Multiobjective Evolutionary Optimization of Quadratic Takagi-Sugeno Fuzzy Rules for Remote Bathymetry Estimation

Marco Cococcioni, *IEEE, Senior Member*, and Beatrice Lazzerini, *IEEE, Member*
Department of Information Engineering
University of Pisa
Pisa, 56122 - Italy
{m.cococcioni, b.lazzerini}@iet.unipi.it

***Abstract*—In this work we tackle the problem of bathymetry estimation using: i) a multispectral optical image of the region of interest, and ii) a set of *in situ* measurements. The idea is to learn the relation that between the reflectances and the depth using a supervised learning approach. In particular, quadratic Takagi-Sugeno fuzzy rules are used to model this relation. The rule base is optimized by means of a multiobjective evolutionary algorithm. To the best of our knowledge this work represents the first use of a quadratic Takagi-Sugeno fuzzy system optimized by a multiobjective evolutionary algorithm with *bounded complexity*, i.e., able to control the complexity of the consequent part of second-order fuzzy rules. This model has an outstanding modeling power, without inheriting the drawback of complexity due to the use of quadratic functions (which have complexity that scales quadratically with the number of inputs). This opens the way to the use of the proposed approach even for medium/high dimensional problems, like in the case of hyper-spectral images.**

***Index Terms*—Bathymetry estimation; quadratic Takagi-Sugeno fuzzy rules; remotely sensed optical images; multiobjective evolutionary algorithms.**

I. INTRODUCTION

The rapid assessment of the bathymetry of a coastal area is more and more demanded by both civilian and military sectors. As an example, assessing the bathymetry near the mouth of a river after a flood could significantly help civil defense agencies in conducting search and rescue operations. In the military context, rapid bathymetry estimation is very useful in amphibious landing operations, especially for uncharted or rapidly changing coastal regions [23].

In recent years several studies [15][23][25][27] have proven that a rapid assessment of bathymetry can be remotely achieved, provided that i) a multi/hyper-spectral optical image is available, acquired by satellite/airborne sensors, and ii) a sufficient number of *in situ* measurements is available, acquired by unmanned vehicles (USVs, AUVs, gliders, etc.).

The underlying idea for remote bathymetry estimation consists in employing supervised computational intelligence tools, like neural networks or fuzzy systems, trained using those spatial points having both an *in situ* depth measurement and an optical spectral signature attached to them. What we can expect in the near future is that such an approach will be more and more successful, due to the combination of higher spatial and spectral resolution optical sensors and to cheaper vehicles able to gather the *in situ* measurements. In the meantime we can still improve bathymetry estimations by working on the algorithmic side, i.e., by using more powerful techniques able to achieve a better accuracy using the current quality of satellite images and quantity of *in situ* measurements.

In this work we propose a different approach with respect to the algorithm we used in the past to solve the bathymetry estimation problem [15]. While in our previous work we used a first-order Takagi-Sugeno (TS) fuzzy rule-based system [31] (i.e., a linear function in the consequent clause of the rule), in this work we use a quadratic one. The new model uses quadratic functions of the inputs, which enables higher modeling power and expressiveness, but with the obvious drawback of a greater computational complexity. Indeed, the number of explaining variables (the regressors) increases quadratically with the number of inputs (i.e., with the number of spectral bands available in the optical image used). To overcome this issue, we present a Multiobjective Evolutionary Algorithm (MEA) able to control (and bound) the complexity of the system, in order to search for the least complex structure able to achieve the same accuracy level. The complexity is measured in terms of number of free parameters, which is influenced by the number of rules and, for each rule, by the number of inputs and the number of regressors involved. Our goal has been achieved by introducing a specially tailored chromosome representation, able to model which subset of the regressors has to be used for each rule in the system, among the $(M+1)\cdot(M+2)/2$ available ones (M being the number of spectral bands).

In the experimental section we prove that not only in the final Pareto front approximation, obtained at the end of the evolutionary optimization process, some solutions are more accurate than the solution found by our previous linear approach [15], but also that these solutions are less complex. This remarkable result has been possible due to the fact that here we are using a more powerful and expressive modeling system (fuzzy rules with quadratic terms) combined with an improved algorithm for selecting only a subset of the available regressors, rule by rule.

II. BATHYMETRY ESTIMATION: PROBLEM STATEMENT

Bathymetry estimation using remotely sensed images has been widely studied since the 1970's, when the first empirical models were proposed [28][29].

Thus the remotely sensed radiance, at specific wavelengths, can be related to the depth, especially when adopting a supervised learning method coupled with the availability of some *in situ* measurements (the latter collected by ships, AUVs or underwater gliders [22]).

In this paper we propose the use of neuro-fuzzy rules [24], as done in [15], as a supervised technique able to automatically create a model that predicts the depth from the reflectances measured by sensors positioned on-board of satellites. Examples of good reflectances (the inputs to our model) are those provided by high resolution (both in space and in frequency) remotely sensed optical images, such as QuickBird, IKONOS, or WorldView-2, to name a few.

Under this approach, the problem consists in deriving a fuzzy model from a training set made of pairs $\{{}^p\mathbf{l}, {}^p\hat{d}\}_{p=1}^{P}$, where ${}^p\mathbf{l} \in \Re^M$ is the vector of the M reflectances associated with the pth pixel in the image for which an *in situ* depth is available (${}^p\hat{d}$). In this study we have used QuickBird images, which provide three spectral bands, so M=3 (${}^p\mathbf{l} = [{}^pl_1, {}^pl_2, {}^pl_3]^T$ for the pth sample in the training set).

Thus the three radiance bands of the image corresponding to wavelengths centered in the visible spectrum were used as input for the fuzzy inference system. The output of the system represents the estimated bottom depth, while the *in situ* measured depth represents the desired output from the system, that is the target.

III. THE QUADRATIC TS MODEL

Fuzzy rule-based regression is usually approached using zero-order or first-order TS fuzzy rules. The use of second-order polynomials in the consequent part of the rules has started to be used only recently [4][6][17][18][19]. The motivation is their higher complexity, which was intractable with the hardware of decades ago. Recently, even high-order TS models have been investigated ([2][20][30]). Generally speaking, these studies prove that the use of more complex consequent functions allows reaching the same accuracy using fewer rules. In this study we focus our attention on quadratic TS models, since they can be a meaningful choice when one needs a more powerful modeling tool but without sensibly increasing the search space.

Let $\{L_1 ... L_M\}$ be a set of variables and $\mathbf{l} = [l_1 ... l_M]^T$ a generic input vector ($l_m \in \Re, \ \forall m = 1...M$). Let U_m $(m = 1...M)$ be the universe of variable L_m and let $P_m = \{A_1^m ... A_{T_m}^m\}$ be a fuzzy partition of U_m made of T_m fuzzy sets. To simplify the discussion, in the following we will use $T_m = T$, i.e., the same granularity for all the inputs. The ith rule of a quadratic TS fuzzy model made of I rules has the form:

r_i: **if** L_1 **is** $A_{z_{i,1}}^1$ **and** … L_M **is** $A_{z_{i,M}}^M$

$$\textbf{then } d_i(\mathbf{l}) = \alpha_i + \sum_{j=1}^{M} \varepsilon_{i,j} l_j + \sum_{j=1}^{M} \sum_{h=j}^{M} \sigma_{i,j,h} l_j l_h$$

where $A_{z_{i,m}}^m$ is the $z_{i,m}th$ fuzzy set defined on the mth variable, and α_i, $\varepsilon_{i,j}$ and $\sigma_{i,j,h}$ are real numbers.

The output of the model $d(\mathbf{l})$, associated with input $\mathbf{l}$, can be computed as $d(\mathbf{l}) = \sum_{i=1}^{I} v_i(\mathbf{l}) \cdot d_i(\mathbf{l})$, where $v_i(\mathbf{l}) = \dfrac{w_i(\mathbf{l})}{\sum_{i=1}^{I} w_i(\mathbf{l})}$ and $w_i(\mathbf{l}) = \prod_{m=1}^{M} A_{z_{i,m}}^m(\mathbf{l})$.

Thus when the pth input ${}^p\mathbf{l}$ of a training set made of P points $\left\{{}^p\mathbf{l}, {}^p\hat{d}\right\}_{p=1}^{P}$ (where ${}^p\hat{d}$ is the corresponding desired output) is fed to the system, it provides the estimated output ${}^pd$. The identification problem consists in optimizing the parameters (the fuzzy sets involved, the parameters in the *then part*, etc…) in order to minimize the error between the predicted output ${}^pd$ and the given desired output ${}^p\hat{d}$.

Please notice that the indexes $z_{i,m}$ can be arranged into the matrix $\mathbf{Z} = \{z_{i,m}\}$, $\mathbf{Z} \in \Re^{I \times M}$. We allow $z_{i,m}$ to be zero, meaning that the corresponding variable plays no role in the associated rule (it is a *don't care* clause). For example, the following matrix $\mathbf{Z}$:

$$\mathbf{Z} = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 1 \\ 3 & 0 & 0 \end{bmatrix}$$

in an M=3 scenario is associated with this antecedent part:

r_1: **if** L_2 **is** A_1^2 **and** L_3 is A_3^3 **then** …

r_2: **if** L_2 **is** A_2^2 **and** L_3 is A_1^3 **then** …

r_3: **if** L_1 **is** A_3^1 **then** ...

We can add a constraint on the maximum number of non *don't care* conditions, as done in [5], in order to bound the complexity of the antecedent part. In the rest of the paper this constraint has been named $\Gamma_{\max}$.

By denoting $q_{i,0} = \alpha_i$, $q_{i,1} = \varepsilon_{i,1}$, …, $q_{i,M} = \varepsilon_{i,M}$, $q_{i,M+1} = \sigma_{i,1,1}$, …, $q_{i,\frac{(M+1)(M+2)}{2}-1} = \sigma_{i,M,M}$, the consequent function can be rewritten as

$${}^pd_i(\mathbf{l}) = \mathbf{q}_i^T \cdot {}^p\mathbf{l}_{se}$$

where $\mathbf{q}_i^T = [q_{i,0}, q_{i,1}, ..., q_{i,M}, q_{i,M+1}, ..., q_{i,\frac{(M+1)(M+2)}{2}-1}]$ and and $\mathbf{l}_{se}^T = [1, l_1 ..., l_M, l_1^2, l_1 l_2, ..., l_1 l_M, l_2^2, l_2 l_3, ..., l_{M-1} l_M, l_M^2]$.

It follows that $d_i(\mathbf{l})$ is linear in $\mathbf{l}_{se}$. Thus standard linear least-squares methods can be applied to estimate parameters $\mathbf{q}_i$.

As in the first-order case, locally meaningful sub-models are determined by using the locally weighted objective function $J_i = \sum_{p=1}^{P} v_i({}^p\mathbf{l})\left({}^p\hat{d} - \mathbf{q}_i^T\, {}^p\mathbf{l}_{se}\right)^2$, which can be written involving vectors and matrices only, as:

$$J_i = \left(\hat{\mathbf{d}} - \mathbf{\Delta}_{se}\mathbf{q}_i\right)^T \mathbf{V}_i \left(\hat{\mathbf{d}} - \mathbf{\Delta}_{se}\mathbf{q}_i\right)$$

where:

$$\mathbf{\Delta}_{se} = \begin{bmatrix} 1 & {}^1l_1 & \dots & {}^1l_M & {}^1l_1^2 & {}^1l_1\,{}^1l_2 & \dots & {}^1l_{M-1}\,{}^1l_M & {}^1l_M^2 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & {}^Pl_1 & \dots & {}^Pl_M & {}^Pl_1^2 & {}^Pl_1\,{}^Pl_2 & \dots & {}^Pl_{M-1}\,{}^Pl_M & {}^Pl_M^2 \end{bmatrix}$$

$\mathbf{V}_i = diag\{[{}^1v_i \dots {}^Pv_i]\} \in \Re^{P\times P}$ and $\hat{\mathbf{d}} = [{}^1\hat{d} \dots {}^P\hat{d}]^T$.

The vector $\mathbf{q}_i$ that minimizes J_i can be obtained by pseudo-inversion [1] :

$$\mathbf{q}_i = [\mathbf{\Delta}_{se}^T \mathbf{V}_i \mathbf{\Delta}_{se}]^{-1} \mathbf{\Delta}_{se}^T \mathbf{V}_i \hat{\mathbf{d}} \qquad i = 1 \dots I.$$

We have called the columns of $\mathbf{\Delta}_{se}$ *consequent regressors*. Differently from [4][6], where all the consequent regressors were used for each rule in the genetic optimization of the quadratic fuzzy model, in this work we have also let the MEA select the subset of consequent regressors to use for each rule (different rules could involve different consequent regressors).

Doing so we have been able to control the complexity of the whole quadratic TS model, as described in the next section.

IV. The Multiobjective Evolutionary Algorithm Use for the Rule Learning

The use of MEAs for optimizing TS rules has been proposed in the last decade [5][8][10][12][13][14][21][33]. An extensive list of contributions to this research area (falling under the umbrella of multiobjective genetic fuzzy systems) can be found in [3], where multiobjective genetic fuzzy systems of the Mamdani type are also reported.

As regards MEAs, many algorithms are available today, like NSGA-II [16], PAES [26] and SPEA2 [35], among many more recent ones [9][32][34]. In this study we have used the same MEA used in [13], namely the $(2+2)$M-PAES, using accuracy and complexity as the two conflicting objectives.

$(2+2)$M-PAES maintains an archive containing the non-dominated solutions generated so far. At each epoch, two solutions randomly selected from the archive are recombined using a crossover operator. With a given probability, each offspring is also mutated before computing its fitness.

We have used $(2+2)$M-PAES to learn the quadratic fuzzy rules starting from scratch (no initial rule base generation tool has been used here). We have assumed that the user is able to decide how many fuzzy terms T should be used in the antecedent part, for the input partitions. Thus the $(2+2)$M-PAES has to identify both the matrix $\mathbf{Z}$ and which consequent regressors to use in the consequent part. The latter part has been modeled by introducing a string of bits $\mathbf{s}_i = [s_{i,1}, s_{i,2}, \dots, s_{i,K}]$, where $s_{i,k} \in \{0,1\}$ and $K = (M+1)(M+2)/2$. When $s_{i,k} = 1$ it means that the kth consequent regressor has been used in the ith rule (i.e., it is an *active regressor*), while when it equals 0 it means that it will not be used in the model. The $\mathbf{s}_i$ vectors can be stored into the matrix $\mathbf{S} \in \{0,1\}^{I\times K} = \left[\mathbf{s}_1^T \dots \mathbf{s}_I^T\right]^T$.

As regards the antecedent part, i.e., the matrix $\mathbf{Z}$, the crossover operator and the three mutation operators are the ones used in [13]. Concerning the consequent part (matrix $\mathbf{S}$), a standard mutation operator for bit strings is adopted. In particular, when applying the mutation (with probability 0.1), we have flipped every bit with probability 0.05. After crossover and possibly mutation, we ensure that no more than $\tau_{\max}$ regressors are active, for each rule. In case of excess, we randomly set to zero some of them until the constraint is met. A three-input fuzzy model with three rules and with $\Gamma_{\max} = \tau_{\max} = 2$ is described by the following $\mathbf{Z}$ and $\mathbf{S}$ matrices:

$$\mathbf{Z} = \begin{bmatrix} 0 & 1 & 3 \\ 0 & 2 & 1 \\ 3 & 0 & 0 \end{bmatrix}, \qquad \mathbf{S} = \begin{array}{cccccccccc} & L_1 & L_2 & L_3 & L_1^2 & L_1L_2 & L_1L_3 & L_2^2 & L_2L_3 & L_3^2 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{array}$$

which correspond to the rule base:

r_1: **if** L_2 **is** A_1^2 **and** L_3 is A_3^3 **then** $d_1 = q_{1,0} + q_{1,2}L_2 + q_{1,9}L_3^2$

r_2: **if** L_2 **is** A_2^2 **and** L_3 is A_1^3 **then** $d_2 = q_{2,7}L_2^2$

r_3: **if** L_1 **is** A_3^1 **then** $d_3 = q_{3,0} + q_{3,6}L_1L_3$

Once the structure of the fuzzy system (matrices $\mathbf{Z}$ and $\mathbf{S}$) has been decided by the genetic operators, the membership functions of the involved fuzzy sets and the consequent parameters are refined by transforming the TS system into its equivalent `anfis` neuro-fuzzy system [24]. The latter is then trained using the gradient descent on the antecedent part and the Kalman filtering on the consequent part.

At the end of the optimization both the accuracy of the model (the mean square error between $\hat{\mathbf{d}}$ and $\mathbf{d}$) and its complexity (the sum of number of non-zero elements in $\mathbf{Z}$ and $\mathbf{S}$) are computed. The new solution enters the archive only if it is non-dominated by any other solution in the archive itself (if the achieve is full, after the insertion, one solution is removed according to a crowding metric [26]).

A. Computational complexity

The computational complexity of the whole MEA essentially coincides with the complexity of the fitness evaluation (multiplied by the number of its evaluations, which is proportional to the population size and the number of epochs). The complexity of each fitness evaluation is mainly due to the estimation of the consequent parameters $\mathbf{q}_i$ for each rule. In the worst case, i.e., when using all the available regressors ($\tau \equiv \tau_{\max}$), such complexity is $P \cdot \tau_{\max}^2$, as shown in [13]. This computational complexity is linear with the number of points P and quadratic with the number of consequent regressors τ, which has been bounded exactly to limit the complexity of the whole identification process.

The interesting feature here is that this complexity does not depend on the fact that we are using first-order or second-order polynomials. Indeed, it would be the same even using high-order polynomials.

V. RESULTS

Figure 1 shows the coastal area of interest for which we had both a remotely sensed image and *in situ* measurements. The area is Castiglione della Pescaia (Grosseto, Italy), and the Quickbird image was acquired on April 27th, 2007 at 10:32 UTC.

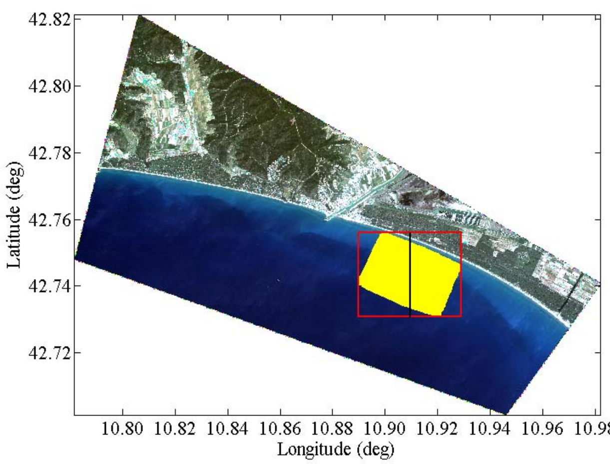

Fig. 1. Quickbird image (RGB) of the area of Castiglione della Pescaia (Grosseto, Italy), acquired on April 27th, 2007 at 10:32 UTC. The red rectangle represents the bounding box enclosing the area covered by *in situ* depth measurements, which are represented as yellow dots. The black line represents a test transept.

Figure 2 shows the *in situ* depth measurements available for the a subset of the area covered by the Quickbird image. Each pixel in this figure represents a value of *in situ* measured depth ${}^p\hat{d}$, which has been used to build the dataset pairs.

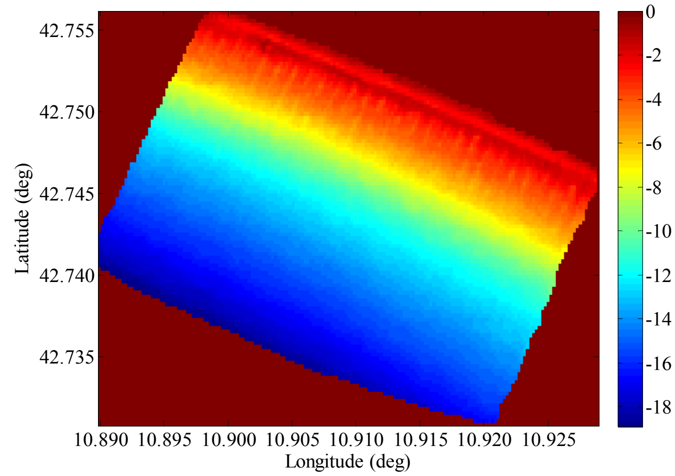

Fig. 2. *in situ* measured depths.

We have used the pixels in the QuickBird image for which we had the corresponding *in situ* measurement. The obtained dataset $\{{}^p\mathbf{I}, {}^p\hat{d}\}$ has been randomly partitioned into three subsets: training, validation and test sets, as in [15]. The validation set is useful for early stopping during the `anfis` learning ($h_{\max}$ being the maximum number of `anfis` iterations).

Then we have run the MEA algorithm using the parameters specified in Table I. We have run it with two objectives: the mean square error (as a measure of accuracy) and the total number of free parameters (as a measure of model complexity).

TABLE I: PARAMETERS USED BY THE MEA AND THE QUADRATIC TS

Parameter name	Symbol	Value
# of objectives	O	2
# of fuzzy sets for each input radiance	T	7
Max # of rules	$I_{\max}$	30
Max # of non *don't care* clauses per rule	$\Gamma_{\max}$	2
Max # of cons. regressors	$\tau_{\max}$	3
# of `anfis` iterations	$h_{\max}$	300
Archive size for the $(2+2)$M-PAES	A	50
# of epochs for the $(2+2)$M-PAES	G	10000

In Table II we compare the results of two different approaches. The first is the linear TS described in [15], while the second is a quadratic TS obtained with the new method in this work, both using Gaussian fuzzy sets. The first and the second column contain the mean squared error on the training and test set, respectively. The last column contains the complexity.

TABLE II: ACCURACY AND COMPLEXITY

	MSE Tr	MSE Ts	Complexity
Linear TS [15]	33.8	36.7	320
Quadratic TS	28.6	29.5	153

As we can see, the quadratic TS model achieves both a better accuracy on the test set (19.6% of improvement) and a significantly lower complexity. The huge complexity of the linear TS used in [15] stems from the fact that that approach used all the input variables (no use of *don't care* conditions) and all the consequent regressors (M+1=4), together with a high number of rules (32). Thus the linear TS made use of 192 antecedent parameters (32 rules × 3 inputs × 2 parameters for each fuzzy set). The number of consequent parameters was: 32 (the number of rules) × 4 (the number of consequent parameters for each rule) = 128. Thus the total number of parameters was 192+128 = 320 (see Table I).

In the quadratic approach, we have selected one solution from the final Pareto front approximation made of 27 quadratic rules. The average number of non *don't care* condition is $\cong 1.7$ (having constrained it to be less than or equal to 2 per rule), while the average number of consequents per rule is equal to $\cong 2.26$ (the maximum $\tau_{\max}$ being 3). In total, the selected quadratic system is made of 92 ($\cong 27 \cdot 2 \cdot 1.7$) antecedent parameters and 61 ($\cong 27 \cdot 2.26$) consequents parameters, for a total of 92+61 = 153.

This means that we have obtained a better accuracy (with respect to the linear TS model) using less than half of the parameters $(153 < 320/2 = 160)$.

VI. Discussion

In [6] we have found the non intuitive result that a quadratic TS fuzzy system was both more accurate and less complex (in terms of the total number of parameters) than a linear counterpart in another regression problem (namely, ocean color properties estimation from remotely sensed multispectral images). That was possible due to the fact that in the quadratic version fewer rules were needed to achieve a very good accuracy. In this study we have further proven that this situation can happen quite frequently, especially when only a subset of all the consequent regressors (whose number grows quadratically) is considered.
Furthermore, the same actions used in [5][13] to speed up the fuzzy identification can be applied here. As a future work we are also considering to speed up our multiobjective evolutionary fuzzy system by using parallel MEAs (as discussed in [7]) and by moving the fitness evaluation on the GP-GPU, as done in [11]. All these actions would allow to approach bathymetry estimation even when using upcoming high-resolution, hyper-spectral images.

VII. Conclusions

TS fuzzy systems of order higher than one are increasingly attracting research attention. In this paper we have employed quadratic TS fuzzy systems to model the relation between the reflectance (obtained from remotely sensed optical images) and the depth, exploiting a set of *in situ* measurements. We have used an MEA to learn the rules from the data. A remarkable feature of our MEA algorithm is that it can bound the complexity of the TS model, in order to control both the degrees of freedom and the training time. We have shown how a quadratic TS model can achieve better accuracy with less than half of the number of parameters used by a linear TS. This opens the way to exploiting the next generation of hyper-spectral and high spatial resolution imagery, along with the large datasets of *in situ* measurements that are going to be collected by next generation of AUVs/gliders.

Acknowledgment

The authors are grateful to Flyby srl for providing the data used in this study.

References

[1] Angelov P.P., and D.P. Filev, "An approach to online identification of Takagi–Sugeno fuzzy models," *IEEE Trans. Syst. Man Cybern.—Part B: Cybernetics*, vol. 34, no. 1, 2004

[2] Bede, B., and Rudas, I.J. "Approximation properties of higher order Takagi-Sugeno fuzzy systems," in Proc. of the 2013 Joint IFSA World Congress and NAFIPS Annual Meeting (IFSA/NAFIPS), 24-28 June 2013, pp.368–373

[3] Cococcioni M, The Evolutionary Multiobjective Optimization of Fuzzy Rule-Based Systems Bibliography Page, http://www.iet.unipi.it/m.cococcioni/emofrbss.html

[4] Cococcioni M., Lazzerini, B., and Marcelloni, F. "Second-order Takagi-Sugeno Model to Identify Sea Water Optically Active Constituent Concentrations from Meris Data," in Proc. of the 10th International Fuzzy Systems Association World Congress, 2003, pp.208–211

[5] Cococcioni, M., Lazzerini, B., and Marcelloni, F., "Towards Efficient Takagi-Sugeno Multi-objective Genetic Fuzzy Systems for High Dimensional Problems", in *Computational Intelligence in Expensive Optimization Problems*, Springer Series in Studies in Evolutionary Learning and Optimization, Editors: Tenne, Y. and Goh, C.-K., 2009, pp.397–422

[6] Cococcioni, M, Lazzerini, B., and Marcelloni, F. "Estimating the Concentration of Optically Active Constituents of Sea Water by Takagi-Sugeno Models with Quadratic Rule Consequents", *Pattern Recognition*, vol. 40, no. 10, pp.2846–2860, 2007

[7] Cococcioni, M., "mcspMEA: the Microcones Separation Parallel Multiobjective Evolutionary Algorithm and its Application to Fuzzy Rule-Based Ship Classification", in *Recent Advances in Computational Intelligence in Defense and Security*, Springer Series in Studies in Computational Intelligence, Editors: Abielmona, R., Falcon, R., Zincir-Heywood, N., and Abbass, H. (*in press*)

[8] Cococcioni, M., Corsini, G., Lazzerini, B., and Marcelloni, F., "Solving the Ocean Color Inverse Problem by Using Evolutionary Multi-Objective Optimization of Neuro-Fuzzy Systems", *Int. J. of Knowledge-based and Intelligent Engineering Systems*, vol. 12, no. 5-6, pp.339–355, 2008

[9] Cococcioni, M., Ducange, P., Lazzerini, B., and Marcelloni, F., "A New Multi-Objective Evolutionary Algorithm based on Convex Hull for Binary Classifier Optimization", in Proc. of the 2007 IEEE Congress on Evolutionary Computation (IEEE-CEC'07), Singapore, September 25-28 2007, pp.3150–3156

[10] Cococcioni, M., Grasso, R., and Rixen, M. "A Hybrid Continuity Preserving Inference Strategy to Speed Up Takagi-Sugeno Multiobjective Genetic Fuzzy Systems", in Proc. of the 5th IEEE Int. Work. on Genetic and Evolutionary Fuzzy Systems (GEFS'11), Paris, 11-15 April 2011, pp.66–72

[11] Cococcioni, M., Grasso, R., and Rixen, M., "Rapid Prototyping of High Performance Fuzzy Computing Applications using High Level GPU Programming for Maritime Operations Support", in Proc. of the 2011 IEEE Symposium on Computational Intelligence for Security and Defense Applications, Paris, 11-15 April 2011, pp.17–23

[12] Cococcioni, M., Guasqui, P., Lazzerini, B., and Marcelloni, F., "Identification of Takagi-Sugeno Fuzzy Systems based on Multi-Objective Genetic Algorithms", in Proc. of the 6th Int. Workshop on Fuzzy Logic and Applications, Crema, Italy, vol. LNCS 3849, September 15-17 2005, pp.172-177

[13] Cococcioni, M., Lazzerini, B., and Marcelloni, F. "On Reducing Computational Overhead in Multi-Objective Genetic Takagi-Sugeno Fuzzy Systems", *Applied Soft Computing*, vol. 11, no. 1, pp.675–688, 2011

[14] Cococcioni, M., Lazzerini, B., Marcelloni, F., "Fast Multiobjective Genetic Rule Learning Using an Efficient Method for Takagi-Sugeno Fuzzy Systems Identification", in Proc. of 8th Int. Conf. on Hybrid Intelligent Systems (HIS'08), Barcelona, Spain, September 10-12 2008, pp. 272–277

[15] Corucci. L, Masini A., and Cococcioni, M., "Approaching bathymetry estimation from high resolution multi-spectral satellite images using a neuro-fuzzy technique", *Journal of Applied Remote Sensing*, vol. 5, no. 053515, pp.1–15, 2011

[16] Deb, K., Pratap, A., Agarwal, S., and Meyarivan R., "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. on Evolutionary Computation*, vol. 6, no. 2, pp.182–197, 2002

[17] Delgado, M.R., Von Zuben, F. and Gomide, F., "Hierarchical genetic fuzzy systems," *Information Sciences*, vol. 136, no. 1–4, pp.29–52, 2001

[18] Delgado, M.R., Von Zuben, F. and Gomide, F., "Optimal Parameterization of Evolutionary Takagi-Sugeno Fuzzy Systems", in Proc. of 8th Information Processing and Management of Uncertainty in Knowledge-Based Systems Conf., Madrid, Spain, 2000, pp.650–657

[19] Delgado, M.R., Von Zuben, F., and Gomide, F., "Coevolutionary genetic fuzzy systems: a hierarchical collaborative approach," *Fuzzy Sets and Systems*, vol. 141, pp.89–106, 2004

[20] Demirli K., and Muthukumaran, P., "Higher Order Fuzzy System Identification Using Subtractive Clustering", *Journal of Intelligent and Fuzzy Systems*, vol. 9, PP. 129–158, 2000

[21] Gacto, M.J., Galende, M., Alcala, R., and Herrera, F., "Obtaining accurate TSK Fuzzy Rule-Based Systems by Multi-Objective Evolutionary Learning in high-dimensional regression problems," in Proc. of the 2013 IEEE International Conference on Fuzzy Systems, 2013, pp.1–7

[22] Grasso, R., Cecchi, D., Cococcioni, M., Trees, C., Rixen, M., Alvarez, A. and Strode, C., "Model based decision support for underwater glider operation monitoring", in Proc. of the 2010 Oceans MTS/IEEE Seattle Conf. (OCEANS/Seattle'10), Seattle, Washington, USA, 20-23 September, 2010, pp.1–8

[23] Grasso, R., Trees, C.C., Spina, F., and Legac, J.C. "Bathymetry estimation in littoral areas by fusing multispectral satellite data and depth samples from AUVs," Maritime Rapid Environmental Conference, 25–27 September 2007, Lerici, Italy

[24] Jang, J.-S.R. "ANFIS: adaptive-network-based fuzzy inference systems," *IEEE Trans. Syst. Man Cybernet*ics, vol. 23, no. 3, pp.665–685, 1993

[25] Kanno, A., Tanaka, Y., Shinohara, Kurosawa, R.A., and M. Sekine, "Which Spectral Bands of WorldView-2 Are Useful in Remote Sensing of Water Depth? A Case Study in Coral Reefs," *Marine Geodesy*, vol. 27, no. 3, pp.283–292, 2014

[26] Knowles, J., and Corne, D.W., "Approximating the nondominated front using the pareto archived evolution strategy," *Evolutionary Computation,* vol. 8, no. 2, pp.149–172, 2000

[27] Leon, J.X., Phinn, S.R., Hamylton, S., and Saunders, M.I. "Filling the «white ribbon» - a multisource seamless digital elevation model for Lizard Island, northern Great Barrier Reef", *International Journal of Remote Sensing*, vol. 34, no. 18, pp.6337–6354, 2013

[28] Lyzenga D.R. "Passive remote sensing techniques for mapping water depth and bottom features," *Appl. Opt.*, vol. 17, no. 3, pp.379–383, 1978

[29] Polcyn F.C., and Sattinger, I.J. "Water depth determination using remote sensing techniques," in Proc. of the 6th Int. Symp. Remote Sens. Environ., Ann Arbor, MI, pp.1017–1028, 1969

[30] Ren, Q., Balazinski, M., and Baron, L. "High-order interval type-2 Takagi-Sugeno-Kang fuzzy logic system and its application in acoustic emission signal modeling in turning process," *Int. J. of Advanced Manufacturing Technology*, vol. 63, pp.1057–1063, 2012

[31] Takagi, T., and Sugeno, M., "Fuzzy identification of systems and its applications to modeling and control," *IEEE Trans. on Systems, Man and Cybernatics*, vol. SMC-15, no. 1, pp.116–132, 1985

[32] Tiwari, S., Fadel, G., and Deb, K., "AMGA2: Improving the performance of the archive-based micro-genetic algorithm for multi-objective optimization," *Engineering Optimization*, vol. 43, no. 4, pp.377–401, 2011

[33] Wang H., Kwong, S., Jin Y., Wei W., and Man, K, "Multi-objective hierarchical genetic algorithm for interpretable fuzzy rule-based knowledge extraction," *Fuzzy Sets and Systems*, vol. 149, no. 1, pp.149–186, 2005

[34] Zhang, Q., and Hui L., "MOEA/D: A Multiobjective Evolutionary Algorithm Based on Decomposition," *IEEE Trans. on Evolutionary Computation*, vol.11, no.6, pp.712,731, 2007

[35] Zitzler, E., Laumanns, M., and Thiele, L., "SPEA2: Improving the Strength Pareto Evolutionary Algorithm," Technical report 103, Computer Engineering and Networks Laboratory (TIK), ETH Zurich, Zurich, Switzerland, 2001