blood meal fertilizer was chosen for the top-dressing fertilization. It is a by-product of the slaughtering deriving from meat packing plants (most often cow blood) and its chief benefit lies in the high nitrogen content (13% w/w of soluble organic N) and in the rather quick mineralization rate of organic N (up to 70% in two weeks, Hartz & Johnstone, 2006). For this reason, we considered that the entire amount of organic-N supplied was completely mineralized 1.5 months after its application.

We used as YG the estimation of the biomass produced 1.5 months after the top-dressing fertilization rather than at the harvest time. This choice was dictated by the need to exclude any other factor that could have influenced the behavior of the crop during the last part of its life cycle (water stress, plant diseases or parasitic attacks), which could hinder the right evaluation of the experimental treatments.

For the not manured treatments (ALG_{NM} and TRF_{NM}), N_F was fixed equal to 50 kg ha^{-1} (5 g m^{-2}) resulting from the product of the predicted amount of crop biomass ($YG = 500$ g

Table 2 Main operations carried out in the fields

Type	Date
Cattle manure (20 t ha^{-1}, fresh weight). Only on the cattle manured sub-areas.	May 5, 2021
Plowing (25 cm deep)	June 1, 2021
Harrowing	June 1, 2021
Sowing of hybrid sorghum Avance Grazer (40 kg ha^{-1} of seed)	June 3, 2021
Top dressing fertilization (ALG or TRD) with blood meal ($N = 13\%$ w/w)	June 21, 2021
Harvest (all above ground biomass)	September 19, 2021

m^{-2} of dm) at the verification time (about 1.5 months after the top-dressing fertilization) by its estimated nitrogen content (N_C = 10 g kg^{-1}). The value of YG was obtained by calibrating the classic logistic function (8) according to the measured value of above ground biomass at fertilization time (Y_{FT} = 75 g m^{-2}); the final production at harvest time was estimated based on the past yields obtained in the cultivation site (Y_{HT} = 1000 g m^{-2} of dm) and the more likely duration of the hybrid growing cycle (D_{GC} = 110 days) (Fig. 2). The value of N_C was set equal to the value of N concentration in sorghum biomass measured at the fertilization time (10 g of N per kg of dm).

$$Y(t) = Y_{HT} / 1 + q\,e^{-r\,(t - t0)} \tag{8}$$

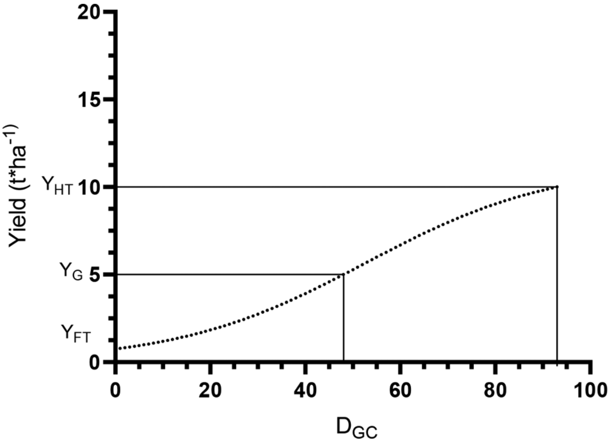
where : Y(t) = above ground biomass (g m^{-2} of dm) at time *t* (days); Y_{HT} = above ground biomass (g m^{-2} of dm) at harvest time (*t* = 110 days); *q* = (Y_{HT} – Y_0) / Y_0 (dimensionless); Y_0 = ground biomass (g m^{-2} of dm) at the initial time t_0 (t_0 = 17 days), that is the ground biomass at fertilization time (Y_{FT}); *r* = the logistic growth rate (dimensionless).

Finally, we used the same N_F value (50 kg ha^{-1}) also for the manured fields, in order to verify the effects of the two methods (ALG_M and TRF_M) on the crop growth, by using a total rate of nitrogen able of satisfying the needs for the entire growing cycle of the crop.

Equipment for acquisition of NDVI data and VRA of fertilizer

The sensor used in our experimentation was the Augmenta Field Analyzer of Second Generation (Raven, 2024), which combines a 5-spectrum, 4 K-camera system and multiple environmental/light sensors to acquire NDVI data at defined time intervals (one each 10 milliseconds) by generating an high volume of point data, which are subsequently reclassified as a TIFF map with a pixel size of 0.25 m^2. The sensor is also able to connect itself directly on the ISOBUS network and to interact with Task Controller Server of the Tractor and Task Controller Client of the spreader, in order to modify in real time the fertilization rate according to NDVI. Unfortunately, it was not possible to modify the VRA strategy implemented by the Augmenta company and, therefore, we were forced to process separately the NDVI map acquired by the sensor on the tractor and upload data to the cloud.

Fig. 2 Logistic function, used to identify the yield goal (Y_G), calibrated on the value of above ground biomass at fertilization time (Y_{FT}). The final production at harvest time is estimated based on the past yields obtained in the cultivation site (Y_{HT}) and the duration of the hybrid growing cycle (D_{GC})



Afterwards, we downloaded data from the cloud in a personal computer (tablet or smartphone) to apply the proposed algorithm through a specific script written in Python (a multiplatform programming language) and to create the VRA map according to the LI strategy.

The fertilization was performed by using a SAME Deutz 6205 Agrotron ISOBUS tractor, equipped with Topcon Universal Terminal and Task Controller (TC-GEO and TC-SC) with variable rate (GEO) and section control (SC) capabilities and a spreader ISOBUS M42ISO by DCM Spreaders (Italy), also equipped with Topcon Electronics and a Task Controller client (TC-GEO and TC-SC), fully compatible with the TC Server of the tractor. The system of tractor and spreader was able to apply fertilizers according to the VRA techniques by treating up to a 42 m width. The spreader M42ISO was equipped with two disks (one right and one left) and two shutter slides, whose opening can be set up in 14 different spreading amplitude adjustments, which can be implemented independently between the right and left side during the fertilizer application. Under the working conditions (10 km/h at constant speed), the Task Controller sent to the spreader the application rate of fertilizer 10 times per second (10 Hz). With this frequency and by considering the limits of the electro-mechanical system, the spreader was able to change the fertilizer amount to be applied every 0.28 m along the direction of movement. There was no overlap as the field width was always smaller than the spreader width. Before carrying out fertilization, we set up the spreader according to specific weight of the blood meal, by using both the calibrations available in the ISOBUS application of the spreader and the DCM app, which allows to double check the settings of the agricultural machine. Subsequently we tested the proper functioning of the spreader on sampling areas of 25 m^2 (5×5 m) and verified the substantial agreement between the sum of established rates for each pixel and the amount of fertilizer really applied by the spreader.

In detail, the steps to be passed were reported in Fig. 3 and summarized below:

i) To translate the NDVI map file in csv format (text format) and, then, in a standard geojson format, where each record is translated in an element (point) with some associated attributes (GPS coordinates and NDVI value);
ii) To create an high definition TIFF map, where each NDVI value is attributed to the pixel of equal area ($0.25\ m^2 = 0.50 \times 0.50$ m). If more than one NDVI piece of data is in the same pixel, the average NDVI value was used. Because of the non-regular distance among points acquired by the Augmenta sensor, due to the fixed acquisition timing coupled to the variable speed and non-constant steering angle of tractor, the map was created by using a bilinear interpolation according to the directions of the terrestrial parallels and meridians. Each pixel of the map represented a MZ, that is an area that can be treated with a different fertilizer rate;
iii) To produce a rectangular TIFF map oriented North-South and East-West, containing the whole field, in order to make compatible the map with the ISOBUS Task Controller requirements. The pixel outside the field margins had a NDVI value equal to -1 (invalid) to be processed as non-treated zones by the algorithm;
iv) To determine the average value of NDVI for the whole field ($NDVI_F$) (3);
v) To calculate for each MZ the difference between its NDVI value and the $NDVI_F$ previously defined ($\Delta NDVI_{MZ}$) (4);
vi) To apply the formulas (5), (2) and (6) and to produce the high resolution TIFF map whose pixels were geo-referenced and associated to the fertilizer rate value (F_{MZ});

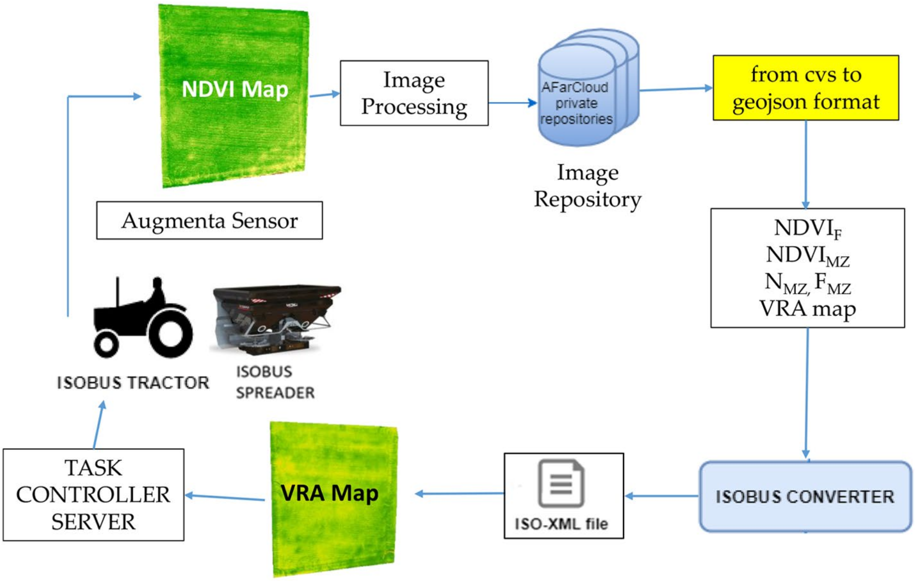


Fig. 3 Flow chart of the functional process for VRA nitrogen fertilizer (see text for acronyms)

vii) To generate a geojson file composed of regular elements (MZs) with associated the GPS point of the center of the area and the average value of the fertilizer rate, calculated by applying a bilinear interpolation of the F_{MZ} values reported in the high resolution TIFF map;
viii) To translate the geojson file in a ISO-XML file format, directly manageable by ISOBUS systems, by using the ISOBUS Converter function from the Python program.

The ISO-XML file is the standard format for tractor and spreader, both equipped with a mutually compatible version of Task Controller (Server and Client, respectively). The ISO-XML file in its simplest form, is composed by a couple of files: a binary file containing all GPS points of the field and its associated fertilizer rate (F_{MZ}) and an xml file containing all data used by the TASK Controller to manage the task in the field and the parameters to decode the binary file.

In particular, the xml file contains both the field limits, the dimension of the standard area element (MZ area), the number of MZ elements in both North-South and East-West directions, to completely fill the square area containing the field to be fertilized, and the standard ISOBUS DDI (Data Dictionary Identifier), which describes how to interpret the treatment value contained in the binary file.

With reference to the ISOBUS standard for Data Dictionary objects definition, using the online reference database resource for Data Dictionary objects (ISOBUS, 2024), the Data Dictionary Identifier (DDI) used in this work to produce the variable rate fertilization map is the DDI number 6, called “Setpoint Mass per Area Application Rate”.

Data collection and processing

The NDVI sensing of the experimental fields was carried out twice: right before the top-dressing fertilization to implement the proposed algorithm and 1.5 months after to evaluate the effects of VRA.

The fertilizer rates supplied to the fields were continuously recorded by the spreader during its operation and allowed to produce a fertilization map reporting the nitrogen rates actually applied in each area unit (0.50×0.50 m $= 0.25$ m^2) and to calculate the average value for each treatment.

Crop samples were collected from four elementary plots of about 100 m^2 established in each field. The samples of above ground crop biomass were taken before the top-dressing fertilization (18th June) and 1.5 months later (5th August) over a 2 m^2 area in each plot, in order to evaluate the growth of sorghum. The plant material was oven-dried at 60° C until constant weight, in order to determine the dry matter (dm) and, then, was ground to 1 mm and digested by H_2SO_4/H_2O_2. The Kjeldahl method was used to determine the total N concentrations.

Over the sorghum growing season, nitrate content was monitored at two depths (0–10 cm and 10–30 cm) on two different dates: 9th and 28th July. We collected four soil sub-samples for each plot, which were pooled together before to be directed to the analysis. Nitrate content was determined by using the ion chromatography method (Dx-500 ion chromatograph; Dionex, Sunnyvale, CA, USA).

The evaluation of the NUE for the four elementary plots established in each field was based on two indices: the Nitrogen Productivity (NP_{PLOT}, amount of crop biomass produced for N unit applied) and Nitrogen Efficiency (NE_{PLOT}, amount of nitrogen adsorbed by the crop for N unit applied). The former (8) was the ratio of the biomass produced in each plot (g m^{-2}) to the corresponding average N rate (g m^{-2}) and the latter (9) was the ratio of the nitrogen adsorbed by the crop in each plot (g m^{-2}) to the corresponding value of N rate (g m^{-2}). Both the used indices were obtained by modifying the classic indices of Agronomic Efficiency of applied Nitrogen (AE) and Apparent Recovery Efficiency (ARE), according to the lack of an unfertilized control (Congreves et al., 2021; Fageria et al., 2005).

$$\mathrm{NP_{PLOT}} = \mathrm{Y_{PLOT}} / \mathrm{N_{PLOT}} \tag{9}$$

$$\mathrm{NE_{PLOT}} = \mathrm{NA_{PLOT}} / \mathrm{N_{PLOT}} \tag{10}$$

where: Y_{PLOT} = biomass produced in each plot under the ALG or TRD method (g m^{-2}); N_{PLOT} = nitrogen rate applied in each plot according to the ALG or TRD method (g m^{-2}); NA_{PLOT} = nitrogen adsorbed by the crop in each plot under the ALG or TRD method.

Data from elementary plots were separately processed for the two sub-areas and the comparison between ALG and TRD treatments was made by using the Student t-test. Data of nitrogen concentration in crop and nitrate content in soil were transformed in arcsine or natural logarithm, in order to fulfil the assumptions of traditional application (version 9.1; SAS Institute Inc., Cary, NC, USA).

The Average (AV), the Standard Deviation (SD) and the Relative Standard Deviation (RSD) obtained from their ratio (ST/AV) were calculated from the two-stage NDVI maps by considering all values associated to each pixel.

Results

The comparison between ALG and TRD methods at field level

The summary of nitrogen rates applied to the experimental fields for the two different fertilization methods are reported in Table 3.

The nitrogen saving achieved with ALG in comparison with TRD method was equal to 1.4 and 0.5 g m^{-2} (14 and 5 kg ha^{-1}), for the manured and not-manured fields, respectively. In percentage terms, these reductions were equal to -28 and – 10%, if compared to the top-dressing rate used for TDR (4.9 g m^{-2}), while the saving for ALG_M dropped to 14%, if we considered the total nitrogen applied with manure at sowing time (8.5 vs. 9.9 g m^{-2}).

The two-stage (before the top-dressing fertilization and 1.5 months later) NDVI maps are reported in Fig. 4 and their descriptive statistics in the Tables 4 and 5.

At the time of top-dressing fertilization (Table 4), the mean value of NDVI was higher for the manured sub-area (0.50 as average of ALG_M and TRD_M) than for the not-manured one (0.40 as average of ALG_{NM} and TRD_{NM}), whereas the differences between the two fertilization methods within the same sub-area were more limited (0.04) and not related to one of the two methods ($ALG_M < TRD_M$ and $ALG_{NM} > TRD_{NM}$). Conversely, the spatial variability of the NDVI values was higher in the not-manured sub-areas (19% of relative standard deviation as average of ALG_M and TRD_M), compared to the manured one (31% of relative standard deviation as average of ALG_{NM} and TRD_{NM}).

A month and half later (Table 5), a high reduction of the variability of NDVI values was observed for ALG method. SD passed from 0.11 to 0.04 for ALG_M and from 0.14 to 0.06 for ALG_{NM}; a similar trend was observed for RSD (from 21.9 to 9.0% and from 33.2 to 13.5%, for ALG_M and ALG_{NM}, respectively). The variability of NDVI values also lowered for TRD_{NM}, although to a lower extent, compared to ALG (RSD passed from 29.0 to 18.0%), whereas for TRD_M we recorded a slight increase in RSD values (from 16.0 to 17.1%).

Some standardization was observed for the average NDVI values, which ranged from 0.44 to 0.47, regardless of the fertilization method and manure used.

Comparison between ALG and TRD methods at plot level

The nitrogen applied with fertilization, the production of above ground biomass, the N content in biomass, the amount of N adsorbed by the crop and the NUE values were calculated for each experimental plot (Table 6). The above ground biomass produced by sorghum 1.5 months after the top-dressing fertilization was significantly higher only for ALG_{NM}, compared to TRD_{NM}. In particular, ALG_{NM} gained + 73 g m^{-2} (+ 22%), compared to TRD_{NM},

Table 3 Mean rate of nitrogen applied at sowing time, top-dressing and in total for the two fertilization methods

Fertilization method	mineral-*N* from manure (sowing) (g m^{-2})	mineral-*N* from blood meal (top dressing) (g m^{-2})	total mineral-*N* (sowing + top dressing) (g m^{-2})
ALG_M	5.0	3.5	8.5
TRD_M	5.0	4.9	9.9
ALG_{NM}	0.0	4.4	4.4
TRD_{NM}	0.0	4.9	4.9

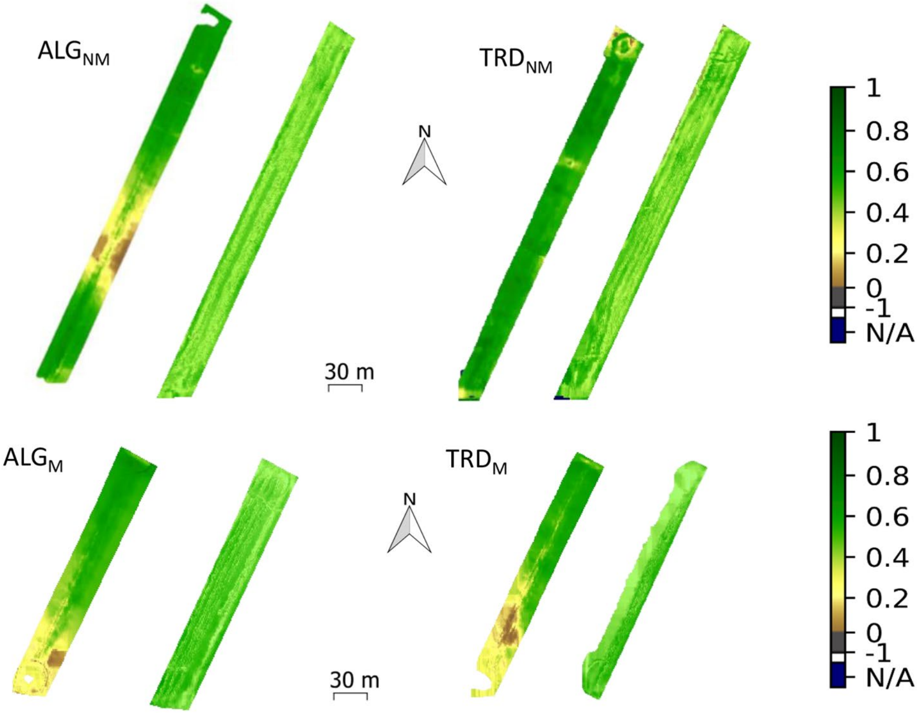


Fig. 4 NDVI maps of the fields for VRA (ALG_{NM} and ALG_{M}) and traditional application (TRD_{NM} and TRD_{M}) of nitrogen fertilizer. On the left: before top-dressing fertilization and on the right: 1.5 months after top-dressing fertilization, for each treatment. Coordinate Systems WGS 84 (EPSG: 4326)

Table 4 Main statistical parameters (average, standard deviation and relative standard deviation) related to the NDVI maps produced before top-dressing fertilization

Fertilization method	Mean NDVI value (-)	Standard deviation (-)	Relative Standard Deviation (%)	Number of values (*n*)	Field area (m^2)
ALG_{M}	0.48	0.11	21.9	30,991	7748
TRD_{M}	0.52	0.08	16.0	42,945	10,736
ALG_{NM}	0.41	0.14	34.1	24,503	6776
TRD_{NM}	0.38	0.11	29.0	19,609	4902

Table 5 Main statistical parameters (average, standard deviation and relative standard deviation) related to the NDVI maps created 1.5 months after top-dressing fertilization

Fertilization method	Mean NDVI value (-)	Standard deviation (-)	Relative Standard Deviation (%)	Number of values (*n*)	Field area (m^2)
ALG_{M}	0.44	0.04	9.0	31,811	7953
TRD_{M}	0.47	0.08	17.1	39,852	9963
ALG_{NM}	0.44	0.06	13.5	25,508	6377
TRD_{NM}	0.44	0.08	18.0	19,806	4952

Table 6 Nitrogen from fertilizer (NF_{PLOT}), above ground biomass (Y_{PLOT}), nitrogen content (NC_{PLOT}), nitrogen adsorbed (NA_{PLOT}), nitrogen productivity ($NP_{PLOT} = YG_{PLOT}/NF_{PLOT}$) and nitrogen efficiency ($NE_{PLOT} = NA_{PLOT}/NF_{PLOT}$) of the two fertilization methods for the experimental plots (student t-test)

Fertilization method	NF_{PLOT} ($g\ m^{-2}$)		YG_{PLOT} ($g\ m^{-2}$ of dm)		NC_{PLOT} ($g\ kg^{-1}$)		NA_{PLOT} ($g\ m^{-2}$)		NP_{PLOT} (-)		NE_{PLOT} (-)	
ALG_M	8.7	-	491	ns	7.1	ns	3.3	ns	56	ns	0.38	ns
TRD_M	10.1		459		6.8		3.2		45		0.32	
	-		$p=0.660$		$p=0.889$		$p=0.898$		$p=0.178$		$p=0.481$	
ALG_{NM}	4.6	-	402	*	8.1	ns	3.3	ns	87	*	0.71	ns
TRD_{NM}	4.9		328		8.0		2.6		67		0.53	
	-		$p=0.025$		$p=0.928$		$p=0.329$		$p=0.013$		$p=0.243$	

whereas for ALG_M the increase in biomass production was equal to +32 g m^{-2} (+7%), compared to TRD_{NM}. Only for the manured treatments (ALG_M and TRD_M) the biomass production approached the expected yield target (YG = 500 g m^{-2}), whereas in the not manured plots the gap with YG was more marked (-98 and −172 g m^{-2} for ALG_{NM} and TRD_{NM}, respectively).

There were no significant difference neither in terms of the N content in biomass (NC_{PLOT}) nor in terms of N adsorbed by the crop (NA_{PLOT}), although the difference between the two fertilization methods in the not manured plots was remarkable (0.7 g m^{-2} for ALG_{NM}).

As far as the Nitrogen Use Efficiency, although the differences between ALG and TRD reached the statistical significance only in one case (NP_{PLOT} = 87 and 67 for AGL_{NM} and TRD_{NM}, respectively), all the calculated indices showed better use of N for ALG, compared to TRD method. ALG was found to be more efficient than TRD, in terms of both biomass produced per unit of N applied (NP_{PLOT}) and the amount of N actually adsorbed per unit of N supplied (NE_{PLOT}). The gap between two methods was more evident in the not manured plots (+20 g of biomass per kg of N applied and +18% of N adsorbed related to the N applied), while in the manured plots the differences were more limited (+11 g of biomass per kg of N applied and +6% of N adsorbed related to the N applied).

Soil nitrate content (Table 7) was always higher for TRD than ALG plots. The differences ranged from +8 ppm (TRD_{NM} on July 9 at 0–10 cm depth) to +45 ppm (TRD_M on July 28 at 0–10 cm depth). The values observed in the manured plots resulted statistically different for each date and soil thickness, whereas the statistical significance between the two fertilization methods was never reached in not manured plots.

Discussion

The VRA of fertilizer according to the proposed algorithm led to a significant reduction in the NDVI variability within the two fields. This result was confirmed by the higher reduction of SD and RSD values for ALG, compared to TRD. A month and half after fertilization (Table 5), the RSD of NDVI values have more than halved (-59 and −62% for ALG_M and ALG_{NM}, respectively), whereas the reduction observed for TRD_{NM} was definitely much smaller (-38%) and for TRD_M there was even an increase in RSD values (+7%). However, the increase of the NDVI homogeneity translated into an increase in mean NDVI values measured one and a half months after fertilization only for ALG_{NM} (from 0.41 to 0.44), whereas we observed an opposite trend for ALG_M (from 0.48 to 0.44).

The NDVI values observed 1.5 months after the fertilization time did not increase (manured sub-area) or increased slightly (not manured sub-area) compared to the values recorded at the fertilization time. A possible explanation may be that the sorghum, at the time of fertilization, had already reached a notable degree of saturation due to the high sowing density and, therefore, the variation of NDVI values recorded 1.5 months later is mainly attributable to the nutritional status of the crop rather than its level of ground cover. In fact, the maximum NDVI values measured at the time of fertilization were already rather high (0.78 for TRD_M and TRD_{NM}, 0.73 for ALG_M and 0.68 for ALG_{NM}) and therefore the effect of top-dressing fertilization was particularly appreciable on the less productive areas of the fields. Overall, it appears that the main effect of nitrogen fertilizer was on homogeneity rather than on a generalized increase in NDVI values. In this regard, it may be that the

Table 7 Soil nitrate contents measured at different soil depths on July 9 and 28 for the tested fertilization methods (student t-test)

Fertilization method	soil nitrate content on July 9 (ppm)				soil nitrate content on July 28 (ppm)			
	0–10 cm		10–30 cm		0–10 cm		10–30 cm	
ALG_M	73	***	75	***	33	*	54	***
TRD_M	102		106		78		88	
	$p=0.000$		$p=0.003$		$p=0.026$		$p=0.009$	
ALG_{NM}	95	ns	114	ns	52	ns	78	ns
TRD_{NM}	103		136		75		96	
	$p=0.464$		$p=0.072$		$p=0.077$		$p=0.056$	

action of other ecological factors (water, nutrients, temperatures, etc.) did not allow the crop to reach high yield levels for the MZs where the crop showed the best growth conditions.

The different effect of the nitrogen fertilization on ALG_{NM} and ALG_M is explained for the former by the improvement of the nutritional conditions of the crop, which had not received any fertilization, and for the latter by the inability to satisfy the higher nitrogen demands due to the higher development reached by the crop at the time of fertilization.

A similar pattern was also observed for TRD_{NM} and TRD_M, although the result was differently obtained. For TRD_M the broadcast application of fertilizer rewarded the MZ, where nutritional conditions were already better and penalized the others, by increasing spatial heterogeneity. For TRD_{NM} fertilization only partially homogenized the nitrogen availability in the field at the sowing time, because of the uniform application of the fertilizer not tailored to compensate for the existing differences.

Although in the absence of a yield map, it is reasonable to assume that the higher homogeneity of the NDVI values found for ALG led to a more efficient and profitable use of the applied nitrogen to the crop (Ma et al., 2014).

At plot level, this behavior was further confirmed by the values of nitrogen productivity (NP_{PLOT}) and efficiency (NE_{PLOT}) that always were higher for ALG than TRD (Shanahan et al., 2008). It is worth noting that the best results in terms of NUE were reached for ALG_{NM}, in compliance with the law of diminishing returns under which the increase of a single factor of production causes a decrease in marginal (incremental) output of a production.

The results of the soil nitrate content provided an indirect confirmation of what was previously discussed. Indeed, the lower concentrations observed in the ALG plots, compared to the TDR ones, can be explained by the lower fertilizer rate and by the higher nitrogen uptake. The fact that statistical significance was detected only for the manured plots seems to be more related to a higher homogeneity of the measured data (right after manuring) than to a higher gap between the two treatments.

Based on our results, the suitability of VRA seems to be particularly recommended for low-input cropping systems, where the limited use of fertilizers increase the importance of their more rational application able to meet the different spatial needs of the crops. Conversely, the use of high fertilizer rates can induce a saturation effect on the crop growth and limit the advantages of the VRA adoption.

The comparison with literature highlights that the nitrogen savings achieved through VRA are consistent with those reported by Fan et al. (2020) (8 to 10%), whereas other authors stated for cereal crops a higher reduction in top-dressing fertilizer rate (20–25%), while obtaining higher yields than traditional methods of application (Zhong et al., 2019; Denora et al., 2022; Vizzari et al., 2019).

We found substantial confirmations also in relation to the evaluation of the NUE. Generally, the calculated indices showed that VRA results in a higher amount of the biomass produced per unit of N applied (NP) and of the N adsorbed per unit of N applied (NE) (Diacono et al., 2013; Ivanov et al., 2021).

From a methodological point of view, the choice to use a low-input cropping system and to evaluate the effects of fertilisation only after 1.5 months, instead of at the time of crop harvest, contributed to stress and clarify the effects of the experimental treatments. High fertilizer rates can mask the benefits of VRA (Denora et al., 2022), both because nitrogen may no longer be a limiting factor and because it may be subjected to natural losses able to affect the obtained results. Likewise, planning the timing of the evaluation of different fertilization methods as soon as these have had their effect allows for minimizing all other external factors (water stress, parasite attacks, etc.), which could affect the results.

The unsuitability of VRA techniques relying only on vegetation indices and the difficulty in using crop and weather forecast simulation models has highlighted the need for calibrating the algorithm used for producing application maps on the site-specific conditions. In our case study, the knowledge of some simple agronomic parameters (Y_{FT}, Y_{HT} and N_C) allowed us to tailor the algorithm on the real needs and conditions of the crop at the experimental site, thus improving the reliability and the feasibility of the proposed method although its validation requires a much greater volume of replicated and repeated experimental data.

Conclusions

The use of algorithms for calculating the optimal fertilizer rate for each MZ plays a key role in the success of VRA techniques. Generally, the available procedures have a broad scope of application but they should only be used after overcoming a careful calibration phase to fit to a specific context. The evaluation of the site-specific conditions can significantly improve the suitability of VRA methods and allow farmers to obtain higher economic advantages.

This implies the engagement of the farmers in the set-up of the rules to follow and the knowledge of some simple parameters about the crop behavior on the field. For this reason, the possibility to adjust and tailor the algorithm to the specific conditions of application can pave the way for a spread diffusion of these techniques in everyday farming.

Moreover, the evaluation of the consistency of the VRA algorithms often is based on the yield results, that however can be affected by extraneous factors not dependent on the fertilization methods. Therefore, the reliability of the comparison between the VRA methods and the broadcasting application of fertilizer can be improved by an early verification of results before other limiting factors can act.

Finally, the affordability of VRA adoption depends on many factors (cost of fertilizers, prize of products, investments for the equipment, etc.) but not the least of which is the type of agricultural system adopted. The best agronomic results are obtained for low-input cropping systems (organic farming, conservation agriculture, agro-ecological systems), where low fertilizer rates are able to exploit the effects of higher efficiency in the use of nitrogen by the crops.

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Data availability The data used to support the findings of this study are available from the corresponding author upon request.

Declarations

Competing interests The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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