An analytical approach to determine the optimal length of paired drip laterals in uniformly sloped fields

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Abstract

Microirrigation plants, if properly designed, allow to optimize water use efficiency and to obtain quite high values of emission uniformity in the field. Disposing paired laterals, for which two distribution pipes extend in opposite directions from a common manifold, can contribute to reduce the initial investment cost, that represents a limiting factor for small-scale farmers of developing countries where, in the last decade, the diffusion of such irrigation system has been increasing.
Objective of the paper is to propose an analytical approach to evaluate the maximum lengths of paired drip laterals for any uniform ground slope, respecting the criteria to maintain emitter flow rates or the corresponding pressure heads within fixed ranges in order to achieve a relatively high field emission uniformity coefficient.
The method is developed by considering the motion equations along uphill and downhill sides of the lateral and the hypothesis to neglect the variations of emitters' flow rate along the lateral as well as the local losses due to emitters' insertions.
If for the uphill pipe, the minimum and the maximum pressure heads occurs at the upstream end and at the manifold connection respectively, on the downhill side, the minimum pressure head is located in a certain section of the lateral, depending on the geometric and hydraulic characteristics of the lateral, as well as on the slope of the field; a second relative maximum pressure head could also exist at the downstream end of the pipe.

The proposed methodology allows in particular to determine separately the number of emitters in uphill and downhill sides of the lateral and therefore, once fixing emitter's spacing, the length of the uphill and downhill laterals and the position of the manifold.
Applications and validation of the proposed approach, considering different design parameters, are finally presented and discussed.

Key-words
Microirrigation, Paired laterals, Optimal length

Introduction

Microirrigation is considered a convenient and efficient system allowing to keep the crop water demand to a minimum, while maintaining current levels of crop production; for this reason it is mostly used in arid regions where water resources for irrigation are limited.
The adoption and diffusion of microirrigation technology, in developed and developing countries, is consequent to economic factors (water price, cost of equipment, crop price), farm organization (size of the farm, experience of the farmer) and environmental conditions (precipitation, soil quality) (Genius et al, 2012).
Mainly in developing countries, small-scale farmers, have been sometimes reluctant to adopt this system due to the initial investment cost required for the equipment, that may be higher than those of other irrigation options.
In order to optimize water use efficiency and to reduce the initial investment cost, the design of the submain unit and its proper management play a key role to maximizing the emitter uniformity and the profitability of the investment. When using non pressure compensating emitters, the first step for designing a submain considers a range of pressure variation along the lateral, which can contribute to obtain the desired uniformity of water distribution in the entire submain. In fact, limiting the range of pressure head makes it possible to reduce the variability of flow rates discharged by the installed emitters.
The criterion of limiting the variation of emitter discharge to about ± 5% of the nominal flow rate or, alternatively, the variation of pressure head to about ± 10% of its nominal value, in order to obtain reasonable high values of distribution uniformity coefficients has been widely used to design drip irrigation single laterals or entire submains. Provenzano (2005) demonstrated that when the exponent x of the flow rate-pressure head relationship is equal to 0.5 and emitters are characterized by a good quality (emitters' manufacturer's variation

coefficient CV ≤ 0.03), such variation of discharge corresponds to a pressure variations of about 20% of the nominal value, and determines values of emission uniformity coefficient EU, as defined by Karmeli and Keller (1975), equal to EU = 90% or higher. Of course, the higher the emitter' CV value, the larger the interval of variability of the flow rates around the average value whereas, for a fixed CV, a lower variability of emitter flow rates is always related to a higher distribution uniformity.

Moreover, using paired laterals for which two distribution pipes extend in opposite directions from a common manifold, as represented in fig. 1, for a fixed pipe diameter, can allow maximizing the lateral length while maintaining the pressure variations within the considered range, so that the initial investment cost of the system can be reduced. Al-Samarmad (2002), considering two design criteria to determine lateral and manifold lengths for a given subunit and using local prices for installing and operating micro irrigation systems, found that the subunit cost decreases as lateral length increases up to a certain limit and then it starts to increase again.

The importance of an adequate analysis of trickle lateral hydraulics aimed to find the optimal length or diameter of laterals laid on sloping fields has been emphasized by Kang et al., (1996). In particular, the forward Step by Step (SBS) procedure, as unanimously recognized, represents the most affordable method to evaluate pressure heads and actual flow rates corresponding to all the emitters in the lateral even if, when applied from the uphill end to the downhill end of the lateral, allows to find the solution after tedious and time consuming iterations.

Despite a detailed analysis should require the evaluation of local losses due to emitter's insertion, whose importance has been emphasized by several Authors (Al Amoud, 1995, Bagarello et al., 1997, Juana et al., 1992, Provenzano et al., 2007), in all the cases when the number of emitter in the lateral and/or the variations of flow velocity due to the emitter connections are limited, such losses can be neglected. In fact, considering that local losses are usually evaluated as an α fraction of flow kinetic head, Provenzano and Pumo (2004) verified that local losses result less than 10% of the total losses for in-line emitters characterized by $\alpha \leq 0.3$ and spaced 1.0 m or more. More recently, Provenzano et al. (2014) on the basis of experiments carried out on five different commercial lay-flat drip tapes, due to the generally low values of α characterizing the emitters, evidenced that neglecting local losses generates an overestimation of the lateral lengths with differences equal to 8.9%, 3.6% and 1.6%, when emitter spacing is equal to 20 cm, 50 cm and 100 cm respectively.

When designing paired laterals, it is fundamental to evaluate the best position of the submain pipe (BSP), which was defined by Keller and Bliesner (2001) as the location of the manifold determining the same minimum pressure in uphill and downhill laterals. On level ground the length of both laterals is identical, whereas for any other field slope, the manifold has to be shifted uphill, in a position that balances the differences in elevation and pressure losses in both sides of the laterals. Based on their definition, Keller and Bliesner (2001) developed graphical and numerical solution methods.

In order to obtain the required uniformity of water application, Kang and Nishiyama (1996) proposed a method for design single and paired laterals laid on both flat and sloped fields based on the finite element method and the golden section search (Gill et al., 1989). For paired laterals, the method allows to obtain the operating pressure head and the BSP at which the maximum uniformity is produced for a fixed emitter discharge, once the lateral length or pipe diameter and other field conditions are given.

Recently, Jiang and Kang (2010), using the energy gradient line approach (Wu, 1975; Wu and Gitlin, 1975, Wu et al., 1986), proposed the best equation form aimed to evaluate the BSP according to the definition provided by Keller and Bliesner (2001) and developed a simple procedure to design paired laterals on sloped fields.

In this study, an analytical approach to design the optimal length of paired drip laterals laid on uniformly sloped fields and to determine the position of the manifold, under the hypotheses to neglect local losses due to the emitters' connections, is presented and discussed. Application and validation of the proposed approach, covering a combination of different design parameters, is finally presented and discussed.

Theory

Fig. 1 illustrates the typical layout of a submain in which the manifold, placed in a generic position, divides each lateral into two sections - uphill and downhill - of different length (paired lateral). Fig. 2 shows the scheme of a single paired lateral characterized by a length L and multiple outlets spaced S, laid on an uniformly sloped field. In the figure, the connection between the manifold and the lateral, the hydraulic grade line and the pressure head distribution are schematically illustrated. As can be observed, n_u and n_d indicate the number of emitters along the uphill and the downhill sides of the lateral, with n the total number of emitters, whereas i_{min}, represents the number of emitters installed in the downhill side of the lateral, from the manifold connection to the pipe section with the minimum pressure head.

For the uphill pipe, the minimum pressure head, $h_{\min}^{(u)}$, arises at the upstream end, whereas the maximum pressure head, $h_{\max}^{(u)}$, is at the manifold connection. On the other side, according to the geometric and hydraulic characteristics of the lateral, as well as to the slope of the field, the minimum pressure head for the downhill pipe, $h_{\min}^{(d)}$, can be located in a certain section of the lateral, whereas a second relative maximum pressure head, $h_{\max}^{(d)}$, could also exist at the downstream end of the pipe.

In order to achieve a relatively high field emission uniformity coefficient along the lateral, it is necessary to limit the variations of pressure head due to elevation changes and head losses. Therefore, indicating h_n the nominal pressure head of the emitter, the hydraulic design criteria of the lateral here considered, assumes that the working pressure heads of the generic emitter, h_i, in both uphill and downhill sides, have to be in the range between 0.9 h_n and 1.1 h_n.

For a lateral with given geometric and hydraulic characteristics, laid on an uniformly sloped field, according to the fixed maximum variations of pressure heads and to the elevation changes, an optimal (maximum) length, L_{opt}, can be identified.

In small diameter polyethylene pipes (PE), friction losses per unit pipe length, J, can be evaluated with the Darcy-Weisbach equation:

$$J = \frac{f}{D}\frac{V^2}{2g} \tag{1}$$

where f [-] is the friction factor, V is the mean flow velocity [m/s], D [m] is the internal pipe diameter and g [m^2/s] is the acceleration of gravity. According to the Blasius equation, friction factor can be expressed, as a function of Reynolds number R:

$$f = 0.316\, R^{-0.25} \tag{2}$$

For a single lateral (n_u = 0) with n emitters, under the hypothesis to neglect the variation of flow rates discharged by the emitters, the total friction losses between the first and the last emitter of the lateral, $\Delta h_f^{(d)}$, can be easily calculated according to Provenzano et al., (2005):

$$\Delta h_f^{(d)} = 0.0235 \frac{\nu^{0.25} S\, q_n^{1.75}}{D^{4.75}} \sum_{i=1}^{n-1} i^{1.75} \tag{3}$$

where ν [m^2 s^{-1}] is the water kinematic viscosity, S [m] is the emitter spacing, q_n [m^3 s^{-1}] is the average emitter discharge corresponding to h_n and i [-] is the generic emitter installed along the lateral.

In order to find analytical solution to design sloping laterals, the generalised harmonic number can be introduced into eq. (3):

$$\Delta h_f^{(d)} = K\ S\ H_{n-1}^{(-1.75)} \tag{4}$$

where H(.,.) is the generalised harmonic number in power -1.75, truncated at n, and K (-) is a parameter that, for the selected resistance law, depends on pipe diameter and emitter flow rate, as following:

$$K = 0.0246 \frac{\nu^{0.25} q_n^{1.75}}{D^{4.75}} \tag{5}$$

For a given lateral K is constant and assumes value ranging in the interval between 1.00e-05 and 1.00e-03, as evaluated according to the common ranges of variability of q_n (4 l/h $< q_n <$ 25 l/h) and D (0.012 m $< D <$ 0.020 m).

Accounting for the differences in emitters elevation and neglecting the kinetic head, the motion equation allows to determine the pressure head of the *i-th* emitter, h_i, along the uphill side, $h_i^{(u)}$, as well as along the downhill side of the lateral, $h_i^{(d)}$, as:

$$h_i^{(u)} = h_{\max}^{(u)} - \Delta h_f^{(u)} + K\ S\ H_{n_u - i}^{(-1.75)} + i\ S\ S_0 \tag{6a}$$

$$h_i^{(d)} = h_{\max}^{(u)} - \Delta h_f^{(d)} + K\ S\ H_{n_d - i}^{(-1.75)} - i\ S\ S_0 \tag{6b}$$

in which S_0 [-] is the field slope (negative downhill). Moreover, according to eq. (4), the total head losses in the uphill, $\Delta h_f^{(u)}$, and in the downhill, $\Delta h_f^{(d)}$, laterals can be evaluated as:

$$\Delta h_f^{(u)} = K\ S\ H_{n_u}^{(-1.75)} \tag{7a}$$

$$\Delta h_f^{(d)} = K\ S\ H_{n_d}^{(-1.75)} \tag{7b}$$

If considering the uphill side of the lateral, by imposing equal to 0.9 h_n the minimum allowed pressure head, $h_{\min}^{(u)}$, at the end of the lateral, and equal to 1.1 h_n the maximum pressure head at the manifold connection, eq. 6a, for $i = n_u$, can be rewritten as:

$$0.9\,h_n = 1.1\,h_n - \Delta h_f^{(u)} + n_u\ S\ S_0 \tag{8}$$

By introducing eq. (7a) into eq. (8) and by normalising the pressure head respect to S, the number of emitters in the uphill lateral, n_u, corresponding to the optimal (maximum) value, can be implicitly expressed as:

$$n_u = \frac{K}{S_0}\, H_{n_{u,opt}}^{(-1.75)} - 0.2\,\frac{h_n}{S_0 S} \tag{9}$$

Contrarily to eq. (6a) in which $h_i^{(u)}$ monotonically decreases with increasing i, and therefore the lowest pressure head occurs at the uphill end of the lateral, eq. (6b) admits a minimum value of pressure head, $h_{min}^{(d)}$, in a certain section of the downhill lateral. In order to know the exact location of this minimum, it is necessary to derive eq. (6b) with respect to i. The derivative of a discrete variable, as i was denoted, exists for any i value under the assumption that di = dS/S. Thus, the partial derivative of eq. (6b) respect to i, yields:

$$\frac{\partial h_i^{(d)}}{\partial i} = -S\ S_0 + 1.75K\ S\left(\zeta\left(-0.75\right) - H_{n_d - i}^{(-0.75)}\right) \tag{10}$$

in which $H_{n_d-i}^{(-0.75)}$ is the generalised harmonic number and ζ (.,.) is the Riemann Zeta function of argument (.), equal respectively to:

$$H_{n_d-i}^{(-0.75)} = \sum_{i=1}^{n_d-i} i^{0.75} \tag{11}$$

$$\zeta\left(-0.75\right) = -\frac{1}{1.75} + \sum_{n=0}^{\infty}\left(-1\right)^n \frac{\gamma_n\left(-1.75\right)^n}{n!} = -0.1336 \tag{12}$$

where γ_n are the Stieltjes constants. The Riemann Zeta function of eq. (12) is a particular case of the more general Hurwitz–Lerch Zeta function (Agnese et al., 2014). By imposing eq. (10) equals to zero, the emitter, i_{min}, in which the minimum pressure head, $h_{min}^{(d)}$ is located, can be determined by solving the following implicit equation:

$$H_{n_d - i_{\min}}^{(-0.75)} = \zeta(-0.75) - \frac{S_0}{1.75K} \tag{13}$$

As expected, eq. (13) shows that i_{min} only depends on the number of the emitters along the downhill side of the lateral, n_d, on the value of K, as well as on the slope of the lateral, S_0, but it is interesting to notice that it does not depend on the spacing S.

Fig. 3 shows, for different K values, the distance $n_d - i_{min}$, between the point (emitter) characterized by the minimum pressure head ($h_i = h_{min}^{(d)}$) and the downhill end of the lateral, as a function of the lateral slope. As can be observed, the value n_d - i_{min} increases with increasing S_0, whereas for a fixed S_0, the position n_d - i_{min} increases with decreasing K.

In the particular case of a lateral laid on a level field ($S_0 = 0$), as evident, the minimum pressure head is located at the downstream end of the lateral ($i_{min} = n_d$), for any K value. On the other hand, for a fixed K, the position of the emitter with the minimum pressure in the downhill lateral head, at rising S_0, shifts uphill.

In order to determine the maximum number of emitters in the downhill lateral, it could be possible i) to fix the minimum allowed pressure head at $i = i_{min}$ and to control that $h_{max}^{(d)} \leq 1.1\, h_n$ or alternatively ii) to fix the maximum allowed pressure head at the end of the downhill lateral and verifying that $h_{min}^{(d)} \geq 0.9\, h_n$. However, according to the results of application (not showed), the former option provides a maximum number of emitters always higher than the latter. Thus, in order to determine the maximum number of emitters in the downhill lateral, the relative minimum admissible pressure head (0.9 h_n) at $i = i_{min}$, has be imposed into eq. (6b):

$$-0.2\frac{h_n}{S} = -K\, H_{n_d}^{(-1.75)} + K\, H_{n_d - i_{\min}}^{(-1.75)} - i_{\min}\, S_0 \tag{14}$$

To find the value n_d satisfying the imposed condition for any fixed slope of the lateral, the system of eqs. (13) and (14) has to be solved. However, the solution in terms of the pairs (n_d, i_{min}) could determine, for $i > i_{min}$, pressure heads higher than 1.1 h_n. This last condition occurs for ground slope higher than a threshold value, $|S_0^{th}|$, representing the maximum value for which operating pressure heads along the entire downhill lateral are in the desired range.

In order to find $|S_0^{th}|$ and the associated optimal number of emitters in the downhill lateral, $n_{d,opt}^{th}$, the maximum pressure head at the end of the lateral has also to be fixed to the maximum admitted value (i.e. for $i = n_d$, $h_{max}^{(d)} = 1.1\ h_n$). Thus, by using eq. (7b) and by considering that for $i = n_d$, $H_{n_d-i}^{(-1.75)} = 0$, eq. (6b) can be rearranged as:

$$n_{d,opt}^{th}\ S_0^{th} + K\ H_{n_{d,opt}^{th}}^{(-1.75)} = 0 \tag{15}$$

The system represented by eqs. (13), (14) and (15) can be solved in terms of $n_{d,opt}^{th}$, $i_{\min}$ and $|S_0^{th}|$, so that, once $n_{d,opt}^{th}$ is known, the optimal length of the entire lateral, corresponding to the threshold ground slope, can be determined as $n_{opt}^{th} = n_{u,opt}^{th} + n_{d,opt}^{th}$.

Examples of application

In the following examples the proposed procedure is applied in order to determine the maximum number of emitters in a paired lateral, under different internal pipe diameters, D, nominal pressure heads, h_n, emitter spacing, S, and flow rates, q_n, for two different ground slopes, S_0.

The first case is related to a lateral with D = 20 mm, q_n = 20 l/h and considers two values of the ratio h_n/S (h_n/S = 20 and h_n/S = 40). According to eq. (5), K value is equal to 5.82e-05.

In Fig. 4a-b the number of emitters in the uphill lateral, n_u, evaluated with eq. 9, the pairs n_d, i_{min}, obtained by solving eqs. (13) and (14), as well as the sum, $n_d + n_u$, are represented as a function of the lateral slope $|S_0|$, for h_n/S = 20 (Fig. 4a) and for h_n/S = 40 (Fig. 4b). In the secondary vertical axes, the dimensionless nominal pressure head at the end of the downhill lateral, $h_{\max}^{(d)}/S$, as well as the minimum and the maximum, 0.9 h_n/S and 1.1 h_n/S, are also showed. As expected, with increasing $|S_0|$, n_u decreases whereas n_d increases, being the values n_u and n_d equals for S_0 = 0, and therefore when the manifold connection is placed in the middle of the lateral. As an example, for h_n/S = 20 (Fig. 4a), the optimal number of emitters along the entire lateral, $n_{opt} = n_u + n_d$, results maximum (n_{opt} = 165) for S_0 = 0 and decreases with increasing $|S_0|$, until reaching a minimum value, n_{opt}^{th} = 158, for $|S_0| = |S_0^{th}|$, being $|S_0^{th}|$ =

9.4 %. As can be observed in Fig. 4a, even if for any $|S_0| > |S_0^{th}|$, an optimal number of emitters n_{opt} higher than n_{opt}^{th} could be evaluated, the solution cannot be accepted because the pressure head at the downhill end of the lateral results higher than the maximum allowable. In fig. 4a, it can also be noticed that, at increasing $|S_0|$, the location of the minimum pressure head (dashed curve) shifts upstream, as a consequence of the results illustrated in fig. 3, passing from i_{min} = 83 (downhill end of the lateral) for S_0 = 0 to i_{min} = 53 for $S_0 = S_0^{th}$ = - 9,4%. Similar observations can be evidenced in the case of h_n/S = 40 (Fig. 4b), to which correspond an optimal number of emitters n_{opt} = 212 for S_0 = 0 and n_{opt}^{th} = 204 ($n_{u,opt}^{th}$ = 48, $n_{d,opt}^{th}$ = 156) evaluated for the threshold slope S_0^{th} = -14,6 %.

Moreover, the value of the normalized pressure head at the end of the lateral, $h_{\max}^{(d)}/S$, increases with the slope, becoming higher than 1.1 h_n/S for $|S_0| > |S_0^{th}|$, as can be analytically quantified by solving the system of eqs. (13), (14) and (15). Of course, all the solutions obtained for $|S_0| > |S_0^{th}|$ cannot be accepted.

The second examined case corresponds to a lateral having internal diameter D = 16 mm and nominal emitters discharge, associated to the pressure head h_n, q_n = 4 l/h (K = 1.00e-05).

Similarly to Fig. 4a-b, Fig. 5a-b shows the number of emitters in the uphill, n_u, and downhill n_d, lateral, the values $i_{\min}$, as well as the sum, $n_d + n_u$, as a function of the lateral slope $|S_0|$, and allows one to evaluate the optimal lateral length for h_n/S = 20 (Fig. 5a) and for h_n/S = 40 (Fig. 5b).

As an example, for h_n/S = 20 and a field slope equal to – 2.0 %, the number of emitters in the uphill and in the downhill sides of the lateral result of 115 and 190 (n_{opt} = 305), respectively, to which corresponds acceptable values of the ratio $h_{\max}^{(d)}/S$ that, at the end of the lateral, is equal to 19.0, whereas for $S_0 = S_0^{th}$ = - 5.0 %, n_{opt}^{th} = 300 is obtained by summing $n_{u,opt}^{th}$ = 71 and $n_{d,opt}^{th}$ = 229.

If comparing the results of the two considered examples, it can be observed that to the lower K value (second example) corresponds, for any field slope, an optimal number of emitters systematically higher than that obtained in the first example. In particular, for K = 5.82e-05 and a field slope of - 2%, the optimal number of emitters results equal to 163.

By the analysis of Fig. 4 and Fig. 5, it is possible to verify that n_{opt}^{th} corresponds to the maximum number of the emitters in a lateral laid on a ground having slope equal to $|S_0^{th}|$, for which operating pressure heads are in the admissible range (0.9 h_n/S ÷ 1.1 h_n/S); in particular, for $n = n_{opt}^{th}$, the minimum pressure head, 0.9 h_n/S, is imposed at $h_{\min}^{(u)}$ and $h_{\min}^{(d)}$, whereas the maximum, 1.1 h_n/S, is imposed at $h_{\max}^{(u)}$ and $h_{\max}^{(d)}$ (Fig. 2). Thus, the knowledge of n_{opt}^{th} and $|S_0^{th}|$ has interesting implications when the optimal length of paired laterals in uniformly sloped ground has to be evaluated. In fact, for a lateral of fixed geometric and hydraulic characteristics, any field slope lower than $|S_0^{th}|$ determines acceptable solutions in terms of maximum number of emitters to be installed along the entire lateral, with pressure heads always within the admitted range. The contemporary knowledge of the corresponding number of emitters in the uphill lateral, allows one to establish the position of the manifold connection. On the other hand, if field slope $|S_0|$ is higher than $|S_0^{th}|$, the corresponding n_d determines unacceptable pressure heads at the end of the lateral, higher than the maximum allowed.

To generalize the results to the usual values of discharges and internal diameters, i.e. K = 1.00e-05 ÷ 1.00e-03, the system of eqs. (13), (14) and (15) has been solved in terms of $n_{d,opt}^{th}, i_{\min}$ and S_0^{th}, in order to obtain, as a function of K, the optimal length of the entire lateral, $n_{opt}^{th} = n_{u,opt}^{th} + n_{d,opt}^{th}$, corresponding to the particular case for which $|S_0|=|S_0^{th}|$.

Fig. 6 shows, as a function of K, the number of the emitters in uphill, n_u^{th} and downhill n_d^{th} laterals, the location of the emitter with the minimum pressure head, i_{min}, as well as the optimal number of emitters in the entire lateral $n_{opt}^{th} = n_u^{th} + n_d^{th}$, for h_n/S = 20 (Fig. 6a) and for h_n/S = 40 (Fig. 6b). In the secondary vertical axes, the threshold value of the slope, $|S_0^{th}|$, is also represented as a function of K. The black dots indicate the threshold values of $|S_0^{th}|$, for both K = 5.82e-05 and K =1.00e-05, for h_n/S = 20 (Fig. 6a) and h_n/S = 40 (Fig. 6b).

Analysis of Fig. 6a,b evidences, as expected, that parameter K determines a noticeable influence on the number of emitters (optimal lateral length). In particular, for both the selected values of h_n/S (20 and 40), the higher the value of K (higher q_n or lower D) the lower the number of emitters. Moreover, for a fixed K, the threshold ground slope increases with h_n/S. As an example, for K = 1.00e-4, $|S_0^{th}|$ is equal to -11.4 % and -17.7 %, for h_n/S = 20 and h_n/S = 40, respectively. Finally, for any K value, increasing h_n/S from 20 to 40, determines a constant increment, equal to 29%, of the optimal number of emitters to be installed and therefore of the optimal length of the lateral.

Validation of the proposed approach

The validity of the proposed approach has been assessed on terms of its ability to predict the variations of pressure heads along the lateral and consequently, for a certain model of emitter, to estimate the distribution of discharged flow rates, according to the actual flow rate-pressure head relationship. In particular, using the iterative forward step-by-step (SBS) procedure, starting from the manifold connection to the end of both the downhill and the uphill sides of the lateral, it was possible to evaluate the differences on operating pressure heads and the subsequent errors in emitter flow rates, associated to the hypothesis of a constant emitter discharge (x = 0) assumed to derive eq. (3).

Towards this aim, the SBS procedure has been applied for a lateral characterized by D = 20 mm and q_n = 20 l/h (K = 5.82e-05) laid i) on a field slope $S_0 = S_0^{th}$ = - 9.4%, as obtained for h_n/S = 20 (case A, n_u^{th} = 38, n_d^{th} = 120) and ii) on a field slope $S_0 = S_0^{th}$ = -14.6% as evaluated for h_n/S = 40 (case B, n_u^{th} = 48, n_d^{th} = 156). In the former case an emitter spacing S = 1.0 m was considered, whereas in the latter S = 0.5 m, so that in both cases h_n resulted equal to 20 m. Moreover, two different flow rate-pressure head relationships ($q=k\ h^x$) expressed by k = 1.24e-06 m^2/s and x = 0.5 (case A1 and B1), and by k = 2.87-07 m^2/s and x = 1.0 (case A2 and B2), were examined.

Fig. 7a,b shows the distributions of pressure heads along the lateral evaluated for case A and B respectively, under the hypothesis of constant emitter flow rates (x = 0) or assuming the other two flow rate-pressure head relationships obtained for x = 0.5 and x = 1.0. According to the results, on both the uphill and downhill sides of the lateral, the value of pressure head corresponding to the generic emitter tends to rise at increasing x, with maximum differences, for x = 0.5 and for x = 1.0, equal respectively to -1.12 % and -1.74 % for case A, and to -1.47 % and -2.24 % for case B. Therefore, the assumption of a constant emitter flow rate determines a quite slight underestimation of the operating emitter pressure heads along the entire lateral. It is also interesting to observe that the position where the minimum pressure head occurs does not depend on the value of the exponent of the flow rate-pressure head relationship. Fig. 8a,b shows, for case A and case B, as a function of the lateral length, the errors on flow rates calculated by considering the pressure head distribution obtained with the proposed approach (x = 0) and the corresponding actual values determined by using the SBS procedure for x = 0.5 and x = 1.0, expressed as a percentage of the latter. As can be observed, for case A, the errors associated to the discharged flow rates result lower than -0.56 % and -

1.74 % for x = 0.5 and x = 1.0, whereas, for case B, lower than -0.74 % and -2.24 % for x = 0.5 and x = 1.0, and therefore always insignificant for practical applications.

Conclusions

The paper presents an analytical approach to evaluate the optimal length of paired drip laterals placed on uniformly sloped grounds. In particular, once fixed the geometric and hydraulic characteristics of the lateral, the maximum number of emitters in the uphill and downhill sides and therefore the optimal lateral length and the position of the manifold, can be determined by considering a simplified friction losses evaluation procedure, that assumes constant emitter flow rates and the criteria to fix the variation of pressure head to ± 10% of its nominal value along the entire lateral. The methodology neglects local losses, so that it can be applied when the morphology of emitter connections do not produce significant reductions of the lateral cross section.

Two examples of application of the proposed approach, covering different values of nominal flow rates and internal pipe diameters (summarized in a single variable, K) and for different combinations of the nominal pressure head and emitter spacing (h_n/S), are presented and discussed. Application of the procedure evidenced that, for any field slope, the optimal number of emitters in the paired lateral increases at decreasing K. Moreover, by fixing K and h_n/S, it exists a threshold ground slope according to which operating pressure heads along the entire downhill lateral are in the desired range, assuming its maximum admissible value at the manifold connection and at the end of the lateral and its minimum admissible in a generic section of the lateral. This threshold ground slope tends to increase at increasing h_n or at decreasing S.

The validation of the proposed approach has been then assessed in terms of its ability to predict the variations of pressure heads along the lateral and consequently to estimate the distribution of emitter flow rates, according to the actual flow rate-pressure head relationship. In particular, application of the iterative forward step-by-step (SBS) procedure, evidenced that the value of pressure head corresponding to the generic emitter tends to rise at increasing values of the exponent x, of the flow rate-pressure head relationship. However, the maximum differences of operating pressure heads along the entire lateral, for x=0.5 and x=1.0 resulted respectively equal to -1.12 % and -1.74 % for the first examined case, and to -1.47 % and -2.24 % for the second.

According to the recognized pressure head, the maximum error associated to the discharged flow rates in the first case resulted always lower than -0.56 % (*x* = 0.5) and -1.74 % (*x* = 1.0), whereas in the second case, lower than -0.74 % (*x* = 0.5) and -2.24 % (*x* = 1.0) and hence in both the examined examples insignificant for practical applications.

Acknowledgements

Research was co-financed by Ministero dell'Istruzione, dell'Università e della Ricerca (MIUR) and FFR 2012-2013 granted by Università degli Studi di Palermo. The contribution to the manuscript has to be shared between authors as following: Theory and applications of the proposed procedure were carried out by Giorgio Baiamonte. All the authors analyzed results and wrote the text. The Authors wish to thank the anonymous reviewers for the helpful comments and suggestions during the revision stage.

List of symbols

D [m] internal pipe diameter

f [-] friction factor

g [m^2/s] acceleration of gravity

h_i [m] pressure head of the generic emitter i

$h_i^{(u)}$ [m] pressure head of the *i-th* emitter in the uphill lateral

$h_i^{(d)}$ [m] pressure head of the *i-th* emitter in the downhill lateral

$h_{min}^{(u)}$ [m] minimum pressure head in the uphill lateral

$h_{max}^{(u)}$ [m] maximum pressure head at the manifold connection

$h_{min}^{(d)}$ [m] minimum pressure head in the downhill lateral

$h_{max}^{(d)}$ [m] maximum pressure head at the downhill end of the lateral

h_n [m] nominal emitter's pressure head

H(.,.) generalised harmonic number

i [-] generic emitter of the lateral counted from the manifold connection

i_{min} [-] number of emitters in downhill lateral, from the manifold connection to the section with minimum pressure head

J [-] friction losses per unit pipe length

K (-) parameter

L [m] length of the lateral

L_{opt} [m] optimal (maximum) length of the lateral

n [-] total number of emitters in the entire lateral

n_u [-] number of emitters in the uphill lateral

n_d [-] number of emitters in the downhill lateral

$n_{d,opt}^{th}$ [-] optimal number of emitters in the downhill lateral corresponding to S_0^{th} [%]

n_{opt}^{th} [-] optimal number of emitters in the entire lateral corresponding to S_0^{th} [%]

n_{opt} [-] optimal number of emitters in the lateral

n_x [-] generic emitter of the lateral counted from the uphill end of the lateral

q_n [m^3 s^{-1}] nominal emitter discharge

R [-] Reynolds number

S [m] emitter spacing

S_0 [%] slope of the lateral

S_0^{th} [%] threshold ground slope for which operating pressure head at the end of the downhill lateral is equal to 1.1 h_n

V [m/s] mean flow velocity

x [-] exponent of the flow rate-pressure head relationship

$\Delta h_f^{(d)}$ [m] total friction losses in the downhill lateral

$\Delta h_f^{(u)}$ [m] total friction losses in the uphill lateral

γ_n Stieltjes constants

ν [m^2 s^{-1}] kinematic water viscosity

ζ Riemann Zeta function

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Figure_1
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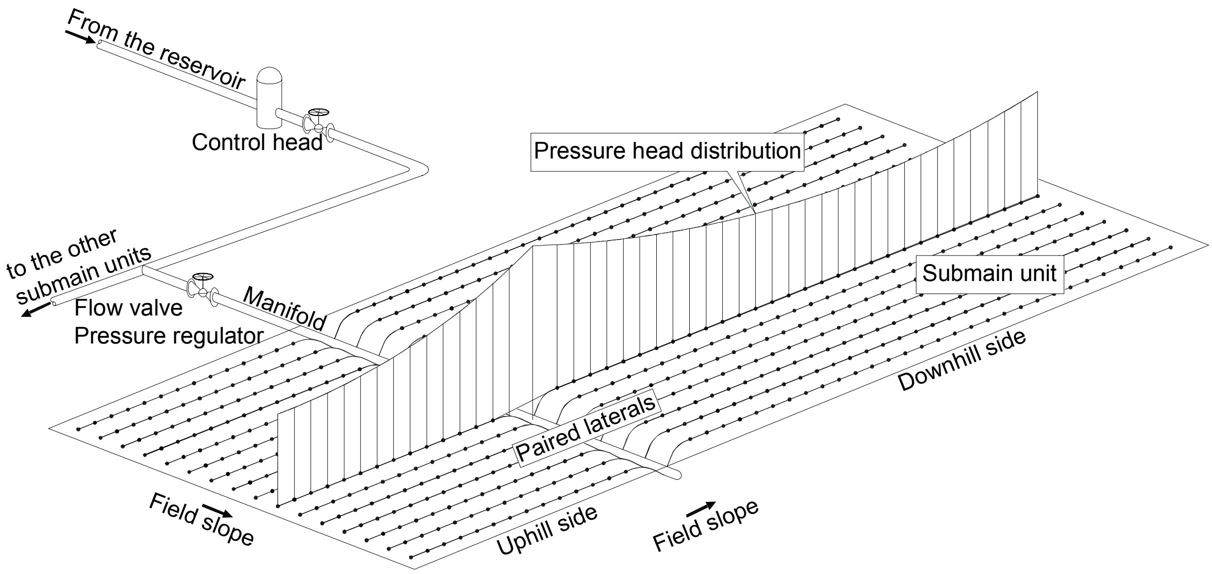

Fig. 1 – Schematic layout of a submain unit with paired laterals. The pressure head distribution line for a generic lateral is also indicated.

Figure_2

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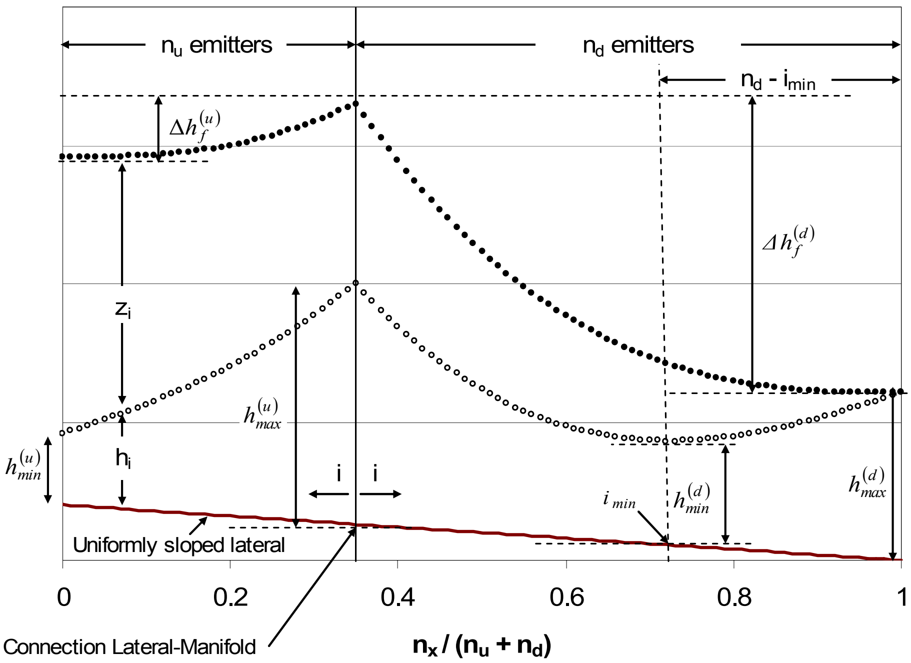

Fig. 2 – Scheme of a microirrigation paired lateral laid on a uniformly sloped field. White and black dots indicate the pressure head distribution and the hydraulic grade line, respectively.

Figure_3
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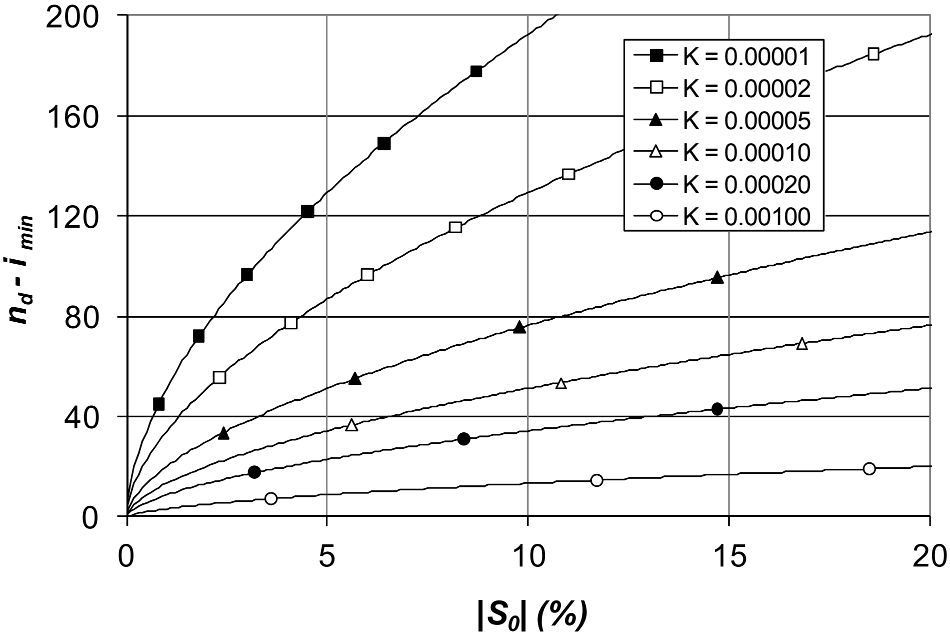

Fig. 3 – Relative position of the emitter characterized by the minimum pressure head along the lateral as a function of $|S_0|$, for different values of the constant K.

Figure_4

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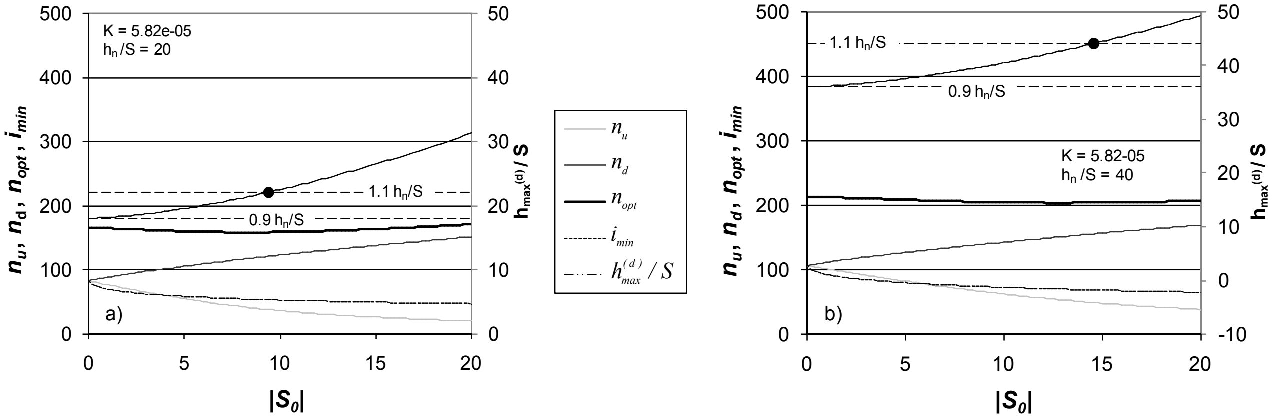

Figure 4 – Number of emitters in the uphill lateral, n_u, evaluated with eq. 9, pairs (n_d, i_{min}) obtained by eqs. (13) and (14), and sum $n_{opt} = n_d + n_u$, as a function of the lateral slope $|S_0|$, for K = 5.82e-05, h_n/S = 20 (a) and h_n/S = 40 (b). In the secondary vertical axes, the dimensionless nominal pressure head at the end of the downhill lateral, hn_d/S, as well as the minimum and the maximum admissible, 0.9 h_n/S and 1.1 h_n/S , are also indicated. Black dots indicate the slope threshold value, $\left|S_0^{th}\right|$.

Figure_5
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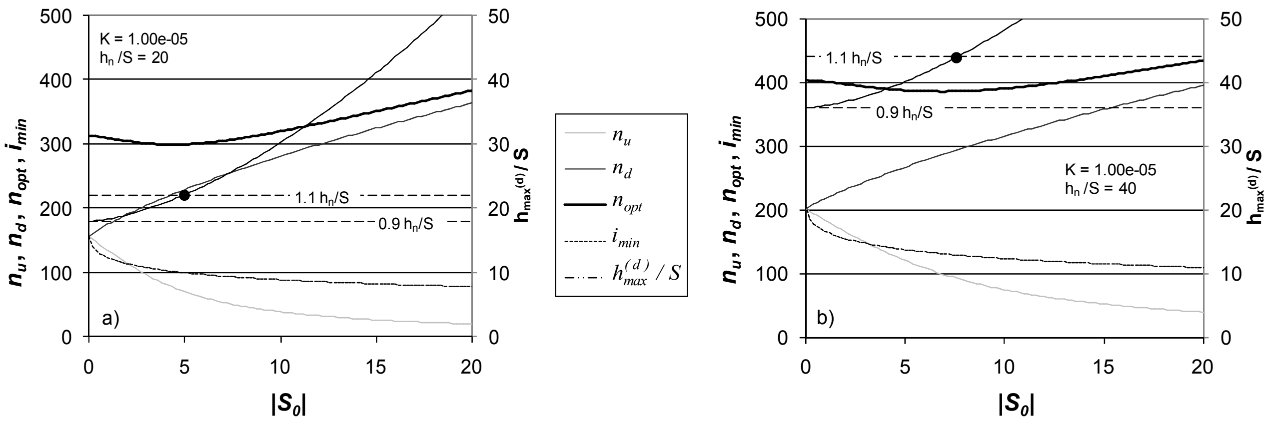

Figure 5 – Number of emitters in the uphill lateral, n_u, evaluated with eq. 9, pairs (n_d, i_{min}) obtained by eqs. (13) and (14), and sum $n_{opt} = n_d + n_u$, as a function of the lateral slope $|S_0|$, for K = 1. 00e-05, h_n/S = 20 (a) and h_n/S = 40 (b). In the secondary vertical axes, the dimensionless nominal pressure head at the end of the downhill lateral, hn_d/S, as well as the minimum and the maximum admissible, 0.9 h_n/S and 1.1 h_n/S , respectively. Black dots indicate the slope threshold value, $\left|S_0^{th}\right|$.

Figure_6

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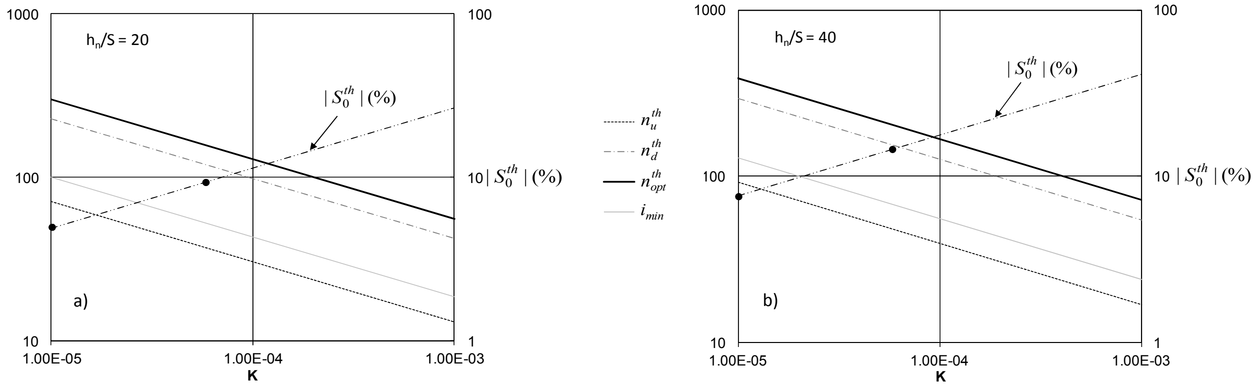

Figure 6 – Number of the threshold emitters in the uphill lateral, n_u^{th}, and in the downhill lateral n_d^{th}, corresponding location of the emitter with the minimum pressure head, i_{min}, and optimal number of emitters in the entire sloped lateral $n_{opt}^{th} = n_u^{th} + n_d^{th}$, as a function of K, for h_n/S = 20 (a) and for h_n/S = 40 (b). In the secondary vertical axes, the slope threshold $|S_0^{th}|$ is also represented. Black dots indicate the slope thresholds corresponding to $h_n/S = 20$ (Figs. 4a and 5a), and to $h_n/S = 40$ (Figs. 4b and 5b).

Figure_7

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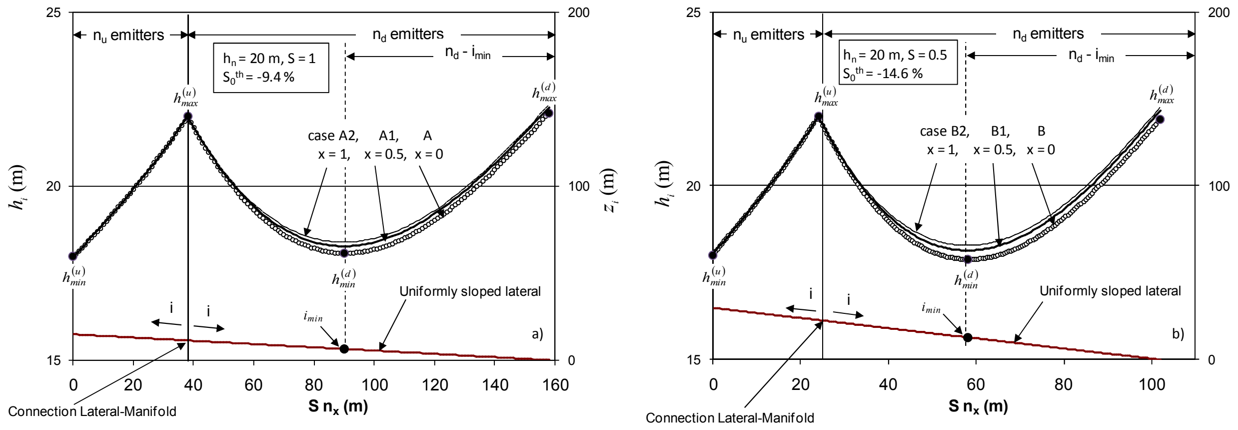

Figure 7 – Distributions of pressure heads along the lateral for case A (a) and B (b), under the hypothesis of constant emitter flow rates (x = 0) or assuming the other two flow rate-pressure head relationships obtained for x = 0.5 and x = 1.0.

Figure_8
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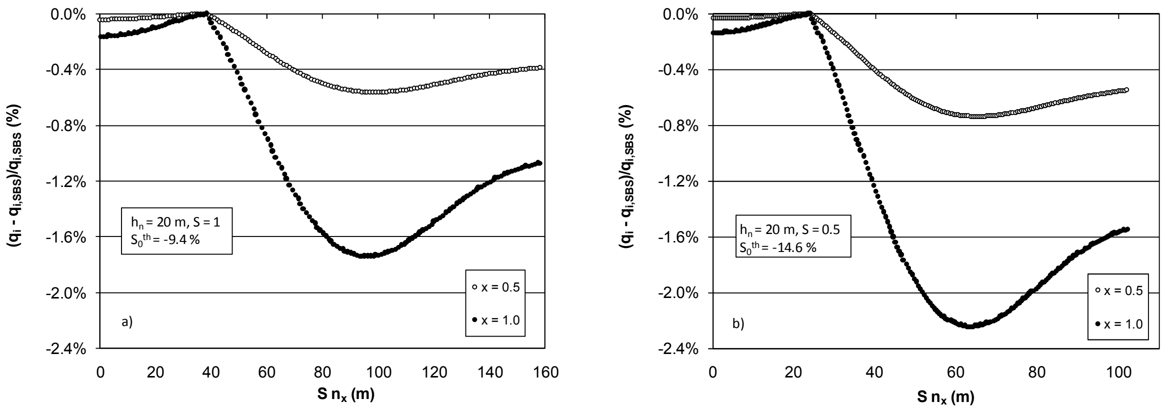

Figure 8 – Errors on flow rates, as a function of the lateral length, calculated by considering the pressure head distribution obtained with the proposed approach (x = 0) and the corresponding actual values determined by using the SBS procedure with exponents of the flow rate-pressure head relationship equal to 0.5 and = 1.0.

FIGURE CAPTIONS

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