Incrementality and Hierarchies in the Enrollment of Multiple Synergies for Grasp Planning

Giuseppe Averta, Franco Angelini, Manuel Bonilla, Matteo Bianchi, and Antonio Bicchi

***Abstract*—Postural hand synergies or eigenpostures are joint angle covariation patterns observed in common grasping tasks. A typical definition associates the geometry of synergy vectors and their hierarchy (relative statistical weight) with the principal component analysis of an experimental covariance matrix. In a reduced complexity representation, the accuracy of hand posture reconstruction is incrementally improved as the number of synergies is increased according to the hierarchy. In this work, we explore whether and how hierarchy and incrementality extend from posture description to grasp force distribution. To do so, we study the problem of optimizing grasps w.r.t. hand/object relative pose and force application, using hand models with an increasing number of synergies, ordered according to a widely used postural basis. The optimization is performed numerically, on a data set of simulated grasps of four objects with a 19-DoF anthropomorphic hand. Results show that the hand/object relative poses that minimize (possibly locally) the grasp optimality index remain roughly the same as more synergies are considered. This suggests that an incremental learning algorithm could be conceived, leveraging on the solution of lower dimensionality problems to progressively address more complex cases as more synergies are added. Second, we investigate whether the adopted hierarchy of postural synergies is indeed the best also for force distribution. Results show that this is not the case.**

***Index Terms*—Grasping, multifingered hands, compliant joint/mechanism.**

I. Introduction

THE human hand is an extremely complex system, composed by many joints, muscles and sensory receptors, which constitute a highly sophisticated and dexterous apparatus of our body. Such an abundance, classically referred to as Bernsteins problem [1], would require a remarkable calculating capacity for its control. Nevertheless, several neuroscientific studies suggested that the human nervous system is able to cope with such complexity and organize it in a simple environment - as it happens for other natural phenomena according to the Simplexity concept [2] - leveraging on a control space of reduced dimensionality [3]–[6], usually defined as synergistic control space.

This synergistic behavior seems to be used by the Central Nervous System to generate coordinated movements, simultaneously activating different Degrees of Freedom (DoFs), instead of acting separately on each joint or muscle. The existence of these patterns (aka synergies) was observed in different motor tasks and at different levels of the motor control architecture, i.e., neural ([7]), muscular ([8], [9]), kinematic ([10]–[12]). For a review on these topics see e.g., [13].

Considering the kinematic level, such observations supported the idea that few combinations of the hand DoFs, e.g., described in terms of main principal components (PCs, i.e., postural synergies) of hand joint angles recorded in grasping tasks, can take into account large part of hand pose variability. Higher order PCs are likely involved to describe more complex tasks such as haptic exploration [14], or manipulation with the environment [15]. In geometrical terms, synergies can be regarded as a basis of hand principal movements: the more the elements are used (or "enrolled"), the more complex tasks can be executed [10].

While postural synergies provide a geometrical description of hand control in the kinematic space, such a model cannot be directly applied to explain force control, generation and distribution in grasping and manipulation tasks. To this goal, we need to consider also the mechanical compliance of the hand musculotendinous system, as introduced in the soft synergy model [16], inspired by the Equilibrium Point Hypotesis [17]. More specifically, according to this model, postural synergies are considered as a reference configuration and the hand is in equilibrium between the reference attraction and the repulsion forces exerted by the object. In [18], authors numerically demonstrated that for a paradigmatic human hand the same postural synergies that are important for pose generation are also involved in the optimal distribution of contact forces during the grasp. The study of optimal contact forces distribution for robotic hands has been tacked in licterature, as for example in [19], where the authors presented an algorithm for the evaluation of optimal contact forces distribution from tactile measurements, and [20], in which the authors faced the problem using dual theorem of non-linear programming. However, to the best of authors' knowledge, there are no works that investigate the role that postural synergies play for the improvement of the grasp quality in terms of optimal contact force distribution.

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Taking inspiration from the observation that the human development seems to progressively increase the hand control capabilities [21], [22] - which could be read as an increment of the available synergies - in this work, we pursue this investigation by analyzing through numerical simulations, the effect of an incremental enrollment of postural synergies for the execution of successful grasping strategies.

The proposed analysis takes also into account, for the first time, hand/object relative configuration. This investigation aims, firstly, at verifying that hand/object relative poses minimizing the force distribution are approximately invariant w.r.t. an incremental enrollment of synergies, and, secondly, at providing evidence on the existence of a correspondence between the hierarchy of hand posture synergies and the one of synergies controlling contact forces. An exhaustive inspection of these aspects requires an analysis that spans several hand grasp configurations. For this reason, to include a large part of of the main graping hand configurations (e.g., referring to [23]), we simulated the grasp of four different objects with cubic and parallelepiped-like shape of variable dimensions. Two cubic objects are used to drive the hand shape for power and precision circular grasps, while two prismatic objects let the hand assume power and precision prismatic configurations. The simulations we performed resulted in a large dataset, which allows a thorough analysis of the contact forces applied to the considered objects, varying the number and the types of the enrolled synergies. In particular, we focused on the grasp success rate and the value of contact forces to assess the quality and the effectiveness of the grasp w.r.t. the modification of the hand/object configuration.

For illustration of the general results of our study, we provide a worked-out example of a instantiation of a multifingered hand. The model used as example has the kinematic parameters and the CAD model of the anthropomorphic PISA/IIT SoftHand (SH) [24]. However, it differs in that all 19 articular joints are assumed to be independently controlled here (thus generalizing the results by ignoring the specific under-actuation scheme of the SH).

In this work we aim at evaluating if incrementality in the choice of the synergies to be enrolled - and eventually implemented in a robotic device - could be the right path toward a novel generation of robotic hands in which the implementation of the first postural synergy preserves the grasp simplicity, while other PCs are used to enhance the grasp quality and, potentially, the manipulation skills. Furthermore, we believe that our study could have an impact for the development of planning algorithms for highly actuated hands. Indeed, these robotic systems dramatically increase the grasping and manipulations capabilities but decrease the planning and control simplicity. In these cases, we believe that our analysis could be used to justify a first rough evaluation of a good grasping strategy in a lower dimensionality, which could be refined in a second step.

II. Background

A. Modeling a Compliant Hand With Synergies

The problem of grasping with a compliant hand is broadly modelized through balance and congruence equations for hand and object. For a hand with n DoFs, which contacts an object in c_p points, we have that

$$\tau = J^T f, \qquad \xi_f = J\dot{q}, \tag{1}$$

where $J \in \mathbb{R}^{c\times n}$ is the hand Jacobian matrix, $c = 3c_p$ is the dimension of the contact force/torque vector, $\tau \in \mathbb{R}^n$ is the joint torque vector, $f \in \mathbb{R}^c$ is the contact force vector, $\xi_f \in \mathbb{R}^c$ is the twist of the contact points on the fingers and $q \in \mathbb{R}^n$ are the joint angles. Similarly, the balance and congruence equations for the object are

$$w_e = Gf, \qquad \xi_o = G^T \xi_e, \tag{2}$$

where $G \in \mathbb{R}^{6\times c}$ is the grasp matrix, $w_e \in \mathbb{R}^6$ is the object wrench, $\xi_o \in \mathbb{R}^c$ is the twist of the contact points on the object and $\xi_e \in \mathbb{R}^6$ is object twist. The hand elasticity K is defined as

$$K = \left(C_s + JC_q J^T\right)^{-1}, \tag{3}$$

where $C_s = (1/k_{stru}) I_{c\times c} \in \mathbb{R}^{c\times c}$ is the structural compliance matrix and $C_q = (1/k_{ss}) I_{n\times n} \in \mathbb{R}^{n\times n}$ is the joint compliance matrix. K relates the contact forces with the displacement between the twist on the contact points on the fingers and on object as $\delta f = K(\delta\xi_o - \delta\xi_f)$ enabling to solve force indeterminacies in the rigid-body system and allowing to implement the soft synergy paradigm introduced in [16].

In neuroscience there are many studies suggesting that the control of the human hand can be simplified with a lower dimensionality description, usually implemented through Principal Component Analysis (PCA) [13]. Given a set of observations of a specific stochastic phenomenon, PCA calculates the orthonormal transformation that converts the original, possibly correlated, variables into a set of linearly uncorrelated and orthogonal variables called PCs or, in case of joint angles organized in a dataset of hand grasps, postural synergies. These components are usually obtained through Singular Value Decomposition (SVD) of the covariance matrix of the dataset. The eigenvectors (i.e., PCs) hence represent the uncorrelated variables that describe the considered dataset, while the eigenvalues associated to the PCs quantify the importance of each eigenvector in terms of explanation of dataset variability. The mapping between the synergistic displacement input $\delta\sigma \in \mathbb{R}^s$ and the joint reference positions δq_r can be formalized as

$$\delta q_r = S\delta\sigma, \tag{4}$$

where S arranges s PCs per-column, with $s \leq n$. Finally, the soft synergy paradigm relates the reference joint position δq_r with the actual joint position δq through

$$\delta q = \delta q_r - C_q \delta\tau. \tag{5}$$

Of note, numerical results depends on the dataset used to calculate S. There are several studies in literature focusing on different task-dependent datasets, ([15], [25]–[28]). One of the most popular also for robotic applications is [10], where the authors demonstrated that just two PCs account for 80% of the grasp dataset variance, while the first one alone explains more than 50% of pose variability. Findings reported in [10] were also used for the design of robotic devices, as for the Pisa/IIT SH, which embeds the first synergy as single DoA. For

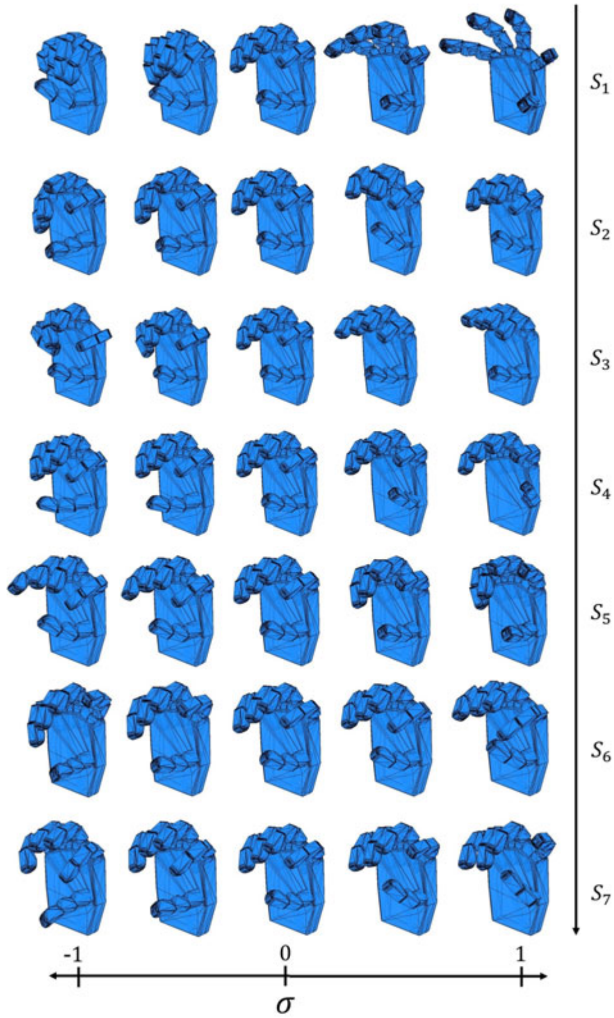

Fig. 1. Different hand shapes using the first 7 synergies introduced in [10] for pose generation. For each row, the central figure shows the mean pose of the hand, while the others report the effect of a specific synergy (from the first to the seventh) modulated by a coefficient $\sigma \in [-1, 1]$.

TABLE I
NOTATION FOR GRASP ANALYSIS WITH POSTURAL SYNERGIES

Notation	Definition
$n \in \mathbb{R}$	number of hand joints
$c_p \in \mathbb{R}$	number of the contact points
$c \in \mathbb{R}$	dimension of the contact force/torque vector
$\tau \in \mathbb{R}^n$	joint torques
$q \in \mathbb{R}^n$	joint angles
$q_r \in \mathbb{R}^n$	joint reference angles
$f \in \mathbb{R}^c$	contact force/torque vector
$w_e \in \mathbb{R}^6$	object wrench
$\xi_e \in \mathbb{R}^6$	object twist
$\xi_f \in \mathbb{R}^c$	twists of the contact points on the fingers
$\xi_o \in \mathbb{R}^c$	twists of the contact points on the object
$J \in \mathbb{R}^{c \times n}$	hand Jacobian matrix
$G \in \mathbb{R}^{6 \times c}$	grasp matrix
$k_{stru} \in \mathbb{R}$	structural stiffness
$k_{ss} \in \mathbb{R}$	joint stiffness
$C_s \in \mathbb{R}^{c \times c}$	structural compliance matrix
$C_q \in \mathbb{R}^{n \times n}$	joint compliance matrix
$K \in \mathbb{R}^{c \times c}$	stiffness matrix
$G_K^R \in \mathbb{R}^{c \times 6}$	$K-$weighted pseudo-inverse of G
$s \in \mathbb{R}$	number of enrolled postural synergies
$\delta\sigma \in \mathbb{R}^s$	synergistic displacement
$S \in \mathbb{R}^{n \times s}$	synergy matrix
$Si \in \mathbb{R}^n$	synergy vector ($i-$th column of S)
$\delta f_p \in \mathbb{R}^c$	particular solution
$\delta f_{hr_s} \in \mathbb{R}^c$	active internal forces
$\delta f_{ho_s} \in \mathbb{R}^c$	passive (preload) internal forces
$F_s \in \mathbb{R}^{c \times s}$	map of $\delta\sigma$ into active internal forces δf_{hr_s}
$e_s \in \mathbb{R}$	rank of F_s
$E_s \in \mathbb{R}^{c \times e_s}$	basis of the range space of F_s
$y \in \mathbb{R}^{e_s}$	parameterizing vector of the active internal forces
$\mu \in \mathbb{R}$	Coulomb friction coefficient
$\mathcal{F}$	composite friction cone
$P \in \mathbb{R}^{c \times c}$	friction limit constraints matrix
$f_n \in \mathbb{R}^{c_p}$	normal components of the contact force
$f_{min} \in \mathbb{R}$	minimum value of f_n

the reader's convenience, we report in Fig. 1 the hand shapes generated using the first seven synergies.

B. Optimization of Grasping Force Distribution

Hand control through dimensionality reduction as suggested by hand synergies requires new approaches to analyze contact forces and grasp quality in terms of force-closure. A grasp has force closure if arbitrary large external wrenches applied to the grasped object can be compensated by the contact forces that the hand is able to apply on the object, see e.g., [29]. Adopting the notation introduced in Table I, we briefly recall here the solution of the force closure problem, with no claim of being exhaustive and specifically targeting grasping through postural synergies and under-actuation. For a more detailed analysis refer to [18], [29], [30].

Given a general synergistic grasping problem of an under-actuated compiant hand, the solution of the force distribution problem is given by

$$f = \delta f_p + \delta f_{hr_s} + \delta f_{ho_s}, \tag{6}$$

where the three terms of the contact force vector $f \in \mathbb{R}^c$ are the particular solution δf_p, the active internal forces δf_{hr_s} and the passive (or preload) internal forces δf_{ho_s}. The latter are here omitted with no loss of generality since they are assumed null (the intersted reader could refer to [30] for a detailed discussion). The particular solution is given by

$$\delta f_p = G_K^R w_e, \tag{7}$$

where $G_K^R = KG^T(GKG^T)^{-1} \in \mathbb{R}^{c \times 6}$ is the K-weighted pseudoinverse of G, and $w_e \in \mathbb{R}^6$ is the wrench applied to the object. The active internal forces are

$$\delta f_{hr_s} = F_s\, \delta\sigma, \tag{8}$$

where $F_s := (I_{c \times c} - G_K^R G)KJS \in \mathbb{R}^{c \times s}$ and $\mathrm{rank}(F_s) = e_s$. Given a basis $E_s \in \mathbb{R}^{c \times e_s}$ for the range space of the matrix F_s, (8) can be rephrased as

$$\delta f_{hr_s} = E_s y, \tag{9}$$

where $y \in \mathbb{R}^{e_s}$ is a vector parameterizing the active internal forces. Thus, the optimal force distribution can be evaluated by minimizing a cost function w.r.t. y. For a complete overview of this problem the reader could refer to [18]. We adopt here the definition of force-closure given in [30].

Definition 1 (Force-Closure): A grasp is defined Force-Closure if and only if the following conditions are satisfied:

1. forces in arbitrary directions are resistible, i.e., $\mathrm{rank}(G) = 6$;

2. the hand configuration is prehensile, i.e., $\exists\, y \in \mathbb{R}^{e_s}$ such that $f(y) \in \mathrm{Int}(\mathcal{F})$.

$\mathrm{Int}(\mathcal{F})$ denotes the internal part of the composite friction cone $\mathcal{F}$. The satisfaction of this friction limit constraint is equivalent to the positive definiteness of the matrix $P \in \mathbb{R}^{c\times c}$, defined as in [31], which contains contact wrenches and friction coefficients.

The problem of finding the optimal distribution of contact forces f in the grasp of an object subject to the external load w_e with regard to the minimization of a suitable cost function $\Psi(y)$ can be formalized as

Definition 2 (Grasping Force Optimization): Given a grasp characterized by G_K^R, E_s, and P, and an object wrench $w_e \in \mathbb{R}^6$, find y^* in (6), such that $f(y^*) \in \mathrm{Int}(\mathcal{F})$, and the cost function $\Psi(f(y^*))$ is minimized.

We assume here zero preload at the beginning of the grasp and an auxiliary constraint on the minimum value $f_{\min}$ for all the normal components f_n of the contact force. Under these hypotheses, the grasping force optimization problem is set up as

$$\begin{aligned} y^* &= \mathrm{argmin}\ \Psi(y) \\ \text{s.t. } f &= G_K^R w_e + E_s\, y, \quad P(f) \succ 0, \quad f_n \succeq f_{\min} \end{aligned} \tag{10}$$

where $f \in \mathbb{R}^c$ are the contact forces and $\Psi(y) = \|f(y)\|_2$.

In this work, we study such a force optimization problem varying the number of enrolled postural synergies and for different hand-object relative poses.

III. Materials and Methods

Given the hand kinematics and the synergy dataset, the analysis described in the previous section does not depend on the particular shape of the object. In fact, the optimization procedure depends only on the joint angles, the position of the contact points, i.e., the 3D points in which the fingers are in contact with the object, and the direction of the contact normals. In this work, we assume that the directions of the contact normals are perpendicular to the object local tangent plane. We obtain these grasp-related quantities through the simulator described in [32], which allows simulating the whole kinematics and dynamics of a generic fully-actuated compliant robotic hand. In this work, we used the kinematic model of the SH [24], which has 19 joints, four for each long finger, and three for the thumb. The synergy set used is the one reported in [10]. The kinematic model of the SH has a good mapping with the human hand model considered in [10] allowing a direct mapping of the postural grasp synergies.

We simulated the grasp of four objects: two cubes (obj #1: $30 \times 30 \times 30$ mm and obj #3: $60 \times 60 \times 60$ mm) and two rectangular bars (obj #2: $30 \times 60 \times 60$ mm and obj #4: $100 \times 60 \times 100$ mm). The shapes and the dimensions of the objects are suitably chosen to include in the analysis several grasp configurations, in accordance with [23]. Indeed, the objects #1 and #2 suggest a precision grip while the objects #3 and #4 a power configuration. The size of the objects allows to generate both circular shaped grasps and prismatic configurations. Furthermore, objects #1 and #2 in conjunction with hand/object relative pose variations allow to generate other grasp approaches, such as lateral pinch. Fig. 2 shows some meaningful grasp strategies simulated. The weight is equal to 1 Kg for all the objects. Finally, we assume a modeling of the contacts as Point Contact With Friction (PCWF) with a friction coefficient $\mu = 1.5$, minimal normal force $f_{\min} = 0.1$ N, structural stiffness $k_{stru} = 1$ N/mm and admissible joint stiffness $k_{ss} = 200$ Nmm/rad, in accordance to [18]. The simulation is intended as a free exploration of the space: the hand shape follows the closure path imposed by the first synergy and adapts around the object according to the soft synergies paradigm. The hand completes its movement when all the fingers are in contact with the object or fully closed. The information about joint angles and the position of the contact points are then stored and passed to the optimization tool for the evaluation of the optimal force distribution. A schematics of the whole procedure is reported in Fig. 3.

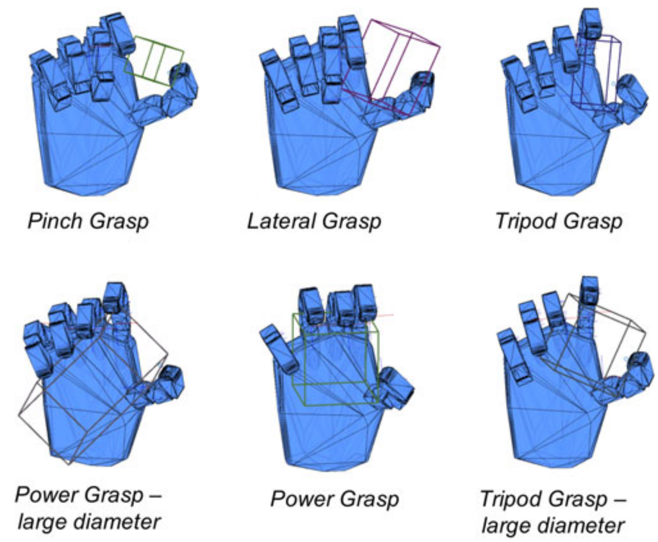

Fig. 2. The particular selection of objects dimensions, in conjunction with the large variation of hand/object relative configuration, allow to simulate a wide set of grasp typologies.

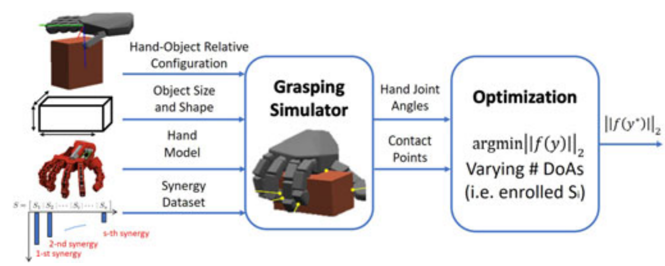

Fig. 3. Block scheme of the simulation and optimization procedure. Simulations take as input the synergy dataset, the hand kinematic model, geometrical properties of the object and the hand/object relative pose. The simulator outputs, i.e., joint angles and contact points, are used as argument of the optimization problem which - given a set of synergies - calculate the optimal distribution of contact forces.

Finding a valuable strategy for grasp success and quality in terms of contact force by changing the hand/object relative configuration represents a key topic for planning and control of robotic hands. This is particularly true considering soft adaptable robotic hands where the importance of hand/object relative pose is crucial to fully take advantage from end-effector adaptability in shaping around different items [33]. This motivates our investigation for different hand poses.

However, the problem of evaluating the best hand configuration is not trivial at all and computationally expensive. The principal parameters that make this issue time-consuming are: (i) the range of the evaluated relative poses, and (ii) the number of DoA enrolled for the analysis. While for (i) the best choice could be a trade-off between a reduced number of samples and an adequate span of the whole workspace, for the number of synergies considered the solution is not obvious. Indeed, the quality of the grasp generally varies w.r.t. the number of enrolled synergies. Relying on that, simulations were performed varying hand/object relative pose, in particular we considered rotations around the axis normal to the palm (abduction-adduction) and rotations around the axis along the long fingers (pronation-supination). The first DoF is spanned in the range $A = [-180, 180]$deg with resolution of 5 deg, while the second DoF is spanned in the range $B = [-60, 60]$deg with resolution of 5 deg. The total number of simulated hand/object poses is 1800 for each object.

In order to clarify the whole optimization procedure, we summarize it in Algorithm 1. First, an initialization procedure is required to load the object geometries, a hand kinematic model, a synergy dataset $\bar{S} \in \mathbb{R}^{n \times n}$ mapped on that model and the hand/object relative configurations, i.e., vectors $A \in \mathbb{R}^{\,\mathrm{a}}$ (abduction-adduction) and $B \in \mathbb{R}^{\,\mathrm{b}}$ (pronation-supination). Then, if the focus is the incrementality analysis we run Algorithm 2. This is composed of two iterative loops. In the external one we iterate on the hand/object relative configurations, simulating each pose $(\alpha, \beta)_i$ to obtain the contact points and hand joint angle values. In the internal loop we incrementally consider the columns of the synergy dataset $\tilde{S} \in \mathbb{R}^{n \times n}$. Indeed, at each step we compute the synergy enrollment set $S \in \mathbb{R}^{n \times s}$ as $S = \tilde{S}(:, 1 : s)$ (4). We then solve the optimization problem (10) for the considered hand pose $(\alpha, \beta)_i$ and the considered synergy enrollment set S. Otherwise, if we focus on the hierarchy analysis, we run Algorithm 3. This cycles over several permutations of the columns of the synergy dataset $\bar{S}$, running Algorithm 2 at each step. It is worth noting that the whole procedure is completely independent on the particular hand model, synergy dataset and object geometry.

Algorithm 1: Grasp Analysis.

procedure INITIALIZATION
  Load Object Geometries
  Load Hand Kinematic Model
  Load Synergy Dataset Matrix $\bar{S} \in \mathbb{R}^{n \times n}$
  Define $A \in \mathbb{R}^{\,\mathrm{a}}$ and $B \in \mathbb{R}^{\,\mathrm{b}}$ ▷Grasp Poses Sets
  Choose *Analysis Type*
procedure GRASP ANALYSIS
  switch *Analysis Type* **do**
    case *Incrementality*
      Compute $\tilde{S} \in \mathbb{R}^{n \times n} = \bar{S}$
      Run *Incrementality Procedure* ▷ Algorithm 2
    case *Hierarchy*
      $Z = perms([1 : n])$
      Run *Hierarchy Procedure* ▷ Algorithm 3

Algorithm 2: Incrementality Procedure.

procedure INCREMENTALITY PROCEDURE
  $i = 0$
  do
    Set $(\alpha, \beta)_i \in \{A, B\}$ ▷Sample of Hand Pose
    Simulate the Grasp, i.e., Compute Contacts
    $s = 0$
    do
      $s = s + 1$
      $S = \tilde{S}(:, 1 : s) \in \mathbb{R}^{n \times s}$ ▷(4)
      Solve Optimization Problem ▷(10)
    while $s \neq n$
    $i = i + 1$
  while $i \neq a \cdot b$

Algorithm 3: Hierarchy Procedure.

procedure HIERARCHY PROCEDURE
  $j = 0$
  do
    $j = j + 1$
    Compute $\tilde{S} \in \mathbb{R}^{n \times n} = \bar{S}(:, Z(j, :))$
    Run *Incrementality Procedure* ▷ Algorithm 2
  while $j \neq size(Z, 1)$

IV. RESULTS AND DISCUSSION

In this section, we evaluate the grasp performances in terms of contact forces. In particular, we analyze the norm of contact forces w.r.t. the hand/object relative poses incrementally enrolling the grasp synergies and focusing on the inspection of local minima (*Analysis Type* = *Incrementality* in Algorithm 1). Secondly, we question the role that each synergy plays in success rate and grasping performances identifying the synergies that give the more dominant contribution (*Analysis Type* = *Hierarchy* in Algorithm 1).

A. Synergy Incrementality and Optimal Hand Configuration

In this first analysis, we evaluate the optimal contact forces w.r.t. the hand/object relative configurations in the case of the enrollment of one or more synergistic DoAs. In particular, we sequentially enroll the synergies from S_1 to S_{15} and evaluate the variations of the optimal grasping forces distribution. Fig. 4 shows the results for the case of object #4 (for the sake of space only cases $\{S_1\}$, $\{S_1, S_2\}$, $\{S_1, S_2, S_3\}$ and $\{S_1, S_2, S_3, \ldots S_{15}\}$ are reported). On the left, we report the force values w.r.t. the relative hand/object pose as colormap. As expected, reading the figure from top to bottom (i.e., increasing the number of enrolled synergies), the value of the contact forces decreases and the grasping success rate increases. Furthermore, results reveal that the local minima are preserved increasing the number of enrolled synergies. To clarify the latter point, we reported on the right - as an example - an exploded view of a local minimum neighborhood.

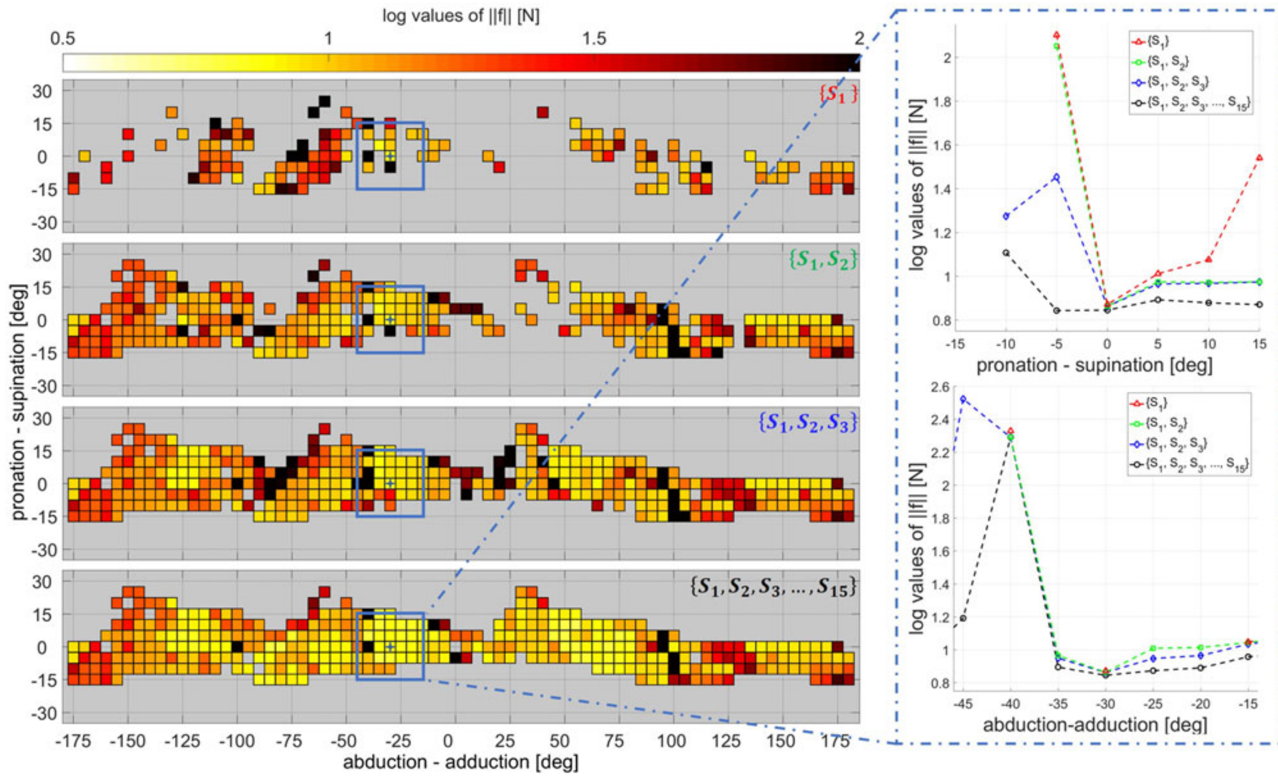

Fig. 4. Norm of the contact forces w.r.t. the hand/object relative abduction and pronation for the case of object #4. On the left, a colormap is used to report the log of contact force values for each 2D hand/object pose. From top to bottom, the analysis reports on an incremental enrollment of synergies. Empty cells refer to poses, which do not provide force closure with the specific synergy set. On the right, we report the exploded view of a local minimum neighborhood. Top figure shows the contact force values w.r.t. variations of hand pronation for a fixed exemplary value of abduction equal to -30 deg (corresponding to a local minimum). Bottom figure shows the contact force values w.r.t. variations of hand abduction for a fixed value of pronation equal to 0 deg. Conclusions that can be drawn are: i) increasing the number of enrolled synergies decreases the contact force values and increases the grasping success rate; ii) the exploded view shows that the local minimum is preserved in a neighborhood for different sets of enrolled synergies.

This could suggest that the poses which represent local minima for the case of a reduced set of enrolled synergies seem to maintain their role increasing the number of DoA, hence remarking a local minimum also for the full actuated hand case. In order to generalize this analysis and to evaluate if this behavior is maintained for all the local minima and for all the objects, we evaluated - for all the objects - the local minima obtained with the set $\{S_1\}$, then we incremented the number of enrolled synergies, i.e., the set $\{S_1, S_2\}$, and evaluated if a local minimum of this second case exists in a neighborhood of $[-10, +10]$deg both for abduction and pronation axis. This procedure can be iterated sequentially increasing the cardinality of the synergy set and evaluating the permanences between $\{S_1, \ldots S_k\}$ and $\{S_1, \ldots, S_k, S_{k+1}\}$, until the case $\{S_1, \ldots, S_{15}\}$. Considering that the data analyzed present missing values - which correspond to no-force closure conditions - we decided to evaluate the local minima using the following policy. Given a synergy set and the correspondent contact force values w.r.t. the relative hand/object pose, we consider as local minimum candidates all the values lower than the median value. Then, for each local minimum candidate, we consider different cases: i) if none of the poses in the considered neighborhood produces force closure, then it is assumed as a local minimum; ii) if in the considered neighborhood there are other relative configurations for which there is force closure but the value of the candidate is lower than the other, then it is assumed as a local minimum; iii) if none of the previous conditions is satisfied, then the candidate is discarded. Numerical results show that the minimum preservation condition is verified in the 96$\%$ of the cases, with a minimum of 92$\%$ for the object #1 and a maximum of 99$\%$ for object #4.

B. *The Role of Synergy Hierarchy*

The conclusions drawn in the previous section assume an incremental enrollment of synergies following the order suggested by their importance for the control of hand shape described in [10]. Notwithstanding, this particular selection results in an arbitrary choice, considering that the control of the hand posture is regulated independently from the control of the contact forces. To evaluate, hence, if there is a one-to-one mapping between the importance of synergies for hand shape and for the control of contact forces, we repeated the optimizations while randomizing the order and the number of the enrolled synergies.

1) A Reduced Set of Synergies: For each hand/object relative configuration the number of combinations to be tested is equal to the sum of the the binomial coefficients $\sum_{k=1}^{N} \binom{N}{k}$. For high values of N, analysis workload would be too high, then to reduce computational complexity, we evaluated the minimum value of N that enables to approximate the grasp capacity of the full synergy set case. For the objects considered in this work, we

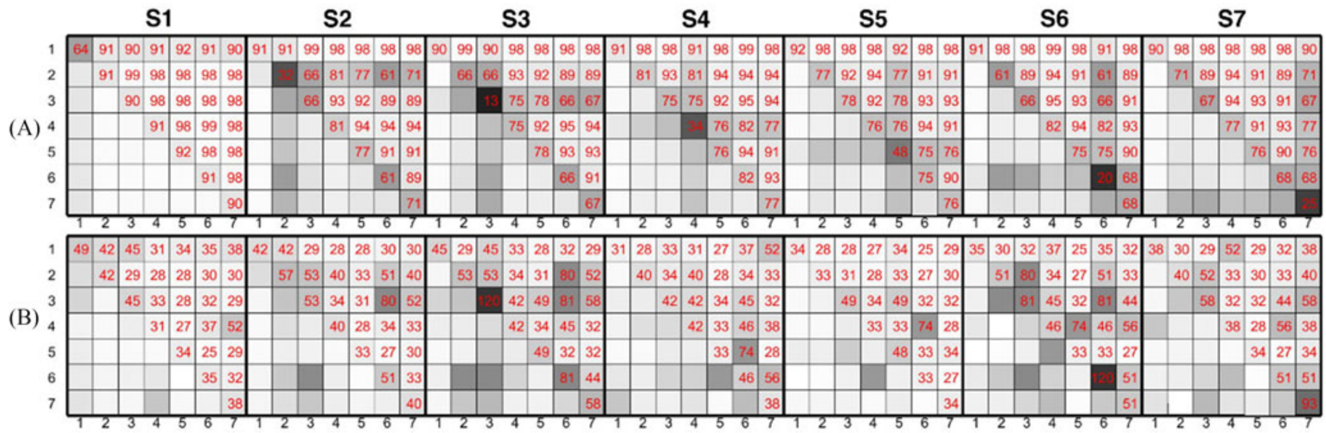

Fig. 5. A: Grasping percentage, averaged for the four objects, while enrolling a 3−permutation of 7 synergies w.r.t. the hand/object relative configurations. Each 7×7 matrix labeled as S_k represents the set of 3−permutations starting with the synergy S_k. The element (i, j) of the S_k matrix contains the round percentage of the successful grasps for the synergies triplet $\{S_k, S_i, S_j\}$. These values are also depicted with a gray scale representation, where black represents 0% and white 100%. Note that, in the 7×7 matrix associated to S_k, the element (k, k) represents the percentage of successful grasps for the case of k as single DoA. The elements on the $k-$row, on the $k-$column and on the diagonal represent the percentage of successful grasps considering two synergies, i.e., S_k and one of the other six respectively. Finally, all the other elements contain the percentage of successful grasps enrolling three synergies (including S_k). B: Optimal contact forces averaged w.r.t. all the hand/object relative configurations and the object considered. The data presentation follows the same paradigm of Fig. 5-A. Note that the matrices are symmetrical.

TABLE II
NORMALIZED PERCENTAGE OF SUCCESSFUL GRASPS PER-OBJECT WITH THE FIRST k SYNERGIES

	1	2	3	4	5	6	7	8
#1	87.16	99.80	100	100	100	100	100	100
#2	62.88	93.51	99.82	100	100	100	100	100
#3	55.67	87.42	97.52	99.38	99.79	99.79	100	100
#4	50.29	83.05	97.41	100	100	100	100	100

obtained that with the first seven PCs the hand is able to produce force closure in all the configurations in which also the full set of synergies does (see Table II). Leveraging on this result, from now on $N = 7$ will be the upper extremity of the dimension of the synergy set considered.

2) The Importance of Synergies in Optimal Contact Forces Distribution: To analyze the actual role that each grasp synergy plays in the minimization of contact forces we evaluated their optimal distribution while randomizing the number and the order of synergies that are incrementally enrolled. To face this problem, leveraging on the result of the previous section, we take into consideration the first 7 synergies, which are selected in randomized sub-sets of one, two and three elements, for a total number of combination equal to 63 and, hence, a maximum number of optimization per-object equal to 113400. For each sub-set and for each object, we calculate the number of successful grasps performed and the optimal distribution of contact forces w.r.t. the hand/object relative configuration. We report the results of this analysis - in a compact form - in Fig. 5. In particular, in Fig. 5-A we show the percentage of successful grasps (normalized w.r.t. the case of the full synergy set), averaged w.r.t. the four object considered. These values are depicted as a confusion matrix in which the white color is associated to 100% and the black to 0%. Each cell of the matrix contains the percentage of successful grasps of a specific triplet. The first index (k) of the triplet is identified by the index reported on the top of the matrix ($S_1, S_2, \ldots, S_k, \ldots, S_7$), which is in common for all the cells under the label S_k . Fixed the matrix associated to the first index of the triplet, the element (i, j) of the matrix contains the round percentage of the successful grasps for the synergies triplet $\{S_k, S_i, S_j\}$. Note that, in the 7×7 matrix associated to S_k, the element (k, k) represents the percentage of successful grasps for the case of S_k as single DoA. The elements on the $k-$row, on the $k-$column and on the diagonal represent the percentage of successful grasps considering two synergies, S_k and one of the other six respectively. Finally, all the other elements contain the percentage of successful grasps enrolling three synergies (including S_k). In Fig. 5-B, the same representation of Fig. 5-A is used to report the optimal contact forces distribution averaged w.r.t. all the possible hand/object relative configurations and all the object considered. Note that the matrices are symmetrical.

These results show that, in the case of single DoA, S_1 is the PC which produces force closure with the highest probability, i.e., in the 65% of the cases. Besides S_1, the PCs with the highest percentages of successful grasps are S_4 (34%) and S_5 (48%). What is also noticeable is that, for the case of two DoAs, the highest percentage of successful grasps is achieved enrolling S_1 and S_5. These values show that S_1, S_4 and S_5 are the components more related to the increasing of the percentage of grasp successful. This observation - as expected - is still true when these synergies are enrolled as second DoA of a pair or triplet. For example, for the triplets starting with S_3, it can be shown that, even if the sub-set containing only S_3 produce force closure only in the 13% of the cases, the pairs $\{S_3, S_1\}$, $\{S_3, S_4\}$ and $\{S_3, S_5\}$ produce force closure in the 90%, 75% and 78% of the cases.

Besides the fundamental role that S_1 plays for the control of contact forces distribution, which was an expected result, we demonstrated that also S_4 and S_5 have a relevant role in increasing the probability to perform force closure. These roles are confirmed by the analysis of the actual optimal contact forces that the hand exert when actuated following the randomized triplets. In fact - with reference to Fig. 5-B - it can be shown that in the case of single DoA the synergies that exert a mean lower value of contact force are, in order, S_4, S_5 and S_1. This confirms the role of S_4 and S_5 in the control of contact forces also under the point of view of forces minimization. Moreover, what can be also shown is that S_4 and S_5 are the components which minimize the averaged value of the contact forces of the couple of synergies in conjunction with S_1.

V. Conclusion

In this work, we have investigated whether an incrementality or a hierarchy for synergy enrollment exist for optimal force distribution at different hand/object relative poses. To achieve this goal we performed simulations where the model of a synergy inspired compliant hand was used to grasp different objects, and we evaluated successful grasp rate and force distribution. This analysis has been performed while testing several hand/object relative poses and synergistic pattern combinations. Main findings of this work are: (i) the minima of contact forces are invariant w.r.t. an incremental enrollment of synergies within the variation of hand/object relative configurations, and, (ii) a hierarchy exists on eigenposture selection. Indeed, the first synergy seems to play the most important role to achieve successful grasps and optimized force distribution and, together with the fourth and fifth, are those mainly devoted to producing compression movements, which are coherent with force closure requirements. This suggests that hierarchy of pre-grasp related synergies can not be directly mapped to contact force distribution. This could be a springboard towards the implementation of novel analysis techniques which can take into consideration different grasp-related quantities.

Note that an experimental validation of these results could be performed employing a highly actuated robotic hand controlled using postural synergies. The control strategy should also implement the hand compliance. Moreover, tactile sensors may be used to estimate the contact forces.

We believe that, taking inspiration from the Uncontrolled Manifold Hypothesis [34], these results could pave the path towards a new generation of soft hands, embedding synergy inspiration, compliance and a minimalistic usage of resources, for more effective robotic grasp strategies. Future works will be devoted to i) extending this analysis to more complex manipulation tasks, and ii) involving a larger set of objects, e.g., taking inspiration from datasets available in literature.

References

[1] N. Bernstein, The Control and Regulation of Movements. New York, NY, USA: Pergamon Press, 1967.

[2] A. Berthoz, *Simplexity: Simplifying Principles for a Complex World (G. Weiss, Trans.)*. New Haven, CT, USA: Yale Univ. Press, 2012.

[3] F. A. Mussa-Ivaldi, "Modular features of motor control and learning," *Current Opinion Neurobiol.*, vol. 9, no. 6, pp. 713–717, 1999.

[4] P. Saltiel, K. Wyler-Duda, A. D'Avella, M. C. Tresch, and E. Bizzi, "Muscle synergies encoded within the spinal cord: Evidence from focal intraspinal NMDA iontophoresis in the frog," *J. Neurophysiol.*, vol. 85, no. 2, pp. 605–619, 2001.

[5] M. L. Latash, *Synergy*. London, U.K.: Oxford Univ. Press, 2008.

[6] P. Stratmann, D. Lakatos, and A. Albu-Schäffer, "Neuromodulation and synaptic plasticity for the control of fast periodic movement: Energy efficiency in coupled compliant joints via PCA," *Frontiers Neurorobotics*, vol. 10, 2016.

[7] M. C. Tresch, P. Saltiel, and E. Bizzi, "The construction of movement by the spinal cord," *Nature Neurosci.*, vol. 2, no. 2, pp. 162–167, 1999.

[8] A. D'avella and F. Lacquaniti, "Control of reaching movements by muscle synergy combinations," *Frontiers Comput. Neurosci.*, vol. 7, 2013, Art. no. 42.

[9] M. C. Tresch and A. Jarc, "The case for and against muscle synergies," *Current Opinion Neurobiol.*, vol. 19, no. 6, pp. 601–607, 2009.

[10] M. Santello, M. Flanders, and J. F. Soechting, "Postural hand synergies for tool use," *J. Neurosci.*, vol. 18, no. 23, pp. 10105–10115, 1998.

[11] C. R. Mason, J. E. Gomez, and T. J. Ebner, "Hand synergies during reach-to-grasp," *J. Neurophysiol.*, vol. 86, no. 6, pp. 2896–2910, 2001.

[12] I. V. Grinyagin, E. V. Biryukova, and M. A. Maier, "Kinematic and dynamic synergies of human precision-grip movements," *J. Neurophysiol.*, vol. 94, no. 4, pp. 2284–2294, 2005.

[13] M. Santello *et al.*, "Hand synergies: Integration of robotics and neuroscience for understanding the control of biological and artificial hands," *Phys. Life Rev.*, vol. 17, pp. 1–23, 2016.

[14] M. T. Turvey, "Action and perception at the level of synergies," *Hum. Movement Sci.*, vol. 26, no. 4, pp. 657–697, 2007.

[15] C. Della Santina *et al.*, "Postural hand synergies during environmental constraint exploitation," *Frontiers Neurorobotics*, vol. 11, 2017, Art. no. 41.

[16] A. Bicchi, M. Gabiccini, and M. Santello, "Modelling natural and artificial hands with synergies," *Phil. Trans. Roy. Soc. B*, vol. 366, no. 1581, pp. 3153–3161, 2011.

[17] A. G. Feldman and M. F. Levin, "The equilibrium-point hypothesis–past, present and future," in *Progress in Motor Control*. New York, NY, USA: Springer, 2009, pp. 699–726.

[18] M. Gabiccini, A. Bicchi, D. Prattichizzo, and M. Malvezzi, "On the role of hand synergies in the optimal choice of grasping forces," *Autonom. Robots*, vol. 31, no. 2–3, pp. 235–252, 2011.

[19] L. Villani, F. Ficuciello, V. Lippiello, G. Palli, F. Ruggiero, and B. Siciliano, "Grasping and control of multi-fingered hands," in *Advanced Bimanual Manipulation*. New York, NY, USA: Springer, 2012, pp. 219–266.

[20] J. Cornellà, R. Suárez, R. Carloni, and C. Melchiorri, "Dual programming based approach for optimal grasping force distribution," *Mechatronics*, vol. 18, no. 7, pp. 348–356, 2008.

[21] B. F. Stanton, J. W. S. Geme III, N. F. Schor, R. E. Behrman, and R. M. Kliegman, *Nelson Textbook of Pediatrics, 19/e*. Gurgaon, Inida: Elsevier, 2007.

[22] M. Asada *et al.*, "Cognitive developmental robotics: A survey," *IEEE Trans. Auton. Mental Develop.*, vol. 1, no. 1, pp. 12–34, May 2009.

[23] M. R. Cutkosky, "On grasp choice, grasp models, and the design of hands for manufacturing tasks," *IEEE Trans. Robot. Autom.*, vol. 5, no. 3, pp. 269–279, Jun. 1989.

[24] M. G. Catalano, G. Grioli, E. Farnioli, A. Serio, C. Piazza, and A. Bicchi, "Adaptive synergies for the design and control of the Pisa/IIT softhand," *Int. J. Robot. Res.*, vol. 33, no. 5, pp. 768–782, 2014.

[25] R. Vinjamuri, M. Sun, C.-C. Chang, H.-N. Lee, R. J. Sclabassi, and Z.-H. Mao, "Temporal postural synergies of the hand in rapid grasping tasks," *IEEE Trans. Inf. Technol. Biomed.*, vol. 14, no. 4, pp. 986–994, Jul. 2010.

[26] P. H. Thakur, A. J. Bastian, and S. S. Hsiao, "Multidigit movement synergies of the human hand in an unconstrained haptic exploration task," *J. Neurosci.*, vol. 28, no. 6, pp. 1271–1281, 2008.

[27] E. Todorov and Z. Ghahramani, "Analysis of the synergies underlying complex hand manipulation," in *Proc. 26th Annu. Int. Conf. IEEE Eng. Med. Biol. Soc.*, vol. 2, 2004, pp. 4637–4640.

[28] V. M. Zatsiorsky and M. L. Latash, "Prehension synergies," *Exercise Sport Sci. Rev.*, vol. 32, no. 2, 2004, Art. no. 75.

[29] D. Prattichizzo and J. C. Trinkle, "Grasping," in *Springer Handbook of Robotics*. New York, NY, USA: Springer, 2016, pp. 955–988.

[30] A. Bicchi, "On the closure properties of robotic grasping," *Int. J. Robot. Res.*, vol. 14, no. 4, pp. 319–334, 1995.

[31] M. Buss, H. Hashimoto, and J. B. Moore, "Dextrous hand grasping force optimization," *IEEE Trans. Robot. Autom.*, vol. 12, no. 3, pp. 406–418, Jun. 1996.

[32] A. Rocchi and K. Hauser, "A generic simulator for underactuated compliant hands," in *Proc. IEEE Int. Conf. Simulation, Modeling, Program. Auton. Robots*, 2016, pp. 37–42.

[33] M. Bonilla *et al.*, "Grasping with soft hands," in *Proc. 2014 14th IEEE-RAS Int. Conf. Humanoid Robots (Humanoids)*, 2014, pp. 581–587.

[34] J. P. Scholz and G. Schöner, "The uncontrolled manifold concept: Identifying control variables for a functional task," *Exp. Brain Res.*, vol. 126, no. 3, pp. 289–306, 1999.