

1 **Optimal integrated sizing and operation of a CHP system with Monte Carlo risk analysis for**
2 **long-term uncertainty in energy demands**

3

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7

8 **Abstract**

9 In this study a probabilistic approach for optimal sizing of cogeneration systems under long-term
10 uncertainty in energy demand is proposed. A dynamic simulation framework for detailed modeling
11 of the energy system is defined, consisting in both traditional and optimal operational strategies
12 evaluation. A two-stage stochastic optimization algorithm is developed, adopting Monte Carlo
13 method for the definition of a multi-objective optimization problem. An Italian hospital facility has
14 been used as a case study and a gas internal combustion engine is considered for the cogeneration
15 unit. The results reveal that the influence of uncertainties on both optimal size and annual total cost
16 is significant. Optimal size obtained with the traditional deterministic approach are found to be sub-
17 optimal (up to 30% larger) and the predicted annual cost saving is always lower when accounting for
18 uncertainties. Pareto frontiers of different CHP configurations are presented and show the
19 effectiveness of the proposed method as a useful tool for risk management and focused decision-
20 making, as tradeoffs between system efficiency and system robustness.

21

22 **Keywords**

23 Combined Heat and Power

24 Optimization

25 Uncertainty

26 Monte Carlo method

27 Multi-objective

28 Decision-making

29

30 **1. Introduction**

31 Cogeneration is the simultaneous production of electric energy and useful heat. Combined
32 Heat and Power (CHP) plants haven been shown to be a reliable, competitive and less polluting
33 alternative to separate generation. The European Union has promoted the use of high-efficiency
34 cogeneration as a measure to save primary energy, avoid electric network losses, reduce emissions,
35 namely greenhouse gases, and improve the security of energy supply [1]. CHP technology is
36 considered an essential means of achieving the European 20% energy efficiency target by 2020 [2].

37 The energy, environmental and economic performances of CHP systems are strongly
38 influenced by prime mover selection, equipment capacity and operational strategy. Undersizing and
39 oversizing of CHP plants are frequent and do not allow the full exploitation of the energy saving of
40 such systems [3]. For this reason, in recent years, many studies have focused on appropriate CHP
41 system design methods [4]. Multi-objective optimization approaches for designing cogeneration
42 systems have been developed both for residential [5] and for large-scale building energy systems
43 [6]. The importance of integrated sizing and operational strategy methods for optimal selection of
44 cogeneration systems has been explicitly addressed [7,8].

45 Different optimization techniques have been used over the years to identify the optimal design
46 of polygeneration systems [9]. Arcuri et al. [10] presented a Mixed Integer Linear Programming
47 (MILP) model for the determination of the design and the running conditions of a trigeneration plant
48 for a hospital complex. Guo et al. [11] carried out a two-stage optimal planning and design method
49 for Combined Cooling Heat and Power (CCHP) microgrid system, using both genetic algorithm and
50 MILP algorithm techniques. Elsidio et al. [12] and Arcuri et al. [13] proposed Mixed-Integer Non-
51 Linear Programming (MINLP) models for determining the most profitable synthesis, design, and
52 annual scheduling of CHP systems.

53 Other works have focused on the optimal exploitation of the CHP potential in existing plants.
54 Franco and Versace [14] defined the optimal operational strategy of a cogeneration plant connected
55 to a District Heating System. Li et al. [15] analyzed the effect of optimized operational strategy on a
56 CCHP system for office and residential buildings. Bischi et al. [16], Ortiga et al. [17] and Ünal et al.
57 [18] investigated the optimal operating schedule of CCHP systems, with a given design.

58 Many of these studies [6,11] have clearly indicated the importance of considering, in future
59 research, the effect of uncertainties in CHP optimal design. Such a task is very challenging, but it is
60 worthwhile for gaining accurate and robust results. In fact, it is well-known how intrinsic
61 uncertainties affecting Distributed Energy Systems (DES), such as energy demands, fuel price
62 fluctuations, regulation, and so on, might undermine the potential profit of such systems [19]. In this
63 regard, several approaches of optimization under uncertainty have been employed, such as general
64 sensitivity analysis [20,21], sensitivity analysis in mathematical programming [22], fuzzy
65 programming [23,24], dynamic programming [25], robust optimization [19], and stochastic
66 programming [26]. Each of these studies focused on specific types of uncertainties and energy
67 systems. Yokoama and Ito [27] proposed a robust optimal design method, through a case study on a
68 cogeneration system, considering uncertain energy demand of a single representative day. Akbari et
69 al. [28] focused on designing a multi-technology distributed energy system in a neighborhood, under
70 demand uncertainty concerning data insufficiency. Momen et al. [29] provided a Monte Carlo method
71 applied to a gas-turbine-based cogeneration system, considering uncertainties in economic
72 parameters. Mavrotas et al. [30] dealt with risk management for uncertainty in fuel costs and discount
73 rate, by means of the combined use of Monte Carlo simulation and MILP algorithm. Li et al. [31]
74 optimized a building CCHP system, considering fluctuations in the hourly energy demands.

75 In the mentioned studies, long-term uncertainties in energy demand are ignored and typical
76 load year data are considered for the whole lifetime of CHP systems. However, fluctuations in energy
77 demand over the years may be significant and their effect on overall performance and optimal sizing
78 must be specifically evaluated.

79 The main purpose and novelty of this study is therefore to accurately investigate the effect of
80 long-term uncertainties in energy demand on CHP systems. For this purpose, an original optimal
81 integrated sizing and operational strategy methodology is defined, which takes analytically into
82 account uncertainties in energy demand. More specifically, this study provides a probabilistic
83 methodology for risk analysis, based on the simulation of the entire life-cycle of the cogeneration
84 project. Such an approach allows to highlight shortcomings and inaccuracies of usual deterministic
85 methods. Moreover, the adopted methodological framework provides results in the form of
86 probability distributions, thus providing fruitful and complete information to decision-makers.

87 The remainder of the paper is organized as follows. In Section 2 the methodological
88 framework is presented in detail. An essential description of the case study follows in Section 3.
89 Section 4 contains a detailed analysis of the results, while the last section contains concluding
90 remarks.

91

92 **2. Methodology**

93 In pursuit of the above-mentioned goals, a specific methodological framework has been
94 developed. Three main tools have been employed: the dynamic simulation based on a full
95 mathematical model of the system, the so-called Monte Carlo sampling Method, and an optimization
96 algorithm.

97

98 **2.1 System Simulation**

99 It is commonly accepted that an extensive and accurate analysis of a CHP unit requires a
100 detailed simulation of the energy system [32]. In fact, preliminary sizing methods, such as the load
101 duration curve, are useful only for assessing orders of magnitude of the project and cannot fully
102 embrace the complexity of a CHP system. Multiple time-varying loads, part-load performances,
103 simultaneous energy balances and various economical features make any rule-of-thumb approach

104 inaccurate. Furthermore, the importance of considering electrical and thermal load fluctuations
105 instead of mean values is recognized [33].

106 For these reasons, a CHP simulation, based on hourly averaged values for load representation,
107 should be adopted [34]. This approach allows several key factors to be considered, such as part-load
108 efficiency, load factor lower bound, hourly time-dependent prices for purchasing and selling
109 electricity, actual operational hours, different operational strategies. In such a way, comprehensive
110 system performances and a detailed CHP operational scheduling can be obtained.

111 In fact, CHP systems can be run by several possible operational strategies. The two most
112 common forms of operational strategies are: *Following the electric load* (FEL) and *Following the*
113 *thermal load* (FTL). Nevertheless, these traditional strategies might not guarantee the best
114 performance of the systems and optimal operating strategies have therefore been investigated in the
115 last few years. [35,36]. Obviously, the adopted operational strategy can significantly influence the
116 optimal sizing of the CHP system [7] and, consequently, it is essential to opt for an integrated
117 methodology, which simulates all the possible operational strategies.

118

119 **2.2 Monte Carlo Method**

120 Uncertainty in model variables and parameters can considerably undermine the accuracy of
121 the results and may lead to erroneous conclusions. Uncertainty in energy load demand plays a key
122 role in the analysis and design of a cogeneration system [27,37,38]. Not only is an accurate estimate
123 of the energy demand frequently difficult to provide, but also evolution and change in loads are likely
124 to occur. In fact, economic analyses concerning CHP installations should deal with a multi-year time
125 horizon, corresponding to the lifetime of the system, which is generally up to 20 years. During such
126 a period, fluctuations in energy demand are highly probable and may be significant.

127 For the above-mentioned reasons, it is necessary to consider uncertainty associated with the
128 energy input data and evaluate its propagation to the results, in the framework of a risk analysis
129 approach. For this purpose, Monte Carlo Method (MCM) is adopted; indeed, Monte Carlo simulation

130 technique is a state-of-the-art methodology in risk analysis and can be employed within the context
131 of risk management of distributed energy infrastructures [39].

132 MCM is a family of numerical methods capable of solving mathematical problems by means
133 of simulation with random variables. Given a deterministic model $y = f(\mathbf{x})$, with k input data
134 $(x_1, \dots, x_i, \dots, x_k)$, MCM operates under the following steps.

- 135 1. To assign a probability density function (pdf_i) to each model input data x_i . It is also
136 possible to specify a correlation between the various input data.
- 137 2. To generate N possible values for each input data, by means of random samples of its
138 probability density function.
- 139 3. To combine the random samples to get N input vectors. If the input data are not correlated,
140 samples can be combined in any order.
- 141 4. To perform the simulation of the model N times, one for each input vector. In such a way,
142 a vector of results is provided, and an input-output mapping of the model is defined, within
143 the input space of the input pdf_i .
- 144 5. The set of N values of the output data $(y_1, \dots, y_j, \dots, y_N)$ defines the probability density
145 function of the result of the simulation.

146 The application of the MCM provides a detailed insight into the probability distribution of the
147 target variable [40], allowing the calculation of typical statistical indicators, such as the mean value,
148 the standard deviation, the skewness, the n -th percentile, the cumulative risk of negative values, etc.
149 This additional information can offer a critical support to decision-makers, providing a probabilistic
150 scenario and guaranteeing a robust decision.

151 A significant element in MCM is the number N of simulations to be performed to get accurate
152 results. In fact, the uncertainty in the statistics obtained by the probability density function of the
153 results drops as N increases. Given a certain confidence level, the margin of error usually decreases
154 as $\sqrt{D/N}$, where D is a constant depending on the different variant of the applied MCM [40].

155

156 2.3 Optimization

157 A detailed optimization procedure, concerning both the long and short-term time frames, has
158 been purposely developed and is described as follows.

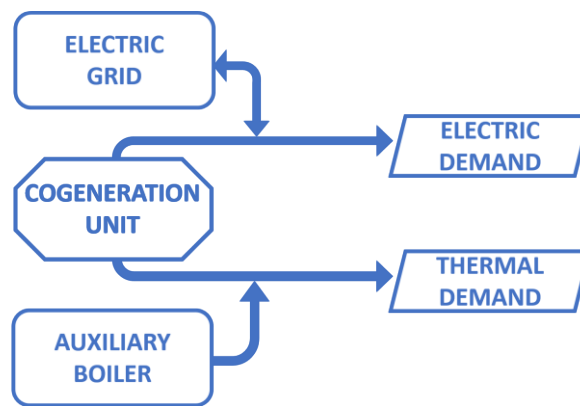
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160 2.3.1 The objective function

161 The optimization problem consists in the determination of the integrated CHP size and
162 scheduling that meet the energy demand with the lowest possible cost. To evaluate the performance
163 of the system, an annual cost saving index performance is defined. It is the annual cost saving with
164 respect to the separate-production energy cost, which represents the reference energy system cost.
165 In the separate-production energy system scenario, the thermal demand is met by a traditional
166 natural gas-fired boiler and the electric demand is covered by the national electrical grid. In such a
167 scenario, the only costs to be accounted for are based on the prices of purchased gas and electric
168 energy required to meet the energy demand:

$$AC_{SP} = \sum_{i=1}^{8760} c_F^i F_{boi,SP}^i + \sum_{i=1}^{8760} c_{PEG} E_{p,SP}^i \quad (1)$$

169



170

171 **Fig. 1** Schematic representation of the Combined Heat and Power system

172

173 In the CHP system scenario (illustrated in Fig. 1), the thermal demand is met partly by the CHP heat
174 production and partly by the gas-fired boiler, and the electric demand is covered partly by the CHP

175 electric production and partly by the national electrical grid. Therefore, the economic features to be
 176 accounted for are: the purchased natural gas price, both for the boiler and the CHP units, the price for
 177 purchasing electricity by the grid, the income for selling electricity to the grid, the cogeneration unit
 178 investment cost, and the cogeneration unit maintenance cost.

$$AC_{CHP} = \sum_{i=1}^{8760} c_F^i F_{boi,CHP}^i + \sum_{i=1}^{8760} c_{PEG} E_{p,CHP}^i \quad (2)$$

$$- \sum_{i=1}^{8760} c_{SEG} E_{s,CHP}^i + \sum_{i=1}^{8760} c_F^i F_{cgu,CHP}^i + C_{I,cgu} + C_{M,cgu}$$

179 It should be noted that both the separate-production and the CHP system costs are annual, which
 180 means they represent the cost needed to meet the energy demand of the reference year. The
 181 cogeneration unit investment cost is considered equally distributed over every year of its design
 182 lifetime.

$$C_{I,cgu} = C_{TI,cgu} / DLT_{cgu} \quad (3)$$

183 Therefore, the optimization problem consists in the maximization of the annual cost saving
 184 percentage:

$$\max\{ACSP\} = \max\left\{\left(\frac{AC_{SP} - AC_{CHP}}{AC_{SP}}\right)\%\right\}. \quad (4)$$

185

186 **2.3.2 Decision variables, demand constraints, capacity constraints, balance equations,** 187 **operational strategy rules**

188 As already indicated, three different operational strategies are considered: FEL, FTL and
 189 minimum cost (MC) operational strategy. For the FEL and the FTL, the only decision variable is the
 190 Co-Generation Unit capacity P_{cgu} , for the design optimization. Instead, five additional decision
 191 variables, for the MC operational strategy of the i -th hour, are defined as follows:

192 $E_{cgu}^i, Q_{cgu}^i, Q_{boi}^i, E_p^i, E_s^i$. In case the simulated operational strategy is either the FEL or the FTL, these

193 operational decision variables are univocally identified by the constraint equations and there is no
 194 need for optimization tools.

195 Demand constraints are defined as follows:

$$E_{CGU}^i + E_p^i - E_s^i - E_d^i = 0 \quad (5)$$

$$Q_{CGU}^i + Q_{boi}^i - Q_d^i = 0 \quad (6)$$

$$E_p^i E_s^i = 0 \quad (7)$$

196 where $i = 1, 2, \dots, 8760$. Equation (7) states that in the i -th timestep electricity is either sold or
 197 purchased.

198 Capacity constraints are defined as follows:

$$E_{CGU}^i - P_{CGU} \delta_{CGU}^i \leq 0 \quad (8)$$

$$E_{CGU}^i - P_{CGU, \min} \delta_{CGU}^i \geq 0 \quad (9)$$

199 where $i = 1, 2, \dots, 8760$, and δ_{CGU}^i is a binary variable equal to 1 when the cogeneration unit is on
 200 and equal to 0 when it is off.

201 The following balance equations are considered.

$$F_{CGU}^i - E_{CGU}^i / \eta_{E, CGU}^i = 0 \quad (10)$$

$$Q_{CGU}^i - E_{CGU}^i \frac{\eta_{Q, CGU}^i}{\eta_{E, CGU}^i} = 0 \quad (11)$$

$$F_{boi}^i - Q_{boi}^i / \eta_{boi}^i = 0 \quad (12)$$

202 where $i = 1, 2, \dots, 8760$.

203 Different operation rules must be additionally implemented for each operational strategy.

204 For the FEL:

$$E_s^i = 0 \quad (13)$$

$$E_p^i = 0, \quad \text{unless } P_{CGU}t - E_d^i < 0 \quad (14)$$

205 where $i = 1, 2, \dots, 8760$.

206 For the FTL:

$$Q_{boi}^i = 0, \quad \text{unless } P_{CGU}t \frac{\eta_{Q,CGU}^i}{\eta_{E,CGU}^i} - Q_d^i < 0 \quad (15)$$

207 where $i = 1, 2, \dots, 8760$.

208 For the MC:

$$\min \left\{ \sum_{i=1}^{8760} c_{F,CHP}^i F_{boi,CHP}^i + \sum_{i=1}^{8760} c_{PEG,CHP} E_{p,CHP}^i - \sum_{i=1}^{8760} c_{SEG,CHP} E_{s,CHP}^i + \sum_{i=1}^{8760} c_{F,CHP}^i F_{CGU,CHP}^i \right\} \quad (16)$$

209 where $i = 1, 2, \dots, 8760$. The 5 decision variables of the operational strategy optimization problem
210 are related to each other by means of Equations (5), (6), (7), and (11). Therefore, there is only one
211 degree of freedom for the minimization.

212

213 2.3.3 Monte Carlo simulation and multi-objective optimization criteria

214 As previously indicated, the MCM consists in a high number of repeated random samples. For
215 each sample a complete year simulation of the system must be run. This procedure allows to consider
216 the whole life of the plant, in terms of energy demand fluctuations, even if an annual index
217 performance is considered. Because of the MCM application, however, the objective function ACSP
218 is not just a single value for each design configuration and operational strategy but is a probability
219 distribution function. Consequently, the definition of a multi-objective optimization criterion is

220 suggested [41]. Such a criterion can represent statistic information contained in the probability density
221 function that are relevant to the selection of the CHP system. For these purposes, we propose to adopt
222 the following two indicators: the expected value $ACSP_{EV}$ and the 2.5-th percentile $ACSP_{2.5th\%ile}$.
223 The expected value of a random variable is the average value of the probability distribution and
224 represents the average performance of the system. On the other hand, the 2.5-th percentile indicates
225 the worst-case scenario. In fact, it is the ACSP value above which 97.5% of the values will occur.
226 Therefore, a Pareto frontier of potentially optimal solutions will be obtained, so that decision-makers
227 will be able to make appropriate tradeoffs within this set of solutions.

228

229 **2.3.4 Solution Method**

230 To solve this problem, we implemented a two-stage optimization algorithm in MATLAB
231 environment:

- 232 - Stage 1: Design Optimization (for the selection of the optimal CHP size);
- 233 - Stage 2: Operational Strategy Optimization (for the determination of the optimal unit
234 commitment).

235 The complete optimization procedure is summarized in the diagram set forth in Fig. 2.

236

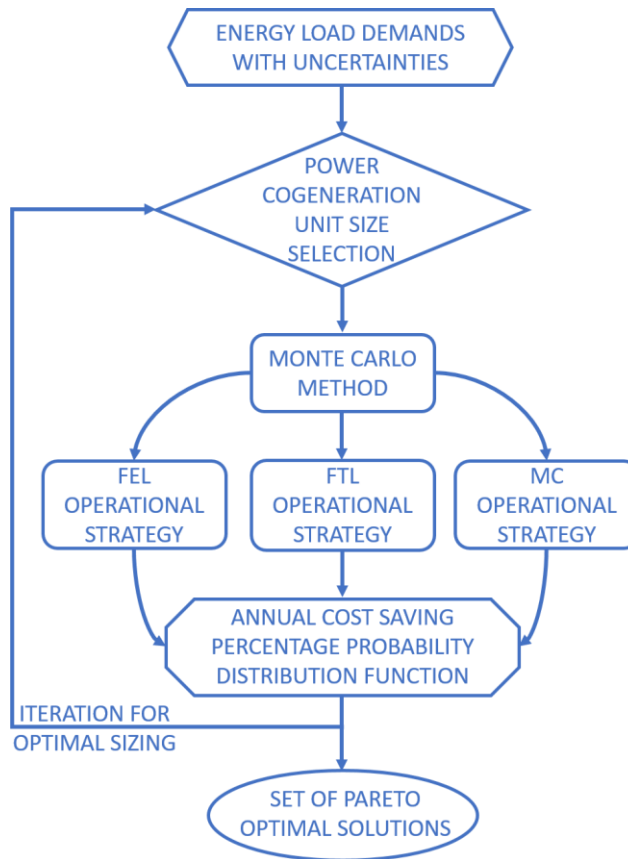


Fig. 2 Optimization procedure

237

238

239

240 The Design Optimization consists in the dynamic simulation of potentially available
 241 cogeneration units with discrete sizes, for each given operational strategy. The comparison between
 242 the annual cost savings of several sizes makes it possible to determine the optimal design. The choice
 243 of a discrete selection of sizes is taken in view of the difficulty in finding commercially available
 244 CHP gas engines with sizes exactly corresponding to optimal numerical solutions [42,43]. For each
 245 selected size, MCM is applied, performing repeated random simulation samples for each operational
 246 strategy. In the event the simulated operational strategy is either the FEL or the FTL, there is no need
 247 for further optimization algorithms, since those strategies can be easily implemented.

248 Conversely, when the performance under optimal unit commitment is investigated, the
 249 Operational Strategy Optimization is required to determine the optimal management of the
 250 cogeneration unit. This optimization algorithm is aimed at identifying, for each simulated timestep,

251 the optimal load factor of the CHP unit that minimizes the overall annual cost saving. For this purpose,
252 several optimization techniques have been used over the years, such as mathematical programming,
253 genetic algorithms, and other methods [4,9]. Usually, these algorithms investigate the entire feasible
254 region, considering a single optimization problem for the whole-time domain (e.g. one year), thus
255 requiring high computational cost. Nevertheless, for the energy system under consideration, the
256 overall optimum coincides with the sum of optimums of every single timestep. We can use the so-
257 called “greedy” approach because the physical system has no “memory” of the previous timesteps.
258 Therefore, the overall problem was split into 8760 subproblems, one for each hourly timestep, and a
259 low computational-cost algorithm, compatible with the high number of simulations required for the
260 MCM, was specifically written. In this way, the problem size for each simulated year is reduced from
261 N_L^{8760} to $N_L \times 8760$, where N_L is the number of the feasible discrete intervals of the CGU load
262 factor L .

263

264 **3. Case study application**

265 The case study used for testing the methodology refers to an operative Italian hospital facility.
266 It is a 500-bed hospital, with a total volume of 230,000 m³. Generally, hospital facilities are
267 particularly suitable to be powered by CHP systems, because of high and constant loads during the
268 year [3].

269 This paragraph summarizes the primary features of the energy system under investigation and
270 the load demand of the case study.

271

272 **3.1 The energy system: technical and economic characterization**

273 In this section, models and features adopted for the components of the simulated energy
274 system are illustrated.

275

276 **3.1.1 Cogeneration unit**

277 The cogeneration unit (CGU) consists in an internal combustion engine (ICE) fueled by
 278 natural gas. ICEs are the most commonly used prime movers for medium scale (100-5000 kW) CHP
 279 applications [36]. The considered nominal electric power capacities P_{cgu} go from 600 kW to 1600
 280 kW, with discrete intervals of 100 kW.

281 The model for the CGU has been taken from [44]. Therefore, the power capacity lower bound
 282 is equal to 50% of the nominal power capacity and part-load efficiencies are considered according to
 283 the following relations:

$$\eta_{E,CGU} = \eta_{E,CGU,nom}(1.1260 L - 0.1260) \quad (17)$$

284

$$\eta_{H,CGU} = \eta_{Q,cguCGU,nom}(0.8253 L + 0.1747) \quad (18)$$

285

286 where the load factor is defined as $L = F_{CGU} \cdot \eta_{E,CGU,nom}/P_{CGU}$. Table 1 reports nominal efficiencies
 287 and the corresponding heat-to-power ratio of the CGU.

288

289 **Table 1**

290 Cogeneration unit main specifications

Parameters	Value
$\eta_{E,CGU,nom}$	38.5%
$\eta_{Q,CGU,nom}$	34.4%
HPR_{nom}	0.894

291

292 The unitary cost of internal combustion engines is significantly influenced by the “scale
 293 effect”. For this reason, a relationship of the CGU cost with respect to size has been considered, based
 294 on [45]:

$$C_{TI,CGU} = 15460 P_{CGU}^{0.7247} \quad (19)$$

295 where $C_{TI,CGU}$ must be expressed in € and P_{cgu} in kW.

296 The design lifetime of the CGU has been considered equal to 20 years, identical for all the accounted
297 sizes; such a duration for cogeneration system projects is commonly accepted [46].

298 The engine maintenance cost per unit of electric kWh produced, as a function of the nominal
299 power capacity, has also been considered, based on [45]. Therefore, the annual maintenance cost is
300 defined as:

$$C_{M,CGU} = \sum_{i=1}^{8760} E_{CGU}^i 0.05604 P_{CGU}^{0.1638} \quad (20)$$

301 where $C_{M,CGU}$ must be expressed in €, E_{CGU}^i in kWh, and P_{CGU} in kW.

302

303 3.1.2 Boiler

304 The nominal power capacity of the natural gas boiler has been considered such as to cover
305 any thermal demand, for each different configuration and operational strategy.

306 It has been modeled with a constant efficiency, with reference to [20]:

$$\eta_{boi} = 0.9 \quad (21)$$

307 Both the boiler and the cogeneration unit are fed by natural gas; the fuel cost per unit of
308 thermal energy, on the lower heating value basis, is:

$$c_F = 0.04 \text{ €/kWh} \quad (22)$$

309

310 3.1.2 Electric grid

311 The electric grid allows for both the sale and the purchase of electric energy. The prices for
312 purchasing and selling electricity have been considered as constant values:

$$c_{PEG} = 0.15 \text{ €/kWh} \quad (23)$$

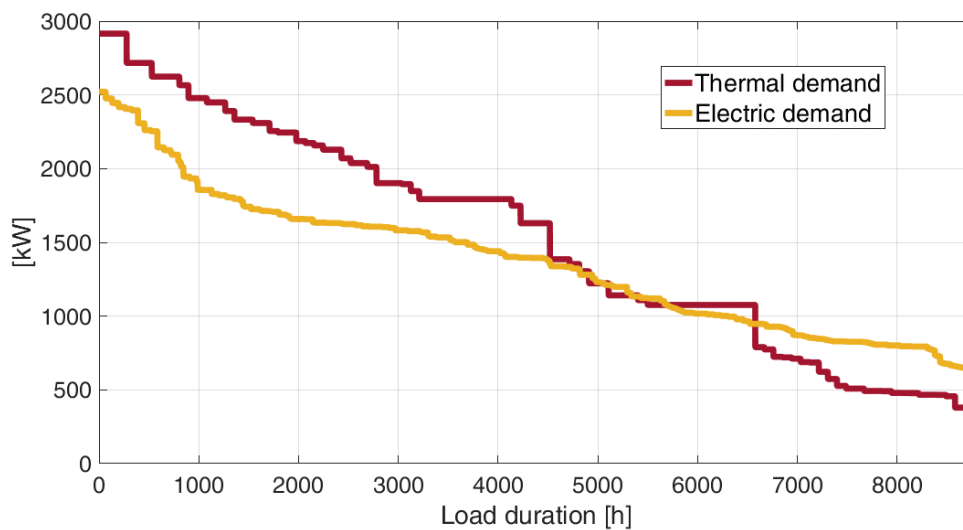
$$c_{SEG} = 0.05 \text{ €/kWh} \quad (24)$$

313 **3.2 Energy load demand**

314 As illustrated above, hourly-averaged values have been adopted for representing the energy
315 load demands. Fig. 3 shows the load duration curves of the electric and thermal demands of the
316 hospital. This data has been obtained from 12 typical days, corresponding to 4 typical weeks. For
317 every week, representing significant seasonal weather periods, one weekday and two weekend days
318 (Saturday and Sunday) have been considered.

319 The uncertainty in the annual energy load demand has been considered through normal
320 distributions. A 20% relative standard deviation has been employed for both electric and thermal
321 demands. Such a value is consistent with the 8-consecutive-year data of energy demand, measured in
322 the test case hospital.

323



324

325 **Fig. 3** Electric and thermal demands: load duration curves

326

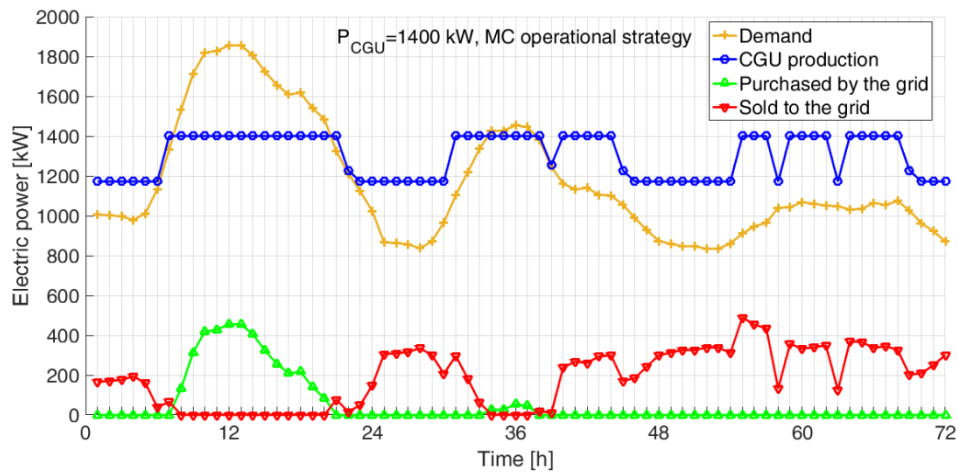
327 **4. Results and discussion**

328 The simulation results for all the potential CGU sizes and operational strategies are shown in
329 this section. 300,000 simulations were performed for each combination of design configuration and

330 operational strategy, so that reliable results and limited uncertainty in the output indicators could be
331 obtained.

332 Figs. 4 and 5 show typical examples of how the simulated energy system works and what kind
333 of detailed outputs are available from the simulations. Fig. 4 shows how the electric demand is met
334 in 72 consecutive hours; Fig. 5 shows the same kind of result, for the thermal demand.

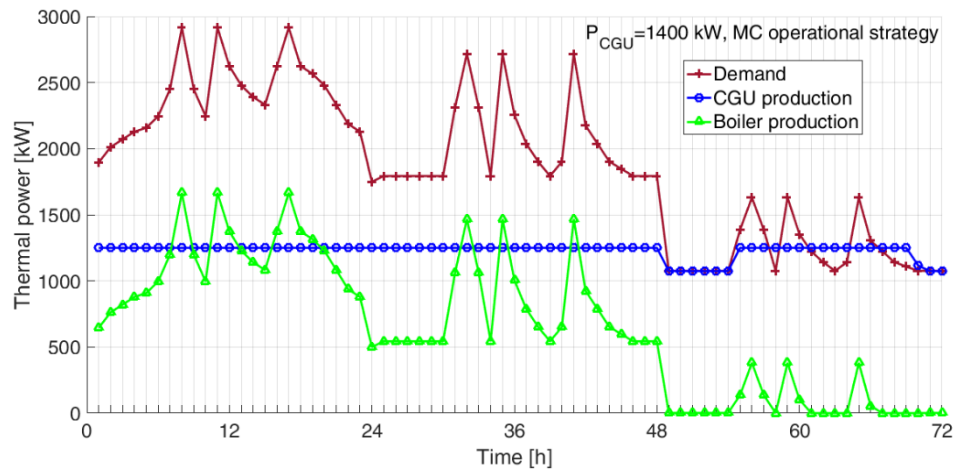
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336

337 **Fig. 4** Example of simulation output: electric power

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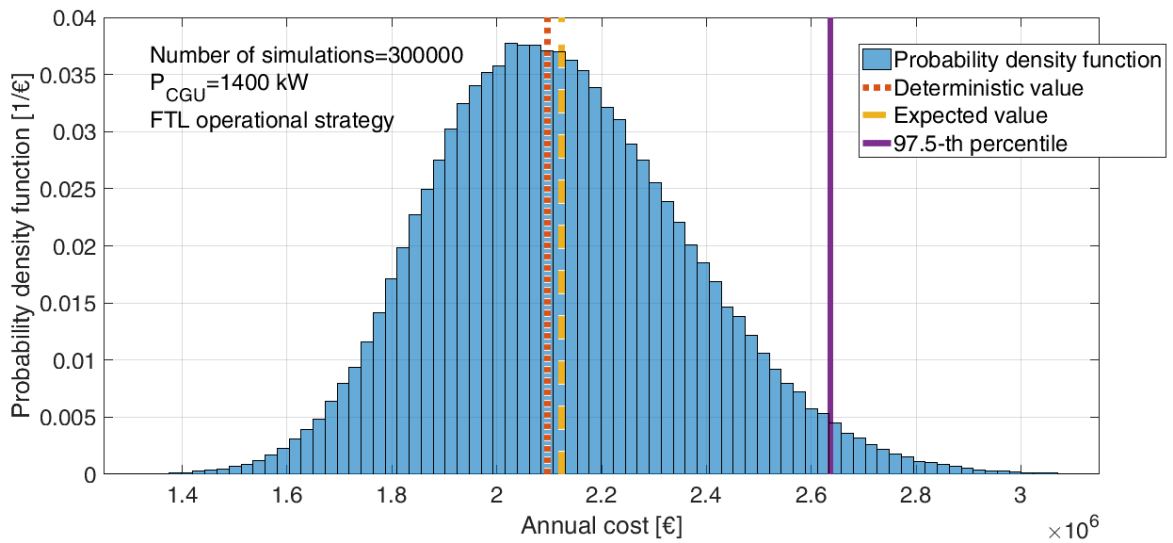


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340 **Fig. 5** Example of simulation output: thermal power

341

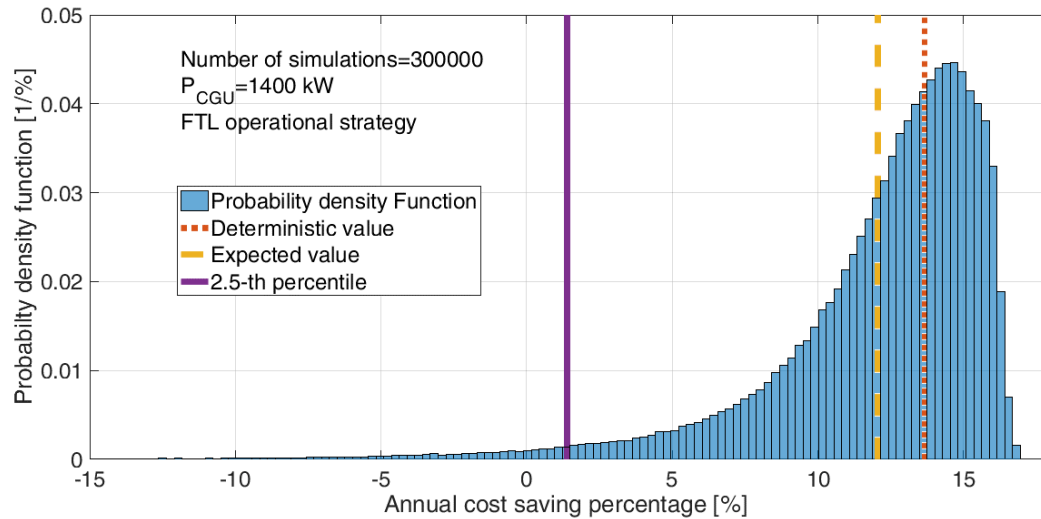
342 In Figs. 6 and 7, examples of the results obtained by the MCM are displayed. Fig. 6 is a
 343 demonstration of one of the probability distribution functions of the annual cost, obtained for a certain
 344 CHP size and operational strategy. Fig. 7, instead, shows the same results in the form of annual cost
 345 saving percentage. In both figures, the vertical dotted line represents the deterministic value of the
 346 index, which corresponds to the simulation result obtained with the most probable (i.e. deterministic)
 347 values of the input data. From these quantitative examples, the importance of assessing the effect of
 348 uncertainties and their propagations to the results is evident. Another interesting consideration arising
 349 from these examples is the asymmetry of the probability distribution function of the annual cost,
 350 despite the symmetry of the input random variables. This aspect, due to the non-linearity of the model
 351 $y = f(\mathbf{x})$, further reinforces the need for a probabilistic approach.
 352



353

354 **Fig. 6** Example of MCM result: annual cost probability density function

355



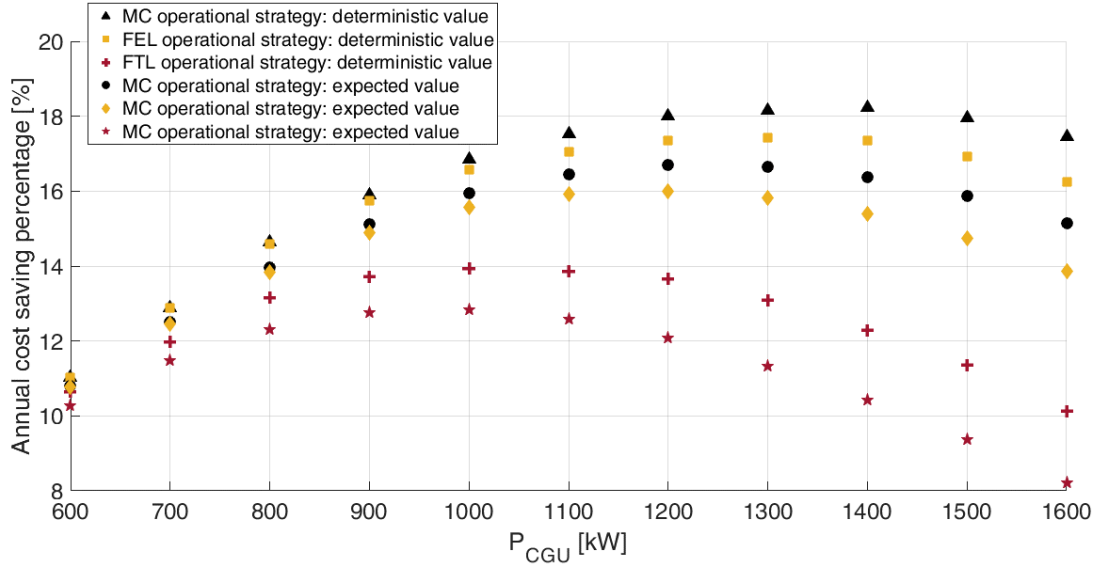
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357 **Fig. 7** Example of MCM result: annual cost saving percentage probability density function

358

359 Fig. 8 shows a comparison between deterministic and probabilistic results, for all the evaluated
 360 operational strategies. The probabilistic results are represented by means of the expected value for
 361 each configuration. The ACSP as a function of the CGU size is shown. As already highlighted, the
 362 main outcome is the numerical gap between the deterministic and the expected values. Moreover, in
 363 this case study, for all the three operational strategies, the expected value of the ACSP is always lower
 364 than the deterministic one, and the gap between these two indicators rises as the CHP size increases.
 365 The method clearly shows how demand uncertainties can significantly affect evaluation of CHP
 366 system performance; therefore, they should always be considered in a thorough analysis. More
 367 specifically, these results reveal that traditional deterministic approaches tend to overestimate the
 368 annual cost saving percentage. In fact, for all three simulated operational strategies, the best annual
 369 cost saving percentage, calculated by means of the deterministic method, is about 10% overestimated
 370 in comparison with the expected value of the probability density functions.

371



372

373

Fig. 8 ACSP for all the CGU sizes and operational strategies: a comparison between the

374

deterministic and the expected value

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Fig. 9 shows the $ACSP_{EV}$ against the $ACSP_{2.5th\%ile}$ values for all the evaluated CGU sizes

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and operational strategies. From this chart, it emerges that some solutions are clearly dominated by

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other more favorable solutions and shall be rejected for this reason. It should also be noted that some

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configurations can even entail a negative cost saving in the worst-case scenario (2.5-th percentile).

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The margin of error in the $ACSP_{EV}$ and in the $ACSP_{2.5th\%ile}$ values, with a confidence level of 95%,

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has been evaluated as $\frac{\sigma(\bar{X})}{\sqrt{N}} t_{0.025,N}$, where σ is the standard deviation, \bar{X} is a sample of size N of a

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random variable (i.e. $ACSP_{EV}$ and $ACSP_{2.5th\%ile}$) and $t_{0.025,N}$ is the value on a t-distribution with N

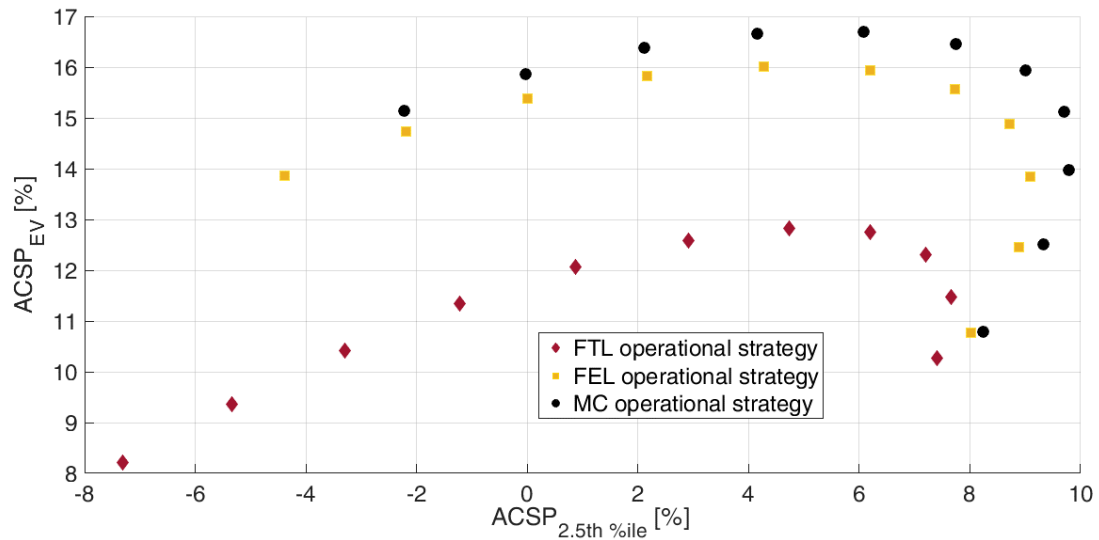
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degrees of freedom for 0.025 right tail probability [47]. The maximum absolute margin of error in the

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$ACSP_{EV}$ and in the $ACSP_{2.5th\%ile}$ values is equal to 0.025% and 0.141%, respectively.

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Fig. 9 $ACSP_{EV}$ vs. $ACSP_{2.5th\%ile}$ for all the CGU sizes and operational strategies

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Finally, Pareto frontiers for each different operational strategy are shown in Figs. 10-12. All these CHP configurations are Pareto efficient with respect to the expected value and the 2.5-th percentile of the annual cost saving percentage. On these same graphics, the best deterministic solutions, namely the highest nominal cost saving solutions, are also represented.

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Thanks to this representation, it is possible to evaluate which sizes are most likely to provide a higher profit, but with a greater risk, and which ones can guarantee an acceptable performance even in the worst-case scenario, at the expense of a lower expected value of the cost saving.

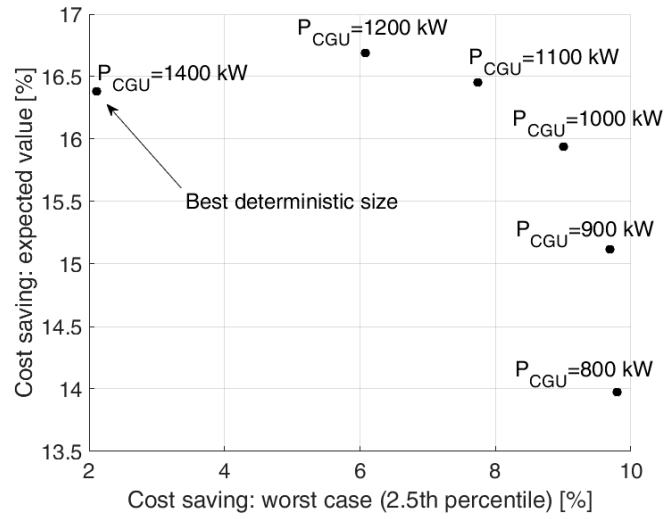
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These charts clearly show the effect of energy demand uncertainties on the overall performance: both in the MC and in the FEL operational strategies, the optimal deterministic sizes are dominated solutions and shall be rejected.

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Furthermore, it is clear how some configurations, despite being Pareto efficient, are reasonably going to be discarded: for example, in Fig. 10, the 800-kW solution provides a substantially lower $ACSP_{EV}$ value, compared to the 900-kW unit, but just a slightly higher $ACSP_{2.5th\%ile}$ value.

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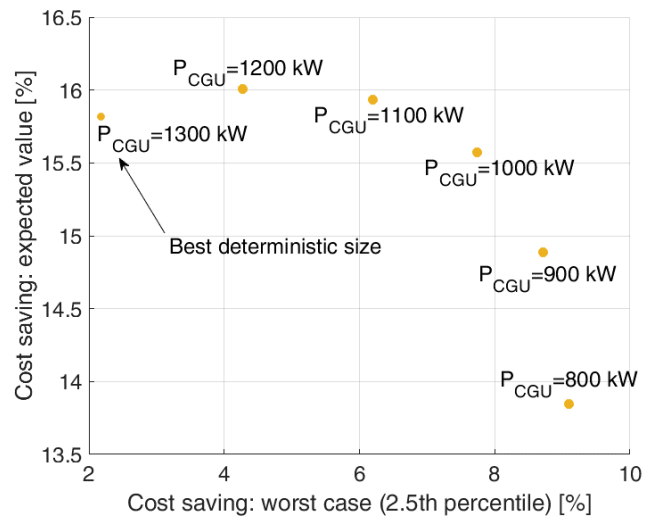


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Fig. 10 MC operational strategy: Pareto frontier of CGU sizes

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Fig. 11 FEL operational strategy: Pareto frontier of CGU sizes

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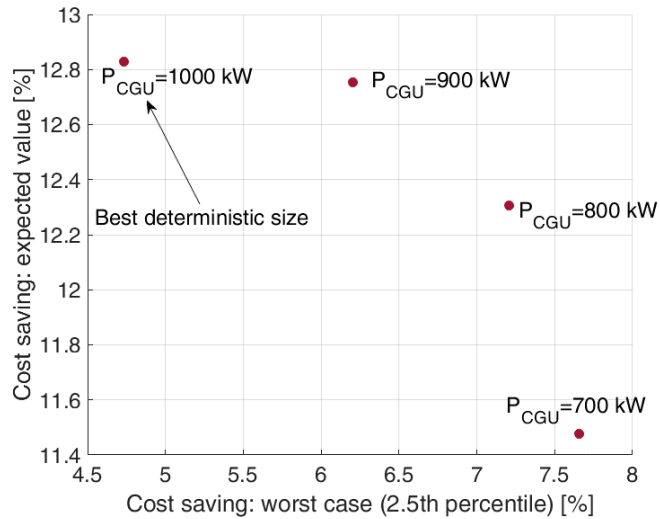


Fig. 12 FTL operational strategy: Pareto frontier of CGU sizes

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425 5. Conclusions

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By comparing the three different operational strategies, it turns out that, in the case study under examination, the FEL strategy always provides better performances than the FTL strategy. In addition, the optimal sizes are larger under the FEL strategy, compared to the optimal ones under the FTL strategy. This is mainly due to the low price of the electricity sold to the grid. Nevertheless, as expected, the MC operational strategy provides the best cost savings and largest optimal sizes.

Furthermore, smaller sizes turn out to be less risky and less unsure than the larger ones, since their performances are less affected by energy demand uncertainties. In any case, the probabilistic approach has clearly shown that the optimal sizes for the case study are significantly smaller than the optimal ones provided by the deterministic approach. For instance, under the MC operational strategy, the Pareto optimal CGU sizes are, on average, almost 30% smaller than the optimal deterministic ones.

In this study, we proposed an original methodology for optimal integrated sizing and operation of cogeneration systems under long-term uncertainty in energy demands. The suggested methodology consists of detailed simulations of the energy system under several operational strategies, a

429 probabilistic analysis based on Monte Carlo method, and a two-stage optimization algorithm. Such
430 an approach allows to analytically and accurately evaluate the effect of energy demand uncertainty
431 and provides a useful tool for robust decision-making.

432 The application of the method has been demonstrated in a case study concerning the
433 implementation of a CHP system for an Italian hospital. First, the influence of uncertainty in energy
434 demands on both optimal cogeneration unit size and annual total cost has been shown. We have
435 clearly highlighted the importance of considering such uncertainties in the evaluation of a CHP
436 system. More specifically, we have shown how traditional deterministic methods tend to oversize
437 cogeneration units and overestimate cost savings. In fact, disregarding long-term uncertainty in
438 energy demand, the optimal size turns out to be about 30% larger and the annual cost saving is
439 overestimated by approximately 10%. Moreover, the implementation of Monte Carlo method has
440 allowed us to define a multi-objective optimization problem. This problem aims at maximizing the
441 expected cost saving while minimizing the risk associated with energy demand uncertainty and can
442 be useful for an accurate assessment of cogeneration plant performance. Pareto frontiers of different
443 CHP configurations have been presented. The simulation results have highlighted that the smaller
444 sizes are less affected by energy demand uncertainties than the larger ones, which, in turn, provide
445 better performance in terms of expected values.

446 Future research may focus on: more complex polygeneration systems (modular cogeneration,
447 energy storage and chillers), effect of combination of uncertainties in several parameters (energy
448 demands at different time scales, fuel and electricity costs, design lifetime, and so on), analysis of
449 correlation between uncertain parameters, and definition of other kinds of multiple criteria
450 (environmental, energetic, exergetic indicators).

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Nomenclature

Parameters

C	Unit cost, €/kWh
C	Cost, €
DLT	Design lifetime, years
t	Timestep, 1 h
η	Efficiency, dimensionless

Continuous variables

AC	Annual cost, €/year
$ACSP$	Annual cost saving percentage, %
E	Electric energy, kWh
F	Energy content of the consumed fuel, kWh
HPR	Heat-to-power ratio, dimensionless
L	Load factor, dimensionless
P	Electric power, kW
Q	Heat, kWh

Binary variables

δ	On-off state for cogeneration units
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Subscripts

$2.5th\ \%ile$	2.5-th percentile
boi	Boiler

<i>CGU</i>	Cogeneration unit
<i>CHP</i>	Combined heat and power production scenario
<i>d</i>	Demand
<i>E</i>	Electric
<i>EV</i>	Expected value
<i>F</i>	Fuel
<i>I</i>	Annualized investment
<i>M</i>	Annual maintenance
<i>min</i>	Minimum
<i>nom</i>	Nominal (L=1)
<i>p</i>	Purchased
<i>PEG</i>	Electricity purchased by the grid
<i>Q</i>	Thermal
<i>s</i>	Sold
<i>SEG</i>	Electricity sold to the grid
<i>SP</i>	Separate-production scenario
<i>TI</i>	Total investment

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456 not-for-profit sectors.

457

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