It is a pleasure and an honor to describe part of the scientific activity of my former advisor Hugo Beirão da Veiga. I first met him in 1993, when I was undergraduate student at the Scuola Normale Superiore di Pisa. At that time it was compulsory for 3rd year-undergraduate students to give a *colloquium*, presenting and reviewing some recent results: A very good way to start handling research papers to taste what really means doing research in mathematics. It was in that occasion that, following the advices of Bruno Rubino –now head of the Department of Mathematics at L'Aquila and who was at that time ending his *Perfezionamento* (Ph.D.) at the SNS– that I took the courage to contact this very polite and elegant man (qualities which make him very peculiar, among the leading scientists in the field) which was known to us as a well-renowned mathematician. He enthusiastically agreed and suggested something about his recent (to my opinion groundbreaking) works on hyperbolic systems, see [58]. The core of the very innovative results have been recently reviewed in Ref. [99]. In that occasion I also received great help from Paolo Secchi, at that time Associate Professor in Pisa (and also with moustaches). In 1994 I started my degree (laurea) thesis under Hugo's direction and in that occasion I learned about the incompressible Euler equations. In particular, I read his joint papers with Alberto Valli [26,27,28,30], but also Ref. [36] and it is surprising that it remained almost ignored by the community, despite being a sharp almost-endpoint in the theory of 2D Euler equations. Similar results have been obtained about 15 years later (in the context of Besov spaces) by Misha Vishik. The Dini continuous space used by Hugo for the vorticity have the same scaling of the critical Besov spaces employed by Vishik for the Cauchy problem, but the approach in [36] is much simpler, based on clever applications of the Ascoli-Arzelà theorem and naturally employed in the initial boundary value problem (see also Ref. [117]). Note that the same results have been independently rediscovered by Herbert Koch in 2002. Immediately after the defense of my degree thesis in 1995, Hugo suggested to make my interests wider and also introduced me to the fascinating field of numerical methods for PDEs. In fact, Alberto Valli (with moustaches too) was the other advisor of my Ph.D. Thesis ended in 2000. Later on I joined the faculty at the former Department of Applied Mathematics, where Hugo has been the chair for several years, providing a new deal in the Department life and organization.

Due to this close relationship, I am describing mainly the results he obtained for the Navier-Stokes equations in the last 20 years and especially those I know better having worked together with him or in related fields. I am not talking about other recent results on singular limits and non-Newtonian fluid, which will be reviewed by Francesca Crispo.

First, I want to say that I always appreciated that Hugo is able to address difficult open problems only with elementary tools and also I appreciate the very peculiar way he has to do research in all his still foregoing career: Go directly to the problem, because if you read too much about what the others have done, you will follow the same paths, without chance of finding something new. He is able to do in this manner, a quality that very few share!

In Ref. [72], which is now his mostly cited work, he extended in a very natural and elementary way the classical criterion linked with the names of Leray, Prodi, and Serrin for regularity of weak solutions to the Navier-Stokes equations. The extensions gives a natural scaling invariant criterion related with the gradient of the velocity and paved the way for extensive research done by many authors. In Ref. [73], which I remember I read the first time as a draft on the train from Florence to Padova, he found an extremely elementary way to show the regularity of the Stokes problem, in the Hilbertian case. He was able to use the contraction fixed point theorem (typical of nonlinear problems) to provide the most elegant and elementary proof of existence of strong solutions for the Stokes system. This paper, which is a sort of mathematical gem, is the one that mostly reflects one of his mottoes: "A simpler proof of a known result is always welcome and useful, because let you understand more about the problem." (Adding also: If the result is true it always is possible to find a simpler solution...)

He also continued the basic research on the Stokes problem in Refs. [83,84], where a very detailed and complete treatment of the system with Navier slip conditions is done (these conditions introduced by Navier are becoming more and more fashionable both from the theoretical and applied point of view, for their special role in describing and modeling the boundary layer.)

In Ref. [74] he studied the problem of the limiting case of criterion of not obtaining the solution to the problem in the full generality (later solved by Seregin, Escauriaza and Sverak) but the extreme simplicity of his approach has to be remarked. In Refs. [75,76] he applied the classical "truncation method" to the Navier-Stokes equations, which is not directly applicable for the presence of the pressure. This seems a very promising and not completely explored tool, yet. Similar ideas have been used also by Caffarelli and Vasseur to obtain well-known results about quasi-geostrophic equations and different proofs of known partial regularity results. The use of this classical method in fluid mechanics seems still prone to future developments.

In Ref. [79,95] we improved the geometric criteria involving the direction of vorticity introduced by Charles Fefferman and Peter Constantin in two directions: Less stringent conditions on the alignment of vorticity near possible singular point and, by using results on Green functions by V.A. Solonnikov, the problem with boundaries. We also came back separately on this problem, see for instance Refs. [81,88,89,112,113,116].

In Ref. [80] Hugo also tackled the problem of fluid structure interactions and this is among the first papers ever appeared on the topic with rigorous analytical results. At that time Hugo was attracted by this problem for possible applications to blood circulation and the theory for simplified models used by Alfio Quarteroni and his group was very demanded. This paper is really technical, dense, and contains a lot of deep ideas. This paper made also a turning in Hugo's scientific life: It was the first paper that he typeset by himself (with some of my help especially for formulæ) in LaTeX.

In Ref. [86] he studied one of Leray's problem in the case of periodic time-evolution. Also this work takes some motivation from pulsatile motion as that of blood flow. The look for a fully-developed solution time-dependent (the counterpart of Hagen-Poiseuille) takes to a very special *inverse problem* which cannot be solved by the standard methods due to the lack of coercivity. He was able to find a proof that seems just "write the Fourier series development and check carefully the terms," but that in reality contains a very subtle reasoning for an inverse problem, which is on the cutting-edge between trivial or ill-posed; the correct track is on the crest between these two valleys, where only professional climbers can walk the first time, to show us the footpath. In the very last few years he came back to capacity, potential theory and so on, presenting in [115] and updated version of some results that he obtained in the seventies about a very popular topic, in which he gave relevant contributions, which have been partially ignored and rediscovered years later by several authors. To end this short summary Hugo is still extremely active, with a lot of energy in doing research.