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Title Page

Nonlinear Dependency of Tooth Movement on Force System Directions

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Abstract:
Introduction: Moment-to-force ratios (M:F) define the type of tooth movement. Typically, the relationship between M:F and tooth movement has been analyzed in a single plane. Here, to increment 3-D tooth movement theory, we test the hypothesis that the mathematical relationships between M:F and tooth movement are distinct depending on force system directions. Methods: A finite element model of a first maxillary premolar, scaled to average tooth dimensions, was constructed based on a CBCT scan. We conducted finite element analysis (FEA) of the M:F and tooth movement relationships, represented by the projected axis of rotation (C.Rot) in each plane, for 510 different Loads. Results: We confirmed that an hyperbolic equation relates the Distance (C.Res-C.Rot) and M:F; however, the constant of proportionality ("k") varied with non-linearly the force direction. With a force applied parallel to the tooth long axis, "k" was 12 times higher than with a force parallel to the mesio-distal direction and 7 times higher than with a force parallel to the bucco-lingual direction. Conclusions: The M:F influence on tooth movement depends on load directions, and it is an incomplete parameter to describe the quality of an orthodontic load system if not associated with force and moment directions.

Introduction

Evaluating the effectiveness of the loads delivered by an orthodontic device onto the teeth is a challenging task. It is expectable to have complex load systems during orthodontic treatment that act simultaneously in all three spatial planes. The analysis of relationships between 3D tooth movement and loads is possible by discrimination of moment-to-force ratios (M:F) in each planar projection. In 3D, each M:F is defined by combinations of the forces contained in the plane and moments perpendicular to it. Unfortunately, even if entire information is provided to an orthodontist about the 3D force system on a specific tooth, extremely limited 3D information is currently available on how a specific tooth will actually move. This occurs because studies on the relationship between M:F and the center (axis) of rotation are typically limited to one plane and one force direction. Because of the morphological asymmetry of teeth, it is very reasonable to hypothesize that each of the 3D permutations of M:F ratios in different directions has a different mathematical relationship with the patterns of tooth movement.

The most common method to describe the type of tooth movement consists of measuring the distance from the tooth's projected Axis of Rotation (Center of rotation, C.Rot) to the virtual intersection of the Axes of Resistance (Center of Resistance, C.Res). Previous authors have evaluated the influence of controlled M:F increments upon the type of tooth movement in one plane. One study focused on applying a force perpendicular to a canine long axis with a parabolic shaped root, obtaining the so-called Burstone formula (M:F=0.068*h^2/D), where h is the distance from the alveolar crest to the apex, and D is the distance between the C.Res and the C.Rot[1]. All previous FE studies[2-4] analyzed the movement in a single plane except one[5] which analyzed loads in the three planes and how they affect the axes of resistance. One paper[3] studied the maxillary premolar and canine, evaluating the effects of different M:F under constant force, and of different forces under constant M:F. They found that the force value influences the type tooth movement, i.e., even with the same M:F, movement is different if the force increases. This happens due to the nonlinear behavior of the PDL (periodontal ligament) which becomes important after a certain strain threshold.[6]

Here, we test the hypothesis that the mathematical relationships between M:F and tooth movement are distinct depending on force direction. To do so, we build a comparative map of the effects of relevant M:F combinations on a maxillary first premolar that can also be useful to help plan tooth movement.

Material and methods

A model composed of tooth, ligaments and alveolar bone structures was created by digital integration of a CBCT scan and a surface structured light scan.[7] An optical scanner was used to reconstruct the tooth crown through the digitalization of plaster casts. The 3D individual dental tissues obtained by the optical scanner and the CBCT sensor were fused to create a multi-body orthodontic model with minimum user interaction.[8] The fusion of the multi-modal data sets allows the most accurate representation for each tissue: i.e., tooth crown by optical scanning and tooth root and alveolar bone by CBCT imaging. The obtained geometries were auto patched to create trimmed NURBS surfaces, which were converted into vendor neutral file format allowing the exchange of CAD models "IGES" (Initial Graphics Exchange Specification). A linear elastic model was used for each structure to test the movements under the assumption of low PDL strains <7.5%.[9]

The PDL was created manually, as it is not always readily discernible in a CBCT volume because of its low thickness. The tooth and bone were dilated uniformly by 0.2 mm, and the dilated parts were intersected to create the PDL as a uniform thick layer. The tooth dimensions were scaled to reflect an average premolar.

The bone and the tooth were modeled as homogenous bodies without discerning between cortical and cancellous bone and enamel, pulp and dentin. This simplification, which was made also in previous studies[10], allowed to save computational time without compromising the testing of our hypothesis under the specified load thresholds. Young's moduli of 20000 MPa and 2000 MPa were assigned to the tooth and the bone respectively. The Poisson's ratio was 0.3 for both.[9]

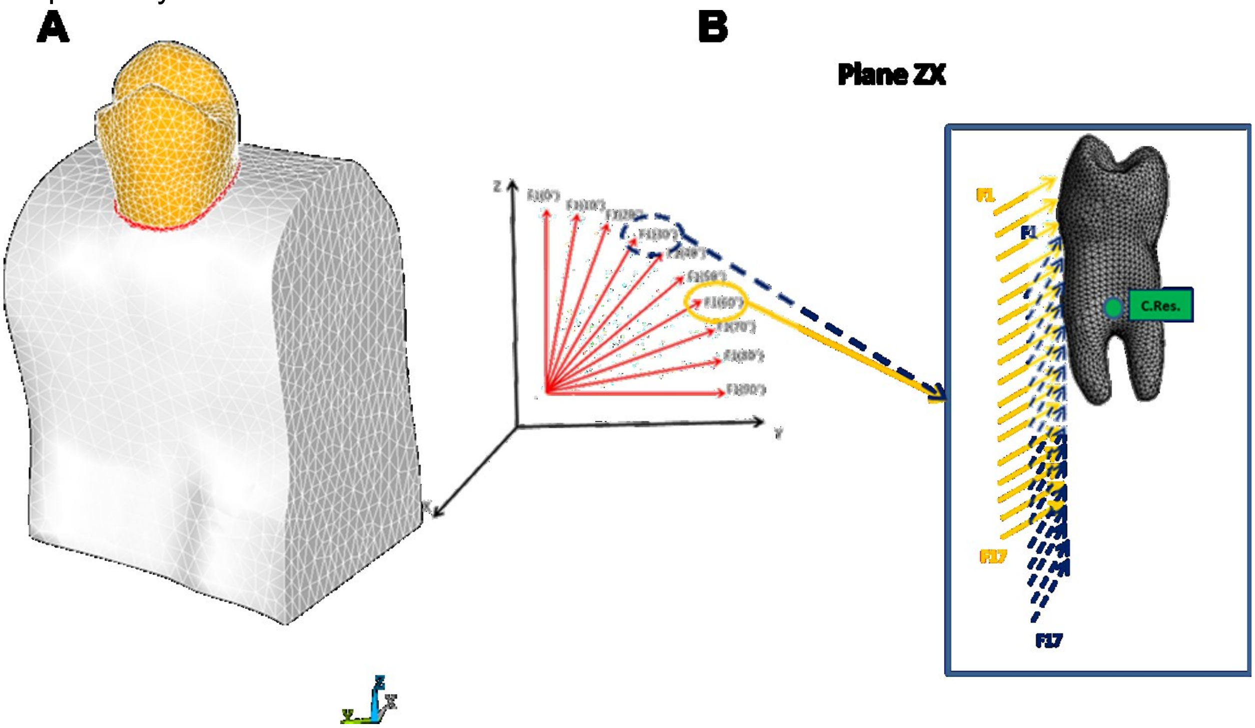

Figure 1. (A) The dentoalveolar complex of the maxillary first premolar meshed with tetrahedral elements. The nodes at the interfaces between tooth-PDL and PDL-Bone were shared between the contiguous bodies. The coordinate system was defined according to the Occlusal Plane. The z-axis is defined as perpendicular to the occlusal plane. The y-axis is parallel to the occlusal plane and approximately congruent with the mesio-distal direction of the tooth. The x-axis is parallel to the occlusal plane and congruent with the palato-buccal direction of the tooth. **(B)** Example of the ZX planar representation of the incremental directional force system changes (left), and correspondent force systems tipped 30 and 60 degrees with respect to the Y axis (right). Each of the different force and moment combinations at the C.Res was equivalent to one of the 17 single forces spaced 2mm or 1mm from each other, where the central force is applied at the C.Res. The higher resolution force increment (1mm) was applied in the region M:F= [-4:4]. The direction of the force for each position was changed in the coordinate plane of interest in 10 degree increments, resulting in 10 different simulations for each M:F value at the C.Res.

The geometries were imported in the Finite Element Software, ANSYS Workbench 16 (ANSYS, Canonsburg, PA, USA), where the three bodies were meshed with solid elements resulting in 140831 tetrahedral elements and 234955 nodes **(Figure 1A)**.

To find the references for tooth translation, three simulations were run applying a moment of 1.5 Nmm parallel to each coordinate system axis to the tooth. The approximate 2D projections of the axes of resistance (center of resistance, C.Res) for the three simulations were recorded according to methodology previously published.[11] After iteratively refining the mesh in region until reaching an edge size of 0.1 mm, the average position of the C.Res was recorded and used for further analysis.

The different M:F tested were applied at the C.Res, because several orthodontic appliances, such as orthodontic aligners, don't use brackets. The force applied at C.Res for every scenario was 0.13N, while the M:F varied from -12mm to 12mm. All the bone's nodes were assigned zero displacement to simulate a rigid body due to the transient nature of tooth displacement solely attributed to bone deformation.[5] The simulations were performed on the three coordinate planes (plane XY, plane YZ, plane ZX). For each plane, 17 force systems were applied at the C.Res of the maxillary first premolar, as shown in **Figure 1B.**

The projected axis of rotation (center of rotation, C.Rot) was evaluated for each scenario through the displacement vectors of two nodes of the tooth.[4] For each plane, a couple of nodes were selected choosing two nodes having as much distance as possible between them on the target plane. This choice was necessary because the tooth is not an ideal rigid body; hence its deformations can add high relative error to the C.Rot location if the nodes are too close. The resulting C.Rot coordinates and the distances from the C.Res were evaluated and analyzed to get a mathematical relationship between M:F and C.Rot. The D (distance C.Res-C.Rot) vs. M:F were fitted using CurveExpert Basic® software (CurveExpert, Madison, AL, USA). The starting model was set as simple hyperbolic[12] and the variable factor characterizing each curve was the "k" value (constant of proportionality of the inverse relationship) in the following expression:

$$Distance\ (C.Res - C.Rot) = \frac{k}{MF}$$

The detailed workflow of the method is shown in **Figure 2**.

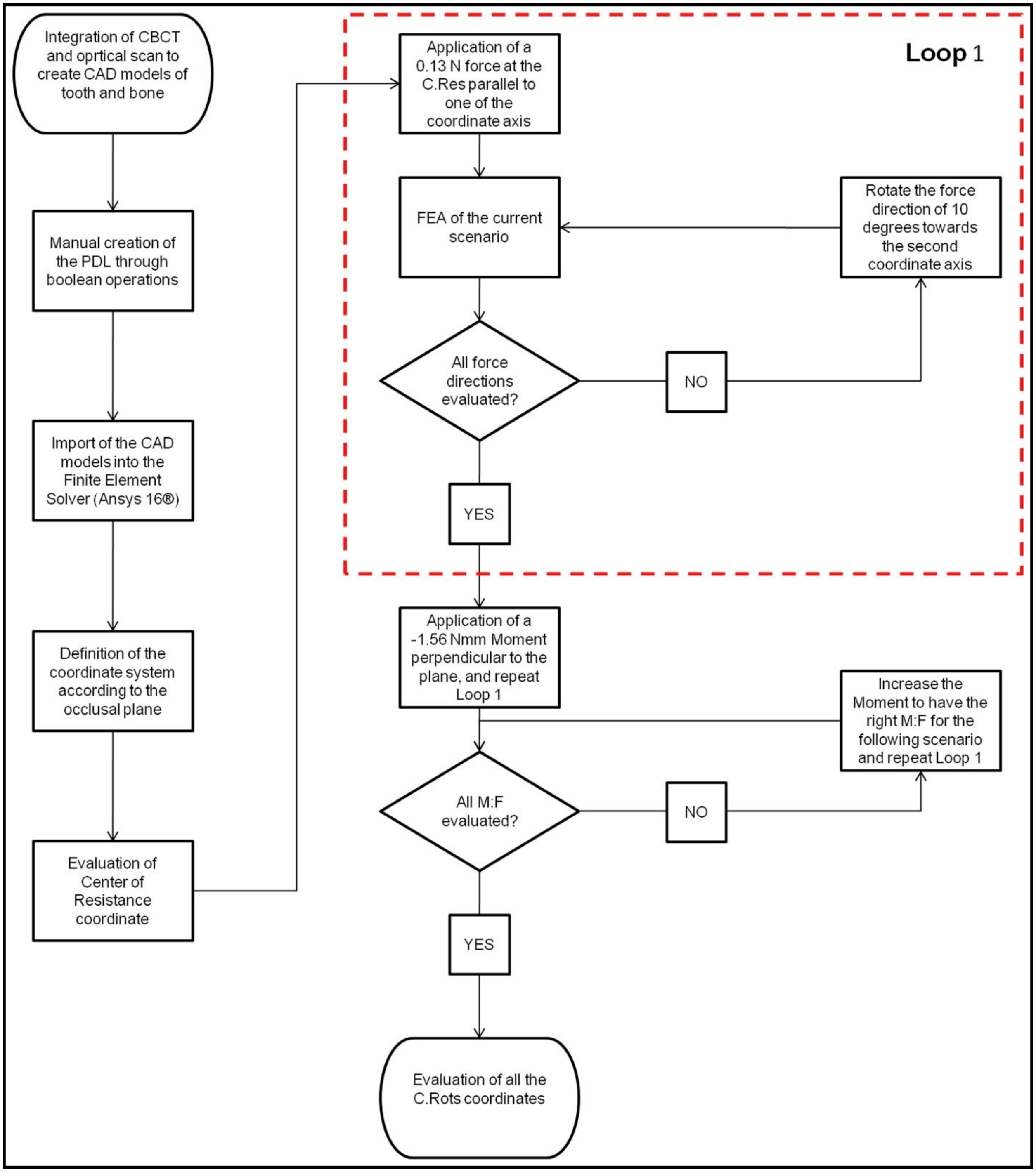

Figure 2. Experiment design.

Results

The relationships between M:F and the distances between C.Rot and C.Res, for a force parallel to each one of the axes, are depicted in **Figure 3**. The curves followed a hyperbolic shape in all planes and with changing directions. However, this occurred with a different constant of proportionality depending on the plane and the force's direction.

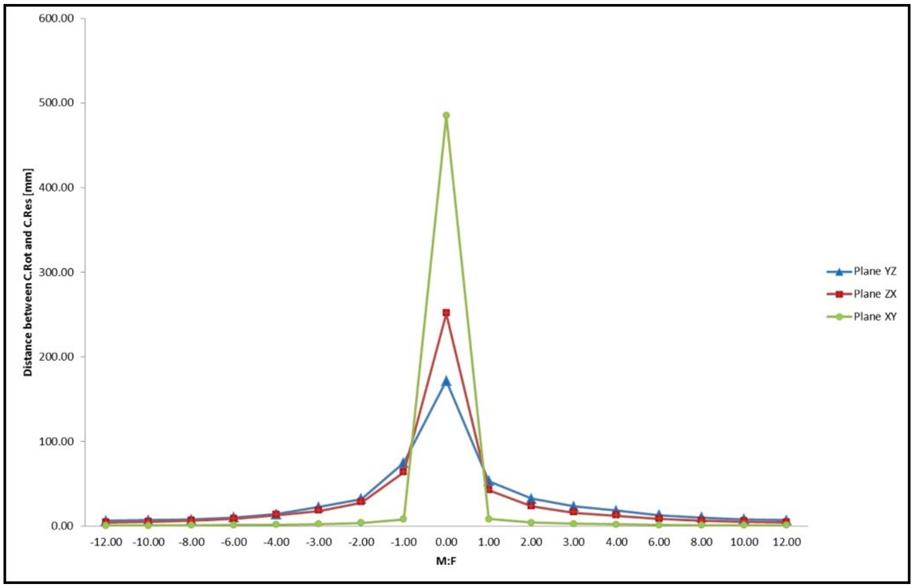

Figure 3. Distances between C.Res and C.Rot when the M:F varies from -12 to 12 for the three planes with the force parallel respectively to: Y for plane XY; Z for plane YZ; Z for plane ZX. The distances are absolute values, thus they are all positive.

Table 1.Value of the "k" factor of the hyperbolic expression for the different scenarios

Angle	PLANE XY	PLANE YZ	PLANE ZX
0	8.8	14.3	44.4
10	8.7	18.7	41.0
20	8.5	25.1	37.2
30	8.1	31.5	33.2
40	7.5	37.5	28.9
50	6.8	43.2	24.6
60	6.1	48.2	20.5
70	5.5	52.3	18.0
80	5.0	56.1	15.5
90	4.9	58.7	15.3

The distance between two neighbor points of the same direction did not increase linearly (**Figure 4**). This is confirmed in 3D on the pictures related to the planes YZ and ZX. Changing the force direction, the distance between two points of the same color was highly different. The different C.Rot for the same force direction lay on a straight line almost perpendicular to the initial applied force direction.

The "k" value of the hyperbolic equation was calculated for all load scenarios. The entire dataset is shown in **Table 1**. **Figure 5** indicates that “k” increased greatly when the force direction became parallel to the tooth long axis (z-axis). Moreover, the "k" values were larger on the YZ and ZX planes than on the XY plane which contained the mesio-distal and linguo-buccal dimensions of the tooth.

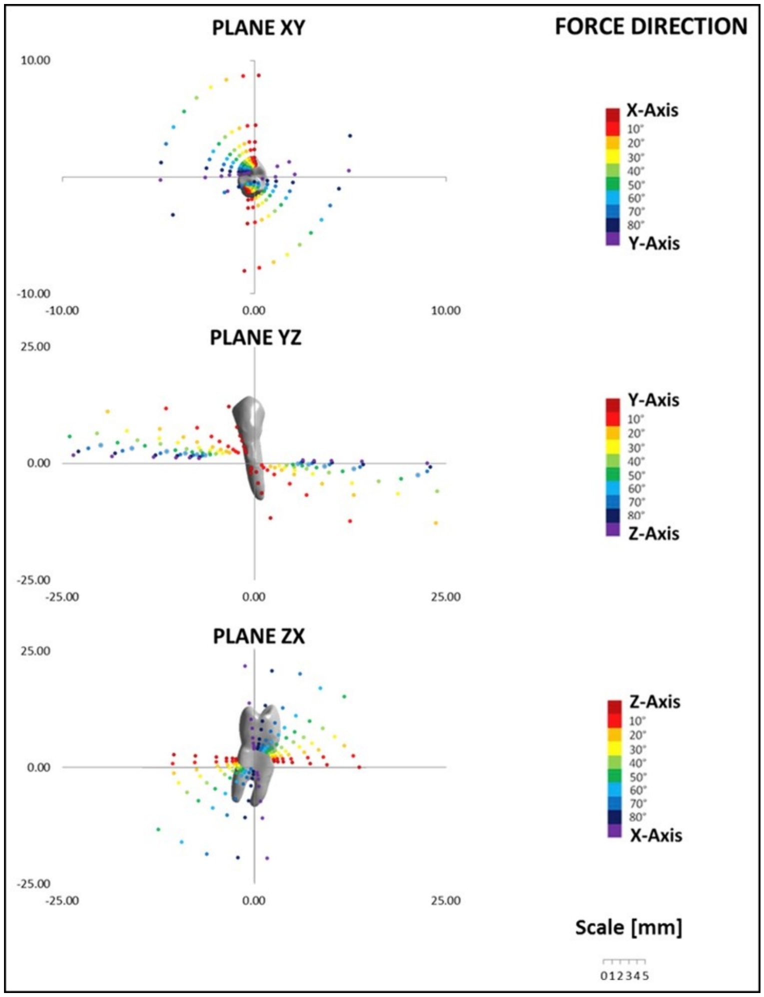

Figure 4. Location of the C.Rots for each force system on the three spatial planes related to the tooth. The coordinate system is centered on the C.Res. The distance of same color’s dots is not constant. It increases going far from the C.Res. Moreover, it can be observed that the C.Rots resulting from forces parallel to “z” are more distant than the one with the force parallel to “y” on the YZ plane. The same result could be observed on the ZX plane, but not all the C.Rots could be shown, because for low M:F, the C.Rot is located very far from the C.Res ($\sim 500\ mm$). Therefore, only a defined ROI (region of interest) around the tooth, with size 25mmx25mm, is displayed for each plane. The three planes are represented with the same scale, thus the scale on the lower right corner of the figure can be used to correct actual figure measurements of the distances between the dots.

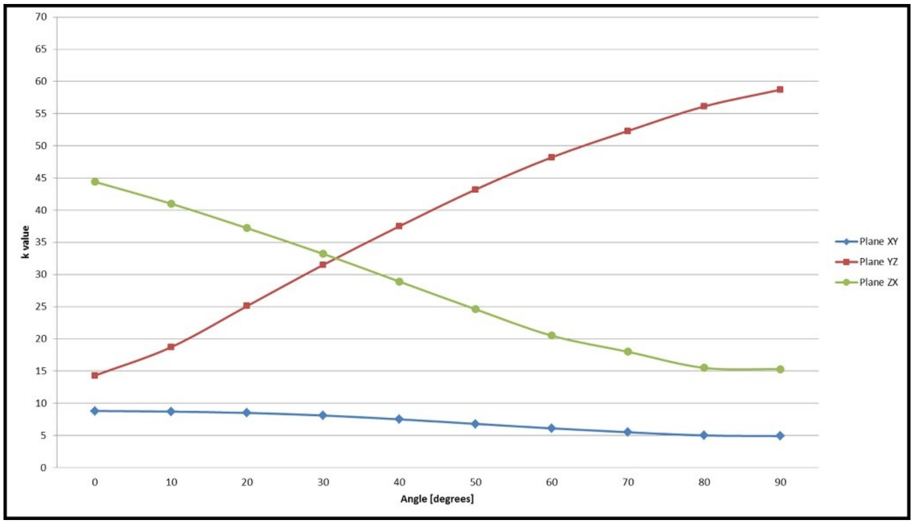

Figure 5. Variation of "k" with the direction of the force vector, represented by the angle between the vector and the first coordinate axis for each plane (x,y and z respectively for planes XY, YZ and ZX).

Discussion

After calculating the C.Rot location for all the different load systems, we found that the M:F and the D (C.Res-C.Rot) have an hyperbolic relationship. The relationship between distance and M:F partially confirms the Burstone formula, but the constant of proportionality of the Burstone formula ($0.068 \cdot h^2$) should be substituted by the parameter "k", which depends by the plane and the force direction. The Burstone formula was proved to be valid only for a parabolic shaped single root tooth. The same formula is not valid in the case of a maxillary first premolar due to non-parabolic and bifurcated root shape.
The FE simulations showed that after changing the force direction in a plane, the distance between two incremental C.Rot positions, as defined by our method, also changed. For instance, in the XY plane, varying the M:F from 1 to 2mm leads to a change in the C.Rot location of 6.79mm if the force is parallel to the linguo-buccal direction and 15.58mm if it's parallel to the tooth axis. Hence the incremental distance increased almost by 3 times when the force direction was parallel to the z-axis, corresponding to the tooth long axis. Therefore, not only the M:F, but also the plane define the quality of the force system. Moreover, the change in direction of the equivalent force at the C.Res also affected the C.Rot distance results within the same plane. When the component parallel to the longer tooth dimension in the plane increases, the "k" value increases too. Consequentially, when the force direction becomes more parallel to the tooth long axis, the D (C.Res-C.Rot) decay rate becomes progressively higher. When the force is applied parallel to the tooth long axis, D (C.Res-C.Rot) can change up to 12 times faster ($k = 58.7 \rightarrow k = 4.9$) compared to when the force is parallel to the mesio-distal tooth axis, and 7 times faster ($k = 58.7 \rightarrow k = 8.8$) when a force becomes parallel to the bucco-lingual direction. It is also important to note that the changes in “k” are non-trivial, i.e., for a 45 degree force,

the “k” is not the average between 0 and 90 degree forces. The "k" values show that the hyperbolic function characterizing each loading scenario is somehow related with the tooth anatomical features as the "k" value is increasing with the tooth dimension. Despite that, it’s still not clear how tooth morphology and “k” are exactly related.
Nevertheless, the slopes of “k” vs. force direction on each plane are not linear **(Figure 4)**, revealing an approximately sigmoid shape. In the XY plane, the "k" values decreases less than 2% from 0 to 10 and 80 to 90 degrees, but it shows a 9.3% decrease from 40 to 50 degrees.

The analysis of the "k" values on the different planes showed that they depend not only on the force direction, but also on the moment direction. If the force is applied parallel to the mesio-distal tooth axis (y-axis) "k" assumes a value of 4.9 if the moment is applied along the z-axis (Plane XY) and a value of 14.3 if the moment is applied along the x-axis (Plane YZ).

The relationship between M:F and tooth movement has been previously investigated by other researchers with limitations. Previous studies[2-4], except one[5] evaluated the effect of M:F variations only on one plane. This study, although providing more comprehensive 3D data, also has a limitation, which is the simplified linear PDL model. This choice was justified because the loads modeled are below the strain threshold where the PDL starts to stiffen.[6] Although our testing hypothesis can be considered accurate only for low forces, it has been shown that the error in D(C.Res-C.Rot) in the plane ZX for a premolar is approximately 2mm when the force varies from 10cN to 200cN with a constant M:F=10 at the bracket.[3] Hence, different force magnitudes should be modeled to take into account for the non-linear PDL behavior since for each force value the C.Rot position would change. Otherwise, the above-mentioned error should be taken into consideration when applying our data for clinical purposes.
Typically, load measurements are conducted with an electromechanical transducer, which can reveal the moments and forces in 3D.[13-15] However, the results obtained by transducers cannot be reliably used to predict or approximate the tooth behavior since limited information is available on the M:F values required for 3D tooth movements. Data such as those obtained in this paper help to clarify what exactly are the different incremental effects of every M:F permutation for a typical patient. The quantification of these differences is important when comparing two sets of 3D force systems to ascertain which appliance, activation or accessory delivers the movement that is closest to the clinical goal. For example, **Figure 6** shows that changing the M:F from 8 to 10mm on plane ZX brings to a D (C.Res-C.Rot) 1.33mm closer to the C.Res. The same change on the plane XY causes a decrease in D (C.Res-C.Rot) of 0.42mm. Clinically, this means that an incremental change in the force system in one plane and with a certain force direction has a differential effect on tooth movement than the same change on another

plane, or on the same plane with a different force direction. Hence, while activating an appliance on different planes, errors in one force system can cause larger differences in tooth displacement depending on the plane. This should be accounted for while designing an orthodontic appliance (for instance, by optimizing wire dimensions, and consequently load-deflection rates in different planes) and evaluating their load systems. Thus, the appliance design should be more precise on the most sensible plane than on the others.

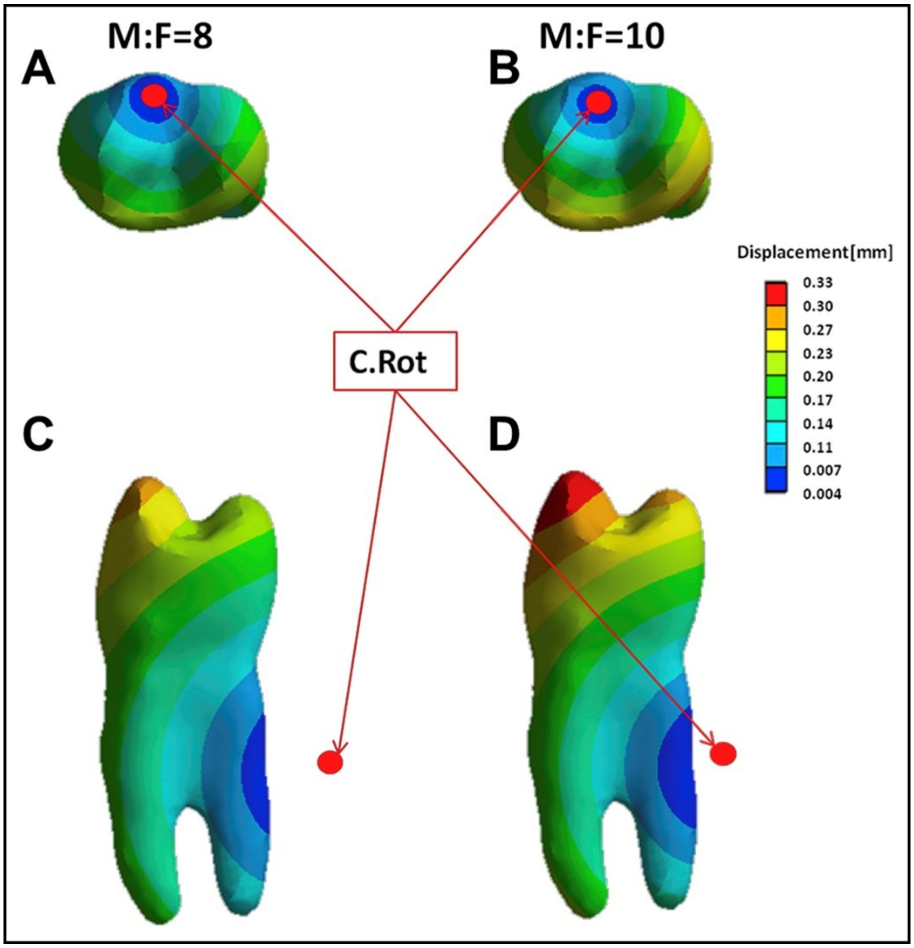

Figure 6. The different incremental changes in C.rot with similar M:F on the planes XY and ZX. The force was directed along x for A&B and along z for C&D. D(C.Res-C.Rot) was respectively 1.83mm;1.41mm;6.00mm;4.77mm for A, B, C and D.

Future studies could attempt to obtain a meaningful mathematical relationship between the "k" value and the tooth morphological features, and the force magnitude. In that case, the "k" values could be evaluated individually to obtain a customized D(C.Res-C.Rot) vs. M:F equation for each patient.

Conclusions

- The relation between Distance C.Res-C.Rot and M:F can be explained through a hyperbolic equation where the factor “k” characterizes the different behavior for each load scenario.
- Equal variations of the M:F have a different influence on each plane; therefore those differences cannot be neglected while measuring the force system delivered by orthodontic appliances to a tooth.
- On the same plane, the force direction influences the hyperbolic relationship between M:F and D (C.Res-C.Rot).
- Further studies are necessary to evaluate a correlation between the “k” value and the tooth morphology. In that case the “k” values could be evaluated individually to obtain a customized D(C.Res-C.Rot) Vs. M:F equation for each patient.

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