

What Schemes Do Preschoolers Develop When Using Multi-Touch Applications To Foster Number Sense (And Why)?

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Abstract: As part of an educational project proposed in Italian preschools, an educator followed a protocol that had been used in a previous study (Baccaglini-Frank and Maracci 2015) proposing two chosen iPad apps to children of ages 5-6. This study investigates the schemes developed by the children in response to the apps, and the role the educator's interventions seem to play in such development. Analyses of the data collected suggest that her interventions privileged and encouraged schemes involving counting, which limited the variety of schemes enacted and the aspects of number sense strengthened through the protocol.

Key words: counting, iPad applications, multi-touch technology, numerical abilities, representation of quantities through fingers, preschool

1. Using multi-touch technology in the classroom

Modern multi-touch technology offers learners new affordances that include recognition of a range of touch and multi-touch gestures as well as voice as inputs. Some studies, though not yet many, have started to analyze these affordances in relation to students' mathematical development, in particular to their development of numerical abilities, or "number sense" (e.g., Baccaglini-Frank and Maracci 2015; Sinclair and Pimm 2015; Sinclair and Baccaglini-Frank 2016). Though it is still an elusive notion, different research communities agree that number sense is a necessary condition for learning formal arithmetic at the early elementary level and it is critical to early algebraic reasoning (English and Mulligan 2013). In particular, literature from different fields of research converges in suggesting that using fingers for counting and representing numbers (Brissiaud 1992), but also in more basic ways (e.g., Gracia-Bafalluy and Noël 2008), can have a positive effect on the development of numerical abilities and of number-sense. Indeed, neuroscience research has shown that there is a neurofunctional link between fingers and number processing. For example, Butterworth (1999) has hypothesized that numerical representations and processes are supported by several component abilities: the innate ability to recognize small numerosities without counting (*subitizing*), fine motor ability (e.g., *finger tapping*), and the ability to mentally represent one's fingers (*finger gnosis*). He argues that through our fingers we construct concrete and abstract representations of number, number words, and number symbols. Fingers are used in all cultures to represent numerosities, and this is why the author believes that finger gnosis is intrinsically linked to numerical representations. Moreover, fingers are always available and they can also be used as

an aid in calculations, and therefore they can work as a bridge between concrete and abstract representations of the notions of “quantity” and “operations”. In a study by Penner-Wilger and colleagues (Penner-Wilger et al. 2007) each component ability was in fact found to be a significant unique predictor of number system knowledge, which in turn was related to calculation skill. Noël has obtained consistent results (2005), and she has also demonstrated how consistent use of fingers positively affects the formation of number sense and thus also the development of calculation skills (Gracia-Baffaluy and Noël 2008). Other researchers have suggested that finger-based counting may facilitate the establishment of number practices (e.g.: Sato et al. 2007; Thompson et al. 2004).

1.1 How does multi-touch technology have the potential to foster the development of certain aspects of students’ numerical abilities?

Literature from the fields of cognitive psychology, neuroscience, and mathematics education has pointed out some fundamental aspects of numerical abilities; we have considered a set of these that can potentially be fostered by multi-touch technology (Baccaglini-Frank and Maracci 2015). Below we list this set and then provide a table with hypotheses on software affordances that might support the development of such aspects (Table 1).

Finger tapping (either simultaneously or sequentially) is considered to be a fine motor ability closely related to finger gnosis and to numerical abilities (Gracia-Baffaluy and Noël 2008).

Subitizing is the rapid, accurate and confident judgment of the number of items in small collections ‘at a glance’, without counting. In developmental psychology it is considered a pre-verbal ability, that is thought to emerge from the ability to allocate attention over multiple individual items in parallel; and it is considered one of the neurocognitive “start-up tools” that numerical abilities are thought to be later founded (Piazza 2010).

Recognizing parts of a whole allows to recognize the complementarity of two numbers with respect to a given one, and therefore to *compose* and *decompose* numbers, for example, “7 plus 8” can be seen in many different ways: $(5+2) + (5+3) = (5+5) + (2+3) = 10 + 5 = 15$; $(5+2) + 8 = 5 + (2+8) = 5 + 10 = 15$; $(8-1) + 8 = 8 \times 2 - 1 = 16 - 1 = 15$; etc. (Resnick et al. 1991). Indeed, a key milestone in children’s numerical development is their understanding of how numbers can be decomposed: e.g. that the number ‘seven’ is not just a word in a sequence but a cardinal amount that can be decomposed into smaller numbers such as 2 and 5 (Fuson 1992).

One-to-one correspondence (with fingers) consists in establishing a one-to-one correspondence between numerosities in analogical form (e.g., dots on a screen) and fingers (not necessarily raised and placed simultaneously); though subitizing may be involved, it is not necessary and the ability

lies in establishing correctly the one-to-one correspondence. Indeed, Margolinas and Wosniak (2012) stress the importance for developing numerical abilities of considering quantities independently of numbers. These processes are intertwined with development of the so-called “finger symbol sets” (Brissiaud 1992) that is the representation of numbers and numbers operations and relations through finger gestures. This ability seems to be an important stepping stone for quickly representing numbers with fingers. Later, building on such ability, adding to analogical representations of numbers verbal or written symbolic representations of the same numbers may foster automatization of the linking between sets of fingers and numbers in symbolical form (Clements 2002; Ladel and Kortenkamp 2013).

Estimation is an ability that has been closely related to numerical abilities (e.g., Sowder 1992).

Counting principles consist of five principles considered to be necessary for children to master for developing number-sense (Gelman and Gallistel 1978); these are a) the one-one-principle that relates every single object to exactly one numeral; b) the stable-order principle prescribing the correct order of numbers; c) the last-word rule that assigns the last said numeral not to the last counted object, but to the quantity as a whole; d) the principle of abstraction, according to which objects of any nature, also abstract, can be counted; e) and the order in which the objects are counted does not matter.

Table 1: Aspects of number abilities associated to multi-touch software affordances that might support their development

<i>Aspect of number abilities</i>	<i>Affordances of multi-touch technology with the potential of fostering development of the aspect</i>
Finger tapping	<ul style="list-style-type: none"> • detecting and differentiating rapid sequences of inputs from different areas of the screen; • accepting input in the form of sequences of rapid taps to identify a target numerosity; • detecting as different inputs the simultaneous presence at a given time (or small interval of time) of two or more fingers; • recording different gestures as separate inputs (swipe with 1 finger, swipe with 2 fingers, lasso, pinch, un-pinch/enlarge...); • manipulating virtual hands by user (to answer questions) or by computer (in proposing questions);

	<ul style="list-style-type: none"> • simulating pianos or string instruments.
Subitizing	<ul style="list-style-type: none"> • showing numerosities on the screen for very brief amounts of time (possibly even fractions of a second), • returning immediate feedback in response to the input given by the user; • the objects to be considered may appear still and placed randomly on the screen or in given arrangements (for example like dots on dice), or they can move all together or one with respect to the other; • input may be given not only as typed numbers (in Arabic code or letters) but also in terms of a number of fingers placed simultaneously on the screen, as a number of sequential taps (possibly on items in the stimulus), or as a “capture” gesture.
Recognizing parts of a whole	<ul style="list-style-type: none"> • detecting as different inputs the simultaneous presence at a given time (or small interval of time) of two or more fingers; • manipulating virtual objects.
One-to-one correspondence (with fingers)	<ul style="list-style-type: none"> • detecting as different inputs the simultaneous presence at a given time (or small interval of time) of two or more fingers; • accurate timing of the user’s performance.
Estimation	<ul style="list-style-type: none"> • providing stimuli with different (large) numerosities which may remain on the screen or disappear after a given time; • providing immediate feedback on the input received as a product of the estimation process.
Counting principles	<ul style="list-style-type: none"> • adding verbal feedback in the form of verbal symbolic number representations to sets of fingers placed on the screen, or to numbers represented in analogical form; • detecting gestures such as simultaneous taps and sequential taps, or their combination, and providing different feedback in response to each of them; • arranging objects on the screen through dragging.

The identification of the aspects of numerical abilities described above, together with the hypotheses on software affordances that might support their development led us to a working hypothesis on what we called *multi-touch potential*:

“Multi-touch technology has the potential to foster important aspects of children’s *development of number-sense*, including the ability to use fingers to represent numbers in an analogical format. We will call this the *multi-touch potential*.” (Baccaglini-Frank and Maracci 2015, p.6).

1.2 From an initial study to investigate the multi-touch potential of two iPad apps to the current study

This hypothesis was explored in an initial study, with the goal of analyzing the multi-touch potential of two iPad apps for fostering preschoolers’ development of number-sense, by 1) investigating the schemes that 4-year-old children develop in their interactions with the software, and how they use their fingers; 2) and relating the schemes enacted with the considered aspects of numerical abilities. The apps are *Ladybug Count* (LBC) and *Fingu* (F) (that we will introduce more in detail in a later section of this chapter) these are environments providing a stimulus (either dots on the back of a ladybug or fruit in groups floating on the screen) to which the user responds placing on the screen a number of fingers that corresponds to the numerosity of the stimulus. We considered the following aspects of numerical abilities: multiple fingers tapping (simultaneously or sequentially), subitizing (simple or double), recognizing parts of a whole, one-to-one correspondence, approximate estimation (of small or large quantities), the counting principles (Baccaglini-Frank and Maracci 2015). Students’ interactions with the apps were analyzed identifying subsets of these aspects that were present in the different strategies used. A total of 15 different strategies were recognized, and all aspects of numerical abilities appeared in at least two strategies. Interestingly, counting strategies, or, more in general, verbal symbolic utterances, were used by very few children. An important finding was confirmation of how the multi-touch potential could be exploited to foster development of the ability to use fingers to represent numbers in an analogical format.

One year later, a research-to-practice group from a university in a different city in northern Italy decided to adopt the same protocol used in the initial study within an educational project aimed at strengthening preschoolers’ numerical abilities, and asked for my supervision. In exchange, I was able to obtain consent forms from the parents of the children in one class to collect videos of the sessions. The 24 children of this class were in the last year of preschool (5-6 years old), from socioeconomic backgrounds comparable to those of the children in the initial study. The two major differences with respect to the initial study were: 1) the age of the children involved, 2) the fact that

the educator in this study, who would be introducing the apps and working with the children, and who was an in-service preschool teacher (not in the same school) with a degree in psychology, and presented to me as an “expert”, had planned to intervene during the preschoolers’ activities with the iPad to “help them learn”. On the contrary, in the initial study the pre-service teacher carrying out the protocol was trained by the research group and intervened minimally during each play session.

My main objective in analyzing these new data was to compare these children’s schemes to those of the younger children in the first study, and gain insight into how they evolved, expecting that such evolution might depend on the interventions of the educator.

The research questions I sought to address were:

- 1) Are there (and if so which) recurrent behaviors in the enactment of the schemes developed by the two groups of children?
- 2) Are there differences (if so, which) between the enactments of the schemes developed by the two groups of children?
 - 2a) If so, what do these differences suggest in terms of aspects of number abilities developed by the two groups of children?
 - 2b) If so, how might these differences be explained in terms of the interventions of the educator?

This chapter addresses, in particular, question 2.

2. Conceptual framework and methodology

I make use of the notion of *scheme* as developed by Vergnaud (1990) to link children’s actions to their goals and intentions in a given situation, and to certain characteristics of the situation itself. This will allow me to relate enactments of schemes to the aspects of number abilities introduced above, and to identify and compare the children’s enactments of the schemes developed, and gain insight into different aspects of numerical abilities developed by the two groups of children. In this section the introduction of the notion of scheme is followed by a presentation of the apps used in the protocol accompanied by summaries of the enactments of schemes developed by 4-year-olds in the initial study.

2.1 The notion of scheme

The notion of scheme as developed by Vergnaud (1990, 2009) elaborates on the Piagetian notion of “scheme”, and characterizes it as *an invariant organization of the activity for a given class of*

situations. The main components of a scheme are: the goal and the anticipated outcomes; the rules of action, of gathering information, of control taking; and the operative invariants (implicit knowledge), including *concepts-in-actions*, that is concepts that are implicitly considered as pertinent, and *theorems-in-actions* that is, propositions believed to be true. We will refer to a (visible) recurring sequence of actions as the “enactment” of a scheme.

Even though all the components of a scheme are important, operational invariants have a prominent role. They consist of the implicit knowledge which structures the whole scheme: they drive the identification of the situation and of its relevant aspects, and allow selecting suitable goals and inferring the rules for generating appropriate sequences of actions for achieving those goals.

2.2 The apps used and the 4-year-old children’s enactments of schemes

The choice of the apps used in the protocol was constrained by limitations during the design of the protocol in the initial study (Baccaglini-Frank and Maracci 2015). In particular, we could use only already published, free or very cheap apps, easy for children to become familiar with, and presenting a strongly structured environment allowing primarily closed conversing-type interactions (Sedig and Sumner 2006; Sinclair and Baccaglini-Frank 2015). Within the constraints, two multi-touch apps, which seemed to have some potential for fostering the development of children’s number-sense, were identified. They are: Ladybug Count and Fingu.

Ladybug Count (Finger Mode¹): The layout of this app is the top view of a ladybug sitting on a leaf, and the aim of each playing turn is to make the ladybug walk off the leaf. This happens when the player places on the screen (in any position) as many fingers as the dots that are on the ladybug’s back. Given a certain number, the dots appear on the ladybug’s back always in the same pattern. As each finger is placed on the screen one of the dots on the ladybug’s back is highlighted (Figure 1), and the iPad makes a “pop” sound.

¹ There is also another mode, called ‘tap mode’, in which success is reached when the user taps the screen (sequentially) as many times as the numerosity of the dots on the ladybug’s back. This mode seems to mostly encourage students to use counting strategies, and to support only to a limited extent children’s development of number abilities. This is why we chose not to use it.



Figure 1: View of the LBC screen with a player that set three fingers on the screen

When all the dots are highlighted a sound is emitted preceding the announcement of the number of dots that were on the ladybug's back. At this time the ladybug walks off the screen and a new one appears. This process repeats as long as the player wants to play. If the player places more fingers on the screen than the dots on the ladybug's back, all the dots become highlighted, but the ladybug does not walk off the leaf and the sound: "Oops!" is emitted. If the player places on the screen fewer fingers than the dots on the ladybug's back, only a number of dots corresponding to the fingers on the screen is highlighted and nothing else happens. This app will be referred to as LBC.

In the initial study we identified 11 enactments used by the children in LBC. These were classified into "general" (6), that is enactments of schemes not apparently linked to a "small" or "large" number of dots on the ladybug's back, and "specific" (5) ones that were sensitive to the number of dots to "count". This was necessary because the children seemed to hold different schemes for a very small number of dots (1-3) or large numbers of dots (7 to 10). Moreover, in several cases the children reacted to the appearance of the ladybug with a large number of dots through verbal expressions such as: "How many!" "That's a lot!". This allowed us to infer that the two situations identified above were different for them, and thus identified different schemes, possibly related to the different aspects of number-sense. For example, the most common enactment of a scheme in the presence of a small number of dots involved the rapid recognition of the small number of dots, apparently through subitizing followed by the placement of the same number of fingers (in a variety of configurations) on the screen simultaneously. This enactment did not contain verbal utterances. The most common enactment in the presence of a large number of dots involved placing all fingers on the screen and then, possibly, removing fingers one at a time until positive feedback was received from the app.

In terms of the aspects of numerical abilities that are in relation with the schemes identified for LBC, we identified the following: multiple finger tapping - simultaneous (6 schemes); multiple finger tapping - sequential (6 schemes, but not all coincident with the 6 for simultaneous tapping); subitizing (for 4 schemes); recognizing parts of a whole (2 schemes); the ability to match numbers of fingers (not necessarily instantaneously) to a number of objects, without counting (7 schemes); the ability to match numbers of fingers (not necessarily instantaneously) to a number of objects counting (2 schemes); estimation (5 schemes); counting principles (3 schemes).

Fingu: The layout of this environment (Barendregt, Lindstrom, Rietz-Leppanen, Holgersson, and Ottosson 2012) looks like a room in which different kinds of floating fruits appear. The objects appear in one group or in two groups that float independently, but within each group the arrangement of the objects remains unvaried. The player has to place on the screen, simultaneously, as many fingers as the objects that are floating within a given amount of time (Figure 2). If s/he succeeds the iPad emits a sound and shows a few dancing happy animations. Otherwise, if the number of fingers is incorrect or time runs out, a different sound is emitted and sad animations appear on the screen. Then the player can go to the next round, until s/he loses or passes the level. The game provides statistics on the performance of the player for each level attempted. For an analysis of cognitive abilities potentially stimulated by this app the reader can refer top 670-671 of Sinclair and Baccaglini-Frank (2016). This app will be referred to as F.

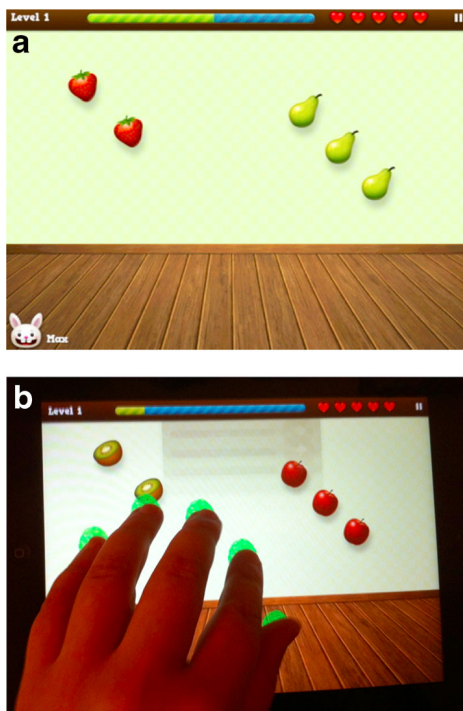


Figure 2: a) View of the screen of F with two groups of floating fruits in fixed arrangements; b) view of the screen of F after the player has set five fingers on the screen, simultaneously

In the initial study we identified 4 enactments used by the children in F. In this environment, selecting the exact number of fingers and placing them simultaneously on the screen within a limited amount of time was a source of difficulties for most children: indeed, it requires, among other skills, the development of advanced fine motor abilities. Typically the children's tendency was to continue to use the first enactment that seemed to be effective in a few cases, despite possible successive failures. Four enactments were identified, two of which involved recognizing the number of objects (be they in a single group or in two groups) without verbally counting, and trying to place on the screen the corresponding number of fingers either of a same hand or of two hands, simultaneously. Many children tried to place their fingers as close as possible to the floating objects, so as to "catch" them, or to reproduce with their fingers the same spatial arrangement of the floating objects (enactment 3). Only a few children quickly counted the floating objects (pointing to them and counting aloud) and then placed a same quantity of fingers on the screen (enactment 4).

In terms of the aspects of numerical abilities that are in relation with the schemes identified for F, we identified the following: multiple finger tapping - simultaneous (3 schemes); multiple finger tapping - sequential (1 scheme); subitizing (1 scheme); recognizing parts of a whole (1 scheme); the ability to match numbers of fingers to a number of objects, without counting (1 scheme); the ability to match numbers of fingers to a number of objects counting (1 scheme); counting principles (1 scheme).

The protocol was administered as follows: the 24 children (ages 5 and 6) worked in groups of 5 and played 5 minutes a day for 2 weeks with the two iPad apps under the supervision of the educator, taking turns while the other children in the group watched. If prompted by the educator, a child from the group could give advice to the child playing, through verbal or gestural utterances. All activities with the apps were video-recorded and transcribed. Identification of students' recurrent behaviors identified as enactments schemes were flagged, as well as all interventions of the educator. Finally, at the end of her work with the children, I interviewed the educator in order to have an additional key of interpretation when analyzing the possible effects of her interventions.

Below we report on two cases that are representative of 20 of the children; then we report on the non-prototypical case of one of the 4 other children.

3. Results: the analysis of two prototypical cases and of one non-prototypical case

The schemes enacted by Giovanna and Sara are quite similar to those of most of the other children in the class. We analyze their development and highlight typical interventions of the educator. In

this section the focus is on the students' schemes, while the interventions of the educator will be analyzed in the next section of the chapter.

3.1 Giovanna (prototypical)

Giovanna (5 years, 3 months) starts her first interaction with LBC very hesitantly, barely showing the fingers she intends to put on the screen and placing them very close together. Every time she hesitates (even for only a few seconds) the educator asks her classmates to “show Giovanna how to do this with her fingers”. After three iterations of this process, Giovanna immediately looks for hints from her classmates, from one in particular, praised by the educator, checking the configuration of fingers he is raising and imitating it. Together with finger configurations, her classmates also shout out the numbers of dots. Giovanna pretends to count them, pointing to a few and then repeats the number pronounced by her classmates. The educator says nothing to stop the classmates from talking, glances at the screen to check it, and makes comments like:

Educator: Seven! How do you do six with your fingers? Come on!

Educator: Come on, you know how to do nine [with your fingers]!

The educator also highly praises children who “know”, as in the following example.

[A ladybug with 7 dots on her back appears]

Giovanna: [Counts up the dots pointing with her finger.] Seven [in a whisper].

Educator: Seven! How do you do seven with your fingers, Giovanna? Come on!

Giovanna: [She timidly raises the fingers on a hand and the index and middle finger of her other hand, imitating the fingers shown by a classmate.]

Educator: Right, very good! See, you *know*!

These interventions by the educator both in LBC and in F seem to lead Giovanna to develop two schemes such as the one described below (Table 2), with the goal of doing what she thinks the educator (seen as a teacher) wants, and seemingly identifying two situations in LBC, based on whether she immediately recognizes the number of dots on the ladybug's back (1a) or not (1b).

Table 2: Two initial schemes enacted by Giovanna while interacting with LBC and F

Two of Giovanna's schemes: 1a-2-3; 1b-2-3
1. Figure out the number of dots/fruits, to do this either 1a. recognize the number immediately

- | |
|--|
| <ol style="list-style-type: none">1b. count them up from one;2. say the number,3. use <i>the</i> fixed configuration approved by the educator for that number. |
|--|

She seems to also develop what we could see as a concept-in-action: every number pronounced verbally corresponds to a *fixed* configuration of fingers.

Giovanna seems to inhibit enactments that involve configurations of fingers other than what she believes to be *the right one*. In fact, in one episode during her first playing session, Giovanna sees the dots on the ladybug's back [there are two on each wing], and she timidly raises two fingers on each hand fingers (here she might have used one-to-one correspondence), waiting for feedback. She sees other children showing four fingers raised on one hand, and copies their finger configuration. The educator says nothing to the other children and simply says "Good!" to Giovanna when she touches the screen and receives positive feedback. A short time after, a ladybug with 9 dots appears. Giovanna does not count, but she seems to over estimates (here she seems to be using estimation), placing all fingers of both hands on the screen, a scheme that was frequently enacted also by children in the initial study. Instead of trying to adjust her fingers (as in schemes identified in the initial study), for example, by lifting one, she takes her hands off and looks for a configuration to copy. After these two episodes Giovanna's behavior can be well described in light of the schemes we hypothesized above.

In general, when Giovanna cannot remember what she believes to be *the approved* finger configuration, she depends on her classmates' hints, and copies, seemingly with no control over the answer that she then gives; or, if she cannot catch a hint quickly, she listens to the number pronounced by the class and counts up from "one", raising her fingers one at a time and always in the same order. Clearly, remembering fixed configurations of fingers associated to a word requires a lot of (otherwise unnecessary) memory and it could even inhibit the development of fundamental aspects of numerical abilities: the child can be successful without ever putting in relation the dots and the fingers raised, other than through a verbal utterance (when a finger configuration is not shown directly), and without developing, for example, awareness of part-whole relations.

In F, Giovanna relies entirely on reaching (either by herself or hearing it from her classmates) a verbal pronunciation of the number of floating fruits from which she produces *the* configuration of fingers if she remembers it. She does not count up her fingers because the game does not give her the time. Her configurations of fingers do not seem to be directly related to the partitions of the fruits into smaller sets when there is more than one. For example, when two floating sets of 2 fruits appear, she hears a classmate say: "four" and puts down the fingers of her right hand excluding her

thumb. The same happens when sets of 3 and 1 appear on the screen. Giovanna seems to rely heavily on her schemes; so much that when possibly perturbing events occur, the scheme remains unchanged. For example, when 4 and 1 fruits appear at a certain point she hears a classmate say (erroneously) “four” and she puts down her usual “four” configuration. When 3 and 2 fruits appear a classmate shows 3 fingers on one hand and 2 on the other; Giovanna sees, but she hesitates and then says: “There are five” and places down all her fingers of a single hand.

Her schemes turn out to be successful in F and Giovanna passes to level II of the game. Now more than 5 fruits can appear. The other children no longer have time to figure out how many fruits are floating around on the screen before they disappear, so Giovanna counts them up each time, starting from “one” and saying the numbers aloud as she points to each fruit. She appears to not know (at least not quickly enough) the configurations for any of the quantities above 5, so she either puts down no fingers or she tries to put down some, frequently less than 5 and loses.

In F her enactments do not seem to change, however in LBC they do. During the later playing sessions, Giovanna proceeds more and more independently with respect to what her classmates say, and the form of her gestures change, as well. For example, she places her fingers on the screen spreading them out on the leaf rather than bunching them up, like during the initial episodes. During the last session with LBC Giovanna has learned to generate fixed configurations quickly for quantities below 5, and for larger quantities she modifies her behavior for figuring out how many fingers to raise. Interestingly, she never counts up her fingers when she recognizes the number of dots. These changes can be seen in terms of new schemes (1a-2a; 1b-2a; 1b-2b), as shown in Table 3:

Table 3: Schemes enacted by Giovanna while interacting with LBC at the end of the administration of the protocol

Giovanna’s final LBC schemes: 1a-2a; 1b-2a; 1b-2b
<ol style="list-style-type: none"> 1. Figure out the number of dots, to do this either <ol style="list-style-type: none"> 1a. recognize the number immediately (if below five) and say it out loud; 1b. (if 1a fails) count them up from one; 2. raise fingers by <ol style="list-style-type: none"> 2a. using <i>the</i> fixed configuration of fingers, when known, corresponding to the number pronounced, 2b. otherwise counting up fingers (in a constant order) starting from one.

For example, when a ladybug with 8 dots appears, Giovanna counts up the dots and counts her fingers, starting with “one” as the thumb of her right hand. The educator gives very positive feedback. The same happens when a ladybug with 10 dots appears, and then when a ladybug with 8 dots appears again. When the next ladybug appears and it presents, again, 8 dots, Giovanna seems to recognize the configuration and remember the configuration for “eight” (as a hand and 3 fingers). This suggests that she still holds valid the concept-in-action we had hypothesized earlier.

This can also be seen in how Giovanna seems to privilege schemes 1a-2a and 1b-2a over ones using 2b. For example, when a ladybug with 4 dots appears she says “four” and in a seemingly automatic way counts up four fingers starting from her thumb on the right hand, but as soon as she sees the four fingers she changes the fingers to her *fixed configuration* for “four”.

The aspects of number abilities that can be related to Giovanna’s most frequently used schemes, which were found to be similar to those of most children in the class, are: multiple fingers tapping (simultaneous), subitizing, and use of the 3 counting principles (one-to-one correspondence, stable order and cardinality, but not order irrelevance or abstraction). We note that one-to-one correspondence does not seem to be directly involved (especially in the early enactments where it would have been important), because the figure configuration is always mediated by pronunciation of a verbal-symbolic numeral that Giovanna directly associates (either because she remembers it, or because she imitates a classmate, or because she carries out a new counting process) to it.

3.2 Sara (prototypical)

Although Sara (5 years, 5 months) is not in Giovanna’s group, with her the educator keeps on intervening in the same way as with Giovanna, proposing to count the dots or fingers immediately at the smallest hesitation, and praising her emphatically whenever she does count. During her initial interactions both with LBC and F, Sara is less insecure than Giovanna: for quantities of 1, 2, 3 or 4 she simply says aloud the number corresponding to the quantity and raises a known configuration of fingers. Sara uses constant configurations for “one”, “two” and “three”, while for “four” she seems to flexibly change the fingers raised and placed on the screen. The educator, in these cases, simply praises Sara for getting positive feedback from the software.

Both for LBC and F the schemes developed by Sara seem to be very similar to Giovanna’s (see Table 4).

Table 4: Schemes enacted by Sara

Sara’s final LBC schemes: 1a-2a; 1b-2a; 1b-2b

1. Figure out the number of dots/fruits, to do this either
 - 1a. recognize the number immediately (if below five) and say it out loud;
 - 1b. (if 1a fails) count them up from one, aloud, pointing to each;
2. raise fingers by
 - 2a. raising *any* known configuration of fingers corresponding to the number pronounced,
 - 2b. otherwise counting up fingers (in a constant order) starting from one.
3. In any case, count up fingers before placing any on the screen.

Interestingly, if Sara has reached 2a in the enactment of her scheme, she still proceeds to step 3, that is, she counts up her raised fingers, seemingly to please the educator. Indeed, although she shows a bit more flexibility than Giovanna in making appropriate finger configurations for quantities up to 4, she, too, seems to be conditioned by the educator's insistence on counting. This results in episodes such as the following. In LBC a ladybug with 4 dots appears:

Sara: Four [and raises two fingers (index and middle) on both hands immediately].
One, two, three, four [she counts the fingers on one hand starting from the thumb].

Educator: Very good!

Sara: [She switches back to her initial configuration of 2 and 2 fingers and places them on the screen, receiving positive feedback from LBC].

Educator: Oh, that's OK, too. Good job!

In this case Sara seems to have recognized the number of dots (1a) and associate correctly a known finger configuration, without counting (2a); so it is surprising that she then counts to four on the fingers of her other hand, to then go back to the configuration with two hands to interact with the app. My conjecture is that she enacts the counting just to please the educator, having picked up on her "counting cues".

All the counting in the enactment of Sara's schemes is too time consuming to be effective in F, so Sara receives negative feedback almost every time. She soon asks to stop playing and to give another classmate a turn. The educator satisfies her request and calls a classmate to play.

The aspects of number abilities that can be related to Sara's most frequently used schemes, are very similar to Giovanna's and to the ones of most children (20 out of 24) in the class; they are: multiple fingers tapping (simultaneous), subitizing, and use of the 3 counting principles (one-to-one correspondence, stable order and cardinality, but not order irrelevance or abstraction). We note that one-to-one correspondence, as before, does not seem to be directly involved, however Sara seems to

be aware of more than one finger configuration for different numbers pronounced verbally. This may be an effect of use of one-to-one correspondence between fingers and objects in experiences prior to her interaction with the apps.

Four (of the 24) children enact schemes that seem quite different from the ones used by the majority of students. Indeed these simply refuse to count, possibly because they have not sufficiently mastered the counting principles. Instead, they either imitate the configurations of fingers shown by their classmates, when they were able to, and failed otherwise, or they try to use enactments that I recognize as similar to those of children in the initial study.

3.3 Amanda (non-prototypical)

One of these four students is Amanda (5 years, 2 months), who never says numbers aloud, but puts down precise numbers of fingers, in a variety of configurations, for quantities up to 4, both in LBC and in F, while for larger numbers she seems to estimate, quickly putting down a hand of fingers and some additional ones, a strategy used by various students in the initial study, as well; or placing all her fingers on the screen and lifting one at a time until she receives positive feedback from the app. This happens repeatedly in LBC for ladybugs with 7 or more dots on their backs. Each time Amanda receives negative feedback from the app and starts adjusting her fingers, the educator intervenes with comments like:

Educator: Remember how [another student who counted the dots and then his fingers] did it? Can you do that, too?

Educator: Sweetie, you need to count...look at the dots. How many are there?

This sort of comment contains reference to a counting strategy, either implicitly as part of another child's enactment, or explicitly. These interventions would interrupt the enactment the Amanda's schemes, making her attempts look like a failure, which would trigger another "counting cue" from the educator, and the vicious cycle would continue, breaking only when the child would give into counting, or somehow be able to place the correct number of fingers on the screen.

While playing F, Amanda seems to be enacting different schemes: when two sets of fruits appear and are small (1, 2, or 3 fruits) she tries to place on the screen the number of fingers, on each hand, corresponding to each floating set; or when up to 4 fruits appear, not necessarily in a same set, she puts down the corresponding number of fingers of her right hand. Amanda responds rather quickly, but frequently receives negative feedback, because she does not seem to double check her raised fingers before placing them on the screen. Sometimes the negative feedback is given by the app also

because she does not wait to place down all fingers on the screen simultaneously. In these cases the educator makes statements like the following.

Educator: Slow down, you need to check your fingers!

Educator: Careful! Your fingers need to go down together!

By the end of the playing sessions Amanda has developed use of fixed configurations, like Giovanna, for small numbers (below 5); and recognizes fixed configurations of dots in LBC for larger numerosities (7, 8 and 10), stating the number aloud, verbally, but without reproducing the numerosity on her hands.

4. Results: what is behind the interventions of the educator?

From the representative excerpts above, and thorough analysis of the whole data set, it is possible to make more general inferences about characteristics of the educator's behavior. Her interventions turn out to be quite explicit about what the children should (or should not) be doing, and most of her interventions are triggered by the student's receiving negative feedback from the app. She tends to not take the time to discuss the children's schemes or enactments, neither collectively, nor individually. Moreover, although she accepts different finger configurations for a same quantity, she does not explicitly comment on how a same quantity can be represented through different finger configurations, for example, putting in one-to-one correspondence two different quantities of fingers for a given quantity of dots or fruits. The only sharing that the educator fosters is in cases in which the child playing appears to be hesitant: she calls on other children to "show your classmate how to do it". These characteristics are not at all aligned with those I had expected.

The educator did not seem to be trying to "access students' thinking", focusing on processes that might have lead the children to the development of a certain strategy; instead she would "assess their thinking", in Crespo's words (2000), and act on the end product, the feedback received from the apps. I had expected that her guidance would have taken into consideration both successful and non-successful (in terms of receiving positive feedback from the apps) enactments, and that she would use these to foster students' sharing of their strategies, and therefore their talking about numbers in verbal and analogical form, perception of numerosity, or representation of numerosity through fingers.

As for the enactments the educator chose to foster, I was curious why she had valued so much, on the one hand, counting, and on the other hand immediate association of finger configurations to verbal-symbolic numerals. More specifically, I was curious why she seemed to be pushing children in two directions (she may have been seeing 2 as an "evolution" of 1): 1) counting the dots or fruits,

counting the fingers, and placing the counted fingers on the screen simultaneously; 2) counting the dots or fruits and immediately making a finger configuration that corresponded to the verbal-symbolic numeral pronounced. From the follow-up interview it became clear that indeed she did have a particular procedure in mind that she thought was best (and should be accomplished quickly), because applicable to all the situations generated by the apps. Below are various claims about this that she made in the interview.

Educator: You need to count up the dots or fruits and then count up your fingers, quickly, and put them down together. [...] Once a kid knows how to do a certain number with his fingers, he doesn't have to count up his fingers any more, and he can use whatever way he wants. [...] which fingers they raise is not important, but they should do it quickly and there are easier ways of doing it. [...] This is how they can always experience success.

5. Answers to the research questions and concluding remarks

Once the schemes of the 24 students in the second study were identified, I compared their enactments to those of the children in the initial study. This comparison showed that the 24 children had behaviors similar to those of the younger children mostly in the first encounters with the apps. By the second activity session with the educator, most of the 24 children's schemes had started to transform into ones like Giovanna and Sara's.

There were significant differences between the enactments of the schemes developed by the two groups of children. The most prominent are: 1) the heavy reliance on (or presence of) counting in the schemes of the students 24 students, while such presence was quite limited in the schemes of the students in the initial study (only 4 of the 15 schemes included counting); 2) most students in the second study seemed to rely on memorized finger configurations corresponding to each verbal-symbolic numeral (with no apparent reference to one-to-one correspondence between the fingers and other analogical representations of the number), while this behavior was not found in students of the initial study.

To me it was particularly surprising to see how almost all the students in the second study had incorporated counting processes into their schemes, especially because the counting processes always started from "one". Indeed knowing how to count from "one" is important, and possibly through these activities the children might have learned to count faster. However, it is not clear how much this enhances other numerical abilities (including those involving other counting strategies) in general: achieving mastery in counting does not simply mean learning to do it *faster*! For example,

it is also important to learn to count on from a number greater than “one”, to count backwards, and to learn to *replace* counting with more effective strategies (e.g., Gray and Tall 1994); but these strategies were not within the multi-touch potential of the apps considered.

These differences can further be analyzed in terms of aspects of number abilities they involve, and thus potentially strengthened in the two groups of children; these are summarized in Table 5.

Table 5: Relationship between students’ enactments of schemes and aspects of number abilities

	<i>Aspect of number sense involved (and n. of schemes) – initial study</i>	<i>Aspect of number sense is involved in most common schemes – second study</i>
<i>Finger tapping</i>		
<i>simultaneous</i>	♦ (9)	♦
<i>sequential</i>	♦ (7)	no
<i>Subitizing</i>	♦ (5)	♦
<i>Recognizing parts of a whole</i>	♦ (3)	no
<i>One-to-one correspondence (with fingers)</i>		
<i>not mediated by verbal-symbolic numerals</i>	♦ (8)	no
<i>mediated by verbal-symbolic numerals</i>	♦ (3)	♦ (cases like Sara)
<i>Estimation</i>	♦ (5)	no
<i>Counting principles</i>		
<i>one-one</i>	♦ (4)	♦
<i>stable order</i>	♦ (4)	♦
<i>cardinality</i>	♦ (4)	♦
<i>abstraction</i>	no	no
<i>order irrelevance</i>	♦ (4)	no

Analyses leading to the construction of this table suggest that the schemes developed by the children in the initial study exploited the multi-touch potential of the apps used to a greater extent than did the schemes that the children in the second study were led to develop. In particular, the abilities to recognize parts of a whole, estimate, or create one-to-one correspondences between fingers and sets of objects, did not seem to be promoted in the group of the 24 children.

Although other factors may have contributed (e.g., the age of the students, or previous classroom experiences), a factor that seemed to be quite influential in determining these differences is how the educator intervened. Indeed, the 24 children initially enacted schemes quite similar to those of the children in the first study, however the educator seemed to vigorously promote *counting* and/or use of *known finger configurations*, influencing the children's strategies. Her interventions were mostly consistent with the claims she made during the follow-up interview. Her main goal was to help children experience success in the apps and, with respect to strengthening numerical abilities she intended to help children learn to count and to represent numbers on their fingers. She did not mention, for example, recognition of part-whole relationships, subitizing, estimating, or even counting on strategies (indeed children's counting always started from "one").

In my opinion, the educator's short-term goal of helping the children experience success, and her narrow-sighted view of how to obtain this while fostering the development of numerical abilities, actually limited such development (at least during this experience) for many children. In most cases the children seemed to develop schemes like those of Sara and Giovanna, whose enactments included trying to memorize fixed configurations of fingers corresponding to verbal numbers (this seemed to be a common interpretation of the children of the educator's comments), or simply copying. The other abilities the educator explicitly intended to promote were to represent a same number in different ways (with fingers), and to count. She failed to promote the former for many children, possibly because she did not taking the time to discuss any of the situations in which children used different representations of a same number.

This brings me to a consideration that goes beyond the scope of this study, but that is closely related to the findings. What might have happened if this educator (and the children she worked with) used a more open digital environment where many different tasks can be proposed and a greater variety of solutions can be given?

For example, let us consider TouchCounts, described in Sinclair (2017), by Sinclair and Jackiw (2011), a very interesting app which exploits the potential of multi-touch screens in innovative ways, offering a wide range of possible interactions (Sinclair and Baccaglini-Frank 2016), especially manipulative interactions (Sedig and Sumner 2006), and encouraging the user to

associate specific gestures to numerical manipulation, promoting children's meaning-making (Goldin-Meadow 2004). TouchCounts seems to have a very high multi-touch potential. In particular, it recognizes a "pinch" gesture to add together sets of floating herds (represented in analogical and symbolical form), generating new larger herds. Such gesture can be seen to embody the fundamental metaphor of addition "collecting together" (Lakoff and Núñez 2000, in Sinclair and Jou 2013), and its symmetry incorporates the commutative property of addition. Another interesting gesture that one of the sub-environments of the software recognizes is a 5-finger-placement together with sequences of one-finger-taps that generate sets of $5+1+1+1+1\dots$ elements. This gesture is associated to the idea of constructing numbers (above 5) as successors of one another; to perform it the child can strengthen the 5-fingers to 5-objects correspondence and finger tapping.

Though with a seemingly very high potential with respect to fostering many aspects of children's numerical abilities, the app has no "built in" assignments, so when it is opened the user finds him/herself in a completely "open" situation. Of course very insightful and rich tasks can be designed and assigned (e.g., Sinclair and Zaskis 2017), however an educator such as the one in this study might find it difficult to come up with any; and even if s/he were given the tasks to assign the students in advance, an expectation to foster rigid interactions making use of a single "good" strategy would inhibit exploitation of the software's multi-touch potential.

Finally, I believe that recognizing and analyzing the role played by the educator in contexts where learning is fostered through software also has important implications for teacher education, as discussed also in Ginsburg et al. (2017). Indeed, such a finding can be used to help teachers (both pre-service and in-service) become more aware of how difficult it is to "hear what children are saying", an ability that "transcends disposition, aural acuity, and knowledge, although it also depends on all of these" (Ball 1993, p. 388). Teacher education, possibly through collective analysis of case studies (e.g., Levin 2002), should foster such ability to hear and respectfully interpret students' contributions, a kind of knowledge for teaching, that Ribeiro, Mellone and Jacobsen (2016) call *interpretative knowledge*.

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