

# Distributed Dynamic Resource Allocation for Cooperative Cognitive Radio Networks with Multi-Antenna Relay Selection

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**Abstract**—A cognitive radio scenario is considered where the signals transmitted by a secondary user (SU) are relayed by multi-antenna relays using an amplify-and-forward cooperation protocol. In this paper, the optimal power allocation and beamforming scheme is derived for the SU transmitters (SU-TXs) which minimizes the exact outage probability of the SU network with relay selection, under both a transmit power constraint and a constraint on the interference power generated at every primary user receiver. After deriving the optimal structure of the beamforming matrix, several distributed resource allocation algorithms are obtained for different levels of channel state information (CSI) at the SU-TXs: perfect CSI and imperfect CSI are considered, along with the case where only channel distribution information (CDI) is available. The numerical results show that the multi-antenna relays can significantly improve the performance of the SU network, which would otherwise be severely limited by the harsh interference constraints. Further, we also investigate how the number of relays, the number of antennas and the level of CSI impact the performance of the SU network. Finally, we point out that the proposed algorithms outperform several algorithms presented in literature.

**Index Terms**—Cognitive Radio, Cooperative Communications, Distributed Power allocation, Optimization

## I. INTRODUCTION

**C**OGNITIVE radio (CR) has been recently recognized as the key to tackle the demanding problems of both scarcity and inefficient utilization of the frequency spectrum [1], [2]. The basic idea behind the CR access framework consists of allowing unlicensed users or secondary users (SUs), to transmit over frequency bands owned by licensed users, or primary users (PUs). One way to achieve this goal is the *underlay* paradigm. In this scenario, the SU transmitter (SU-TX) may occupy the same frequency band and time slot as the PUs. However, the SU-TX has to adapt its transmission parameters (TPs) with the aim of keeping the interference level at the PU receivers (PU-RXs) below a given threshold depending on the required quality of service (QoS) of the PU network [3]–[5]. The underlay paradigm requires only a reduced (or null) cooperation between the SU and PU network and leads to a more efficient utilization of the spectrum, though the price to be paid is that the interference level

constraints to be met at the PU-RX limit the achievable data rate performance.

In [6]–[8], a possible solution to alleviate the effect of the interference level constraints on the achievable data rate is proposed: the SU-TXs are equipped with multiple antennas to balance between optimizing the SU performance and avoiding interference to the PU-RXs, thus paving the way to the so-called *cognitive beamforming* (CB) concept. In [6], the MIMO channels from the SU-TX to the SU-RX and PU-RXs are assumed to be perfectly known at the SU-TX. However, this assumption is quite unrealistic since the PU and SU units belong to independent networks, which implies a loose, or even absence of, cooperation between them. In [7], a more robust CB design is proposed which assumes that only the mean and covariance matrix of the MISO channel between the SU-TX and the PU-RX are available due to the loose cooperation between the SU and PU. Robustness is provided by keeping the interference to the PU-RX below a threshold for all realizations of the channel between the SU-TX and the PU-RX within a given uncertainty set. This robust design is extended to a multi-user scenario in [8], where, in addition to the interfering channels, also the channel state information (CSI) about the SU channels is assumed to be imperfect.

In [9], it is shown that cooperative relaying can also be successfully used in CR networks to reduce the outage probability of the SU network. Although the design of a multi-antenna relay node (RN) in a non-cognitive system was already investigated in [10], [11], the design of multi-antenna RNs in a cognitive setting is still under study. The design of the optimal beamforming matrix for a multi-antenna RN is presented in [12]. Therein, it is assumed that the RNs are equipped with multiple antennas, while the PU-RXs, SU source node (SN) and destination node (DN) are all equipped with only a single antenna. In [13] and [14], the scenario is extended to the case where all the nodes of the PU and SU network are equipped with multiple antennas. In [14], the beamforming matrices for both the SN and RN are jointly optimized in order to maximize the throughput of the SU network. However, the proposed centralized solution requires perfect CSI (PCSI) of all the SU channels, as well as the interfering channels to the PU network. The assumption of PCSI about all the channels in a cooperative network is not realistic, because of estimation errors and feedback delays in a fast-fading channel. A more realistic algorithm is proposed for a multi-antenna RN in [15], where the available CSI about the channels to the DN and the PU-RXs is assumed to be imperfect.

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**Motivation and Contributions.** This paper addresses the design of a distributed resource allocation (RA) algorithm which minimizes the exact outage probability of a single-carrier SU network based on multi-antenna RNs operating in an amplify-and-forward (AF) mode in an underlay scenario. Our aim, is to extend our previous works [16]–[18] by considering fixed RNs equipped with multiple transmit and receive antennas. More specifically, first we show that the optimal CB matrix at the RN, considered in [12], [15], can be reduced to a CB vector. Then, in contrast with [12], [15], the presence of the SN to DN link is exploited as well and the PU-RXs can also be equipped with multiple receive antennas.

Further, an additional contribution of the paper is to optimize the SU network taking into account different levels of CSI available at the SU-TX, (independently) for both the SU-TXs to PU-RXs interference channels and the channels toward the DN; these levels are: i) PCSI, ii) imperfect CSI (ICSI), iii) channel distribution information (CDI). The proposed approach allows to trade off the above levels of CSI, requiring different complexity and message exchange, against the achievable outage probability under the given interference constraints at the PU-RXs. By deriving different algorithms which depend on the available level of CSI, we extend the robust approach used in the single-hop scenarios from [7], [8] to a cooperative multi-antenna multi-relay scenario.

Finally, we stress the following facts about the proposed RA algorithms: i) they minimize the exact outage probability instead of an approximation as in [17]–[19]; ii) they are distributed, thus meaning that each node of the SU network can independently optimize its TPs. The latter results in more practical algorithms when compared to the more general yet centralized algorithms proposed in [14]; iii) in the numerical results, they are shown to have a clear performance benefit over other already existing algorithms, introduced in [15], [20].

**Organization.** Section II describes the system model and the performance metric. Section III introduces the RA problem by presenting the constraints and the objective function. Section IV presents the optimal CB matrix and derives the optimal TPs for the different levels of CSI. Section V shows the numerical results, while Section VI concludes the paper.

**Notations.** Denoting by  $\mathbf{x}$  a column vector, the quantities  $\mathbf{x}^T$ ,  $\mathbf{x}^*$ ,  $\mathbf{x}^H$  and  $\|\mathbf{x}\|$  refer to the transpose, the conjugate, the conjugate transpose and the norm of  $\mathbf{x}$ , respectively.  $\mathbf{z} \sim N_c(\mathbf{0}, \Sigma)$  indicates that  $\mathbf{z}$  is a zero-mean circularly symmetric complex Gaussian random variable (RV) with covariance matrix  $\Sigma$ .  $E[\cdot]$ ,  $\mathbf{I}_n$ ,  $\otimes$  and  $\angle z$  denote statistical expectation,  $n \times n$  identity matrix, the Kronecker product and the phase angle of a complex scalar  $z$ , respectively. The operator  $\text{vec}(\mathbf{X})$  produces a vector that contains the columns of the matrix  $\mathbf{X}$ , stacked below each other.

## II. SYSTEM MODEL

### A. The cooperative network

The CR scenario we consider is based on a single-carrier SU network, consisting of a SN, a DN and  $M$  AF RNs. We also assume that  $N_{\text{PU}}$  PU-RXs are active in the same bandwidth. In Fig. 1, we present a CR network that is typical for device-to-device communications [15]: the SN and the DN are equipped

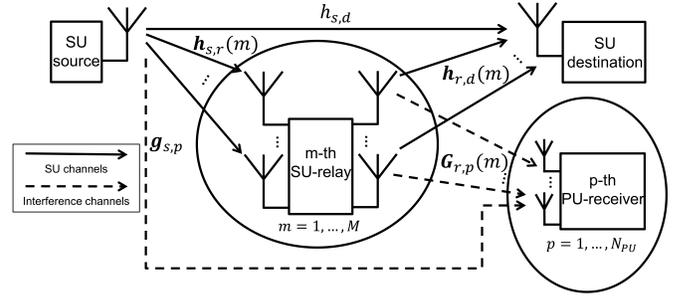


Fig. 1. The cognitive radio relay network.

with a single antenna, while the RNs have  $N_a$  transmit/receive antennas. It represents, for example, a scenario where two mobile phones can communicate directly with the help of the (multi-antenna) basestation of a femtocell. The PU-RXs have  $K$  receive antennas.

We assume that all wireless channels are flat fading. Similarly as in [12]–[15], the channel gains are assumed to be zero-mean circularly symmetric complex Gaussian RVs. The variables  $h_{s,d} \in \mathbb{C}$ ,  $\mathbf{h}_{s,r}(m) \in \mathbb{C}^{N_a \times 1}$  and  $\mathbf{h}_{r,d}(m) \in \mathbb{C}^{N_a \times 1}$ , indicated in Fig. 1, denote the channel gains between the SN and the DN, between the SN and the  $N_a$  receive antennas of the  $m$ th RN, and between the  $N_a$  transmit antennas of the  $m$ th RN and the DN, respectively. In the sequel we make use of the covariance matrix of  $\mathbf{h}_{r,d}(m)$ , defined as  $\mathbf{R}_h(m) \triangleq E[\mathbf{h}_{r,d}(m)\mathbf{h}_{r,d}(m)^H]$ . The coefficients of the Rayleigh-fading interference channels from the SN to the  $p$ th PU-RX and from the  $m$ th RN to the  $p$ th PU-RX will be denoted by  $\mathbf{g}_{s,p} \in \mathbb{C}^{K \times 1}$  and  $\mathbf{G}_{r,p}(m) \in \mathbb{C}^{N_a \times K}$ , respectively, with respective covariance matrices  $\mathbf{R}_{s,p} \triangleq E[\mathbf{g}_{s,p}\mathbf{g}_{s,p}^H]$  and  $\mathbf{R}_{r,p}(m) \triangleq E[\mathbf{g}_{r,p}(m)\mathbf{g}_{r,p}(m)^H]$ , where  $\mathbf{g}_{r,p}(m) = \text{vec}(\mathbf{G}_{r,p}(m))$  ( $m = 1, \dots, M; p = 1, \dots, N_{\text{PU}}$ ).

In the first time slot, the SN transmits the symbol  $x$ , with  $E[|x|^2] = 1$ , to the DN and the  $M$  RNs. The signals received by the DN and the  $m$ th RN, are expressed as<sup>1</sup>

$$y_{s,d} = \sqrt{E_0}h_{s,d}x + n_{s,d} \quad (1)$$

$$\mathbf{y}_{s,r}(m) = \sqrt{E_0}\mathbf{h}_{s,r}(m)x + \mathbf{n}_{s,r}(m), \quad m = 1, \dots, M \quad (2)$$

where  $E_0$  denotes the transmit energy per symbol used by the SN, and the noise terms  $n_{s,d}$  and  $\mathbf{n}_{s,r}(m)$  are distributed as  $N_c(0, \sigma_{s,d}^2)$  and  $N_c(\mathbf{0}, \sigma_{s,r}^2(m)\mathbf{I}_{N_a})$ , respectively.

In the second time slot, the DN selects the RN which yields the highest SNR at the DN [19], and the selected  $m$ th RN,  $m = 1, \dots, M$ , multiplies its received signal by a CB matrix  $\mathbf{F}(m) \in \mathbb{C}^{N_a \times N_a}$  and forwards it to the DN. The signal received by the DN from the  $m$ th RN can be written as

$$y_{r,d}(m) = \mathbf{h}_{r,d}(m)^T \mathbf{F}(m) \mathbf{y}_{s,r}(m) + n_{r,d}(m) \quad (3)$$

where the noise vector  $n_{r,d}(m) \sim N_c(0, \sigma_{r,d}^2(m))$ . Further, the average transmit energy per symbol  $E_m$  of the  $m$ th RN is given by

$$E_m = E_0 \|\mathbf{F}(m)\mathbf{h}_{s,r}(m)\|^2 + \sigma_{s,r}^2(m) \|\mathbf{F}(m)\|^2. \quad (4)$$

<sup>1</sup>The interference from the PU-TXs can be included by increasing the variance of the different noise terms [7].

### B. The performance metric

The metric that will be used to quantify the performance of the SU network is the link outage probability  $P_{\text{out}}$  between the SN and the DN [21], defined as

$$P_{\text{out}} \triangleq \Pr\{C \leq R\} \quad (5)$$

where the rate  $R$  is the average number of information bits per channel use and  $C$  is the instantaneous capacity of the SN-DN channel (including the selected relay channel). We note that the optimization of the outage probability  $P_{\text{out}}$  is beneficial for applications, such as video transmission, where the data rate is fixed, as this increases the reliability of the system, at the expense of additional throughput.

The SNR at the DN associated with the  $m$ th RN is given by

$$\eta_m = \frac{E_0 |\mathbf{h}_{r,d}(m)^T \mathbf{F}(m) \mathbf{h}_{s,r}(m)|^2}{\sigma_{s,r}^2(m) \|\mathbf{h}_{r,d}(m)^T \mathbf{F}(m)\|^2 + \sigma_{r,d}^2(m)}. \quad (6)$$

At the DN, the best RN is selected, i.e., the one yielding the larger SNR  $\eta_m$  [19]. To make this possible, the SU network has to use a periodic training interval, which allows all the RNs to transmit separately. This will allow the SU DN to measure the SNRs  $\eta_m$  of the different RNs ( $m = 1, \dots, M$ ).

Hence, after the second time slot (with only one RN transmitting out of  $M$ ), maximum ratio combining (MRC) of the signals received from the SN and the selected RN is applied. The overall received SNR at the DN of the AF cooperative network after MRC yields

$$\eta = \eta_0 + \max_{m \in \{1, \dots, M\}} \eta_m \quad (7)$$

where  $\eta_0 = E_0 |h_{s,d}|^2 / \sigma_{s,d}^2$ . Thus, the corresponding instantaneous channel capacity (in bit per channel use) can be expressed as  $C = (1/2) \log_2(1 + \eta)$ . This leads to

$$P_{\text{out}} = \Pr\{\eta \leq 2^{2R} - 1\} \quad (8)$$

where  $\eta$  depends on  $E_0$ , the CB matrices  $\{\mathbf{F}(m), m = 1, \dots, M\}$  and all channel gains  $h_{s,d}$ ,  $\{\mathbf{h}_{s,r}(m), m = 1, \dots, M\}$  and  $\{\mathbf{h}_{r,d}(m), m = 1, \dots, M\}$  from the SU network.

## III. RESOURCE ALLOCATION

The RA consists of dynamically selecting the transmit energy  $E_0$  at the SN and the CB matrix  $\mathbf{F}(m)$  at the  $m$ th RN ( $m = 1, \dots, M$ ) such that  $P_{\text{out}}$  (8) is minimized, under transmit energy constraints at the SU-TXs, and interference constraints at the PU-RXs. The objective function and the constraints depend on the level of available CSI.

### A. Available channel information

Considering a generic vector  $\mathbf{f}$  of channel gains, PCSI refers to the case where the realization of  $\mathbf{f}$  is known. In the case of ICSI, one has access only to an estimate  $\hat{\mathbf{f}}$  of  $\mathbf{f}$ , and to the covariance matrix  $\mathbf{R}_e$  of the corresponding estimation error  $\mathbf{e} = \mathbf{f} - \hat{\mathbf{f}}$ ; the error is caused by noise and/or feedback delay (see Appendix B). In the case of CDI, only the channel covariance matrix  $\mathbf{R}_f$  is known.

We assume that the RNs and the DN always have PCSI regarding their receiving channels, i.e., the  $m$ th

RN and the DN know the realizations of  $\mathbf{h}_{s,r}(m)$  and  $(h_{s,d}, \mathbf{h}_{r,d}(m)^T \mathbf{F}(m) \mathbf{h}_{s,r}(m))$ , respectively ( $m = 1, \dots, M$ ). In order to perform the MRC, the DN also has to know the noise variances  $\sigma_{s,d}^2$  and  $\sigma_{s,r}^2(m) \|\mathbf{h}_{r,d}(m)^T \mathbf{F}(m)\|^2 + \sigma_{r,d}^2(m)$  which correspond to the signals received from the SN and from the selected RN, respectively.

PCSI of the interference and transmission channels is much harder to obtain in a time-varying environment, because it requires feedback from the other SU nodes or even from the PU network. Therefore, we will consider the optimization of  $P_{\text{out}}$  for the cases where the SN and the RNs have PCSI, ICSI or CDI regarding their interference channels ( $\mathbf{g}_{s,p}$  and  $\{\mathbf{G}_{r,p}(m), m = 1, \dots, M\}$ , respectively) to the PU-RXs and regarding their transmission channels ( $\{h_{s,d}, \mathbf{h}_{s,r}(m), m = 1, \dots, M\}$  and  $\{\mathbf{h}_{r,d}(m), m = 1, \dots, M\}$ , respectively); later on, we will point out that the SN needs no knowledge about its transmission channels.

### B. Transmit energy constraints

We impose the following constraint on the transmit energy of the SN

$$0 \leq E_0 \leq E_0^{(max)} \quad (9)$$

while the transmit energy of the  $m$ th RN ( $m = 1, \dots, M$ ) is constrained by

$$0 \leq E_m \leq E_m^{(max)} \quad (10)$$

where  $E_m$  is given by (4). In the above,  $E_m^{(max)}$  denotes the maximal transmit energy per symbol that the  $m$ th SU node ( $m = 0, \dots, M$ ) is able or allowed to transmit.

### C. The interference constraints

Denoting by  $\mathcal{I}_{s,p}$  and  $\mathcal{I}_{r,p}(m)$  the interference at the  $p$ th PU-RX ( $p = 1, \dots, N_{\text{PU}}$ ) caused by the SN and the  $m$ th RN, respectively, we have

$$\mathcal{I}_{s,p} = E_0 \|\mathbf{g}_{s,p}\|^2 \quad (11)$$

$$\begin{aligned} \mathcal{I}_{r,p}(m) &= E_0 \|\mathbf{G}_{r,p}(m)^T \mathbf{F}(m) \mathbf{h}_{s,r}(m)\|^2 \\ &+ \sigma_{s,r}^2(m) \|\mathbf{G}_{r,p}(m)^T \mathbf{F}(m)\|^2. \end{aligned} \quad (12)$$

According to the underlay paradigm, the interference at the PU-RXs should be below a given interference threshold  $\Gamma$ . The formulation of these interference constraints depends upon the level of CSI that is available at the transmitting nodes about their channel gains to the PU-RXs. In practice it will be very difficult for the SU network to obtain PCSI about the channel gains towards the PU-RXs. For this reason, we will also consider ICSI and CDI.

Theoretically, the knowledge about the CSI or CDI related to the interference channels can come directly from the PU; practically, it will be provided by a *centralized spectrum manager* which continuously monitors the frequency bandwidth of interest [5].

1) *Interference constraints with PCSI*: When PCSI is available at the corresponding transmitting node, the interference constraints to be met by the SN and the  $m$ th RN,  $m = 1, \dots, M$ , are expressed as

$$\mathcal{I}_{s,p} \leq \Gamma, \quad p = 1, \dots, N_{\text{PU}} \quad (13)$$

$$\mathcal{I}_{r,p}(m) \leq \Gamma, \quad p = 1, \dots, N_{\text{PU}} \quad (14)$$

where  $\mathcal{I}_{s,p}$  and  $\mathcal{I}_{r,p}(m)$  are given by (11) and (12), with  $\mathbf{g}_{s,p}$  and  $\mathbf{G}_{r,p}(m)$  denoting the actual channel realizations.

2) *Interference constraints with ICSI*: Here we assume that the CSI at each SU-TX about its channel gain to the  $p$ th PU-RX is imperfect. Similar to  $\hat{\mathbf{f}}$  in (65), the variables  $\hat{\mathbf{g}}_{s,p}$  and  $\hat{\mathbf{g}}_{r,p}(m)$  denote the estimates of  $\mathbf{g}_{s,p}$  and  $\bar{\mathbf{g}}_{r,p}(m)$ , respectively.

Based on the instantaneous channel gain given by (65), we define for given  $\hat{\mathbf{g}}_{s,p}$ ,  $\mathbf{B}_{s,p}$ ,  $\hat{\mathbf{g}}_{r,p}(m)$  and  $\mathbf{B}_{r,p}(m)$  the following ellipsoid channel uncertainty sets [7]

$$\mathcal{U}_{s,p}(\hat{\mathbf{g}}_{s,p}, \mathbf{B}_{s,p}) \triangleq \{ \mathbf{g}_{s,p} : \mathbf{g}_{s,p} = \hat{\mathbf{g}}_{s,p} + \mathbf{B}_{s,p} \boldsymbol{\epsilon}_{s,p}, \|\boldsymbol{\epsilon}_{s,p}\|^2 \leq 1 \} \quad (15)$$

$$\mathcal{U}_{r,p}(\hat{\mathbf{g}}_{r,p}(m), \mathbf{B}_{r,p}(m)) \triangleq \{ \bar{\mathbf{g}}_{r,p}(m) : \bar{\mathbf{g}}_{r,p}(m) = \hat{\mathbf{g}}_{r,p}(m) + \mathbf{B}_{r,p}(m) \boldsymbol{\epsilon}_{r,p}(m), \|\boldsymbol{\epsilon}_{r,p}(m)\|^2 \leq 1 \} \quad (16)$$

where  $\boldsymbol{\epsilon}_{s,p} \in \mathbb{C}^{K \times 1}$  and  $\boldsymbol{\epsilon}_{r,p}(m) \in \mathbb{C}^{KN_a \times 1}$ . The variables  $\hat{\mathbf{g}}_{s,p}$  and  $\hat{\mathbf{g}}_{r,p}(m)$  denote the center of the ellipsoids, while the variables  $\mathbf{B}_{s,p}$  and  $\mathbf{B}_{r,p}(m)$  determine their shape. We now choose  $\mathbf{B}_{s,p}$  and  $\mathbf{B}_{r,p}(m)$ ,  $m = 1, \dots, M$ , as

$$\mathbf{B}_{s,p} = \sqrt{\frac{\chi_\alpha^2(2K)}{2}} \mathbf{R}_{\mathbf{e},s,p}^{\frac{1}{2}} \quad (17)$$

$$\mathbf{B}_{r,p}(m) = \sqrt{\frac{\chi_\alpha^2(2KN_a)}{2}} \mathbf{R}_{\mathbf{e},r,p}(m)^{\frac{1}{2}} \quad (18)$$

where  $\mathbf{R}_{\mathbf{e},s,p}$  and  $\mathbf{R}_{\mathbf{e},r,p}(m)$  denote the error covariance matrix  $\mathbf{R}_{\mathbf{e}}$  from Appendix B, corresponding to the substitutions  $\mathbf{f} = \mathbf{g}_{s,p}$  and  $\mathbf{f} = \bar{\mathbf{g}}_{r,p}(m)$ , respectively, and  $\chi_\alpha^2(l)$  denotes the  $\alpha$ -percentile of the  $\chi^2$ -distribution with  $l$  degrees-of-freedom. For the above values of  $\mathbf{B}_{s,p}$  and  $\mathbf{B}_{r,p}(m)$ , the actual channel gains  $\mathbf{g}_{s,p}$  and  $\bar{\mathbf{g}}_{r,p}(m)$  belong with probability  $\alpha$  to their respective sets  $\mathcal{U}_{s,p}(\hat{\mathbf{g}}_{s,p}, \mathbf{B}_{s,p})$  and  $\mathcal{U}_{r,p}(\hat{\mathbf{g}}_{r,p}(m), \mathbf{B}_{r,p}(m))$ . Hence, the task of the SU network is that of ensuring that the interference constraints (13)-(14) hold for *every* channel gain in  $\mathcal{U}_{s,p}(\hat{\mathbf{g}}_{s,p}, \mathbf{B}_{s,p})$  and  $\mathcal{U}_{r,p}(\hat{\mathbf{g}}_{r,p}(m), \mathbf{B}_{r,p}(m))$ . The parameter  $\alpha$  denotes the desired level of robustness, namely the minimum probability for which the interference constraints (13)-(14) hold. Thus, the resulting interference constraints are expressed as

$$\mathcal{I}_{s,p} \leq \Gamma, \quad p = 1, \dots, N_{\text{PU}}, \quad \forall \mathbf{g}_{s,p} \in \mathcal{U}_{s,p}(\hat{\mathbf{g}}_{s,p}, \mathbf{B}_{s,p}) \quad (19)$$

$$\mathcal{I}_{r,p}(m) \leq \Gamma, \quad p = 1, \dots, N_{\text{PU}}, \quad \forall \bar{\mathbf{g}}_{r,p}(m) \in \mathcal{U}_{r,p}(\hat{\mathbf{g}}_{r,p}(m), \mathbf{B}_{r,p}(m)). \quad (20)$$

3) *Interference constraints with CDI*: In this case, we follow the same approach as in Section III-C2. The interference constraints are the same as in equations (19)-(20), but the channel uncertainty sets (15)-(16) are slightly modified. The values of the estimated channel gains are now fixed to the

mean of the actual channel gains, i.e.,  $\hat{\mathbf{g}}_{s,p} = \mathbf{E}[\mathbf{g}_{s,p}] = \mathbf{0}$  and  $\hat{\mathbf{g}}_{r,p}(m) = \mathbf{E}[\bar{\mathbf{g}}_{r,p}(m)] = \mathbf{0}$ , while  $\mathbf{B}_{s,p}$  and  $\mathbf{B}_{r,p}(m)$ ,  $m = 1, \dots, M$ , are selected as

$$\mathbf{B}_{s,p} = \sqrt{\frac{\chi_\alpha^2(2K)}{2}} \mathbf{R}_{s,p}^{\frac{1}{2}} \quad (21)$$

$$\mathbf{B}_{r,p}(m) = \sqrt{\frac{\chi_\alpha^2(2KN_a)}{2}} \mathbf{R}_{r,p}(m)^{\frac{1}{2}} \quad (22)$$

where  $\mathbf{R}_{s,p}$  and  $\mathbf{R}_{r,p}(m)$  were introduced in Section II-A.

#### D. The objective function

The aim of the RA is to minimize the link outage probability (8) by dynamically allocating the SN transmit energy  $E_0$  and the RN beamforming matrices  $\mathbf{F}(m)$  ( $m = 1, \dots, M$ ). Let us stack all channel gains from the SU network (i.e.,  $h_{s,d}$ ,  $\{\mathbf{h}_{s,r}(m), m = 1, \dots, M\}$  and  $\{\mathbf{h}_{r,d}(m), m = 1, \dots, M\}$ ) into the vector  $\mathbf{h}_{\text{SU}}$ . Denoting by **CSI** the channel information which is available at the SN and the RNs regarding the channel vector  $\mathbf{h}_{\text{SU}}$ , the minimization of  $P_{\text{out}}$  is equivalent with choosing the value of  $E_0$  and  $\mathbf{F}(m)$  ( $m = 1, \dots, M$ ) which minimizes

$$\Pr\{\eta \leq 2^{2R} - 1 | \mathbf{CSI}\} \quad (23)$$

for a given value of **CSI**. Although we have presented the objective function in a centralized manner, it will be demonstrated in Section IV that for some types of CSI the resulting RA algorithms are distributed: each node only needs CSI about its own channel gains to calculate its TPs; for the other types of CSI where the optimum RA algorithm is centralized, also a (suboptimum) distributed algorithm will be derived.

#### IV. MINIMIZATION OF THE LINK OUTAGE PROBABILITY

In this section several distributed algorithms will be proposed to minimize the outage probability  $P_{\text{out}}$  of the SU network over the transmit energy  $E_0$  and the beamforming matrices  $\mathbf{F}(m)$  ( $m = 1, \dots, M$ ) under the interference and transmit power constraints.

In IV-A, we start by pointing out that the optimum beamforming matrix  $\mathbf{F}(m)$  performs MRC of the received signals followed by beamforming which is described by a beamforming *vector*  $\mathbf{v}(m)$ . This result simplifies the expression of the transmission and interference constraints, and it becomes clear that the constraints on  $E_0$  and  $\mathbf{F}(m)$  ( $m = 1, \dots, M$ ) are independent from each other. The independence of the constraints, together with the observation that the outage probability  $P_{\text{out}}$  (8) is a monotonically decreasing function of  $E_0$ , allows to separate the optimization of  $E_0$  from the optimization of  $\mathbf{F}(m)$  ( $m = 1, \dots, M$ ). It is quite obvious that the optimization problem (OP) of the SN reduces to the constrained maximization of the transmit energy  $E_0$ , irrespective of the level of CSI available at the SN regarding its transmission channel gains. This constrained maximization of  $E_0$  is considered in Section IV-B. Next, the beamforming vector  $\mathbf{v}(m)$  is optimized in Section IV-C, making a distinction between the different levels of channel information available at the RNs about the channel gains towards the DN. For each of these cases, we also consider three levels of CSI (PCSI,

ICSI, CDI) for the interference channels, yielding a total of 9 combinations.

#### A. Structure of optimal beamforming matrix

The following theorem shows that the optimal beamforming matrix  $\mathbf{F}(m)$  has a specific structure, which is valid irrespective of the channel knowledge at the RNs regarding their interference channels and channels to the DN.

**Theorem 1.** *Assuming the  $m$ th RN has a perfect knowledge of its channel vector  $\mathbf{h}_{s,r}(m)$  from the SN, the optimal beamforming matrix  $\mathbf{F}(m)$ , which minimizes the outage probability under the transmit constraint (10) and the interference constraints that correspond to the level of available information on the interference channels, has the following structure*

$$\mathbf{F}(m) = \frac{\mathbf{v}(m)^* \mathbf{h}_{s,r}(m)^H / \|\mathbf{h}_{s,r}(m)\|}{\sqrt{E_0 \|\mathbf{h}_{s,r}(m)\|^2 + \sigma_{s,r}^2(m)}} \quad (24)$$

where  $\mathbf{v}(m) \in \mathbb{C}^{N_a \times 1}$  can be any complex vector.

*Proof:* See Appendix A. ■

It should be noted that the normalization present in the denominator of (24) implies that the RN has to determine the average received energy per symbol for every frame. It follows from (24) that the signal  $\mathbf{F}(m) \mathbf{y}_{s,r}(m)$  transmitted by the  $m$ th RN can be expressed as  $\mathbf{v}(m)^* z_{s,r}(m)$ , where  $\mathbf{v}(m)$  is a beamforming vector (to be further optimized), and

$$z_{s,r}(m) = \frac{\mathbf{h}_{s,r}(m)^H \mathbf{y}_{s,r}(m)}{\|\mathbf{h}_{s,r}(m)\| \sqrt{E_0 \|\mathbf{h}_{s,r}(m)\|^2 + \sigma_{s,r}^2(m)}} \quad (25)$$

results from applying MRC of the signals received by the  $m$ th RN, followed by the proper scaling.

By substituting (24) in (4), (6) and (12), the energy  $E_m$ , the SNR  $\eta_m$  and the interference  $\mathcal{I}_{r,p}(m)$ ,  $m = 1, \dots, M$ , can be expressed as

$$E_m = \|\mathbf{v}(m)\|^2 \quad (26)$$

$$\eta_m = \frac{\eta_{s,r}(m) \eta_{r,d}(m)}{\eta_{s,r}(m) + \eta_{r,d}(m) + 1} \quad (27)$$

$$\mathcal{I}_{r,p}(m) = \|\mathbf{v}(m)^H \mathbf{G}_{r,p}(m)\|^2 \quad (28)$$

where  $\eta_{s,r}(m) = E_0 \|\mathbf{h}_{s,r}(m)\|^2 / \sigma_{s,r}^2(m)$  and  $\eta_{r,d}(m) = |\mathbf{v}(m)^H \mathbf{h}_{r,d}(m)|^2 / \sigma_{r,d}^2(m)$ . The constraints involving the optimum beamforming matrix  $\mathbf{F}(m)$  can be expressed in terms of the beamforming vector  $\mathbf{v}(m)$ . It will be convenient to introduce the rank-1 matrix  $\mathbf{S}(m) \triangleq \mathbf{v}(m) \mathbf{v}(m)^H$ . The matrix  $\mathbf{S}(m)$  is positive semi-definite, which is denoted as  $\mathbf{S}(m) \succeq \mathbf{0}$ .

- The transmit constraint (10) at the  $m$ th RN ( $m = 1, \dots, M$ ) becomes

$$\|\mathbf{v}(m)\|^2 \leq E_m^{(max)} \quad (29)$$

or, equivalently,

$$\text{Tr}(\mathbf{S}(m)) \leq E_m^{(max)}. \quad (30)$$

- The interference constraint (14) to be met by the  $m$ th RN ( $m = 1, \dots, M$ ) in the case of PCSI reduces to

$$|\mathbf{v}(m)^H \mathbf{G}_{r,p}(m)|^2 \leq \Gamma, \quad p = 1, \dots, N_{PU} \quad (31)$$

We transform this constraint into

$$\text{Tr}\left(\mathbf{S}(m) \mathbf{G}_{r,p}(m) \mathbf{G}_{r,p}(m)^H\right) \leq \Gamma, \quad p = 1, \dots, N_{PU}. \quad (32)$$

- In the case of ICSI, the interference constraint (20) related to the  $m$ th RN ( $m = 1, \dots, M$ ) is given by

$$|\mathbf{v}(m)^H \mathbf{G}_{r,p}(m)|^2 \leq \Gamma, \quad p = 1, \dots, N_{PU}, \\ \forall \hat{\mathbf{g}}_{r,p}(m) \in \mathcal{U}_{r,p}(\hat{\mathbf{g}}_{r,p}(m), \mathbf{B}_{r,p}(m)) \quad (33)$$

with  $\mathbf{B}_{r,p}(m)$  given by (18). This can be rewritten as

$$\hat{\mathbf{g}}_{r,p}(m)^H \mathbf{S}_K(m) \hat{\mathbf{g}}_{r,p}(m) \leq \Gamma, \quad p = 1, \dots, N_{PU}, \\ \forall \hat{\mathbf{g}}_{r,p}(m) \in \mathcal{U}_{r,p}(\hat{\mathbf{g}}_{r,p}(m), \mathbf{B}_{r,p}(m)) \quad (34)$$

where  $\mathbf{S}_K(m) = \mathbf{I}_K \otimes \mathbf{S}(m)$ . Note that for given  $(m, p)$  the constraint on  $\mathbf{S}(m)$  must be satisfied for a *continuum* of interference channel gains. However, we can convert this constraint by using the following S-lemma [22].

**Lemma 1.** *Let*

$$f_j(\mathbf{z}) = \mathbf{z}^H \mathbf{A}_j \mathbf{z} + 2\Re(\mathbf{b}_j^H \mathbf{z}) + c_j, \quad j = 1, 2$$

where  $\mathbf{z} \in \mathbb{C}^{N \times 1}$ ,  $\mathbf{A}_j \in \mathbb{C}^{N \times N}$  is a Hermitian matrix,  $\mathbf{b}_j \in \mathbb{C}^{N \times 1}$  and  $c_j \in \mathbb{R}$ . Suppose that there exists a  $\mathbf{z}_0$  such that  $f_1(\mathbf{z}_0) < 0$ . Then the following two statements are equivalent:

- $f_2(\mathbf{z}) \leq 0$  for every  $\mathbf{z} \in \mathbb{C}^{N \times 1}$  for which  $f_1(\mathbf{z}) \leq 0$
- There exists some  $\lambda \geq 0$  such that

$$\lambda \begin{pmatrix} \mathbf{A}_1 & \mathbf{b}_1 \\ \mathbf{b}_1^H & c_1 \end{pmatrix} - \begin{pmatrix} \mathbf{A}_2 & \mathbf{b}_2 \\ \mathbf{b}_2^H & c_2 \end{pmatrix} \succeq \mathbf{0}. \quad (35)$$

Defining  $f_1(\epsilon_{r,p}(m)) = \epsilon_{r,p}(m)^H \epsilon_{r,p}(m) - 1$  and

$$f_2(\epsilon_{r,p}(m)) = \epsilon_{r,p}(m)^H \mathbf{B}_{r,p}(m)^H \mathbf{S}_K(m) \mathbf{B}_{r,p}(m) \epsilon_{r,p}(m) \\ + 2\Re((\mathbf{B}_{r,p}(m)^H \mathbf{S}_K(m) \hat{\mathbf{g}}_{r,p}(m))^H \epsilon_{r,p}(m)) \\ + \hat{\mathbf{g}}_{r,p}(m)^H \mathbf{S}_K(m) \hat{\mathbf{g}}_{r,p}(m) - \Gamma \quad (36)$$

we can use the S-lemma to rewrite the interference constraint (34) for given  $(m, p)$  as

$$\left\{ \begin{array}{l} \text{Equation (35) where} \\ \mathbf{A}_1 = \mathbf{I}_{KN_a}, \quad \mathbf{A}_2 = \mathbf{B}_{r,p}(m)^H \mathbf{S}_K(m) \mathbf{B}_{r,p}(m), \\ \mathbf{b}_1 = \mathbf{0}, \quad \mathbf{b}_2 = \mathbf{B}_{r,p}(m)^H \mathbf{S}_K(m) \hat{\mathbf{g}}_{r,p}(m), \\ c_1 = -1, \quad c_2 = \hat{\mathbf{g}}_{r,p}(m)^H \mathbf{S}_K(m) \hat{\mathbf{g}}_{r,p}(m) - \Gamma, \\ \lambda = \lambda_p(m) \end{array} \right. \quad (37)$$

which has to hold for at least one non-negative value of  $\lambda_p(m)$ .

- In the case of CDI, the constraint related to the  $m$ th RN ( $m = 1, \dots, M$ ) is equal to (33) with  $\mathbf{B}_{r,p}(m)$  given by (22) and  $\hat{\mathbf{g}}_{r,p}(m) = \mathbf{0}$ , which yields

$$|\mathbf{v}(m)^H \mathbf{G}_{r,p}(m)|^2 \leq \Gamma, \quad p = 1, \dots, N_{PU}, \\ \forall \hat{\mathbf{g}}_{r,p}(m) \in \mathcal{U}_{r,p}(\mathbf{0}, \mathbf{B}_{r,p}(m)). \quad (38)$$

This is equivalent to

$$\begin{pmatrix} \lambda_p(m)\mathbf{I}_{KN_a} - \mathbf{B}_{r,p}(m)^H \mathbf{S}_K(m) \mathbf{B}_{r,p}(m) & \mathbf{0} \\ \mathbf{0} & \Gamma - \lambda_p(m) \end{pmatrix} \succeq 0, \quad p = 1, \dots, N_{\text{PU}}. \quad (39)$$

with  $\lambda_p(m) \geq 0$ . The latter can be simplified to

$$\mathbf{B}_{r,p}(m)^H \mathbf{S}_K(m) \mathbf{B}_{r,p}(m) \preceq \Gamma \mathbf{I}_{KN_a}, \quad p = 1, \dots, N_{\text{PU}}. \quad (40)$$

### B. Optimal transmit energy $E_0$

The new formulation of the constraints, shown in Section IV-A, clearly demonstrates that the optimization of the transmit energy  $E_0$  can be separated from the optimization of  $\mathbf{F}(m)$  ( $m = 1, \dots, M$ ) without any performance loss. At the SN, the following OP has to be solved

$$\begin{cases} E_0^{(opt)} = \arg \max_{E_0} E_0 \\ \text{s.t. (9), intf constraints} \end{cases} \quad (41)$$

where ‘‘intf constraints’’ refers to the relevant interference constraints corresponding to the level of available knowledge about the interference channels. As the transmit constraint is given by (9), the SN needs information only about its interference channels.

When PCSI about the channel gains to the PU-RXs is available, the interference constraints (13) yield the following solution

$$E_0^{(opt)} = \min \left\{ \frac{\Gamma}{\|\mathbf{g}_{s,1}\|^2}, \dots, \frac{\Gamma}{\|\mathbf{g}_{s,N_{\text{PU}}}\|^2}, E_0^{(max)} \right\}. \quad (42)$$

When ICSI is available, the interference constraints are given by (19). Using the lemma 1, this interference constraint for a given PU-RX  $p$  ( $p = 1, \dots, N_{\text{PU}}$ ) can be rewritten as

$$\begin{pmatrix} \lambda_p \mathbf{I}_K - E_0 \mathbf{B}_{s,p}^H \mathbf{B}_{s,p} & -E_0 \mathbf{B}_{s,p}^H \hat{\mathbf{g}}_{s,p} \\ -E_0 \hat{\mathbf{g}}_{s,p}^H \mathbf{B}_{s,p} & \Gamma - \lambda_p - E_0 \|\hat{\mathbf{g}}_{s,p}\|^2 \end{pmatrix} \succeq 0, \quad \lambda_p \geq 0. \quad (43)$$

The optimal  $E_0^{(opt)}$  is found by solving

$$\begin{cases} E_0^{(opt)} = \arg \max_{E_0, \lambda_1, \dots, \lambda_{N_{\text{PU}}}} E_0 \\ \text{s.t. (9), (43)} \end{cases} \quad (44)$$

which is a semidefinite program (SDP) that can be solved in polynomial time using the software package CVX [23].

Finally, when only CDI is available, the solution is obtained by substituting  $\hat{\mathbf{g}}_{s,p} = \mathbf{0}$  in (43), which can be simplified to

$$E_0 \mathbf{B}_{s,p}^H \mathbf{B}_{s,p} \preceq \Gamma \mathbf{I}_K, \quad p = 1, \dots, N_{\text{PU}} \quad (45)$$

where  $\mathbf{B}_{s,p}$  ( $p = 1, \dots, N_{\text{PU}}$ ) is given by (21). The optimal  $E_0^{(opt)}$  is found as

$$E_0^{(opt)} = \min \left\{ \frac{\Gamma}{\sigma_{\max}^2(1)}, \dots, \frac{\Gamma}{\sigma_{\max}^2(N_{\text{PU}})}, E_0^{(max)} \right\} \quad (46)$$

where  $\sigma_{\max}(p)$  is the largest singular value of  $\mathbf{B}_{s,p}$  ( $p = 1, \dots, N_{\text{PU}}$ ).

### C. Optimal relay beamforming

In this subsection we will discuss the OPs for the different RNs. A distinction is made between the levels of information (PCSI, ICSI, CDI) at the RNs about their channel gains to the DN. We will first introduce the generic OP

$$\begin{cases} \mathbf{V}^{(opt)} = \arg \min_{\mathbf{V}} \Pr\{\eta \leq 2^{2R} - 1 | \text{CSI}\} \\ \text{s.t. (29), intf constraints, } m = 1, \dots, M \end{cases} \quad (47)$$

where  $\mathbf{V} \triangleq [\mathbf{v}(1), \mathbf{v}(2), \dots, \mathbf{v}(M)]$ . Depending upon the available information on the interference channels, the constraint functions (31), (33) or (38) have to be used, for PCSI, ICSI or CDI, respectively. As the constraints on  $\mathbf{v}(m)$  ( $m = 1, \dots, M$ ) are independent, we will show that we can split OP (47) into  $M$  independent subproblems. In order to demonstrate this, we will assume that the RNs also know the value of  $E_0$  and  $h_{s,d}$ ; however, it will turn out that this requirement is unnecessary for most cases. First, we introduce the conditional cumulative distribution function (CDF)

$$F_m(x) = \Pr\{\eta_m \leq x | \text{CSI}\}. \quad (48)$$

This allows us to rewrite the objective function from OP (47) as

$$\begin{aligned} & \Pr\left\{ \max_{m \in \{1, \dots, M\}} \eta_m \leq 2^{2R} - 1 - \eta_0 | \text{CSI} \right\} \\ &= \prod_{m=1}^M F_m(2^{2R} - 1 - \eta_0) \end{aligned} \quad (49)$$

where the second step comes from the fact that the channels to and from the different RNs are assumed to be independent. The OP for the  $m$ th RN becomes ( $m = 1, \dots, M$ )

$$\begin{cases} \mathbf{v}(m)^{(opt)} = \arg \min_{\mathbf{v}(m)} F_m(2^{2R} - 1 - \eta_0) \\ \text{s.t. (29), intf constraints} \end{cases}. \quad (50)$$

It will become clear that this OP is not necessarily concave [24], which makes it very hard to solve. Therefore, we will rewrite the OP in terms of the rank-1 positive semi-definite matrix  $\mathbf{S}(m) = \mathbf{v}(m)\mathbf{v}(m)^H$ , which yields

$$\begin{cases} \mathbf{S}(m)^{(opt)} \\ = \arg \min_{\mathbf{S}(m) \succeq 0, \lambda_1(m) \geq 0, \dots, \lambda_{N_{\text{PU}}}(m) \geq 0} F_m(2^{2R} - 1 - \eta_0) \\ \text{s.t. (30), intf constraints} \end{cases} \quad (51)$$

where the interference constraint functions are (32), (37) or (40). Note that  $\lambda_p(m)$  only appears as an optimization variable if (37) is used. Further, we have dropped the rank-1 constraint  $\text{Rank}(\mathbf{S}(m)) = 1$ , which makes OP (51) a SDP. However, by dropping the rank constraint, the matrix  $\mathbf{S}(m)^{(opt)}$  from (51) in general can have a rank higher than 1. The optimal solution to OP (50) is only found in the case that  $\text{Rank}(\mathbf{S}(m)) = 1$ .

1) *Beamforming with PCSI*: In the case where the RNs have perfect knowledge of their respective channel gains to the DN, the minimization of the objective function in (51) becomes equivalent with the maximization of  $\eta_m$  for the given channel realizations. As  $\eta_m$  is a monotonically increasing function of  $|\mathbf{v}(m)^H \mathbf{h}_{r,d}(m)|^2$ , the minimization of

$F_m(2^{2R} - 1 - \eta_0)$  in (51) can be substituted by the maximization of  $|\mathbf{v}(m)^H \mathbf{h}_{r,d}(m)|^2 = \text{Tr}(\mathbf{h}_{r,d}(m) \mathbf{h}_{r,d}(m)^H \mathbf{S}(m))$ . In Appendix C, we show that for the three levels of CSI regarding the interference channels, OP (51) has a rank-1 solution. Hence, we can write  $\mathbf{S}(m)^{(opt)} = \mathbf{v}(m)^{(opt)} \mathbf{v}(m)^{(opt)H}$ , and  $\mathbf{v}(m)^{(opt)}$  is the optimal solution of OP (50).

2) *Beamforming with ICSI*: In the case where the RNs are assumed to have ICSI on their channel gains to the DN, the actual channel gain to the DN can be written as

$$\mathbf{h}_{r,d}(m) = \hat{\mathbf{h}}_{r,d}(m) + \mathbf{e}_{r,d}(m) \quad (52)$$

where  $\mathbf{e}_{r,d}(m) \sim N_c(0, \mathbf{R}_e(m))$  and  $\mathbf{R}_e(m)$  is defined as  $\mathbf{R}_e$  in Appendix B, with  $\mathbf{f} = \mathbf{h}_{r,d}(m)$ .

The corresponding objective function to be minimized is shown in (50), where among the variables affecting  $\eta_m$  only the error vectors  $\mathbf{e}_{r,d}(m)$  are considered random, and the remaining variables are assumed to be known; for notational convenience, the conditioning on the known vector CSI will not be shown.

Using (27) we can manipulate the CDF (48) into  $F_m(x) = \Pr[|\mathbf{v}(m)^H \mathbf{h}_{r,d}(m)|^2 \leq \gamma_m(x)]$  when  $0 \leq x < \eta_{s,r}(m)$  with  $\gamma_m(x) = \sigma_{r,d}^2(m) \frac{x(\eta_{s,r}(m)+1)}{\eta_{s,r}(m)-x}$ . If  $x \leq 0$  or  $\eta_{s,r}(m) \leq x$ , the objective function  $F_m(x)$  is equal to 0 or 1, respectively, and independent of the value of  $\mathbf{v}(m)$ . When  $0 \leq x < \eta_{s,r}(m)$ , exploiting the fact that the RV  $|\mathbf{v}(m)^H \mathbf{h}_{r,d}(m)|^2$  is distributed according to a scaled non-central  $\chi^2$ -distribution, the objective function  $F_m(x)$  can be rewritten as [25]

$$F_m(x) = 1 - Q\left(\sqrt{2a(m)}, \sqrt{2b(m,x)}\right) \quad (53)$$

where  $Q(\cdot, \cdot)$  represents the first-order Marcum Q-function [26, eq. (4.33)], and  $a(m)$  and  $b(m, x)$  are both real-valued functions given by  $a(m) = \frac{|\mathbf{v}(m)^H \hat{\mathbf{h}}_{r,d}(m)|^2}{\mathbf{v}(m)^H \mathbf{R}_e(m) \mathbf{v}(m)}$  and  $b(m, x) = \frac{\gamma_m(x)}{\mathbf{v}(m)^H \mathbf{R}_e(m) \mathbf{v}(m)}$ . Using [27, eq. (7),(8)] it is easy to see that  $Q\left(\sqrt{2a(m)}, \sqrt{2b(m,x)}\right)$  is increasing in  $a(m)$  and decreasing in  $b(m, x)$ . Therefore, setting  $\mathbf{v}(m)^H \mathbf{R}_e(m) \mathbf{v}(m) = c$ ,  $c$  being a real-valued variable, the OP (50) can be solved by maximizing  $a(m)$  for each value of  $c$ . Hence, using  $\mathbf{S}(m)$ , the objective function in OP (51) is changed to

$$\begin{aligned} & \mathbf{S}(m, c) \\ &= \arg \max_{\mathbf{S}(m) \succeq 0, \lambda_1(m) \geq 0, \dots, \lambda_{N_{\text{PU}}}(m) \geq 0} \text{Tr}\left(\mathbf{S}(m) \hat{\mathbf{h}}_{r,d}(m) \hat{\mathbf{h}}_{r,d}(m)^H\right) \end{aligned} \quad (54)$$

with an additional constraint:  $\text{Tr}[\mathbf{R}_e(m) \mathbf{S}(m)] = c$ . Ideally, this modified OP then has to be solved for every value of  $c$ , where  $c \in [0, c_{\text{max}}]$ . In practice, we divide this interval in a sufficiently number of points and solve this modified OP for each point. The value of  $c_{\text{max}}$  can be found by solving OP (51), but with the following objective function

$$c_{\text{max}} = \max_{\mathbf{S}(m) \succeq 0, \lambda_1(m) \geq 0, \dots, \lambda_{N_{\text{PU}}}(m) \geq 0} \text{Tr}[\mathbf{R}_e(m) \mathbf{S}(m)]. \quad (55)$$

Finally, the optimal value of the original OP (51) is found by substituting  $\mathbf{S}(m, c)$  in (53) and then minimizing (53) over  $c$ . With  $c^*$  minimizing (53), the optimal value  $\mathbf{S}(m)^{(opt)}$  equals  $\mathbf{S}(m, c^*)$ .

Because of the equality constraint  $\text{Tr}[\mathbf{R}_e(m) \mathbf{S}(m)] = c$ , the results from Appendix C cannot be applied here. This means that in general  $\text{Rank}(\mathbf{S}(m)^{(opt)}) \geq 1$ . In this case, a suboptimal rank-1 solution can be obtained from the matrix  $\mathbf{S}(m)^{(opt)}$  by using the randomization approach described in [28]. First, we generate  $L$  independent random vectors  $\mathbf{v}_l(m) \sim N_c(0, \mathbf{S}(m)^{(opt)})$  ( $l = 1, \dots, L$ ), which are then scaled such that each of them satisfies with equality the most stringent constraint. Finally, the vector  $\mathbf{v}_l(m)$  ( $l = 1, \dots, L$ ) which gives the largest value for the objective function is selected as an approximation for  $\mathbf{v}(m)^{(opt)}$ .

Note that for the minimization of (53) the  $m$ th RN has to know, apart from CSI about its own channel gains to the DN and PU-RXs, the instantaneous SNR value  $\eta_0$  on the direct SN-DN channel. Thus, this solution is not distributed as  $\eta_0$  depends on the realization of the channel gain  $h_{s,d}$  and the transmit energy  $E_0$  selected by the SN. Therefore, we also consider a *distributed* version of this OP, which minimizes (53) over  $c$  under the worst-case assumption on  $\eta_0$ , that is  $\eta_0 = 0$ . The performance of this distributed yet suboptimum solution will be compared to the optimum but centralized solution in Section V-D.

3) *Beamforming with CDI*: In this case, we assume that the RNs only know the distribution of their channel gains to the DN. We follow the same reasoning as in Section IV-C2, but now  $\mathbf{h}_{r,d}(m)$  is assumed to be a RV with mean 0 and covariance matrix  $\mathbf{R}_h(m)$ . This means that in equation (53) we have  $a(m) = 0$  and  $b(m, x) = \frac{\gamma_m(x)}{\mathbf{v}(m)^H \mathbf{R}_h(m) \mathbf{v}(m)}$ . In Section IV-C2, it was shown that  $F_m(x)$  is increasing with  $b(m, x)$ . Hence, the minimization of  $F_m(2^{2R} - 1 - \eta_0)$  in (51) can be substituted by the maximization of  $\text{Tr}(\mathbf{R}_h(m) \mathbf{S}(m))$ . For the scenario with PCSI about the interference channels, we prove in Appendix C that we can always find a rank-1 solution to OP (51). For the case of ICSI and CDI, it is possible that  $\mathbf{S}(m)^{(opt)}$  has a rank higher than 1. In this case, a rank-1 approximation is found by using the randomization approach described in Section IV-C2.

#### D. Computational complexity

In this subsection, we discuss the computational complexity of our proposed algorithms. To calculate the complexity, we use the same approach as in [29]. Using this approach, we can show that the complexity scales linearly in the number of RNs  $M$ . The computational complexity of the algorithm proposed for beamforming with PCSI, ICSI and CDI is the same. However, in the scenario with ICSI the complexity scales linearly in the number of points that we consider for the interval  $c \in [0, c_{\text{max}}]$ . Further there is a difference in complexity depending upon the level of CSI that is available about the interference channels. If PCSI is available about the interference channels, the complexity for each RN to reach an  $\epsilon$ -optimal solution is in the order of

$$\mathcal{O}((\sqrt{N_{\text{PU}}} + N_a N_a^6) \cdot \ln(\epsilon)) \quad (56)$$

when  $N_a$  and  $N_{\text{PU}}$  go to infinity. As the complexity of the algorithm in [12] can be shown to be  $\mathcal{O}((\sqrt{N_{\text{PU}}} + N_a^2 N_a^{12}) \cdot$

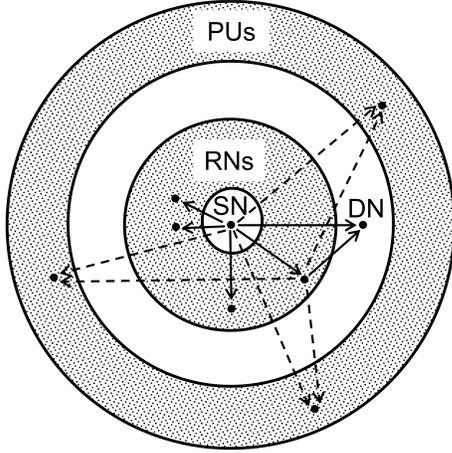


Fig. 2. The topology of the network.

$\ln(\epsilon)$ , it is clear that we were able to significantly reduce the complexity.

In the scenario with ICSI or CDI about the interference channels we get

$$\mathcal{O}((K^{2.5}N_{\text{PU}}^{1.5}N_{\text{a}}^{6.5} + K^{3.5}N_{\text{PU}}^{1.5}N_{\text{a}}^{5.5}) \cdot \ln(\epsilon)) \quad (57)$$

when  $N_{\text{a}}$ ,  $N_{\text{PU}}$  and  $K$  go to infinity.

## V. NUMERICAL RESULTS

We consider the configuration as shown in Fig. 2. The SN and DN are located at coordinates  $(0, 0)$  and  $(1.375, 0)$ , respectively, while the supporting RNs are assumed to be uniformly distributed inside an annulus with outer radius 1.25 and inner radius 0.25. The PU-RXs are uniformly distributed inside an annulus with outer radius 2.5 and inner radius 1.5. The SN broadcasts its message in the first time slot, and in the second time slot only the best RN amplifies and forwards the message to the DN. The solid lines refer to the messages exchanged within the SU network, while the dashed ones refer to the interference from the SN and RNs to the PU-RXs.

The outage probabilities are calculated by means of Monte Carlo simulations. For each channel realization, we randomly select a different location for the RNs and the PU-RXs. We assume that  $\mathbb{E}[|h_{\text{s,d}}|^2] = 1/d_{\text{s,d}}^2$ ,  $\mathbb{E}[\mathbf{h}_{\text{s,r}}(m)\mathbf{h}_{\text{s,r}}(m)^H] = 1/d_{\text{s,r}}^2(m)\mathbf{I}_{N_{\text{a}}}$ ,  $\mathbf{R}_{\text{h}}(m) = 1/d_{\text{r,d}}^2(m)\mathbf{I}_{N_{\text{a}}}$ ,  $\mathbf{R}_{\text{s,p}} = 1/d_{\text{s,p}}^2\mathbf{I}_K$  and  $\mathbf{R}_{\text{r,p}}(m) = 1/d_{\text{r,p}}^2(m)\mathbf{I}_{KN_{\text{a}}}$  ( $m = 1, \dots, M$ ;  $p = 1, \dots, N_{\text{PU}}$ ), where  $d_{\text{s,d}}$ ,  $d_{\text{s,r}}(m)$ ,  $d_{\text{r,d}}(m)$ ,  $d_{\text{s,p}}$  and  $d_{\text{r,p}}(m)$  denote the distances between the corresponding nodes.

We take the same value  $E^{(\text{max})}$  for the maximal transmit energies  $E_m^{(\text{max})}$  ( $m = 0, \dots, M$ ), and set the noise variances on all channels equal to  $\sigma^2$ . This allows to express the outage probabilities of the different scenarios as a function of  $E^{(\text{max})}/\sigma^2$  (dB). In the case of ICSI, the estimation error  $\sigma_e^2$  is chosen equal to  $\sigma^2/(2E^{(\text{max})})$ . For all simulations, we take memory of the predictor  $P = 7$ ,  $R = 0.5$  bits/channel use,  $\alpha = 0.9$ , symbol interval  $T = 50$  ns, Doppler spread  $f_{\text{d}} = 144$  Hz,  $\Gamma = \sigma^2$ . For more information about these channel variables we refer to Appendix B.

In the following, the performance of the relay network will be compared to a SU direct-link network, which optimizes the following expression for the outage probability

$$P_{\text{out}} = \Pr\{\log_2(1 + \frac{E_0 |h_{\text{s,d}}|^2}{\sigma_{\text{s,d}}^2}) \leq R\} \quad (58)$$

where the optimum value of  $E_0$  is obtained from (42), (44) or (46), depending upon the available CSI on the channel gains to the PU-RXs. The main disadvantage of the relay network compared to the direct-link network is the requirement of an additional time slot.

We also note that in the case where  $\text{Rank}(\mathbf{S}(m)) > 1$ , a rank-1 solution is found by applying the randomization approach from Section IV-C2. The number of generated vectors  $L$  is chosen equal to 50. In our numerical results, we have found that the performance loss compared to the multi-rank solution is negligible. Therefore, in the case where  $\text{Rank}(\mathbf{S}(m)) > 1$ , only the performance curves corresponding to the rank-1 approximation will be shown.

Unless mentioned otherwise, in the case where ICSI about the channels to the DN is available, we will show the performance corresponding to the centralized OP.

### A. Influence of the number of antennas per PU-RX

We now consider the performance of a SU network with 2 RNs ( $M = 2$ ); the RNs are each equipped with 3 transmit and receive antennas ( $N_{\text{a}} = 3$ ). We show the performance in the scenario where there are two PU-RXs present ( $N_{\text{PU}} = 2$ ) with a single antenna ( $K = 1$ ) and the scenario where there is a single PU-RX ( $N_{\text{PU}} = 1$ ) with two antennas ( $K = 2$ ). Fig. 3 shows three subfigures, where each subfigure corresponds to a certain level of CSI about the interference channels. Each subfigure then shows the curves for all the levels of CSI about the coefficients between the RNs and the DN: PCSI, ICSI and CDI.

1) *PCSI about interference channel*: Fig. 3a shows the exact outage probabilities in the case where the SN and the RNs have PCSI about their interference channels. The availability of the PCSI will allow the RNs to transmit their beams away from the PU-RXs. The performances in the case of PCSI and ICSI about the channel coefficients to the DN are very similar, because both cases allow the RNs to steer their beams towards the DN and at the same time away from the PU antennas. However when only CDI is available about the channels gains to the DN, the RNs are only able to reduce the interference at the PU-RXs. This explains the large performance gap between the performance curves for CDI and PCSI.

We also notice that the outage probability  $P_{\text{out}}$  is the highest in the case where  $N_{\text{PU}} = 1$  and  $K = 2$ . This is explained by the fact that we consider an interference constraint per PU-RX, which means the interference constraint is harder to satisfy when  $K > 1$ .

Additionally, it is worth emphasizing that the outage probability  $P_{\text{out}}$  will eventually converge to a non-zero value for increasing  $E^{(\text{max})}$ . To explain this, we take a closer look at (7) and (27). In the case of PCSI about the channel gains

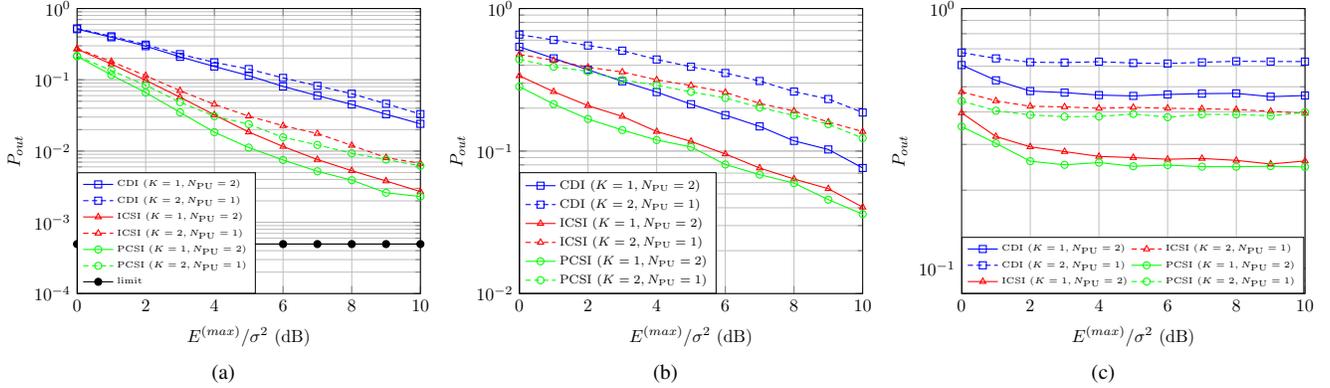


Fig. 3. Outage probability for a single PU-RX with  $K = 2$  and for two PU-RXs with  $K = 1$  ( $M = 2$ ,  $N_a = 3$ ). (a) PCSI of intf channels. (b) ICSI of intf channels. (c) CDI of intf channels.

to the DN, when  $N_a > KN_{PU}$  and  $E^{(max)}$  goes to infinity, there are enough degrees of freedom available to select an infinitely large  $\mathbf{v}(m)$  which is orthogonal to the columns of  $\mathbf{G}_{r,p}(m)$  for  $p = 1, \dots, N_{PU}$ . This means that the optimal solution of (50) will result in a value for  $|\mathbf{v}(m)^H \mathbf{h}_{r,d}(m)|^2$  which is infinitely large, so that (27) can then be written as  $\eta_m = \eta_{s,r}(m) = \frac{E_0}{\sigma_{s,r}^2(m)} \|\mathbf{h}_{s,r}(m)\|^2$ . Hence, using (5) and (27), the outage probability for infinite  $E^{(max)}$  can be written as

$$P_{out} = \Pr \left\{ \frac{E_0^{(opt)} |h_{s,d}|^2}{\sigma_{s,d}^2} + \max_{m \in \{1, \dots, M\}} \frac{E_0^{(opt)}}{\sigma_{s,r}^2(m)} \|\mathbf{h}_{s,r}(m)\|^2 \leq 2^{2R} - 1 \right\} \quad (59)$$

where  $E_0^{(opt)}$  is given by (42). This lower limit can be further decreased by adding more RNs to the SU network or by equipping the RNs with more antennas. We have shown also the limiting value (59) in Fig. 3a.

2) *ICSI about interference channel*: Fig. 3b shows the exact outage probabilities in the case where the SN and the RNs have ICSI about their interference channels. Comparing these curves with those from Fig. 3a clearly shows that the ICSI about the interference channels causes a significant performance loss. Hence, having only ICSI (instead of PCSI) on the channel gains to the PU antennas deteriorates the performance much more than having ICSI on the channel gains to the DN.

3) *CDI about interference channel*: Finally, Fig. 3c shows the exact outage probabilities in the case where the SN and the RNs only have CDI about their interference channels. In this scenario we notice that for increasing  $E^{(max)}/\sigma^2$  the outage probability quickly converges to a non-zero limiting value. This shows that the performance of the SU network is severely limited by the interference constraints: the RNs are unable to steer the transmit beam away from the PU-RXs, which means they do not benefit from the higher allowed transmit energy when  $E^{(max)}/\sigma^2$  is increased. This is even worse for the case where  $N_{PU} = 1$  and  $K = 2$ .

### B. Influence of the number of RNs

We compare the performance of a SU network with a single RN ( $M = 1$ ) and with 2 RNs ( $M = 2$ ). Further, we set  $N_a = 3$ ,  $N_{PU} = 2$  and  $K = 1$ . In Fig. 4a, we consider PCSI for the interference channels and the channels to the DN. As a reference we have also shown the performance of the maximal ratio reception (MRR)-orthogonally projected maximum ratio transmission (OPMRT) and MRR-maximum ratio transmission (MRT) schemes proposed in [20]. For the MRT algorithm, the beamforming matrix is given by  $\mathbf{F}(m) = a \mathbf{h}_{r,d}^* \mathbf{h}_{s,r}^H$ , where  $a \in \mathbb{R}$  is chosen such that the constraint (10) and (12) are satisfied. For the OPMRT algorithm, the beamforming matrix is given by the projection of  $a \mathbf{h}_{r,d}^* \mathbf{h}_{s,r}^H$  into the null space of  $[\mathbf{G}_{r,1}(m), \dots, \mathbf{G}_{r,N_{PU}}(m)]^T$ , where  $a \in \mathbb{R}$  is chosen such that (10) is satisfied. It can be clearly seen that a significant performance improvement can be achieved by using the optimal beamforming algorithm from Section IV-C1, which we labeled PCSI in Fig. 4a. Further we notice that we get a performance gain by increasing the number of RNs  $M$ . This performance gain is explained by the fact that the DN is able to select the RN which has the most favorable channel conditions, which are: strong channel gain between SN and RN, a strong channel gain between the RN and the DN and a weak link between the RN and the PU-RXs.

In Fig. 4b, we consider ICSI for the interference channels and the channels to the DN. We have also shown the performance of the robust beamforming (RB) proposed in [15]. In order to compare both algorithms in a fair way, the parameters  $\mathbf{T}_p$  and  $\mathbf{Q}$  described in [15] for the RB algorithm are set to  $\mathbf{T}_p = \frac{2}{\chi_a^2(2KN_a)} \mathbf{R}_{e,r,p}(m)^{-1}$  ( $p = 1, \dots, N_{PU}$ ) and  $\mathbf{Q} = \frac{2}{\chi_a^2(2N_a)} \mathbf{R}_e(m)^{-1}$ . In Fig. 4b, we show that by specifically optimizing the outage probability  $P_{out}$ , a higher reliability can be achieved compared to the RB algorithm which optimizes the worst-case capacity. Further, we again see a clear gain in performance by increasing the number of RNs from 1 to 2.

### C. Number of antennas

In Fig. 5, we take  $M = 2$ ,  $N_{PU} = 2$ ,  $K = 1$  and we compare the various scenarios for a different number of

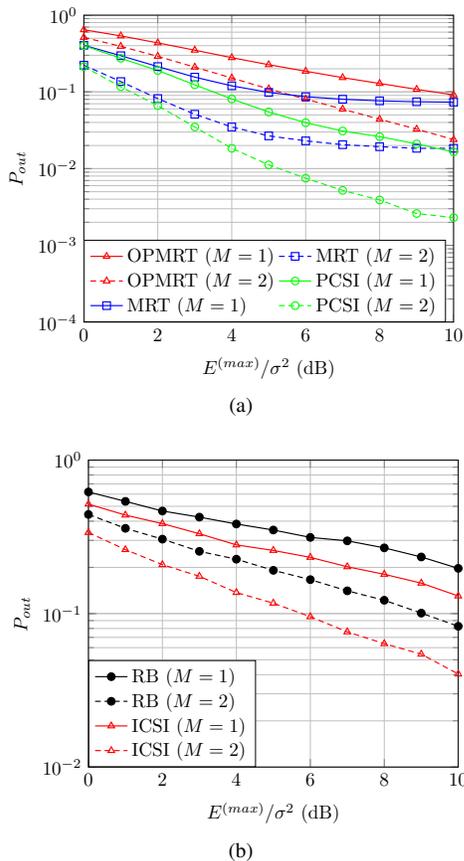


Fig. 4. Comparison of our proposed algorithms, the MRR-OPMRT and MRR-MRT scheme from [20], and the robust beamforming scheme from [15] ( $M = 1$  and  $2$ ,  $N_a = 3$ ,  $N_{PU} = 2$ ,  $K = 1$ ). (a) Beamforming with PCSI (PCSI of intf channels). (b) Beamforming with ICSI (ICSI of intf channels).

transmit and receive antennas at each RN:  $N_a = 2, 3$  and  $4$ . As in Section V-A, we split the discussion according to the level of CSI about the interference channel coefficients.

1) *PCSI about interference channel*: Fig. 5a shows  $P_{out}$  in the case where the SU-TXs have PCSI about their interference channel. We also show the limiting values of  $P_{out}$  when  $E^{(max)}$  goes to infinity and PCSI is available about the channel gains to the DN. For  $N_a = 3, 4$ , the limit is given by (59), which is valid for  $N_a > KN_{PU}$  only; for  $N_a = 2$ , we have solved (41) and (51) without transmit energy constraint. These lower limits clearly show that the largest performance improvement is achieved by going from 2 to 3 antennas. This is because we need at least  $N_a = 3$  to avoid interference at the 2 PU-RXs; for  $N_a < 3$ , the RNs have to limit their transmit energies in order not to violate the interference constraints.

In the case of CDI, we see an improvement by going from  $N_a = 2$  to  $N_a = 3$ , but only a very small improvement by increasing the number of antennas  $N_a$  from 3 to 4. The latter gain is only caused by the MRC at the RN, while the former gain is also caused by the fact that the number of antennas  $N_a$  exceeds the number of PU-RX antennas  $KN_{PU}$ . However, in the case of ICSI and PCSI we keep noticing a large performance improvement by increasing the number of antennas at the RNs. The gain is explained by the MRC and beamforming towards the DN. We note that the performance

loss between the case of ICSI and PCSI is rather small (a difference in  $E^{(max)}/\sigma^2$  of around 1 – 2 dB). Finally, Fig. 5a also shows the performance of the direct link SN-DN in the case of PCSI about the channel gains to the PU-RXs. The relay network outperforms the direct link network in all three cases.

2) *ICSI about interference channel*: In Fig. 5b, the case where the SU-TXs have ICSI about their interference channel is addressed. In the case of CDI, we again notice that there is an improvement by going from  $N_a = 2$  to  $N_a = 3$ , while there is almost no performance difference between  $N_a = 3$  and  $N_a = 4$ , since i) transmit beamforming towards the DN is impossible, when only CDI is available about the channel gains to the DN, and ii) at the RN, the gain offered by the MRC is countered by the loss in performance caused by the uncertainty on the interference channels. Further, from (18), it is clear that the interference constraints are dependent on the factor  $\chi_\alpha^2(2KN_a)$ . This factor increases with the number of antennas  $KN_a$ , which makes the interference constraints more stringent. In Fig. 5b, we also show the performance of the direct link SU network in the case where the SN has ICSI about its interference channel. Again, we can emphasize that the multi-antenna multi-relay network has a significantly better performance than the direct link network.

3) *CDI about interference channel*: In Fig. 5c, we consider the case where the SU-TXs have CDI about the interference channel. We observe that when only CDI is available about the channel gains to the DN, increasing the number of antennas at the RNs does not bring any performance gain and can even lead to a degradation caused by the more stringent interference constraints due to the larger value of the factor  $\chi_\alpha^2(2KN_a)$ . In the case of ICSI and PCSI about the channel gains to the DN, we notice only a very small performance improvement. It is clear that interference is the main limiting factor in this scenario, and increasing the number of antennas does not significantly improve the performance. Finally, in Fig. 5c we have also shown the performance of the direct link SU network which only has CDI about its interference channels. In this case, the multi-antenna multi-relay network does not provide any performance improvement, compared to the direct link, when only CDI on the channels to the DN is available, since beamforming is impossible and the multiple antennas are not able to avoid interference at the PU-RXs; when using the relays, the gain provided by MRC at the DN is offset by the need for two time slots.

#### D. Centralized versus distributed solutions

In this subsection we assume that the SN and the RNs have ICSI about their channel coefficients to the DN. We make a distinction between PCSI, ICSI and CDI for the knowledge about the interference channels. Choosing  $M = 2$ ,  $N_{PU} = 2$ ,  $K = 1$  and  $N_a = 3$ , the performance of the centralized and distributed solutions of (51) are compared in Fig. 6; note that in the centralized solution  $\eta_0$  must be known by the RNs, whereas in the distributed solution the RNs assume  $\eta_0 = 0$ . In order to investigate the influence of the distance  $d_{s,d}$  on the performance, we show the outage

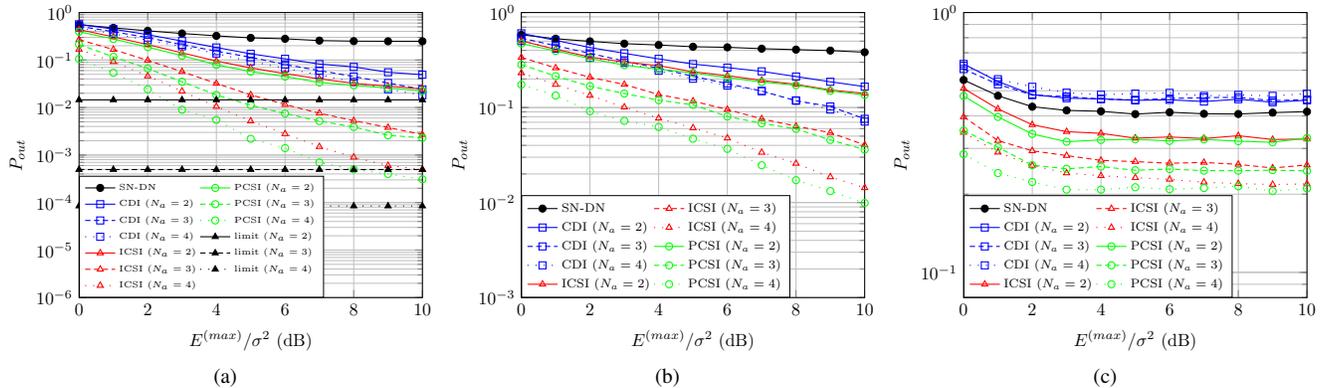


Fig. 5. Outage probability for  $N_a = 2, 3$  and  $4$  ( $M = 2, N_{PU} = 2, K = 1$ ). (a) PCSI of intf channels. (b) ICSI of intf channels. (c) CDI of intf channels.

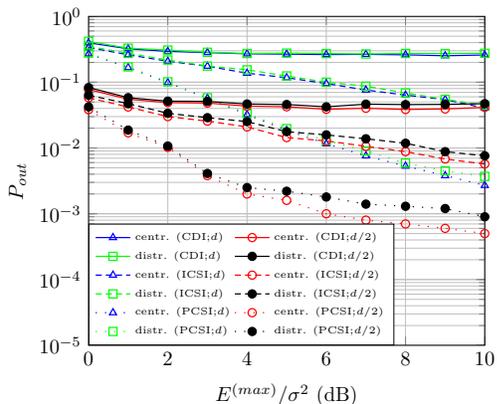


Fig. 6. Comparison between the centralized and distributed solution ( $M = 2, N_{PU} = 2, K = 1, N_a = 3$ , ICSI for channels to DN).

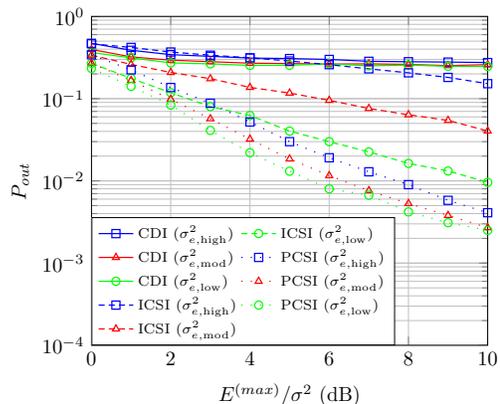


Fig. 7. Outage probability for different estimation error variances ( $M = 2, N_{PU} = 2, K = 1, N_a = 3$ , ICSI for channels to DN).

probability for two scenarios: one where the SN and DN are separated by a distance  $d$ , and one where they are separated by a distance  $d/2$ , with  $d = 1.375$ . Fig. 6 shows that the distributed solution yields only a very small performance loss when  $d_{s,d} = d$ . In the case where  $d_{s,d} = d/2$ , the performance loss is slightly higher. However, as the distributed solution is easier to implement, the slight performance loss of the distributed algorithm is certainly acceptable.

### E. Quality of the ICSI

In Fig. 7 the performances of the SU network are displayed for different values of the channel estimation error  $\sigma_e^2$ . We consider  $\sigma_e^2$  equal to  $\sigma_{e,low}^2 = \frac{1}{5} \frac{\sigma^2}{2E^{(max)}}$ ,  $\sigma_{e,mod}^2 = \frac{\sigma^2}{2E^{(max)}}$  and  $\sigma_{e,high}^2 = 5 \frac{\sigma^2}{2E^{(max)}}$  and assume that the SN and the RNs have ICSI about their channel coefficients to the DN. In the case of PCSI about the interference channels, the RNs can avoid interference at the PU-RXs; the performance differences for the considered values of  $\sigma_e^2$  can be attributed to the different accuracies of RN beamsteering towards the DN. In the case of ICSI about the interference channels, the RNs cannot avoid interference at the PU-RXs, yielding a worse performance compared to PCSI; the performance differences for the considered values of  $\sigma_e^2$  are larger than for PCSI, because the estimation error variance also applies

to the interference channels, hence affecting the amount of interference. Finally, when only CDI about the interfering channels is available, the performance is severely limited by the interference constraints, and the effect of the estimation error variance  $\sigma_e^2$  is very small.

## VI. CONCLUSIONS

In this contribution, several distributed RA algorithms have been derived that minimize the exact outage probability of a cooperative SU network with multi-antenna AF-relay selection, while protecting the QoS of the PU network.

The numerical results show that the introduction of multi-antenna RNs can substantially improve the performance of a SU network which uses the underlay paradigm. The performance of the SU network was investigated for different levels of channel knowledge. The SU network considerably benefits from having at least imperfect channel knowledge of the channel gains to the PU-RXs, and having RNs with a number of antennas larger than the number of PU antennas. When only CDI about the interference channels is available, the interference constraints have a devastating effect on the SU performance. The level of channel state information available about the channels to the DN is shown to have a smaller impact on the overall performance, as compared to the interference

channel state information. But also here substantial gains compared to CDI are achieved when an estimate of the channel gains to the DN is available. The quality of this estimate has less effect than the quality of the estimate of the interference channels.

We have shown that having multiple RNs available can considerably improve the SU performance, compared to the case with a single relay, because the probability of having poor channel conditions is significantly reduced.

Finally, we have pointed out that the proposed algorithms compare favorably to several algorithms presented in literature, in terms of outage probability and/or complexity.

#### APPENDIX A PROOF OF THEOREM 1

The following proof is based on [13]. Assuming that the  $m$ th RN knows  $\mathbf{h}_{s,r}(m)$ , the matrix  $\mathbf{F}(m)$  can without loss of generality be decomposed as

$$\mathbf{F}(m) = [\mathbf{w}_1(m)\mathbf{W}_2(m)] \begin{bmatrix} \mathbf{h}_{s,r}(m) \\ \|\mathbf{h}_{s,r}(m)\| \mathbf{H}_{s,r}^\perp(m) \end{bmatrix}^H \quad (60)$$

where  $\mathbf{w}_1(m) \in \mathbb{C}^{N_a \times 1}$ ,  $\mathbf{W}_2(m) \in \mathbb{C}^{N_a \times (N_a - 1)}$  and  $\mathbf{H}_{s,r}^\perp(m) \in \mathbb{C}^{N_a \times (N_a - 1)}$ . The matrix  $\mathbf{H}_{s,r}^\perp(m)$  makes  $\begin{bmatrix} \mathbf{h}_{s,r}(m) \\ \|\mathbf{h}_{s,r}(m)\| \mathbf{H}_{s,r}^\perp(m) \end{bmatrix}$  a unitary matrix, which means that  $\mathbf{H}_{s,r}^\perp(m)^H \mathbf{h}_{s,r}(m) = \mathbf{0}$ . Using (60), it can be shown that  $\eta_m$  from (6) is given by

$$\eta_m = \frac{\frac{E_0}{\sigma_{s,r}^2(m)} \|\mathbf{h}_{s,r}(m)\|^2 \|\mathbf{h}_{r,d}(m)\|^T \mathbf{w}_1(m)}{\|\mathbf{h}_{r,d}(m)\|^T \mathbf{w}_1(m) + \|\mathbf{h}_{r,d}(m)\|^T \mathbf{W}_2(m) + \frac{\sigma_{r,d}^2(m)}{\sigma_{s,r}^2(m)}}. \quad (61)$$

Combining equation (4) and (60), we can express the transmit power  $E_m$  as

$$E_m = E_0 \|\mathbf{h}_{s,r}(m)\|^2 \|\mathbf{w}_1(m)\|^2 + \sigma_{s,r}^2(m) (\|\mathbf{w}_1(m)\|^2 + \|\mathbf{W}_2(m)\|^2). \quad (62)$$

In the same way, by combining (12) and (60), the interference  $\mathcal{I}_{r,p}(m)$  to be used in the interference constraints from Section III-C can be written as

$$\mathcal{I}_{r,p}(m) = E_0 \|\mathbf{h}_{s,r}(m)\|^2 \|\mathbf{G}_{r,p}(m)\|^T \mathbf{w}_1(m) + \sigma_{s,r}^2(m) (\|\mathbf{G}_{r,p}(m)\|^T \mathbf{w}_1(m) + \|\mathbf{G}_{r,p}(m)\|^T \mathbf{W}_2(m)) \quad (63)$$

where  $\mathbf{G}_{r,p}(m)$  denotes the actual interference channel (PCSI), or a point in the volume  $\mathcal{U}_{r,p}(\hat{\mathbf{g}}_{r,p}(m), \mathbf{B}_{r,p}(m))$  (ICSI) or  $\mathcal{U}_{r,p}(\mathbf{0}, \mathbf{B}_{r,p}(m))$  (CDI). From (61) we see that the highest value of  $\eta_m$  is reached when  $\mathbf{W}_2(m) = \mathbf{0}$ . From (7) and (8) it follows that a larger value for  $\eta_m$  will certainly decrease the outage probability  $P_{\text{out}}$ , irrespective of which information (PCSI, ICSI or CDI) on  $\mathbf{h}_{r,d}(m)$  is available at the  $m$ th RN. Further, we note that  $\mathbf{W}_2(m) = \mathbf{0}$  will also lead to a lower value of the transmit energy  $E_m$  (62) and the interference  $\mathcal{I}_{r,p}(m)$  (63). Therefore

we can conclude that the optimal value of  $\mathbf{W}_2(m)$  will always be  $\mathbf{0}$ . Without loss of generality we can define

$$\mathbf{w}_1(m) = \frac{\mathbf{v}(m)^*}{\sqrt{E_0 \|\mathbf{h}_{s,r}(m)\|^2 + \sigma_{s,r}^2(m)}} \quad (64)$$

which leads to (24).

#### APPENDIX B IMPERFECT CSI

In this section we will describe the case where the CSI available at a SU-TX is imperfect due to feedback delay and estimation errors. The following formulas use a generic channel vector  $\mathbf{f} \in \mathbb{C}^{F \times 1}$ , where  $F$  denotes the number of components of  $\mathbf{f}$ . Its covariance matrix is denoted by  $\mathbf{R}_f$ . We also introduce the vector **ICSI** which denotes the imperfect CSI available at the SU-TX about the actual channel realization  $\mathbf{f}$ . We make the assumption that the channel vector  $\mathbf{f}$  and the **ICSI** are jointly zero-mean circular symmetric Gaussian. It then follows that  $\mathbf{f}$  conditioned on the **ICSI** is Gaussian, with expectation  $\hat{\mathbf{f}} = \mathbf{E}[\mathbf{f}|\mathbf{ICSI}]$  and covariance matrix  $\mathbf{R}_e = \mathbf{E}[\mathbf{f}\mathbf{f}^H|\mathbf{ICSI}] - \hat{\mathbf{f}}\hat{\mathbf{f}}^H$ . Introducing  $\mathbf{e} \sim N_c(\mathbf{0}, \mathbf{R}_e)$ , the instantaneous channel gain  $\mathbf{f}$  can be decomposed as

$$\mathbf{f} = \hat{\mathbf{f}} + \mathbf{e} \quad (65)$$

where  $\hat{\mathbf{f}}$  is the minimum mean-squared error (MMSE) estimate of  $\mathbf{f}$  based on **ICSI**, and the estimation error  $\mathbf{e}$  is independent of  $\hat{\mathbf{f}}$ .

For the numerical results we will make some additional assumptions about **ICSI** as in [16]. We consider the case where the channel  $\mathbf{f}(t)$  is slowly time-varying. According to Jakes' model [30], we take  $\mathbf{E}[\mathbf{f}(t+u)\mathbf{f}(t)^H] = J_0(2\pi f_d u) \mathbf{R}_f$  where  $J_0(\cdot)$  denotes the zeroth-order Bessel function of the first kind, and  $f_d$  denotes the Doppler spread. At the SU-TX, the information about  $\mathbf{f}(0)$  consists of  $P$  delayed channel estimates, i.e.,  $\mathbf{ICSI} = [\hat{\mathbf{f}}(\tau_1)^T, \dots, \hat{\mathbf{f}}(\tau_P)^T]^T$ .

The estimate  $\hat{\mathbf{f}}(\tau_k)$ ,  $k = 1, \dots, P$ , is given by

$$\tilde{\mathbf{f}}(\tau_k) = \mathbf{f}(\tau_k) + \tilde{\mathbf{e}}(\tau_k) \quad (66)$$

where  $\tau_k$  represents the feedback delay relative to the instantaneous channel vector  $\mathbf{f}(0)$ . The noise vector  $\tilde{\mathbf{e}}(\tau_k) \sim N_c(0, \sigma_e^2 \mathbf{I}_F)$ , where  $\sigma_e^2$  denotes the variance of the estimation error. In the numerical results we take  $\tau_k = kDT$ , which indicates that the channel estimates are updated every  $D$  symbol intervals  $T$ .

Introducing the matrix  $\mathbf{J} \in \mathbb{C}^{P \times P}$  as  $\mathbf{J}_{k,l} \triangleq J_0(2\pi f_d(\tau_k - \tau_l))$ ,  $k = 1, \dots, P; l = 1, \dots, P$ , it can now be shown that  $\hat{\mathbf{f}} = \mathbf{X}\mathbf{Y}^{-1}\mathbf{ICSI}$ , where

$$\mathbf{X} \triangleq [J_0(2\pi f_d \tau_1), J_0(2\pi f_d \tau_2), \dots, J_0(2\pi f_d \tau_P)] \otimes \mathbf{R}_f \quad (67)$$

$$\mathbf{Y} \triangleq \mathbf{J} \otimes \mathbf{R}_f + \mathbf{I}_P \otimes \sigma_e^2 \mathbf{I}_F \quad (68)$$

and the covariance matrix is given by  $\mathbf{R}_e = \mathbf{R}_f - \mathbf{X}\mathbf{Y}^{-1}\mathbf{X}^H$ .

APPENDIX C  
RANK-1 SOLUTION OF THE RELAXED OP

The following lemma, which is proven in [31], can be directly applied to OP (51) with PCSI or CDI on the channels to the DN, with PCSI on the interference channels.

**Lemma 2.** *The following OP is convex in  $\mathbf{S}$  and always has solutions with  $\text{Rank}(\mathbf{S}) \leq 1$ :*

$$\begin{cases} \mathbf{S}^{(opt)} = \arg \max_{\mathbf{S} \succeq \mathbf{0}} \text{Tr}(\mathbf{A}\mathbf{S}) \\ \text{s.t. } \text{Tr}(\mathbf{B}_i \mathbf{S}) \leq b_i, \quad i = 1, \dots, I \end{cases} \quad (69)$$

where  $\mathbf{A}$  is a Hermitian matrix, the matrices  $\mathbf{B}_i$  are Hermitian with  $\mathbf{B}_i \succeq \mathbf{0}$  and  $\sum_{i=1}^I \mathbf{B}_i \succ \mathbf{0}$ , and the scalars  $b_i$  satisfy  $b_i \geq 0, \forall i$ .

In the case where PCSI is available for the channels to the DN and with ICSI or CDI on the interference channels, we have to consider a different formulation of OP (51). If we let  $\chi$  denote the optimal value of the objective function  $\text{Tr}(\mathbf{h}_{r,d}(m)\mathbf{h}_{r,d}(m)^H \mathbf{S}(m))$ , we can rewrite the OP as follows

$$\begin{cases} \mathbf{S}(m) = \arg \min_{\mathbf{S}(m) \succeq \mathbf{0}, \lambda_1(m) \geq 0, \dots, \lambda_{N_{PU}}(m) \geq 0} \text{Tr}(\mathbf{S}(m)) \\ \text{s.t. } \text{Tr}(\mathbf{h}_{r,d}(m)\mathbf{h}_{r,d}(m)^H \mathbf{S}(m)) \geq \chi - \delta \\ (37) \text{ or } (40), \quad p = 1, \dots, N_{PU} \end{cases}, \quad (70)$$

where  $\delta$  denotes any small positive value and  $\lambda_p(m)$  is only required when (37) is used. This OP is convex and it can be easily verified that as  $\delta$  approaches 0 the solution will approach the same optimal value as OP (51). It can now be proven that the solution to OP (70) has a rank-1 solution. For more details, we refer to [15] where a similar proof is given in Appendix D.

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