

A two-sector model of economic growth with social capital accumulation

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Abstract

In this paper we analyze a two-sector growth model in which the utility function is not additively separable in consumption and “quality leisure time”. Differently from the main body of theoretical literature on quality leisure, we assume that the “productivity” of leisure is not determined by the stock of human capital but instead by the quality of social environment, which in turn depends on the joint action of the economy-wide average leisure and of the stock of social capital. In this context we show that the time evolution of social capital may exhibit an inverted-U shaped path, according to which the stock of social capital, initially increasing, becomes definitively decreasing. This result is consistent with several empirical studies about the time evolution of social capital in industrialized economies (see, e.g., Robert Putnam 1995,2000). Furthermore, we show that the inverted-U shaped evolution of the stock of social capital can be observed only if the balanced growth path is locally indeterminate.

Keywords: economic growth; social capital; quality leisure; local indeterminacy of equilibrium selection.

JEL Codes: J22; O33; O41; Z13

1 Introduction

In this paper we analyze a two-sector growth model in which “quality leisure time” (see the seminal work of Becker 1975) enters the utility function of economic agents as, among the others, in Heckman (1976), Stokey and Rebelo (1995), Ortigueira (2000), Mino (2002), Gomez Suarez (2008), Azariadis et al. (2013)¹. However, differently from the main body of theoretical literature on quality leisure, we assume that the “productivity” of leisure is not determined by the stock of human capital but instead by the quality of social environment, which in turn depends on the joint action of the economy-wide average leisure and of the stock of social capital.² Individuals allocate their time between the production of a private good and leisure, which is entirely devoted to social participation. Classical work in sociology has long stressed the impact of social interactions on individuals’

¹For a review of empirical literature supporting the relevance of quality leisure time see Gomez Suarez (2008).

²Chou’s (2006) models show that social capital can foster economic growth through various channels, such as financial development and networking between firms.

well-being and actions (Simmel 1972, Weber 1978). Manski (2000) exposit the economic perspective on social interactions and compares it with that of sociology. Broadly speaking, the nature of our micro-macro interactions, with individuals' well-being and choices depending on key features of the macro-environment (such as economy-wide average leisure) they are embedded in, is close in spirit to economics papers such as Cooper and John (1998), dealing with the microfoundations of macroeconomic coordination failures³, and Glaeser et al. (1996), investigating the effects of social interactions on individuals' decisions to engage in criminal activities⁴. In our model, in line with Coleman (1988,1990), we assume that social participation incidentally generates durable ties as a by-product via a learning-by-doing mechanism. In the long run, such ties accumulate in a stock which constitutes the "social capital" of the economy (see e.g. Antoci et al. 2005,2007,2012b). The time allocation choice of each individual has a negligible effect on the evolution of social capital; so, differently from the accumulation process of human capital, the dynamics of social capital are considered entirely as exogenously determined by the representative agent (the framework is that analyzed by Wirl 1997).⁵

We introduce the possibility that the private good and the quality of social environment can be either substitutes or complements. In the last decades several contributions in the literature have set forth the idea that a poor social environment may modify the prevailing consumption patterns, leading individuals to increase the consumption of private goods to defend themselves from social degradation (see, among the others, Putnam 2000, Corneo 2005, Bruni and Stanca 2008, Bartolini and Bonatti 2008, Antoci et al. 2007,2012a,b). In such a context, a low quality of social environment may incentivize behaviors that are perceived as individually rational (that is, utility maximizing for the agents who carry them out), but that may reduce the well-being of the whole population at the aggregate level. The mechanism underlying these perverse effects may be briefly illustrated as follows. In order to defend themselves from the degradation of social environment, economic agents make self-protective choices through the consumption of private goods. The consequent reduction in social participation further deteriorates the social environment and consequently increases the incentive to produce and consume private goods as a self-protection device. This substitution mechanism of social interaction via private goods may thus lead to a vicious circle that ultimately determines an unsustainable growth path, along which the growth of physical capital is associated to a reduction in the stock of social capital.

In economic literature there are some other economic growth models in which the "productivity" of leisure is influenced by the stock of social capital rather than by human capital; see, among the others, Antoci et al. (2005,2007,2012b), Bruni et al. (2008), Bilancini and D'Alessandro (2012). The results obtained by the analysis of our model are consistent with those obtained in the literature dealing with the issue: the expansion of market activities - private consumption and physical capital accumulation - may crowd out the relational sphere of the economy driving it towards a social poverty trap (in the sense of Antoci et al. 2007) characterized by a high level of private consumption and a low quality of social environment. The main difference between our work and those present in the literature cited above is that, due to the simplicity of our model, we are

³Cooper and John (1998) show that strategic complementarities and spillovers can generate both coordination failures and a multiplier process associated with changes in exogenous variables. The inefficiencies are driven by the presence of externalities in payoff functions.

⁴Glaeser et al. (1996) set up a local interactions model where agents' decisions to commit a crime are influenced by their neighbors' decisions and offer empirical evidence that positive covariance across agents' decisions about crime is a key explanation for variance in crime rates across time and space.

⁵Dinda (2008) sets up a one-sector growth model where the engine of growth is capital accumulation and social capital formation depends on the development of human capital.

able to show that the time evolution of social capital may exhibit an inverted-U path, according to which the stock of social capital, initially increasing, becomes definitively decreasing. More specifically, it is important to note that also Antoci et al. (2013) account for the emergence of an inverted-U shaped path in the time evolution of social capital. However, while in Antoci et al. (2013) the agents were supposed to be boundedly rational, the engine of private growth was an exogenous technological progress and there was no accumulation of physical capital, in our study the agents are optimizers and the accumulation processes of physical and social capital are jointly modeled⁶. Our current analysis is therefore more complete, as it can take account of the feedback effects of the accumulation of social capital on the accumulation of physical capital, and indicates that sustainable balanced growth paths can exist along which physical and social capital grow at strictly positive rates. Our main result is consistent with several empirical studies focusing on the time evolution of social capital in industrialized economies. Putnam (1995,2000) has documented how most indicators of social capital followed an inverted-U path in the United States during the twentieth century. In the first two thirds of the century Americans took a more and more active role in the social and political life of their communities and they behaved in an increasingly trustworthy way toward one another (Putnam 2000, p. 183). Then, beginning in the 1960s and 1970s and accelerating in the 1980s and 1990s, an erosion of the stock of American social capital started to take place⁷. According to Putnam (1995), this decline in the level of participation in group activities threatened the quality of democracy and the quality of life. He also looks for the reasons underlying the documented fall in social capital and identifies in generational differences, increases in television viewing, commuting times and female labor-market participation the major culprits. As noted by Sobel (2002), Putnam’s thesis stimulated a broad range of research activities: cross-national studies of social capital, research into the social capital of firms, and work investigating how trust is created in neighborhoods and in transition economies⁸. Similarly, Halpern (2005, p. 210) stated that “by almost all measures, social capital declined in the USA over the period from 1960 to 2000”...“but this decline follows an earlier period of growth in U.S. social capital stretching back to the beginning of the twentieth century.” (citation taken from Sequeira and Ferreira-Lopes 2011). Costa and Kahn (2003), Bjørnskov (2008) and several other scholars also achieve similar results in their empirical works.

The present paper has the following structure. Sections 2 and 3 introduce the model and the related growth dynamics. Section 4 deals with the analysis of the model. Section 5 contains some final remarks. A mathematical appendix concludes the paper.

⁶The accumulation process of the two forms of capital is also analyzed in Antoci et al. (2005). However, in that model there was no endogenous growth of physical capital and the authors did not obtain the inverted-U shaped evolution of social capital result.

⁷One of the main factors, stressed by Putnam, through which economic growth can cause a reduction in social connectedness is technology, which has made news and entertainment increasingly individualized: “Electronic technology allows us to consume hand-tailored entertainment in private, even utterly alone ... the time allocation of Americans massively shifted toward home-based activities (especially watching TV) and away from socializing outside the home” (2000, pp. 216-217 and 238).

⁸Sobel (2002) criticized Putnam’s (2000) work, being unconvinced by the details of his argument. In particular, his critique focuses on the causality issue and on the lack of an analytical framework allowing the reader to evaluate the claim that the apparent trends are related. However, it is worth noting that Putnam (2000) appears to be aware that the direction of causality has not been established. Next, even though we agree that it is unclear whether most of the phenomena presented by Putnam can be accounted for within a common framework, we believe that Putnam’s thesis on the time evolution of social capital is overall correct. Our model’s main prediction is consistent with such thesis, as we show that the time evolution of social capital may exhibit an inverted-U shaped path, according to which the stock of social capital, that is initially increasing, falls as time unfolds.

2 Setup of the model

Let us consider an economy constituted by a continuum of identical economic agents. At each instant of time $t \in [0, \infty)$, the representative agent produces the output Y using a Cobb-Douglas technology:

$$Y = AK^\alpha L^{1-\alpha}, \text{ with } 0 < \alpha < 1, \quad (1)$$

where K is the stock of physical capital accumulated by the representative agent and L is his labor input. The population size is normalized to one. Moreover, for simplicity of notation, the time dependence of all variables is suppressed. The term A represents production externalities. To make the model more tractable, we further specify these externalities as $A = \bar{K}^{1-\alpha}$, where \bar{K} denotes the economy-wide average level of physical capital. Thus capital externalities are strong enough to allow for sustained endogenous growth (see, e.g., Itaya and Mino 2004; Itaya 2008). The representative agent takes \bar{K} as exogenously given. For the sake of simplicity, we have assumed that the production process of the private output Y does not depend on the stock of social capital K_s . In Antoci et al. (2012b, 2013), models in which the production of the private output Y depends on K_s have been analyzed in a context without physical capital accumulation. We leave to future research the analysis of a model with a higher degree of interdependence between the two sectors.

In each instant of time t , the well-being of the representative agent is assumed to depend on the consumption of a private good C , on time devoted by the representative agent to social activities $1 - L$ and on the quality of social environment $Q := (\bar{1} - \bar{L})^{1-\delta} K_s$, determined by the joint action of the economy-wide average social participation $\bar{1} - \bar{L}$ and of the aggregate stock of social capital K_s . More specifically, the utility function is assumed to be given by:

$$U(1 - L, Q, C) = \frac{[(1 - L)^\delta Q C]^{1-\varepsilon} - 1}{1 - \varepsilon} = \frac{[(1 - L)^\delta (\bar{1} - \bar{L})^{1-\delta} K_s C]^{1-\varepsilon} - 1}{1 - \varepsilon}, \quad (2)$$

where $0 < \delta < 1$ and $\varepsilon > 0$, $\varepsilon \neq 1$. The parameter ε denotes the inverse of the intertemporal elasticity of substitution in consumption. We assume that the utility function (2) is concave in C and $1 - L$ (the representative agent considers $\bar{1} - \bar{L}$ and K_s as exogenously determined). As a result, we obtain the parameter restriction $\varepsilon > \delta/(1 + \delta)$, where $\delta/(1 + \delta) \in (0, 1/2)$. It is worth noting that C and Q enter multiplicatively, rather than additively, in the utility function. The reason why we decided to opt for this specification is threefold. First, the multiplicative form allows us to consider both the case in which C and Q are complements and the case in which they are substitutes. Indeed C and Q enter the utility function as complements when $\varepsilon < 1$; in this case, economic agents are willing to slow down their consumption C as a result of a reduction in Q , compared with the context in which Q does not enter the utility function. For instance, attending parties in the city one lives in can be viewed as private goods whose marginal utility increases as the quality of the social environment increases. Vice versa, C and Q are substitutes when $\varepsilon > 1$; in such a context, agents are incentivized to increase their consumption level C to defend themselves from a reduction in Q . In this regard, self-defense tools such as portable weapons or anti-aggression sprays can be viewed as private goods whose marginal utility decreases as the quality of social environment increases: living in a high-crime, low-trust neighborhood increases the marginal utility of self-defense tools⁹. Second, this specification does not rule out, at least a priori, the existence of balanced growth paths (see on this Ladron-De-Guevara et al. 1999). Third, this modelling choice

⁹Notice that $\partial^2 U(1 - L, Q, C)/\partial C \partial Q \geq 0$ for $\varepsilon \leq 1$; that is, if $\varepsilon < 1$ ($\varepsilon > 1$) the marginal utility of C increases (respectively, decreases) when Q increases.

creates an analogy between our model, where agents' utility depends on accumulated social capital, and the models with a similar utility function where the utility that individuals get from leisure depends on the level of accumulated human capital (see e.g. Ortigueira 2000, Mino 2004, Bilancini and D'Alessandro 2012). The accumulation process of physical capital is assumed to be given by:

$$\dot{K} = AK^\alpha L^{1-\alpha} - C, \quad (3)$$

where \dot{K} denotes the time derivative of K . For the sake of simplicity, we have assumed that the stock of physical capital does not depreciate.

The time evolution of K_s is instead assumed to be governed by:

$$\dot{K}_s = (\overline{1-L})K_s - \zeta K_s, \quad (4)$$

where the parameter $\zeta \in (0, 1)$ represents the depreciation rate of K_s and determines the net return on investments into social capital. Its value is strictly positive because social ties need care to be preserved over time and a zero average "investment" in social relations (i.e. $\overline{1-L} = 0$) leads the economy towards a complete depletion of the stock of social capital (i.e. $K_s \rightarrow 0$ for $t \rightarrow +\infty$). The exponent of K_s in $(\overline{1-L})K_s$ is assumed to be equal to 1, so an unbounded growth of K_s can (at least a priori) occur under the equation (4)¹⁰.

The representative agent faces the following intertemporal maximization problem:

$$\max_{L, C} \int_0^\infty \frac{[(1-L)^\delta Q C]^{1-\varepsilon} - 1}{1-\varepsilon} e^{-\eta t} dt \quad (5)$$

subject to (3) and (4), where $\eta > 0$ is the subjective discount rate. Being economic agents a continuum, the impact of each agent on the economy-wide average participation $\overline{1-L}$, and consequently on the dynamics of K_s , is negligible. Accordingly, in solving problem (5), the representative agent takes $\overline{1-L}$ and the dynamics (4) as exogenously given. Since agents are identical, ex post $\overline{1-L} = 1-L$ holds.

3 Dynamics

In the remaining part of this paper, we shall assume that the objective function (5) assumes finite values along the feasible trajectories of the dynamic system (3)-(4) (see e.g. Uzawa 1965, Mulligan and Sala-i-Martin 1993, Ladron de Guevara et al. 1997, Brito and Venditti 2010); in the mathematical appendix we highlight the conditions under which this happens.

To analyze the maximization problem (5), we set up the current value Hamiltonian:

$$\begin{aligned} H &= \frac{[(1-L)^\delta Q C]^{1-\varepsilon} - 1}{1-\varepsilon} + \Omega (AK^\alpha L^{1-\alpha} - C) \\ &= \frac{[(1-L)^\delta (\overline{1-L})^{1-\delta} K_s C]^{1-\varepsilon} - 1}{1-\varepsilon} + \Omega (AK^\alpha L^{1-\alpha} - C), \end{aligned}$$

where Ω is the co-state variable associated to K . The time evolution of K_s is considered as exogenously determined by the representative agent and consequently the dynamic constraint (4) does

¹⁰By posing the exponent of K_s strictly higher than 1 in (4), we could obtain growth paths along which K_s goes to infinity in finite time.

not enter the Hamiltonian function. Such framework was introduced in economic literature by Wirl (1997); in such a context, the trajectories followed by the economy do not describe the social optimum. However, they represent Nash equilibria in the sense that, along them, no agent has an incentive to modify his choices if the others don't modify theirs.

By applying the Pontryagin's Maximum Principle, we obtain the dynamic system:

$$\begin{aligned}\dot{K} &= \frac{\partial H}{\partial \Omega} = AK^\alpha L^{1-\alpha} - C, \\ \dot{\Omega} &= \eta\Omega - \frac{\partial H}{\partial K} = \Omega(\eta - \alpha AK^{\alpha-1} L^{1-\alpha}),\end{aligned}$$

where the values of L and C , in each instant of time, are given by the solutions of the problem:

$$\underset{L, C}{Max} H \quad (6)$$

with $L \in [0, 1]$ and $C \geq 0$.

We focus on the interior solutions of problem (6) in that, as shown in the mathematical appendix, choices with $L = 0, 1$ or $C = 0$ can be excluded. Taking account that (ex post) $A = \bar{K}^{1-\alpha} = K^{1-\alpha}$ and $\bar{1} - \bar{L} = 1 - L$, the optimality conditions on L and C can be written as follows:

$$\frac{\partial H}{\partial L} = (1 - \alpha)\Omega K L^{-\alpha} - \delta C^{1-\varepsilon} (1 - L)^{-\varepsilon} K_s^{1-\varepsilon} = 0, \quad (7)$$

$$\frac{\partial H}{\partial C} = C^{-\varepsilon} (1 - L)^{1-\varepsilon} K_s^{1-\varepsilon} - \Omega = 0. \quad (8)$$

From (7) we obtain the value of C as a function of L and K :

$$C = C_{opt} := \frac{1 - \alpha}{\delta} (1 - L) K L^{-\alpha} > 0. \quad (9)$$

By substituting C_{opt} in (8), we obtain the equation:

$$\Omega = \left(\frac{1 - \alpha}{\delta} \right)^{-\varepsilon} L^{\alpha\varepsilon} (1 - L)^{1-2\varepsilon} K^{-\varepsilon} K_s^{1-\varepsilon}, \quad (10)$$

which determines the value L_{opt} of L satisfying the conditions (7) and (8).

A solution of the dynamic system:

$$\dot{K} = K L_{opt}^{1-\alpha} - C_{opt}, \quad (11)$$

$$\dot{\Omega} = \Omega(\eta - \alpha L_{opt}^{1-\alpha}), \quad (12)$$

$$\dot{K}_s = K_s (1 - L_{opt} - \zeta), \quad (13)$$

satisfying the transversality condition:

$$\lim_{t \rightarrow +\infty} K \Omega e^{-\eta t} = 0, \quad (14)$$

and along which the objective function in problem (5) assumes a finite value, is a solution of problem (5) in that such a problem meets the assumptions of Mangasarian's Theorem.

To analyze the dynamic system (11)-(13), let us assume that an interior solution of problem (6) exists; by evaluating the time derivatives of the logarithms of both sides of the equation (10), we obtain:

$$\frac{\dot{\Omega}}{\Omega} = -(1 - 2\varepsilon)\frac{\dot{L}}{1 - L} + \alpha\varepsilon\frac{\dot{L}}{L} - \varepsilon\frac{\dot{K}}{K} + (1 - \varepsilon)\frac{\dot{K}_s}{K_s},$$

that is:

$$\left[\frac{2\varepsilon - 1}{1 - L} + \frac{\alpha\varepsilon}{L} \right] \dot{L} = \frac{\dot{\Omega}}{\Omega} + \varepsilon\frac{\dot{K}}{K} - (1 - \varepsilon)\frac{\dot{K}_s}{K_s}. \quad (15)$$

The substitution in (15) of the values of $\dot{\Omega}/\Omega$, \dot{K}/K and \dot{K}_s/K_s , expressed by the equations (11)-(13), gives the equation:

$$\left(\frac{2\varepsilon - 1}{1 - L} + \frac{\alpha\varepsilon}{L} \right) \dot{L} = \frac{\varepsilon}{\delta} \left[\left(1 + \delta - \alpha - \frac{\alpha\delta}{\varepsilon} \right) L + \alpha - 1 \right] L^{-\alpha} + (1 - \varepsilon)L + \eta - (1 - \varepsilon)(1 - \zeta), \quad (16)$$

which, for:

$$L \neq L^* := \frac{\alpha\varepsilon}{1 - (2 - \alpha)\varepsilon} \quad (17)$$

can be written as follows:

$$\dot{L} = f(L) := \frac{\frac{\varepsilon}{\delta} \left[\left(1 + \delta - \alpha - \frac{\alpha\delta}{\varepsilon} \right) L + \alpha - 1 \right] L^{-\alpha}}{\frac{2\varepsilon - 1}{1 - L} + \frac{\alpha\varepsilon}{L}} + \frac{(1 - \varepsilon)L + \eta - (1 - \varepsilon)(1 - \zeta)}{\frac{2\varepsilon - 1}{1 - L} + \frac{\alpha\varepsilon}{L}}. \quad (18)$$

Remark 1 The assumption (17) does not reduce the generality of our analysis in that $L = L^*$ is not a solution of (16); more specifically, for $L = L^*$, the left side of (16) is equal to 0 while the right side is generically different from 0 (that is, it is equally to 0 only if an equality condition on parameter values holds).

4 Indeterminacy and inverted-U growth paths

The equilibrium dynamics of K , K_s and L can be analyzed by studying the differential equation (18). Notice that, according to the equations (11)-(13), every stationary state L_{ss} of (18) is associated to a *balanced growth path (BGP)* along which the growth rates of K , K_s , Ω , L and C (respectively γ_K , γ_{K_s} , γ_Ω , γ_L and γ_C) are constant. In particular, the following result holds.

Lemma 2 *Let L_{ss} be a stationary state of the equation (18); then, for $L = L_{ss}$, it holds:*

$$\begin{aligned} \gamma_K &= \gamma_C = L_{ss}^{-\alpha} \left[\frac{(1 + \delta - \alpha)L_{ss} + \alpha - 1}{\delta} \right], \\ \gamma_{K_s} &= 1 - L_{ss} - \zeta, \\ \gamma_\Omega &= -\varepsilon\gamma_K + (1 - \varepsilon)\gamma_{K_s}, \\ \gamma_L &= 0. \end{aligned}$$

Proof. These values are obtained by substituting L_{ss} in the equations (11)-(13) and (15). ■

Remark 3 Notice that $\gamma_K > 0$ holds if and only if $L_{ss} > (1 - \alpha)/(1 + \delta - \alpha)$, while $\gamma_{K_s} > 0$ holds if and only if $L_{ss} < 1 - \zeta$.

The following proposition gives the condition on L_{ss} under which the *BGP* associated to a stationary state L_{ss} of (18) satisfies the transversality condition (14).

Proposition 4 *Let L_{ss} be a stationary state of the differential equation (18); then the *BGP* associated to L_{ss} satisfies the transversality condition (14) if and only if $L_{ss} < 1/(1 + \delta)$.*

Proof. See the mathematical appendix. ■

It is worth stressing that both K and K_s grow at a strictly positive rate along the *BGP* associated to L_{ss} if and only if (see Remark 3):

$$\frac{1 - \alpha}{1 + \delta - \alpha} < L_{ss} < 1 - \zeta, \quad (19)$$

where $(1 - \alpha)/(1 + \delta - \alpha) < 1/(1 + \delta)$ always holds (see Proposition 4). If instead:

$$L_{ss} > \max \left\{ \frac{1 - \alpha}{1 + \delta - \alpha}, 1 - \zeta \right\},$$

then the stock of physical capital K grows at a strictly positive rate while K_s decreases at a strictly negative rate and therefore $K_s \rightarrow 0$ for $t \rightarrow +\infty$.

Notice that such a “sustainable” *BGP*, along which both K and K_s grow without bound, can exist only if $1 - \zeta > (1 - \alpha)/(1 + \delta - \alpha)$; that is, if the depreciation rate ζ (which determines the net return on investments into social capital K_s) is low enough. The interactions allowed by Internet use may generate an abatement of the depreciation rate ζ of social capital and consequently an increase in the net return on investments into social capital. The growing literature on Facebook (see Antoci et al. 2012a, for a review of such a literature) and other social networks suggests that these social participation devices may play a relevant role in the preservation of existing social relations (see, among the others, Ellison et al. 2007, Pénard and Poussing 2010).

The following analysis deals with the existence conditions of a balanced growth path and its stability properties. We define “locally determinate” a balanced growth path associated to a repulsive stationary state L_{ss} under dynamics (18). In such a context, being the labour input L a jumping variable, the representative agent can follow a growth path approaching the *BGP* associated to L_{ss} only by choosing the initial labor input $L(0)$ equal to the stationary state value L_{ss} . Thus, the choice of $L(0)$ is “determined” and the growth path followed by the economy coincides with the *BGP* associated to the stationary state value L_{ss} . When, instead, a stationary state L_{ss} is attractive, then the associated *BGP* is called “locally indeterminate” in that there exists a continuum of initial choices $L(0)$ according to which the path followed by the economy can approach the *BGP* associated to L_{ss} . In such a context, the time evolution of K and K_s depends on the expectations of economic agents (see, e.g., Krugman 1991, Benhabib and Farmer 1999, Itaya 2008). If individuals expect a low initial value $\frac{1 - L(0)}{1 - L(0)}$ of the average social participation $1 - L$, then they will choose $L(0)$ such that $1 - L(0) = \frac{1 - L(0)}{1 - L(0)}$ and the economy will follow a growth path (approaching the *BGP* associated to L_{ss}) characterized by a “low” accumulation of social capital and a “high” accumulation of physical capital (see the system (11)-(13)); the opposite holds if individuals expect

a high initial value of $\overline{1-L}$. Economies starting from the same initial values of K and K_s , $K(0)$ and $K_s(0)$, may therefore follow rather different transition paths.

To analyze the dynamics under the differential equation (18), we have to take account of the following preliminary result.

Lemma 5 *The denominator of $f(L)$ (see (18)):*

$$Den(L) := \frac{2\varepsilon - 1}{1 - L} + \frac{\alpha\varepsilon}{L},$$

satisfies the following properties:

- 1) If $\varepsilon > 1/2$, then $Den(L) > 0 \quad \forall L \in (0, 1)$ holds.
- 2) If $\varepsilon < 1/2$, then $Den(L) > 0 \quad \forall L \in (0, L^*)$ and $Den(L) < 0 \quad \forall L \in (L^*, 1)$ hold, where $L^* := \alpha\varepsilon/[1 - (2 - \alpha)\varepsilon]$ (see (17)).

Let us now consider the numerator of $f(L)$ (see the equation (18)):

$$Num(L) := \frac{\varepsilon}{\delta} L^{-\alpha} [(1 + \delta - \alpha)L + \alpha - 1] - (1 - \varepsilon)(1 - L - \zeta) + \eta - \alpha L^{1-\alpha}. \quad (20)$$

The stationary states L_{ss} of (18) coincide with the zeros of the function $Num(L)$. The following proposition deals with the existence conditions of stationary states L_{ss} and with their stability properties. Remember that the transversality condition (14) is satisfied if and only if $L_{ss} < 1/(1 + \delta)$ (see Proposition 4) and that the utility function (2) is concave in C and $1 - L$ if and only if $\varepsilon > \delta/(1 + \delta)$, where $\delta/(1 + \delta) \in (0, 1/2)$.

Proposition 6 *The differential equation (18) admits a unique stationary state $L_{ss} < 1$ if:*

$$\varepsilon > \frac{\alpha - \eta - \zeta}{1 - \zeta}, \quad (21)$$

where $(\alpha - \eta - \zeta)/(1 - \zeta) < 1$. No stationary state $L_{ss} < 1$ exists if the condition (21) does not hold. When the condition (21) is satisfied, we have that:

- 1) If $\varepsilon > 1/2$, then the stationary state L_{ss} is repulsive and therefore the associated BGP is locally determinate.
- 2) If $\varepsilon < 1/2$ and $L_{ss} < L^* := \alpha\varepsilon/[1 - (2 - \alpha)\varepsilon]$ (see (17)), then the stationary state L_{ss} is attractive and therefore the associated BGP is locally indeterminate.
- 3) If $\varepsilon < 1/2$ and $L_{ss} > L^*$, then the stationary state L_{ss} is repulsive and therefore the associated BGP is locally determinate.

Proof. We have to analyze the graph of $Num(L)$.

I) Let us start by proving that $Num(L)$ is strictly concave. It holds:

$$sign [Num''(L)] = sign [(1 - \alpha)(\alpha\delta + \alpha\varepsilon - \delta\varepsilon - \varepsilon)L - \varepsilon(1 - \alpha^2)]. \quad (22)$$

The expression (22) is linear in L ; furthermore:

$$sign [Num''(0)] = sign [-\varepsilon(1 - \alpha^2)], \quad sign [Num''(1)] = sign [\alpha(1 - \alpha)(\alpha\delta - (\delta + 2)\varepsilon)],$$

where $-\varepsilon(1-\alpha^2) < 0$ and $\alpha(1-\alpha)(\alpha\delta - (\delta+2)\varepsilon) < 0$ ¹¹. Consequently, $Num''(L) < 0$ always holds in $(0, 1)$. The strict concavity of $Num(L)$ implies that there exist at most two zeros of $Num(L)$.

II) Let us consider the behavior of $Num(L)$ for L near to 0 and to 1. It is easy to check that $\lim_{L \rightarrow 0^+} Num(L) = -\infty$, while $Num(1) = \varepsilon(1-\zeta) + \eta - \alpha + \zeta$, where:

$$Num(1) > 0 \iff \varepsilon > \frac{\alpha - \eta - \zeta}{1 - \zeta}. \quad (23)$$

So, a unique stationary state $L_{ss} < 1$ exists if $Num(1) > 0$.

Let us consider the case $Num(1) \leq 0$. In such a context, a necessary condition to have a zero of $Num(L)$ in the interval $(0, 1)$ is:

$$Num'(1) = 1 - \alpha(1 - \alpha) + \frac{\varepsilon}{\delta} [(1 - \alpha)(1 + \delta) - \delta] < 0. \quad (24)$$

Condition (24) is never satisfied if $(1 - \alpha)(1 + \delta) - \delta \geq 0$ (i.e. $1 - \alpha \geq \delta/(1 + \delta)$) while, if $(1 - \alpha)(1 + \delta) - \delta < 0$, it is satisfied if:

$$\varepsilon > \frac{\delta - (1 - \alpha)\alpha\delta}{\delta - (1 - \alpha)(1 + \delta)}. \quad (25)$$

However, condition (25) is never satisfied in that $[\delta - (1 - \alpha)\alpha\delta] / [\delta - (1 - \alpha)(1 + \delta)] > 1$ always holds, while $Num(1) \leq 0$ holds for $\varepsilon \leq (\alpha - \eta - \zeta) / (1 - \zeta)$, where $(\alpha - \eta - \zeta) / (1 - \zeta) < 1$ (see (23)).

The stability properties of the unique stationary state $L_{ss} < 1$, when existing, follow from Lemma 5. ■

Figure 1a shows the graph of the function $f(L)$ for parameter values according to which the *BGP* is determinate ($\varepsilon = 0.61$) while Figure 1b illustrates the graph of $f(L)$ in a context in which the *BGP* is indeterminate ($\varepsilon = 0.381$). Figures 2a and 2b represent the time evolution of the stock of social capital K_s and L along one of the trajectories approaching the locally indeterminate *BGP*¹². Notice that K_s , initially increasing, becomes definitively decreasing and therefore the stock of social capital follows an inverted-U path. The case illustrated in Figures 2a and 2b occurs when the following conditions hold:

- (a) $L_{ss} > 1 - \zeta$ (see (19)), that is, if the stationary state L_{ss} of (18) is such that along the associated *BGP* the growth rate of K_s is strictly negative (see the system (11)-(13)).
- (b) The initial choice $L(0)$ of the labour input is such that $L(0) < 1 - \zeta$.

If condition (b) holds, then the value of \dot{K}_s is initially positive and consequently K_s is increasing (see (13)). However, if condition (a) holds, the value of L crosses (in finite time) the threshold value $1 - \zeta$; when this happens, \dot{K}_s becomes definitively negative and K_s becomes definitively decreasing. Notice that, in our model, the existence of inverted-U paths is strictly linked with the existence of an indeterminate *BGP*. In fact, if the *BGP* is determinate, the initial value $L(0)$ is

¹¹The latter inequality holds if and only if $\varepsilon > \alpha\delta/(\delta+2)$. Such a condition is always satisfied in that the concavity assumption on the utility function requires $\varepsilon > \delta/(\delta+1)$, where $\delta/(\delta+1) > \alpha\delta/(\delta+2)$ always holds.

¹²Figure 2 was obtained by the time re-scaling $t \rightarrow t/\varphi$, with $\varphi = 0.01$, to express the inverted-U path in a more "realistic" time scale. An analogous result could be obtained by assuming a non-unitary total factor productivity in the production functions $AK^\alpha L^{1-\alpha}$ and $(1-L)K_s$.

posed equal to the stationary value L_{ss} and, consequently, the time evolution of K and K_s is always monotonic, either always increasing or always decreasing (see (11)-(13)). It is worth stressing that the indeterminacy of the *BGP* can be observed only if $\varepsilon < 1/2$ (case 2 of the above proposition); under such a condition, the utility function (2) is concave in C and $1 - L$, considering $\overline{1 - L}$ as exogenously determined, but it is not concave posing $\overline{1 - L} = 1 - L$ (that is, it is not concave from the social point of view). This is the context in which indeterminacy can occur and therefore agents' expectations can play a key role in determining the transition path of the economy towards the locally indeterminate *BGP*.

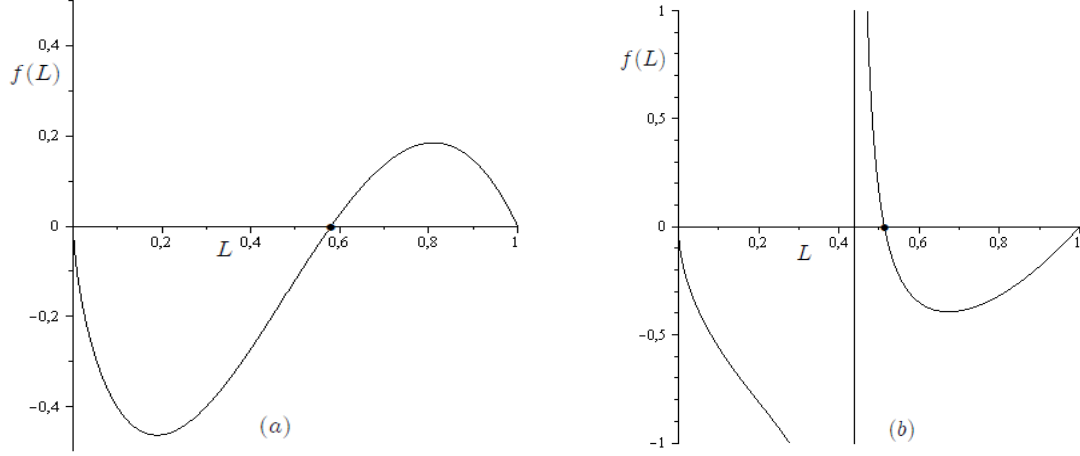


Figure 1. (a) The graph of $f(L)$ for parameter values according to which the *BGP* is determinate: $\alpha = 0.359$, $\delta = 0.901$, $\varepsilon = 0.61$, $\eta = 0.2$, $\zeta = 0.02$. (b) The graph of $f(L)$ in a context in which the *BGP* is indeterminate: $\alpha = 0.359$, $\delta = 0.901$, $\varepsilon = 0.381$, $\eta = 0.2$, $\zeta = 0.02$.

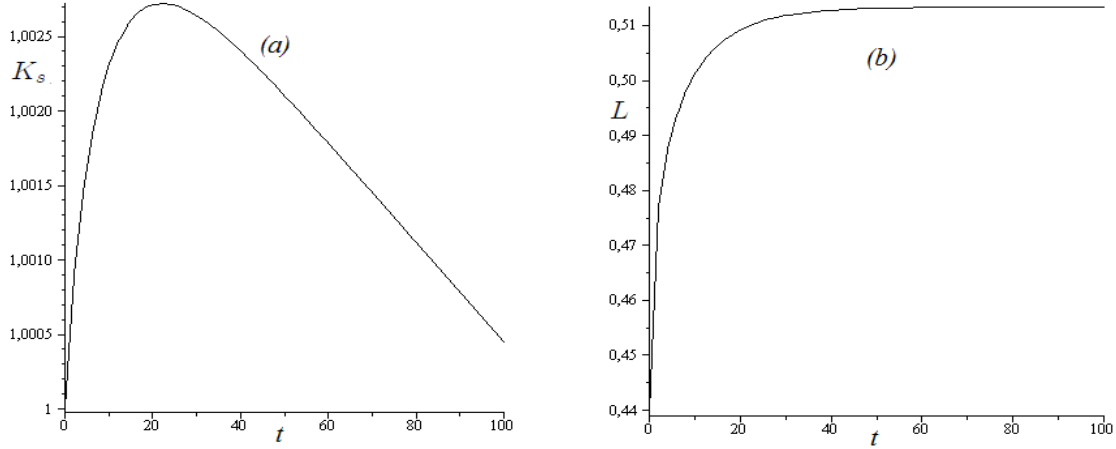


Figure 2. The time evolution of the stock of social capital K_s (a) and L (b) along one of the trajectories approaching the locally indeterminate *BGP*. The parameter values are: $\alpha = 0.49$, $\delta = 0.6$, $\varepsilon = 0.381$, $\eta = 0.294$, $\zeta = 0.49$, and time re-scaling $t \rightarrow t/\varphi$, with $\varphi = 100$.

The objective of the remaining part of our analysis is to highlight the effect on L_{ss} and on its stability properties due to changes in the value of the more relevant parameters of the model. It is easy to check that:

$$\frac{\partial L_{ss}}{\partial \zeta} > 0 \quad \text{for } \varepsilon > 1, \quad (26)$$

$$\frac{\partial L_{ss}}{\partial \eta} < 0, \quad (27)$$

$$\frac{\partial L_{ss}}{\partial \delta} < 0, \quad (28)$$

while the comparative statics analysis on the parameters α and ε does not give clear cut results.

Remember that the parameter ε measures the inverse of the intertemporal elasticity of substitution; furthermore, the quality of social environment Q and the private consumption C enter the utility function as complements when $\varepsilon < 1$ and as substitutes when $\varepsilon > 1$. So, the result in (26) can be interpreted as follows.

If Q and C are substitutes (that is, economic agents can defend themselves from a reduction in Q by increasing their consumption level C), an increase in the depreciation rate ζ (which implies a reduction in the net return on investments into social capital) generates an increase in the labor employed in the production of private goods at the expenses of social participation. This, according to the equations (11) and (13), generates an increase in the growth rate of physical capital (and, consequently, of private consumption) and a reduction in the growth rate of social capital, which becomes strictly negative for high enough values of ζ (see (19)). The opposite holds when Q and C are complements.

In such a context, a reduction in Q has the effect to reduce the marginal utility of private consumption C ; this leads to a reduction in the *BGP* value of the labour employed in the production process of private goods. Consequently, when Q and C are complements, the reduction in the growth rate of K_s due to an increase in ζ is compensated, at least partially, by an increase in social participation (see (13)).

The results (27) and (28) can be interpreted as follows. The parameter η represents the discount rate; when η increases, individuals are less incentivated to accumulate physical capital to obtain higher private consumption levels in the future; consequently, as a by-product, social participation increases along the *BGP*. Finally, if the parameter δ increases, then the externality due to the economy-wide average leisure $1 - \bar{L}$ becomes less relevant (see (2)); this leads each economic agent to a higher evaluation of his time devoted to leisure with a consequent reduction in L_{ss} .

Figures 3 and 4 illustrate the effects on the sustainability and stability properties of the *BGP* due to variations in parameters ε , η and ζ . We define "sustainable" (respectively, "unsustainable") a *BGP* along which $K, K_s \rightarrow \infty$ (respectively, $K \rightarrow \infty, K_s \rightarrow 0$).

The two-parameter bifurcation diagram in Figure 3 evaluates the effects of variations in ε and ζ . In the white region of such figure, either the transversality condition is violated or the objective function (5) does not assume a finite value along the *BGP*; in the blue (respectively, yellow) region, the *BGP* is determinate (respectively, indeterminate) and sustainable; finally, in the red (respectively, grey) region, the *BGP* is determinate (respectively, indeterminate) and unsustainable.

Figure 4 illustrates the effects of variations in δ and ζ (the meaning of colors is the same). These numerical simulations confirm our analytical results; in particular, a *BGP* can be sustainable only

if the depreciation rate ζ of K_s is low enough; further, an increase in the parameter δ favours the existence of sustainable *BGP*; finally, an indeterminate *BGP* can be observed only if ε is low enough.

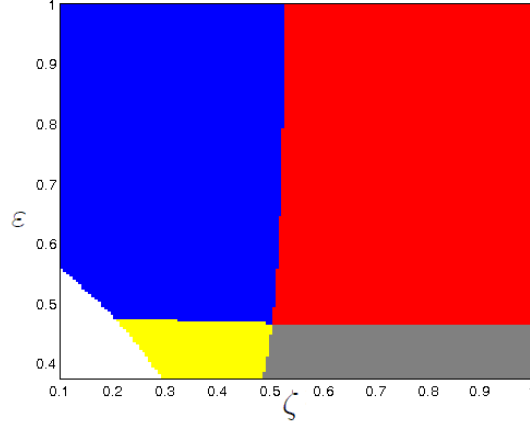


Figure 3. Bifurcation diagram with respect to parameters ε and ζ ; the remaining parameters are fixed at the values $\alpha = 0.49$, $\delta = 0.6$, $\eta = 0.294$.

Legenda: blue→the *BGP* is determinate and sustainable; yellow→the *BGP* is indeterminate and sustainable; red→the *BGP* is determinate and unsustainable; grey→the *BGP* is indeterminate and unsustainable; white→either the transversality condition is violated or the objective function (5) does not assume a finite value.

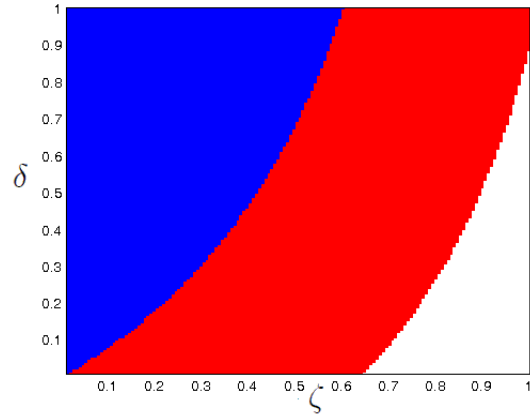


Figure 4. Bifurcation diagram with respect to parameters δ and ζ ; the remaining parameters are fixed at the values: $\alpha = 0.3$, $\varepsilon = 1.6$, $\eta = 0.2$.

Legenda: blue→the *BGP* is determinate and sustainable; red→the *BGP* is determinate and unsustainable; white→either the transversality condition is violated or the objective function (5) does not assume a finite value.

5 Concluding remarks

We have analyzed a two-sector growth model in which the “productivity” of leisure is determined by the quality of social environment, which in turn depends on the joint action of the economy-wide average leisure and of the stock of social capital. The accumulation process of social capital is influenced by two factors:

- 1) Aggregate social participation which, according to Coleman (1988,1990), generates durable ties as a by-product via a learning-by-doing mechanism; in the long run, such ties accumulate in a stock which constitutes the “social capital” of the economy.
- 2) The depreciation rate ζ of the stock of social capital, which depends on the ability of economic agents to preserve existing social connections. The parameter ζ may be, for example, positively affected by an increase in mobility and negatively affected by the participation to Facebook and to other social networks (see Antoci et al. 2012a).

In such a context, we have shown that sustainable balanced growth paths can exist along which physical capital and social capital grow at strictly positive rates. However, the sustainability of the growth path followed by the economy is a frail scenario which, according to the condition (19), can be observed only if the depreciation rate ζ of social capital is low enough and if social participation $1 - L_{ss}$, evaluated along the balanced growth path, is high enough.

When $1 - L_{ss} < \zeta$, and therefore the condition (19) is not satisfied, economic growth is not sustainable and the stock of social capital K_s approaches 0 for $t \rightarrow +\infty$. In such a context, the time evolution of social capital may exhibit an inverted-U path, according to which the stock of social capital, initially increasing, becomes definitively decreasing. This result is consistent with several empirical studies about the time evolution of social capital in industrialized economies (see, among the others, Putnam 1995,2000). In this regard, it is important to note that the inverted-U shaped path of social capital result had already been presented in a previous paper by Antoci et al. (2013). However, while in that paper the agents are supposed to be boundedly rational, the engine of private growth is exogenous technological progress and there is no accumulation of physical capital (but only accumulation of social capital), in this study the economic agents are engaged in an intertemporal maximization problem and we find that the inverted-U shaped relationship can arise also in the presence of perfect foresight. Further, we explicitly model the accumulation process of both physical and social capital, showing that sustainable balanced growth paths can exist along which physical and social capital grow at strictly positive rates. It is worth to stress that also the work of Antoci et al. (2005) deals with the accumulation of physical and social capital. However, the authors do not obtain the inverted-U shaped evolution of social capital result. Moreover, in Antoci et al. (2005), private and relational goods are perfectly substitutable and the production technology does not allow for the existence of a balanced growth path. Next, the dynamics’ stationary states are all saddle-point stable, so that indeterminacy never occurs. By contrast, in the present paper the quality of social environment Q and the consumption of the private good C can be either (imperfect) substitutes or complements¹³. Furthermore, we have created a link between the strand of literature on social capital accumulation and the huge macroeconomic literature on indeterminacy, as we have

¹³The current study also markedly differs from other work by Antoci and co-authors, such as Antoci et al. (2007,2008,2012b). The models outlined in Antoci et al. (2008,2012b) exclusively focus on social capital accumulation, without considering physical capital, and suppose that players are boundedly rational. In these articles, an inverted-U shaped relationship does not emerge. Finally, our paper also substantially departs from Antoci et al. (2007), as the latter focuses on an evolutionary game exclusively considering social capital accumulation.

shown that an inverted-U path can be observed only if a locally indeterminate balanced growth path exists. In such a context, there exists a continuum of growth paths that the economy can follow and the equilibrium selection depends on individuals' expectations about the economy-wide average social participation.

Finally, we have shown that the substitutability between the quality of social environment Q and the consumption of the private good C plays a key role in determining the social participation $1 - L_{ss}$. In a context in which Q and C are substitutes, a poor social environment may lead individuals to increase the production and consumption of private goods to defend themselves from social degradation. This substitution process may generate a self-enforcing mechanism according to which the deterioration of social environment fuels the expansion of private consumption (analogous results are obtained in Bartolini and Bonatti 2008, Bruni et al. 2008, Bilancini and D'Alessandro 2012, Antoci et al. 2005, 2007, 2012b). By contrast, when Q and C are complements, we have shown that the opposite mechanism operates; that is, a poor social environment stimulates an increase in social participation. This is the case in that, in such a context, economic agents defend themselves against social degradation by increasing the time spent in the relatively less productive sector, that is, by increasing social participation.

The main objective of our paper was the introduction of a modelling framework by which the accumulation process of social capital can be analyzed. Such a framework could be used to build more complex models. For example, the production process of the private output Y could be assumed to depend on the stock of social capital K_s (as in Antoci et al. 2012b, 2013). We leave these possible generalizations of our model to future research.

6 Mathematical appendix

6.1 Growth paths with $L = 0, 1$ or $C = 0$

In the main text we have only considered interior solutions of problem (6), that is solutions with $L \in (0, 1)$ and $C > 0$. In this appendix we show how solutions with $L = 0, 1$ or $C = 0$ of problem (6) can be excluded, if $K, K_s > 0$. Let us consider the Hamiltonian function:

$$H = \frac{[C(1-L)^\delta(\overline{1-L})^{1-\delta}K_s]^{1-\varepsilon} - 1}{1-\varepsilon} + \Omega(AK^\alpha L^{1-\alpha} - C).$$

Notice that, in the context $1 - \varepsilon < 0$, $H \rightarrow -\infty$ holds for $C \rightarrow 0$ or $L \rightarrow 1$; this implies that H cannot be maximized by choosing $C = 0$ or $L = 1$. In the context $1 - \varepsilon > 0$, we have that:

- a) If the representative agent expects that $\overline{1-L} > 0$, then $\lim_{C \rightarrow 0} \partial H / \partial C = +\infty$ holds and therefore $C = 0$ cannot maximize H (remember that the economy-wide average value $\overline{1-L}$ is taken as exogenously determined by the representative agent, however it coincides (ex post) with $1-L$).
- b) If the representative agent expects that $\overline{1-L} = 0$, then $1-L = 0$ and $C = 0$ maximize the function H , which in such a case becomes $H = \Omega(AK^\alpha L^{1-\alpha} - C)$. Since the solution $C = 0$ has no economic meaning, we rule out this solution by assuming that the representative agent expects $\overline{1-L} > 0$ and consequently he always chooses $1-L > 0$ and $C > 0$.

Let us now prove that solutions with $1-L = 1$ (i.e. with $L = 0$) can also be ruled out. The above analysis has excluded solutions with $C = 0$, therefore the equation $\partial H / \partial C = 0$ must be

satisfied and this implies that $\Omega > 0$; consequently:

$$\lim_{L \rightarrow 0} \frac{\partial H}{\partial L} = +\infty.$$

This excludes a solution with $L = 0$.

6.2 Proof of Proposition 4

By (11), (12) and Lemma 1, along a balanced growth path associated to a stationary state L_{ss} of (18) we have:

$$\frac{\dot{\Omega}}{\Omega} = \eta - \alpha L_{ss}^{1-\alpha}, \quad \frac{\dot{K}}{K} = L_{ss}^{1-\alpha} - \frac{C}{K} \Big|_{ss},$$

where $C/K|_{ss}$ is the (constant) value of C/K evaluated along the balanced growth path. Consequently:

$$K = K(0)e^{(L_{ss}^{1-\alpha} - \frac{C}{K}|_{ss})t}, \quad \Omega = \Omega(0)e^{(\eta - \alpha L_{ss}^{1-\alpha})t},$$

and the transversality condition (14) can be rewritten as follows:

$$\begin{aligned} \lim_{t \rightarrow +\infty} \Omega K e^{-\eta t} &= 0 \iff \lim_{t \rightarrow +\infty} \Omega(0)e^{(\eta - \alpha L_{ss}^{1-\alpha})t} K(0)e^{(L_{ss}^{1-\alpha} - \frac{C}{K}|_{ss})t} e^{-\eta t} = 0 \iff \\ &\iff \lim_{t \rightarrow +\infty} \Omega(0)K(0)e^{[(1-\alpha)L_{ss}^{1-\alpha} - \frac{C}{K}|_{ss}]t} = 0 \iff (1-\alpha)L_{ss}^{1-\alpha} - \frac{C}{K} \Big|_{ss} < 0. \end{aligned}$$

Notice now that, by (9), $C/K = [(1-\alpha)/\delta](1-L)L^{-\alpha}$ holds and therefore:

$$\begin{aligned} (1-\alpha)L_{ss}^{1-\alpha} - \frac{C}{K} \Big|_{ss} < 0 &\iff (1-\alpha)L_{ss}^{1-\alpha} - \frac{1-\alpha}{\delta} \frac{(1-L_{ss})L_{ss}^{1-\alpha}}{L_{ss}} < 0 \\ &\iff \frac{1-\alpha}{\delta} [\delta L_{ss} - (1-L_{ss})] < 0 \iff L_{ss} < \frac{1}{1+\delta}. \end{aligned}$$

This completes the proof.

6.3 A finite value of the objective function

In this appendix, we will specify the conditions under which the integral in the objective function (5) assumes finite values along the feasible BGP of the system (11)-(13).

Proposition 7 *The objective function (5) assumes finite values along the BGP corresponding to a stationary state L_{ss} of the equation (18) if and only if:*

$$(1-\varepsilon) \left\{ \left[L_{ss}^{1-\alpha} - \frac{(1-\alpha)}{\delta} (1-L_{ss})L_{ss}^{-\alpha} \right] + 1 - L_{ss} - \zeta \right\} - \eta < 0$$

holds.

Proof. The objective function (5) assumes a finite value along a *BGP* corresponding to a stationary state L_{ss} if the following integral assumes a finite value:

$$\int_0^\infty \frac{(CQ)^{1-\varepsilon} - 1}{1-\varepsilon} e^{-\eta t} dt = \int_0^\infty \frac{\left[\frac{C}{K} K K_s (1-L) \right]^{1-\varepsilon}}{1-\varepsilon} e^{-\eta t} dt + \int_0^\infty \frac{-1}{1-\varepsilon} e^{-\eta t} dt. \quad (29)$$

Posing $L = L_{ss}$ and taking into account that the rate C/K is constant along the *BGP* associated to L_{ss} , we have that the former of the integrals in (29) can be written as:

$$\frac{\left[\frac{1-\alpha}{\delta} (1-L_{ss})^2 L_{ss}^{-\alpha} K_s(0) K(0) \right]^{1-\varepsilon}}{1-\varepsilon} \cdot \int_0^\infty e^{(1-\varepsilon) \left[L_{ss}^{1-\alpha} - \frac{1-\alpha}{\delta} (1-L_{ss}) L_{ss}^{-\alpha} \right] t} e^{(1-\varepsilon)(1-L_{ss}-\zeta)t} e^{-\eta t} dt,$$

where $K_s(0)$ and $K(0)$ are the initial values of K_s and K . This completes the proof.

Remark 8 If the assumption of convergence of the integral in (5) is satisfied, then its maximized value V_{Max} , evaluated along the *BGP* associated to L_{ss} , is given by:

$$V_{Max} = \frac{\left[\frac{1-\alpha}{\delta} (1-L_{ss})^2 L_{ss}^{-\alpha} K_s(0) K(0) \right]^{1-\varepsilon}}{\left\{ (1-\varepsilon) \left[\left[L_{ss}^{1-\alpha} - \frac{1-\alpha}{\delta} (1-L_{ss}) L_{ss}^{-\alpha} \right] + 1 - L_{ss} - \zeta \right] - \eta \right\} (\varepsilon - 1)} + \frac{1}{\eta(\varepsilon - 1)}.$$

■

Remark 9 According to Proposition 7, the following results can be pointed out:

- 1) If the instantaneous utility function (2) is not bounded from above (i.e. if $1 - \varepsilon > 0$) and the product KK_s is increasing, then the increase in KK_s must be low enough; that is, if the following conditions hold:

$$\begin{cases} \left[L_{ss}^{1-\alpha} - \frac{1-\alpha}{\delta} (1-L_{ss}) L_{ss}^{-\alpha} \right] + 1 - L_{ss} - \zeta > 0, \\ 1 - \varepsilon > 0, \end{cases}$$

then it must hold:

$$(1-\varepsilon) \left[L_{ss}^{1-\alpha} - \frac{1-\alpha}{\delta} (1-L_{ss}) L_{ss}^{-\alpha} + 1 - L_{ss} - \zeta \right] - \eta < 0.$$

- 2) If the instantaneous utility function (2) is bounded from above (i.e. if $1 - \varepsilon < 0$) and the product KK_s is increasing, then the integral in (5) always assumes a finite value.
- 3) If the instantaneous utility function (2) is not bounded from above and the product KK_s is decreasing, then the integral in (5) always assumes a finite value.

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