On economic growth and minimum wages

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Abstract We offer an analysis of the existence of a positive relationship between minimum wages and economic growth in a simple one-sector overlapping generations economy à la Romer (1986), in the case of both homogeneous and heterogeneous labour and without considering any growth-sustaining externalities which the minimum wage can generate. Assuming also the existence of unemployment benefits financed with balanced-budget consumption taxes not conditional upon age, we show that the minimum wage can promote economic growth and welfare despite the occurrence of unemployment. There may also exist a growth- and welfare-maximising minimum wage.

Keywords Endogenous growth • Minimum wage • Unemployment • OLG model

JEL Classification H24 • J60 • O41

1. Introduction

An important and widely debated argument in the economic literature deals with the effects of minimum wages in aggregate macroeconomic models in both static and dynamic contexts. Opponents view the minimum wage as a misguided social policy, essentially because it reduces

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employment and output.[1] Proponents, instead, typically focus on redistributive goals to raise the incomes of the low-paid. Moreover, minimum wage legislation can also have important interactions with the social welfare system (e.g., the unemployment benefit system, see Shimer and Werning, 2007), especially in Europe where labour market rigidities represent major aspects of real phenomena (see, e.g., Blanchard, 1998).

Even if minimum wages and unemployment compensations cannot in all likelihood be expected to greatly reduce household poverty (see, e.g., OECD, 1998; Müller and Steiner, 2008), their effectiveness in reducing income inequality among households is greater, that is minimum wages have been chiefly used for equity reasons, thus trading off with efficiency goals (see Tamai, 2009 for a theoretical analysis of the effects of the minimum wage on income inequality.[2] As regards empirical evidence, see DiNardo et al., 1996; Fortin and Lemieux, 1997[3]).

An insight into the effects of the minimum wage in aggregate macroeconomic models was first gained through the seminal paper by Stigler (1946). The basic (one-sector, static, partial equilibrium) model of the minimum wage effects on employment and unemployment focuses on a single competitive labour market with homogeneous workers (all covered by the legislated wage),

[1] The debate on the effects of the minimum wage on employment has seen renewed interest starting from the works by Neumark and Wascher (1995) and Card and Krueger (1994, 1995, 1998). See also the recent paper by Falk et al. (2006) and the survey by Neumark and Wascher (2006).

[2] In particular, Tamai (2009) uses a continuous-time endogenous growth model à la Romer (1986) with heterogeneous households (divided by ability) and political determination of the minimum wage to analyse how inequality, unemployment and growth are related. He finds a positive correlation between inequality, unemployment and growth.

[3] The former authors find that the fall in the minimum wage and the de-unionisation process in the U.S. contributed to explain the rise in wage inequality and conclude that institutions "are as important as supply and demand considerations [see Katz and Murphy, 1992] in explaining changes in the U.S. distribution of wages from 1979 to 1988." (DiNardo et al., 1996, p. 1001). The latter, instead, analyse the effects of institutional changes in the U.S. in the 1980s, and conclude that the minimum wage is especially important for women but "[w]hen men and women are considered together, institutions have an even larger impact on inequality" (Fortin and Lemieux, 1997, p. 75–76).

and concludes for both employment and output reductions. In dynamic contexts, however, the impact of legislated wage minima on employment, economic growth and welfare is controversial. In particular, it has been shown that if the minimum wage generates some positive externalities it can be growth-improving under certain conditions.

Generally speaking, economic growth models with minimum wages may be divided into at least three categories: (*i*) two-sector closed-economy models with overlapping generations (OLG) and growth-promoting externalities of the minimum wage; (*ii*) minimum wage effects in an open economy either in a two-period OLG context or in an infinite horizon continuous-time growth model; (*iii*) Schumpeterian growth models with labour market imperfections.

Models of class (i) (growth-promoting externalities): Cahuc and Michel (1996) and Ravn and Sørensen (1999). The former build an endogenous growth model à la Lucas (1988) with skilled and unskilled workers, and introduce the minimum wage in the market for raw labour which causes a positive external effect on the accumulation of human capital, given increasing demand for skilled labour and the consequent desire of unskilled workers to improve their skills to avoid unemployment. Therefore, through this channel the minimum wage may promote economic growth and welfare. However, Cahuc and Michel (1996), in line with the traditional negative view of labour market rigidities, also remark that in an exogenous (neoclassical) growth context the minimum wage increases unemployment and reduces growth (see Cahuc and Michel, 1996, Proposition 1, p. 1469). Ravn and Sørensen (1999) assume that a minimum wage for unskilled workers affects labour productivity growth through two sources of accumulation of skills: schooling before entering the labour market and training on the job. They show that the final effect of a rise in the minimum wage on growth is potentially uncertain because it can induce skill formation through schooling while also reducing training.

Models of class (ii) (minimum wage in open economies with and without growth-promoting externalities): Askenazy (2003) and Irmen and Wigger (2006). The former considers an open-economy continuous-time endogenous growth model à la Ramsey (1928) and finds that the minimum wage may cause a shift of effort from the production sector to the R&D sector and thus may stimulate growth through this channel. The latter consider a two-country OLG growth model à la Romer (1986) with capital mobility and no growth-promoting externalities induced by the minimum wage. They find that, depending on the elasticity of substitution between capital and efficient labour, the output elasticity of efficient labour and the differences between the propensity to save in both the domestic and foreign countries, introducing a minimum wage in the domestic country may stimulate global economic growth. Their model, therefore, implies that with the Cobb-Douglas production function and/or uniform propensity to save, the minimum wage would be harmful to the global economic growth, thus confirming that in the absence of a positive external effect the conventional belief about the growth-depressing role of minimum wages holds in a double Cobb-Douglas context.

Models of class (iii) (Schumpeterian growth models with labour market imperfections): Aghion and Howitt (1994) and Meckl (2004). Due to the well-known Schumpeterian idea of creative-disruption, the former authors find, in a model where the *intrasectoral* allocation of labour within the intermediate goods sector causes unemployment, that economic growth and unemployment can be positively related. By contrast, the latter finds that, due to minimum wages, growth and unemployment are ambiguously related depending on the sign of the wage differentials between sectors (the final-good sector; the intermediate-good sector; the R&D sector), i.e. the *intersectoral* allocation of labour matters. The higher the wage in the R&D sector, the likelier economic growth and unemployment will be positively correlated.

Unlike previous studies, in this paper we use the basic one-sector OLG growth model à la Romer (1986) with Cobb-Douglas utility and production functions to assess the role the minimum wage can play on economic growth and welfare when the government also finances unemployment benefits at a balanced budget and without assuming growth-promoting externalities which the minimum wage can generate. We provide necessary and sufficient conditions for a regulated-wage economy with unemployment to grow faster than the *laissez-faire*, in the case of both homogeneous and heterogeneous labour. While in the former case the minimum wage covers all workers, in the latter the minimum wage is introduced only in the market for raw labour. Interestingly, a growth-maximising minimum wage can exist in both cases. Moreover, we show that in the long run, individuals may be better off in an economy with regulated wages and the highest possible welfare level is achieved when growth is maximised.

The remainder of the paper is organised as follows. Section 2 develops the baseline model with homogeneous labour and consumption taxation not conditional on age. Section 3 and 4 analyse the growth and welfare effects, respectively, of the minimum wage. Section 5 presents some extensions and analyses, in particular, the case of heterogeneous labour. Section 6 concludes.

2. The model

2.1. Individuals

Consider an OLG closed economy populated by a continuum of N identical two-period lived individuals (Diamond, 1965). When young, each individual is endowed with one unit of labour which is inelastically supplied to firms. When old, he/she is retired.

The lifetime (logarithmic) utility function of agent j born at t ($U^j{}_t$) is defined over young-aged and old-aged consumptions, $c^j{}_{1,t}$ and $c^j{}_{2,t+1}$, respectively, that is:

$$U^j{}_t = \ln\left(c^j{}_{1,t}\right) + \beta \ln\left(c^j{}_{2,t+1}\right), \tag{1}$$

where $0 < \beta < 1$ represents the degree of individual (im)patience to consume over the life cycle.

Individuals at t can either be employed ($j = e$) or unemployed ($j = u$). If employed, they earn a minimum wage, $w_{m,t}$, per unit of labour fixed by law as a constant mark-up $\mu > 1$ over the prevailing competitive wage, $w_{c,t}$, see, e.g., Irmen and Wigger (2006), that is:

$$w_{m,t} := \mu w_{c,t}\,. \tag{2.1}$$

If unemployed, they receive an unemployment benefit b_t defined as a fraction of the competitive wage,[4] that is:

$$b_t := \gamma w_{c,t}, \tag{2.2}$$

where $0 < \gamma < 1$ is the replacement rate. We assume that unemployment benefits are financed with *ad valorem* consumption taxes ($\tau_t > 0$). The budget constraint of the young at t therefore reads as

$$c^j{}_{1,t}(1+\tau_t) + s^j{}_t = x^j{}_t, \tag{3.1}$$

where $s^j{}_t$ is the saving rate of agent j and $x^j{}_t = \{w_{m,t}, b_t\}$ is an income-when-young variable equal to: (*i*) the minimum wage if the individual is employed; (*ii*) the unemployment benefit if she is unemployed. The budget constraint of the old instead is

$$c^j{}_{2,t+1}(1+\tau_{t+1}) = (1+r_{t+1})s^j{}_t, \tag{3.2}$$

where r_{t+1} is the interest rate from t to $t+1$.

Employed and unemployed individuals choose how much to save out of their disposable income to maximise Eq. (1) subject to Eqs. (3). The first order conditions for an interior solution are:

$$\frac{c^j{}_{2,t+1}}{c^j{}_{1,t}} = \beta(1+r_{t+1})\frac{1+\tau_t}{1+\tau_{t+1}}. \tag{4}$$

Combining Eq. (4) with Eqs. (3), when individual j is alternatively employed or unemployed, gives young-aged consumption, old-aged consumption and the saving rate of both the employed and unemployed of generation t, which are respectively given by:

$$c^j{}_{1,t} = \frac{x^j{}_t}{(1+\beta)(1+\tau_t)}, \tag{5.1}$$

$$c^j{}_{2,t+1} = \frac{\beta(1+r_{t+1})x^j{}_t}{(1+\beta)(1+\tau_{t+1})}, \tag{5.2}$$

[4] Note that the results of the present paper would be qualitatively the same if unemployment benefits were proportional to the actual minimum wage, rather than the competitive wage.

$$s^j{}_t = \frac{\beta\, x^j{}_t}{1+\beta}. \tag{5.3}$$

Defining L_t as the number of people employed at t, aggregate saving in the economy ($S_t = Ns_t$) is defined as the sum of savings of both employed and unemployed, that is $Ns_t = L_t s^e{}_t + (N - L_t)s^u{}_t$, that can alternatively be written as

$$s_t = (1-u_t)s^e{}_t + u_t s^u{}_t, \tag{6.1}$$

where $u_t := (N - L_t)/N$ is the aggregate unemployment rate at t.[5] Therefore, exploiting Eqs. (5.3) and (6.1), the economy-wide saving rate is expressed as:

$$s_t = \frac{\beta}{1+\beta}\left[w_{m,t}(1-u_t) + b_t u_t\right]. \tag{6.2}$$

Using the same line of reasoning, summing up young-aged and old-aged consumptions of both the employed and unemployed from Eqs. (5.1) and (5.2) gives

$$c_{1,t} = \frac{1}{(1+\beta)(1+\tau_t)}\left[w_{m,t}(1-u_t) + b_t u_t\right], \tag{7.1}$$

$$c_{2,t+1} = \frac{\beta}{(1+\beta)(1+\tau_{t+1})}(1+r_{t+1})\left[w_{m,t}(1-u_t) + b_t u_t\right]. \tag{7.2}$$

2.2. Firms

As in Romer (1986), Daveri and Tabellini (2000) and Irmen and Wigger (2002), we assume the technology of production faced by each firm $i = 1, \ldots, I$ as:

$$Y_{i,t} = K_{i,t}{}^{\alpha}(A_{i,t}L_{i,t})^{1-\alpha} = B k_t{}^{1-\alpha} K_{i,t}{}^{\alpha} L_{i,t}{}^{1-\alpha}, \tag{8}$$

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[5] Eq. (6.1) reveals that separating with employed and unemployed people is the same as assuming a representative individual that can be employed for the fraction $1-u_t$ of time and unemployed for the remaining fraction u_t.

where $Y_{i,t}$, $K_{i,t}$ and $L_{i,t}$ are, respectively, the output produced, and capital and labour hired by firm i, $A_{i,t} := a\frac{K_t}{N}$ is an index of labour productivity of each single firm, which is assumed to depend on the average per capita stock of capital in the economy, $k_t = K_t / N$, and it is taken as given by firm i, $B := a^{1-\alpha} > 0$ is a scale parameter and $0 < \alpha < 1$. Since all firms are identical, setting $L_{i,t} = L_t$, $K_{i,t} = K_t$ and $Y_{i,t} = Y_t$, then aggregate production at t is $Y_t = B k_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$, where $L_t = (1-u_t)N$ is the total labour force employed. Therefore, per capita output is $y_t = B k_t (1-u_t)^{1-\alpha}$, where $y_t = Y_t / N$. Assuming that capital totally depreciates at the end of each period and output is sold at unit price, profit maximisation implies:

$$r_t = \alpha B (1-u_t)^{1-\alpha} - 1, \tag{9.1}$$

$$w_{m,t} = (1-\alpha) B k_t (1-u_t)^{-\alpha}. \tag{9.2}$$

Combining Eqs. (2.1) and (9.2), and knowing that $w_{c,t} = (1-\alpha) B k_t$ is the equilibrium competitive wage, the (constant) unemployment rate is

$$u_t = u(\mu) = 1 - \mu^{\frac{-1}{\alpha}}. \tag{10.1}$$

From Eq. (10.1) it is easy to verify that a rise in μ (γ) monotonically increases (does not affect) unemployment. Moreover, the interest rate is lower than under *laissez-faire*, that is:

$$r(\mu) = \alpha B \mu^{\frac{\alpha-1}{\alpha}} - 1 < r(1), \tag{10.2}$$

where $r(\mu) > 0$ for any $\mu < (\alpha B)^{\frac{\alpha}{1-\alpha}} := \mu_{\Omega}$, which is assumed to be always fulfilled.

Notice that in an infinite horizon growth model à la Ramsey (1928), where economic growth is driven by the interest rate rather than the saving rate, the minimum wage (even if used together with unemployment benefits) reduces the growth rate of consumption if the technological externality is defined in per capita terms, because the interest rate is below the *laissez-faire* level due to the unemployment occurrence in such a case, as revealed by Eq. (10.2) (see, e.g., Hellwig and Irmen, 2001). However, if we alternatively assume, in line with Corneo and Marquardt (2000), that the production externality is defined in terms of capital per employed worker, then the minimum wage is growth-neutral in a Ramsey-type growth model because the interest rate does not depend on the unemployment rate in such a case. We also note in an OLG model with unemployment, assuming the production externality to be defined in either per capita or per worker terms may lead to different final outcomes as regards the effects of the minimum wage on economic growth and the unemployment dynamics (see Ono, 2007).

2.3. Government

The unemployment benefit expenditure at t $(b_t u_t)$ is entirely financed with (time-adjusted)[6] *ad valorem* taxes levied on the first and second period consumptions of *all* individuals. Therefore, the (per capita) government budget reads as:

$$b_t u_t = \tau_t \left(c_{1,t} + c_{2,t}\right). \tag{11}$$

Exploiting Eqs. (2.1), (2.2), (7.1), the one-period backward (7.2),[7] (9.1) and (10.1), the budget-balancing tax rate is constant and given by:

$$\tau_t = \tau(\mu) = \frac{\gamma(1-\alpha)(1+\beta)\left(\mu^{\frac{1}{\alpha}} - 1\right)}{\mu(1+\alpha\beta) - \gamma(1-\alpha)\beta\left(\mu^{\frac{1}{\alpha}} - 1\right)}, \tag{12}$$

[6] In Section 5.1 we study the opposite case of endogenous replacement rate and fixed consumption tax rate.

[7] Since both the minimum wage and unemployment benefit are introduced at the beginning of time t, the consumption of the old born at $t-1$ is $c_{2,t} = \frac{\beta(1+r_t)w_{c,t-1}}{(1+\beta)(1+\tau_t)}$, where $w_{c,t-1} = \frac{w_{c,t}}{1+g_c}$ and $g_c = \frac{\beta}{1+\beta}(1-\alpha)B - 1$ is the growth rate of per capita income under *laissez-faire*.

where $\tau(1)=0$ and $\tau'(\mu)>0$. Moreover, the unemployment benefit expenditure is viable ($\tau>0$) if and only if $1<\mu<\mu_{\mathrm{M}}$, where μ_{M} is an upper feasibility bound on the wage mark-up.

3. Balanced growth

Given Eq. (11), market clearing in goods and capital markets is expressed as

$$k_{t+1}=s_t\,, \tag{13}$$

which is combined with Eq. (6.2) to obtain:

$$k_{t+1}=\frac{\beta}{1+\beta}\left[w_{m,t}(1-u_t)+b_t\,u_t\right]. \tag{14}$$

We now analyse how the minimum wage affects capital accumulation and, hence, economic growth. To this purpose, let us first rewrite Eq. (14) as a generic function of the wage mark-up as

$$k_{t+1}=s\left[w_{m,t}(\mu),u(\mu)\right]. \tag{15}$$

The total derivative of Eq. (15) with respect to μ gives:

$$\frac{dk_{t+1}}{d\mu}=\underbrace{\overbrace{\frac{\partial s}{\partial w_{m,t}}}^{+}\cdot\overbrace{\frac{\partial w_{m,t}}{\partial\mu}}^{+}\cdot\overbrace{(1-u)}^{+}}_{+}+\underbrace{\overbrace{\frac{\partial s}{\partial u}}^{-}\cdot\overbrace{\frac{\partial u}{\partial\mu}}^{+}\cdot\overbrace{(w_{m,t}-b_t)}^{+}}_{-}, \tag{16}$$

Eq. (16) reveals that the final effect of a rise in the wage mark-up is ambiguous on growth as it increases both wage income and unemployment. The economic intuition is simple. Since in an OLG context economic growth is driven by savings, how the minimum wage affects the saving rate will be crucial. While a rise in the minimum wage causes a positive effect on savings because the income of the employed is now greater, the unemployment rate rises as well. The rise in unemployment affects savings through two channels of opposite sign: first, the amount of resources saved by the employed is now lower because the employment rate is reduced, so that the saving rate shrinks through this channel; second, the amount of resources saved by the unemployed is now larger, and this, in turn, positively affects savings. However, since the minimum wage is higher than unemployment benefit, increased unemployment always reduces savings. In sum, there exist a positive wage effect and a negative unemployment effect on growth when μ is raised, and this clearly explains why the final effect of a rise in the minimum wage on capital accumulation is potentially uncertain in this simple OLG model.

Exploiting Eqs. (2.1), (2.2), (10.1) and (14), the dynamic evolution of capital is described by:

$$k_{t+1} = (1+g_c)\mathrm{H}(\mu)k_t\,, \tag{17}$$

where $\mathrm{H}(\mu) := \mu^{\frac{-1}{\alpha}}\left[\mu + \gamma\left(\mu^{\frac{1}{\alpha}} - 1\right)\right]$, $\mathrm{H}(1) = 1$ and g_c is the growth rate under *laissez-faire* (see Footnote 8). Therefore, the growth rate of the economy is:[8]

$$g(\mu) = (1+g_c)\mathrm{H}(\mu) - 1\,. \tag{18}$$

Now, let

$$\bar{\gamma} := 1 - \alpha\,, \tag{19}$$

be a threshold value of the replacement rate (equal to the weight of labour in production). Then, the following proposition holds.

Proposition 1. *Let $1 < \mu < \mu_{\mathrm{M}}$ hold to guarantee feasibility of the unemployment benefit policy. (1) If $\gamma \leq \bar{\gamma}$, then $g(\mu) < g_c$. (2) If $\gamma > \bar{\gamma}$, then $g(\mu) > g_c$ for any $1 < \mu < \mu^{\circ}$, $g(\mu)$ is maximised at $\mu = \hat{\mu}$ and $g(\mu) < g_c$ if $\mu > \mu^{\circ}$, where*

$$\hat{\mu} := \frac{\gamma}{\bar{\gamma}} > 1\,, \tag{20}$$

and $\mu^{\circ} > \hat{\mu}$ is the value of the wage mark-up such that $g(\mu^{\circ}) = g_c$.

[8] The growth rate of capital and output per capita is the same because the unemployment rate is constant. Moreover, since $g(\mu)$ is independent of time, the model does not show transitional dynamics.

Proof. Differentiating Eq. (18) with respect to μ gives $g'(\mu)=(1+g_c)\mathrm{H}'(\mu)$, where $\mathrm{H}'(\mu)=\alpha^{-1}\mu^{\frac{-(1+\alpha)}{\alpha}}(\gamma-\mu\bar{\gamma})$ and thus $\mathrm{sgn}\{g'(\mu)\}=\mathrm{sgn}\{\mathrm{H}'(\mu)\}$. Then $\mathrm{H}'(\mu)\gtreqless 0$ if $\mu\lesseqgtr\hat{\mu}$. Therefore Proposition 1 follows, since (1) if $\gamma\le\bar{\gamma}$, $\mathrm{H}'(\mu)<0$ for any $1<\mu<\mu_{\mathrm{M}}$, and (2) if $\gamma>\bar{\gamma}$, the facts that $\mathrm{H}'(\mu)=0$ only at $\mu=\hat{\mu}$ and $\mathrm{H}''(\mu)=0$ only at $\mu=\hat{\mu}(1+\alpha)>\hat{\mu}$ complete the proof because $\mathrm{H}(\mu)=1$ twice at $\mu=1$ and $\mu=\mu^{\circ}$. **Q.E.D.**

Corollary 1. *Let* $\mu=1$ *hold. If* $\gamma<\bar{\gamma}$ ($\gamma>\bar{\gamma}$) *then the introduction of minimum wages reduces (promotes) economic growth.*

Proof. The proof is straightforward since $g'(1)<0$ (>0) for any $0<\gamma<\bar{\gamma}$ ($\bar{\gamma}<\gamma<1$). **Q.E.D.**

Proposition 1 shows that the growth rate in a regulated-wage economy with unemployment may be higher than under *laissez-faire*. If the replacement rate exceeds the weight of the labour input in production, the positive growth effect of the increased workers' income is greater than the negative unemployment effect, and this eventually spurs economic growth beyond the *laissez-faire* level. Moreover, it is important to note that (*i*) a large enough replacement rate always exists to guarantee positive effects of minimum wages on growth, (*ii*) the higher the output elasticity of capital, i.e. the higher the weight of capital in production, the lower the size of the replacement rate required for the minimum wage to foster growth, and (*iii*) a growth-maximising minimum wage can exist. Moreover, since both the growth rate and unemployment rate increase along with the wage mark-up, at least for any $1<\mu<\hat{\mu}$, then a positive link between unemployment and growth indirectly exists in such a case.

Of course, exactly how the unemployment benefit expenditure is financed is crucial for the results. With consumption taxes, unemployment benefits are not completely retrieved by the amount of resources needed to finance it (as, instead, would be the case with either wage income taxes or lump-sum taxes on the young). Therefore, consumption, saving and economic growth are effectively raised.

We now illustrate Proposition 1 by taking the following configuration of technological parameters: $\alpha = 0.45$,[9] which generates $\bar{\gamma} = 0.55$, and $B = 20$. Then, in line with the unemployment benefit legislation in several European countries, we choose $\gamma = 0.7$. Therefore, the growth-maximising wage mark-up is $\hat{\mu} = 1.272$ (which corresponds to a minimum wage of 27.2 higher than the competitive wage) and $\mu^{\circ} = 1.778$. As regards preferences, we consider that every period consists of 30 years and assume $\beta = 0.3$, i.e. the discount factor is 0.96 per annum (see de la Croix and Michel, 2002, p. 50).[10] Therefore we obtain $g(\hat{\mu}) = 1.627$, $u(\hat{\mu}) = 0.414$, $\tau(\hat{\mu}) = 0.26$ and $g_c = 1.538$. It should be noted that in this example the unemployment rate at the growth-maximising wage mark-up is huge (41.4 per cent). In this stylised economy, however, labour is homogeneous and the minimum wage is fixed as a mark-up over the market wage (rather than computed as a fraction of average earnings, as in several real economies). Therefore, in order to evaluate our results for drawing policy conclusions, it may be relevant to introduce labour heterogeneity (e.g., skilled and unskilled labour) and then compute the minimum wage as a percentage of the average wage. This exercise will be presented in Section 5.2 after showing that considering skilled (high-paid) and unskilled (low-paid) labour, and introducing the minimum wage in the market for raw labour, does not alter the main conclusions of Proposition 1.

4. Welfare

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[9] This value may be considered as an average between the output elasticities of capital in developed and developing countries, which, according to, e.g., Kraay and Raddatz (2007), are $\alpha = 0.33$ and $\alpha = 0.5$, respectively.

[10] Note that with this parameter set the upper feasibility bound on the wage mark-up is $\mu_{\mathrm{M}} = 6.568$.

While we have shown that economic growth can actually be fostered by the minimum wage, another crucial aspect regards its welfare effects. This section deals with this subject and contrasts welfare levels in both the regulated-wage economy with unemployment and competitive-wage economy with full employment. Although the introduction of wage minima cannot represent a Pareto improvement, because the current old would not opt for it as (*i*) the interest rate at t shrinks due to the occurrence of unemployment (see, Eq. 10.2), and (*ii*) the rise in the tax rate to finance unemployment benefits reduces consumption,[11] it could be instructive to study whether the minimum wage can make individuals better off along the balanced growth path (BGP).

Let us begin the welfare analysis by noting that Eq. (1), applying a positive monotonic transformation, can also be expressed as $V_t = (c_{1,t})^{\frac{1}{1+\beta}}(c_{2,t+1})^{\frac{\beta}{1+\beta}}$, where $V_t := e^{\frac{U_t}{1+\beta}}$. Since both young-aged and old-aged consumption grow without transition at the constant rate $g(\mu)$,[12] the time evolution of individual welfare in an economy with minimum wages is expressed by the indirect utility function:

$$W_t(\mu) = W_0(\mu)[1+g(\mu)]^t = W_t(1)\Pi(\mu)\left[\frac{1+g(\mu)}{1+g_c}\right]^t, \tag{21}$$

where

$$W_0(\mu) = W_0(1)\Pi(\mu), \tag{22}$$

is the welfare level at $t=0$,

[11] Let the minimum wage be introduced at t. Comparison of the consumption of the old born at $t-1$ in both the regulated-wage and competitive-wage economies then implies $c_{2,t}(\mu) < c_{2,t}(1)$, where $c_{2,t}(\mu) = \frac{\beta[1+r(\mu)]w_{c,t-1}}{(1+\beta)[1+\tau(\mu)]}$, $c_{2,t}(1) = \frac{\beta[1+r(1)]w_{c,t-1}}{1+\beta}$, $r(\mu) < r(1)$ and $\tau(\mu) > 0$.

[12] This follows because the unemployment rate, the interest rate and the budget-balancing consumption tax rate are constant.

$$\Pi(\mu) := \frac{\mathrm{H}(\mu)}{1+\tau(\mu)}\mu^{\frac{(\alpha-1)\beta}{\alpha(1+\beta)}} = \frac{\mu^{\frac{1+\beta(2-\alpha)}{\alpha(1+\beta)}}\left[\mu+\gamma\left(\mu^{\frac{1}{\alpha}}-1\right)\right]\left[\mu(1+\alpha\beta)-\gamma(1-\alpha)\beta\left(\mu^{\frac{1}{\alpha}}-1\right)\right]}{\mu(1+\alpha\beta)+\gamma(1-\alpha)\left(\mu^{\frac{1}{\alpha}}-1\right)}, \tag{23}$$

is a constant that crucially depends on the level of the existing minimum wage,

$$c_{1,0}(1) = \frac{(1-\alpha)Bk_0}{1+\beta}, \tag{24.1}$$

$$c_{2,1}(1) = \frac{\beta\alpha(1-\alpha)B^2k_0}{1+\beta}, \tag{24.2}$$

$$W_0(1) = \left[c_{1,0}(1)\right]^{\frac{1}{1+\beta}}\left[c_{2,1}(1)\right]^{\frac{\beta}{1+\beta}}, \tag{22'}$$

are, respectively, the initial consumption when young and old and the initial welfare level under *laissez-faire*, with $k_0 > 0$ given, and In an economy with full employment, therefore, the welfare function is:

$$W_t(1) = W_0(1)(1+g_c)^t, \tag{21'}$$

is the welfare function in an economy with full employment.

From Eqs. (21) and (21') it is clear that BGP welfare generically depends on (*i*) the growth rate of per capita income, and (*ii*) the initial consumption levels. As regards the former, to the extent that a rise in the wage mark-up is beneficial to economic growth, the welfare effects of the minimum wage are positive through this channel. As regards the latter, comparing Eqs. (22) and (22') makes clear that the consumption and welfare levels at $t=0$ can be higher or lower than under *laissez-faire*, depending on whether $\Pi(\mu)$ is higher or lower than unity. A rise in the wage mark-up increases the income of the young, which tends to raise consumption. However, the increased minimum wage also implies a higher consumption tax rate to finance a larger amount of unemployment benefits. This, in turn, tends to reduce consumption.

Therefore, if $\Pi(\mu)<1$ (that is, consumption at $t=0$ in the regulated-wage economy is lower than under *laissez-faire*, because the weight of consumption taxation is relatively high), then the final effect of a rise in the wage mark-up μ on welfare is *a priori* uncertain because of the existence of two opposite forces at work: (*i*) the positive growth effect, and (*ii*) the negative effect due to the reduced consumption at $t=0$. In this case, therefore, the minimum wage causes welfare losses for the current old-aged, for the generation born at time $t=0$ and for some generations $t>T>0$. However, we will show below that beyond T the minimum wage implies welfare gains because the positive growth effect always dominates in the long run, i.e. when Point (2) of Proposition 1 holds, consumption grows at a higher rate than under *laissez-faire* and, thus, there exists a threshold generation beyond which welfare becomes larger despite the rise in unemployment.

In contrast, if $\Pi(\mu)>1$ (that is, consumption at $t=0$ in the regulated-wage economy is larger than under *laissez-faire* because the weight of consumption taxation is relatively low), then we obtain the important result that the generation born at $t=0$ as well as *all* the (infinite) subsequent generations will be better off in such a case.

We now proceed to show the results discussed above. First, using Eq. (23) we identify the parametric conditions for which $\Pi(\mu)>1\,(<1)$. Second, since the analytical treatment of Eq. (21) is cumbersome, we resort to numerical simulations to show that in the long run: (*i*) the minimum wage can effectively produce welfare gains either for all generations $t\geq 0$ or for all generations t born beyond the threshold $T>0$, and (*ii*) in the long run the highest possible welfare level is achieved when economic growth is maximised.

We now identify the conditions for which the introduction of the minimum wage evaluated at the margin ($\mu=1$) may or may not be welfare-improving for all generations $t\geq 0$ but the current old-aged, i.e. those born at $t=-1$.

Define $F(\mu):=\Pi(\mu)-1$, $\alpha_W:=\dfrac{\beta}{1+2\beta}<\dfrac{1}{3}$ and $\gamma_W:=\dfrac{(1-\alpha)(1+\alpha\beta)(1+2\beta)}{(1+\beta)[\alpha(1+2\beta)-\beta]}$, with $0<\gamma_W<1$ if and only if $\alpha<\alpha_{W,1}$ or $\alpha>\alpha_{W,2}$, where

$$\alpha_{W,1} := \frac{-(1+2\beta)-\sqrt{2\beta^4+7\beta^3+9\beta^2+5\beta+1}}{\beta(1+2\beta)} < 0,$$

$$\alpha_{W,2} := \frac{-(1+2\beta)+\sqrt{2\beta^4+7\beta^3+9\beta^2+5\beta+1}}{\beta(1+2\beta)}, \quad 1/2 < \alpha_{W,2} < 1.$$

Since $\alpha_{W,1} < 0$, it can be ruled out. Then we have the following proposition.

Proposition 2. *(1) Let* $0 < \alpha < \alpha_W$ *hold. Then the introduction of minimum wages (evaluated at* $\mu = 1$*) is welfare-worsening at* $t = 0$*. (2.1) Let* $\alpha_W < \alpha < \alpha_{W,2}$ *hold. Then* $\gamma_W > 1$ *and for any* $0 < \gamma < 1$ *the introduction of minimum wages (evaluated at* $\mu = 1$*) is welfare-worsening at* $t = 0$*. (2.2) Let* $\alpha_{W,2} < \alpha < 1$ *hold. Then for any* $0 < \gamma < \gamma_W$ *[*$\gamma_W < \gamma < 1$*] the introduction of minimum wages (evaluated at* $\mu = 1$*) is welfare-worsening [welfare-improving] at* $t = 0$*.*

Proof. Differentiating $F(\mu)$ with respect to μ and evaluating it at $\mu = 1$ gives

$$F'_\mu(1) = \frac{(1+\beta)[\alpha(1+2\beta)-\beta]-\gamma(1-\alpha)(1+\alpha\beta)(1+2\beta)}{\alpha(1+\beta)(1+\alpha\beta)^2}.$$

Therefore, if $0 < \alpha < \alpha_W$, then $F'_\mu(1) < 0$. In contrast, if $\alpha_W < \alpha < \alpha_{W,2}$, then since $\gamma_W > 1$, $F'_\mu(1) < 0$; if $\alpha_{W,2} < \alpha < 1$, then for any $0 < \gamma < \gamma_W$ [$\gamma_W < \gamma < 1$], $F'_\mu(1) < 0$ [$F'_\mu(1) > 0$]. **Q.E.D.**

Proposition 2 reveals that the lower the weight of employed labour in production and the higher the replacement rate, the more likely is the introduction of the minimum wage to increase consumption at $t = 0$. In this case, the weight of the increased budget-balancing tax rate on consumption is relatively low with respect to the increased income when young, and thus welfare rises even at $t = 0$.

We now consider how a rise in the minimum wage above the existing level ($\mu > 1$) affects the function Eq. (23) in the case $\Pi(\mu) > 1$. Although the analysis of Eq. (23) is cumbersome, it can be shown that, in general, $\Pi(\mu)$ is an inverted U-shaped function of the wage mark-up μ. The economic reason is that there exists a range between the competitive wage (i.e. $\mu = 1$) and an upper bound of the minimum wage (i.e. $\mu = \mu_{\Pi}$) such that all generations $t \geq 0$ can obtain welfare gains (of course if Point (2.2) of Proposition 2 holds). This also means that if the wage mark-up is fixed beyond the threshold μ_{Π}, some initial generations $0 \leq t < T$ are harmed. An illustration of this finding is provided in Table 4.

We now assume that the conditions of Point (2) of Proposition 1 hold and proceed with some numerical exercises to analyse the welfare effects of μ for the cases $\Pi(\mu) < 1$ and $\Pi(\mu) > 1$. Using the same parameter set as in Section 3,[13] Table 2 summarises the evolution of individual welfare for the wage mark-ups reported in Table 1 (Row 1), which also shows the corresponding values of the unemployment rate, the budget-balancing consumption tax rate and the growth rate (Rows 2, 3 and 4, respectively).

Table 1. Wage mark-up and other macroeconomic and policy variables

μ	1	1.1	1.272	1.4
$u(\mu)$	0	0.19	0.414	0.526
$\tau(\mu)$	0	0.096	0.26	0.381
$g(\mu)$	1.538	1.598	1.627	1.618

Table 2. Evolution of individual welfare when μ varies ($k_0 = 1$, $W_t/10000$)

	$t=0$	$t=1$	$t=3$	$t=5$	$t=7$	$t=9$
$W_t(1)$	0.001	0.0027	0.017	0.112	0.722	4.657
$W_t(1.1)$	0.0009	0.0025	0.016	0.114	0.773	5.222
$W_t(1.272)$	0.0008	0.0021	0.0148	0.102	0.706	4.875
$W_t(1.4)$	0.0007	0.0018	0.0129	0.088	0.609	4.178

	$t=11$	$t=13$	$t=15$	$t=18$	$t=30$	$t=50$

[13] Note that with this configuration of parameters we get $\alpha_{W,2} = 0.572$ and hence $\gamma_W > 1$ is satisfied (see Point (2.1) of Proposition 2). Since $\alpha = 0.45 < 0.572$, then the minimum wage is welfare-worsening at $t = 0$ (see Table 2, Column 1).

$W_t(1)$	30.01	193.38	1246.10	20382.87	$1.45 \cdot 10^9$	$1.80 \cdot 10^{17}$
$W_t(1.1)$	35.26	238.11	1607.78	28209.37	$2.67 \cdot 10^9$	$5.26 \cdot 10^{17}$
$W_t(1.272)$	33.65	232.40	1604.57	29110.33	$3.15 \cdot 10^9$	$7.76 \cdot 10^{17}$
$W_t(1.4)$	28.64	196.34	1345.94	24156.56	$2.50 \cdot 10^9$	$5.74 \cdot 10^{17}$

Since the budget-balancing tax rate is higher as the wage mark-up increases (see Table 1), the more μ is increased, the stronger the negative welfare effect due to the reduction in initial consumption. Thus Table 2 shows that the generations born at $t=0$ as well as in the subsequent periods $0<t<T$ are made worse off, i.e. the minimum wage policy cannot represent a Pareto improvement. However, in the long run welfare gains are obtained irrespective of the size of the minimum wage, that is a threshold generation $T>0$ exists beyond which all the future generations $t>T$ are made better off because the positive growth effect asymptotically dominates. Indeed, from Table 2 we see that when $\mu=1.1$ (1.272) [1.4] individuals of the fifth (ninth) [thirteenth] generation, as well as those born in all future periods, are better off in the regulated-wage economy with unemployment than under *laissez-faire*. Moreover, and most important, Table 2 also shows that the highest possible long-run welfare levels are achieved at the growth-maximising wage mark-up.[14]

We now take $0.7=\alpha>\alpha_{W,2}=0.572$ to show that the minimum wage can actually be welfare-improving for all generations $t \geq 0$. We then obtain $\bar{\gamma}=0.3$, $\gamma_W=0.544$, $\hat{\mu}=2.333$, $\mu^\circ=14.82$ and $\mu_{\mathrm{M}}=988$. Moreover, there is a whole range of values of the wage mark-up, i.e. $1<\mu<1.341$, such that $\Pi(\mu)>1$, and then $W_0(\mu)>W_0(1)$ in such a case. The results are summarised in Tables 3 and 4.

[14] This has been ascertained through extensive numerical simulations for $t>50$ not reported in Table 2 for economy of space. Moreover, it can be shown (numerically) that the growth-maximising wage mark-up coincides with the welfare-maximising one as t becomes large enough (see Result 2).

Table 3. Wage mark-up and other macroeconomic and policy variables

μ	1	1.1	2.333	3
$u(\mu)$	0	0.127	0.701	0.791
$\tau(\mu)$	0	0.03	0.24	0.306
$g(\mu)$	0.384	0.452	0.643	0.632

Table 4. Evolution of individual welfare when μ varies ($k_0 = 1$, $W_t / 100$)

	$t=0$	$t=1$	$t=3$	$t=5$	$t=7$	$t=9$
$W_t(1)$	0.0642	0.0889	0.17	0.327	0.627	1.202
$W_t(1.1)$	0.0648	0.0941	0.198	0.419	0.884	1.866
$W_t(2.333)$	0.0565	0.0929	0.25	0.677	1.83	4.943
$W_t(3)$	0.052	0.0849	0.226	0.602	1.605	4.276

	$t=11$	$t=13$	$t=15$	$t=18$	$t=30$	$t=50$
$W_t(1)$	2.304	4.418	8.471	22.48	1116	$7.49 \cdot 10^5$
$W_t(1.1)$	3.938	8.311	17.53	53.74	4742	$8.29 \cdot 10^6$
$W_t(2.333)$	13.34	36.05	97.35	432	$1.67 \cdot 10^5$	$3.45 \cdot 10^9$
$W_t(3)$	11.39	30.34	80.83	351.4	$1.25 \cdot 10^5$	$2.26 \cdot 10^9$

Table 4 shows that when the output elasticity of capital is high enough, i.e. $\alpha_{W,2} < \alpha < 1$, and the wage mark-up is not fixed at too high a level, i.e. $1 < \mu < \mu_{\Pi}$, where $\mu_{\Pi} = 1.341$, the minimum wage is welfare-improving for all generations $t \geq 0$. In this case, the weight of the increased income when young more than counterbalances the negative effect on consumption due to the increase in the budget-balancing tax rate so that welfare rises. Of course, when the wage mark-up increases further on, the weight of the higher tax rate becomes larger and, hence, for any $\mu > \mu_{\Pi}$ there exist some generations $0 \leq t < T$ that incur welfare losses because consumption shrinks. To this purpose, Table 4 shows that when $\mu = 2.333$ (3) the positive growth effect dominates at $t = 1$ (3) and welfare gains can then effectively be obtained. Moreover, in the long run the highest possible welfare level is still achieved when the government maximises growth.

To sum up, from Eq. (21) the following results hold as regards the welfare effects of the minimum wage:

Result 1. *If Point (2) of Proposition 1 holds, then for any* $1 < \mu < \mu^{\circ}$, $\lim_{t \to +\infty} W_t(\mu) = \lim_{t \to +\infty} W_t(1)$. *Therefore, there exists a threshold generation* $T > 0$ *such that* $W_t(\mu) > W_t(1)$ *for any* $t > T$. *Moreover, if Point (2) of Proposition 1 holds,* $\alpha_{W,2} < \alpha < 1$ *and* $\gamma_W < \gamma < 1$, *then for any* $1 < \mu < \mu_{\Pi}$, $W_t(\mu) > W_t(1)$ *for every* $t \geq 0$.

Result 2. *The necessary condition to maximise economic growth defines the criterion to maximise welfare because* $\frac{W_t(\mu)}{\partial \mu}\Big|_{\mu = \hat{\mu}} = 0$ *when* t *becomes large.*

In the numerical example presented in Tables 1–4 we have studied the effects of a *once-and-for-all* minimum wage policy introduced at time $t = 0$, for instance at the growth-maximising level, and then kept unchanged in all future periods. In this case the welfare level of the first T generations is drastically reduced because of the need to finance a large amount of unemployment benefits immediately. Could the welfare losses actually be smoothed across the first T generations if, alternatively, the minimum wage is *gradually* raised across periods up to the growth-maximising level?[15] Comparison of Tables 5.A (once-and-for-all) and 5.B (progressive rise in the minimum wage) makes it clear that a gradual increase in the minimum wage is better than fixing it directly at the growth-maximising value at time $t = 0$, because: (*i*) the welfare losses of the seventh generation are smaller in such a case, and (*ii*) the welfare gains are obtained starting from the ninth generation in both cases. The economic reason is simple: if the minimum wage is increased progressively, the rise in both unemployment and consumption taxes to finance the benefit expenditure is gradual

[15] This argument is relevant especially because the introduction of the minimum wage in several European countries followed this design even when the process was rapid, as recently occurred in the UK or Ireland (see, e.g., Dolado et al., 1996 for evidence of the impact of different minimum wage policies in Europe). We thank an anonymous referee for suggesting this point be clarified.

across periods. This in turn implies that the negative impact of the minimum wage on consumption of the first T generations is smaller in such a case.

Table 5.A. The case of the once-and-for-all minimum wage policy ($k_0 = 1$, $W_t/10000$)

	$t=0$	$t=1$	$t=3$	$t=5$	$t=7$	$t=9$
$W_t(1)$	0.001	0.0027	0.017	0.112	0.722	4.657
$W_t(1.272)$	0.0008	0.0021	0.0148	0.102	0.706	4.875

Table 5.B. The case of a progressive rise in the minimum wage ($k_0 = 1$, $W_t/10000$)

	$t=0$ $\mu=1.05$	$t=1$ $\mu=1.1$	$t=3$ $\mu=1.15$	$t=5$ $\mu=1.2$	$t=7$ $\mu=1.25$	$t=9$ $\mu=1.272$
$W_t(1)$	0.00106	0.0027	0.017	0.112	0.722	4.657
$W_t(\mu)$	0.00101	0.0025	0.016	0.108	0.721	4.875

5. Extensions

In this section we present some modifications and extensions of the baseline model. In particular, in Section 5.1 we assume the replacement rate, rather than the consumption tax rate, as the endogenous variable that balances the government budget. In Section 5.2 we assume the existence of skilled and unskilled labour and then introduce the minimum wage in the market for raw labour. Finally, in Section 5.3 a capital income tax is used to finance the unemployment benefit system.

5.1. Endogenous replacement rate

In this section we assume the replacement rate γ_t, rather than the consumption tax rate τ, is endogenous and adjusted from period to period to balance out the unemployment benefit budget Eq. (11).

Exploiting Eqs. (2.1), (2.2), (7.1), the one-period backward (7.2), (9.1), (10.1) and (11), the (constant) budget-balancing replacement rate is obtained as

$$\gamma_t = \gamma(\mu,\tau) = \frac{\tau\mu(1+\alpha\beta)}{\left(\mu^{\frac{1}{\alpha}}-1\right)[1+\beta(1+\tau)](1-\alpha)}, \tag{25}$$

where $\gamma'_\mu(\mu,\tau)<0$ and $\gamma'_\tau(\mu,\tau)>0$. Now, the unemployment benefit policy is viable if $\gamma(\mu,\tau)<1$, and this would alternatively require

$$\tau < \hat{\tau} := \frac{(1-\alpha)(1+\beta)\left(\mu^{\frac{1}{\alpha}}-1\right)}{\mu(1+\alpha\beta)-(1-\alpha)\beta\left(\mu^{\frac{1}{\alpha}}-1\right)}. \tag{26}$$

Exploiting Eqs. (2.1), (2.2), (10.1), (14) and using (25) to eliminate the replacement rate, the growth rate in the regulated-wage economy is

$$g(\mu,\tau) = (1+g_c)\mathrm{P}(\mu,\tau)-1, \tag{27}$$

where $\mathrm{P}(\mu,\tau) := \frac{\mu^{\frac{\alpha-1}{\alpha}}(1+\beta)(1-\alpha+\tau)}{(1-\alpha)[1+\beta(1+\tau)]}$, $g'_\mu(\mu,\tau)<0$ and $g'_\tau(\mu,\tau)>0$, that is the rate of economic growth is a monotonic decreasing (increasing) function of the wage mark-up (consumption tax).

Now, let us define

$$\tilde{\tau} := \frac{(1-\alpha)(1+\beta)\left(1-\mu^{\frac{1-\alpha}{\alpha}}\right)}{\mu^{\frac{1-\alpha}{\alpha}}\beta(1-\alpha)-(1+\beta)}, \tag{28}$$

as the threshold value of the consumption tax such that $\mathrm{P}(\mu,\tau)=1$, where $\hat{\tau}>\tilde{\tau}$, and

$$\tilde{\mu} := \left[\frac{1+\beta}{\beta(1-\alpha)}\right]^{\frac{\alpha}{1-\alpha}}, \tag{29}$$

as the threshold value of the wage mark-up below which $\tilde{\tau}>0$.

Although a rise in μ is now always growth-reducing, the following proposition shows that minimum wages used in conjunction with unemployment benefits can stimulate growth.

Proposition 3. *Let* $1 < \mu < \tilde{\mu}$ *and* $\tilde{\tau} < \tau < \hat{\tau}$ *hold. Then* $g(\mu, \tau) > g_c$ *and the unemployment benefit policy is feasible.*

Proof. From Eq. (27) it is easy to verify that $g(\mu, \tau) = g_c$ if and only if $\mathrm{P}(\mu, \tau) = 1$. Since $\mathrm{P}(\mu, \tau) > 1$ for any $\tau > \tilde{\tau}$, $\tilde{\tau} > 0$ for any $1 < \mu < \tilde{\mu}$, and the unemployment benefit policy is feasible if and only if $\tau < \hat{\tau}$, then Proposition 2 follows. **Q.E.D.**

5.2. Skilled and unskilled labour

In this section we assume the existence of two types of labour: skilled (S) and unskilled (U). The market for skilled labour is competitive and the market for unskilled labour is regulated by law. We take the division of labour exogenously and assume that each young is endowed with θ units of unskilled labour and $1-\theta$ units of skilled labour, where $0 < \theta < 1$ (see, e.g., Martínez and Iza, 2004), which are supplied inelastically to firms. Therefore, $N = N^U + N^S$, where $N^U = \theta N$ and $N^S = (1-\theta)N$. For skilled labour, individuals earn the competitive wage w_t^S. For unskilled labour, they earn a minimum wage $w_{m,t}^U := \mu w_{c,t}^U$ fixed by law if employed, where $w_{c,t}^U$ is the competitive wage of the unskilled, while receiving a benefit $b_t := \gamma w_{c,t}^U$ if unemployed. Of course, $w_t^S > w_{m,t}^U$. Preferences are still determined by Eq. (1). Therefore, the aggregate saving rate is:

$$s_t = \frac{\beta}{1+\beta}\left\{(1-\theta)w_t^S + \theta\left[w_{m,t}^U(1-u_t) + b_t u_t\right]\right\}, \tag{30}$$

where $u_t = \left(\theta N - L_t^U\right)/\theta N$ is the unemployment rate in an economy with heterogeneous labour and L_t^U is the demand for raw labour.

There are three factors of production: physical capital (K), skilled (L^S) and unskilled (L^U) labour. The production function of the single firm i at time t is now

$Y_{i,t} = K_{i,t}^{\alpha_1}\left(L_{i,t}^{S}\right)^{\alpha_2}\left(L_{i,t}^{U}\right)^{\alpha_3}\left(A_{i,t}\right)^{\alpha_2+\alpha_3} = \overline{\overline{B}}k_t^{1-\alpha_1}K_{i,t}^{\alpha_1}\left(L_{i,t}^{S}\right)^{\alpha_2}\left(L_{i,t}^{U}\right)^{\alpha_3}$, where $\alpha_1+\alpha_2+\alpha_3=1$, $\alpha_2>\alpha_3$ and the labour productivity index $A_{i,t}$ is defined as in Section 2.2, with $\overline{\overline{B}} := a^{1-\alpha_1} > 0$. Aggregate production at t takes place according to $Y_t = \overline{\overline{B}}k_t^{1-\alpha_1}K_t^{\alpha_1}\left(L_t^{S}\right)^{\alpha_2}\left(L_t^{U}\right)^{\alpha_3}$, where $L_t^{S} = N^S = (1-\theta)N$ and $L_t^{U} = (1-u_t)N^U = (1-u_t)\theta N$.[16] The intensive form production function is $y_t = \overline{\overline{B}}k_t(1-u_t)^{1-\alpha_1-\alpha_2}(1-\theta)^{\alpha_2}\theta^{1-\alpha_1-\alpha_2}$. Profit maximisation implies:

$$r_t = \alpha_1\overline{\overline{B}}(1-u_t)^{1-\alpha_1-\alpha_2}(1-\theta)^{\alpha_2}\theta^{1-\alpha_1-\alpha_2} - 1, \tag{31}$$

$$w_t^{S} = \alpha_2\overline{\overline{B}}k_t(1-u_t)^{1-\alpha_1-\alpha_2}(1-\theta)^{\alpha_2-1}\theta^{1-\alpha_1-\alpha_2}, \tag{32}$$

$$w_{m,t}^{U} = (1-\alpha_1-\alpha_2)\overline{\overline{B}}k_t(1-u_t)^{-(\alpha_1+\alpha_2)}(1-\theta)^{\alpha_2}\theta^{-(\alpha_1+\alpha_2)}. \tag{33}$$

Since $w_{c,t}^{U} = (1-\alpha_1-\alpha_2)\overline{\overline{B}}k_t(1-\theta)^{\alpha_2}\theta^{-(\alpha_1+\alpha_2)}$, then from Eq. (33) the unemployment rate is

$$u_t = u(\mu) = 1-\mu^{\frac{-1}{\alpha_1+\alpha_2}}. \tag{34}$$

Therefore, a rise in μ increases unemployment and reduces both the interest rate and skilled wage, and thus it also decreases the ratio of the skilled wage to the unskilled one (see, e.g., Sener, 2006). Moreover, since a rise in γ does not affect unemployment it leaves the ratio $w_t^{S}/w_{m,t}^{U}$ unaltered. Therefore, the ratio of skilled to unskilled wage is greater when skilled labour is scarcer (see, e.g., Acemoglu and Zilibotti, 2001), i.e. when θ is high.

The government budget is $\theta b_t u_t = \tau_t(c_{1,t} + c_{2,t})$ and thus the budget-balancing consumption tax rate now becomes:

[16] This production function implies limited substitutability between all the production factors (see, e.g., Lindh and Malmberg, 1999; Fanti and Manfredi, 2005). The question of the degree of substitutability between inputs is still controversial in the economic literature. Although recent contributions provide evidence in favour of capital-skill complementarities (e.g., Hamermesh, 1993; Duffy et al., 2004), in this context such an assumption would lead to a lack of analytical tractability.

$$\tau_t = \tau(\mu) = \frac{\gamma(1-\alpha_1-\alpha_2)(1+\beta)\left(\mu^{\frac{1}{\alpha_1+\alpha_2}}-1\right)}{\mu(1+\alpha_1\beta)-\gamma(1-\alpha_1-\alpha_2)\beta\left(\mu^{\frac{1}{\alpha_1+\alpha_2}}-1\right)}. \tag{35}$$

which is similar to Eq. (12) and where $\tau(1)=0$, $\tau'(\mu)>0$ and $\tau>0$ for any $1<\mu<\mu_{\mathrm{MM}}$, where μ_{MM} is an upper feasibility bound on the wage mark-up.

Since equilibrium in goods and capital markets is still determined by Eq. (13), the growth rate of the economy can now be expressed as:

$$\overline{\overline{g}}(\mu) = \left(1+\overline{\overline{g}}_c\right)\overline{\overline{\mathrm{H}}}(\mu) - 1, \tag{36}$$

where $\overline{\overline{g}}_c = \frac{\beta}{1+\beta}\overline{\overline{B}}(1-\alpha_1)(1-\theta)^{\alpha_2}\theta^{1-\alpha_1-\alpha_2}-1$ is the growth rate under *laissez-faire* in an economy with labour heterogeneity, $\overline{\overline{\mathrm{H}}}(\mu) := \mu^{\frac{\alpha_1+\alpha_2-1}{\alpha_1+\alpha_2}} + \gamma\frac{1-\alpha_1-\alpha_2}{1-\alpha 1}\left(1-\mu^{\frac{-1}{\alpha_1+\alpha_2}}\right)$ and $\overline{\overline{g}}(1)=\overline{\overline{g}}_c$.

Let

$$\overline{\overline{\gamma}} := 1-\alpha_1, \tag{37}$$

be a threshold value of the replacement rate (equal to the sum of the weight of skilled and unskilled labour in production). The following proposition shows that with heterogeneous labour the same parametric conditions of an economy with homogeneous labour are involved in determining whether the minimum wage can foster growth (compare Eqs. 19 and 20 with Eqs. 37 and 38).

Proposition 4. *Let* $1<\mu<\mu_{\mathrm{MM}}$ *hold to guarantee feasibility of the unemployment benefit policy. (1) If* $\gamma \le \overline{\overline{\gamma}}$*, then* $\overline{\overline{g}}(\mu)<\overline{\overline{g}}_c$*. (2) If* $\gamma > \overline{\overline{\gamma}}$*, then* $\overline{\overline{g}}(\mu)>\overline{\overline{g}}_c$ *for any* $1<\mu<\mu^{\circ\circ}$*,* $\overline{\overline{g}}(\mu)$ *is maximised at* $\mu=\hat{\hat{\mu}}$ *and* $\overline{\overline{g}}(\mu)<\overline{\overline{g}}_c$ *if* $\mu>\mu^{\circ\circ}$*, where*

$$\hat{\hat{\mu}} := \frac{\gamma}{\overline{\overline{\gamma}}} > 1, \tag{38}$$

and $\mu^{\circ\circ}>\hat{\hat{\mu}}$ *is the value of the wage mark-up such that* $\overline{\overline{g}}(\mu^{\circ\circ})=\overline{\overline{g}}_c$.

Proof. Differentiating Eq. (36) with respect to μ gives $\bar{\bar{g}}'(\mu)=(1+\bar{\bar{g}}_c)\bar{\bar{\mathrm{H}}}'(\mu)$, where $\bar{\bar{\mathrm{H}}}'(\mu)=\frac{1-\alpha_1-\alpha_2}{(1-\alpha_1)(\alpha_1+\alpha_2)}\mu^{\frac{-(1+\alpha_1+\alpha_2)}{\alpha_1+\alpha_2}}(\gamma-\mu\bar{\bar{\gamma}})$ and thus $\mathrm{sgn}\{\bar{\bar{g}}'(\mu)\}=\mathrm{sgn}\{\bar{\bar{\mathrm{H}}}'(\mu)\}$. Then $\bar{\bar{\mathrm{H}}}'(\mu)\gtreqless 0$ if $\mu\lesseqgtr\hat{\hat{\mu}}$. Therefore, Proposition 3 follows, since (1) if $\gamma\le\bar{\bar{\gamma}}$, $\bar{\bar{\mathrm{H}}}'(\mu)<0$ for any $1<\mu<\mu_{\mathrm{MM}}$, and (2) if $\gamma>\bar{\bar{\gamma}}$, the facts that $\bar{\bar{\mathrm{H}}}'(\mu)=0$ only at $\mu=\hat{\hat{\mu}}$ and $\bar{\bar{\mathrm{H}}}''(\mu)=0$ only at $\mu=\hat{\hat{\mu}}(1+\alpha_1+\alpha_2)>\hat{\hat{\mu}}$ complete the proof because $\bar{\bar{\mathrm{H}}}(\mu)=1$ twice at $\mu=1$ and $\mu=\mu^{\circ\circ}$. **Q.E.D.**

The models presented in Sections 2 and 5.2 represent a theoretical abstraction on how the minimum wage can affect economic growth and welfare. Indeed, the minimum wage is computed as a mark-up over (*i*) the competitive wage, in the case of homogeneous labour (Section 2), and (*ii*) the competitive wage in the market for raw labour, in the case of labour heterogeneity (Section 5.2). In the real world, however, the minimum wage is fixed as a fraction of average earnings, and beneficiaries are essentially the poorest among low-wage workers. On the basis of the Kaitz index, minimum wages in Europe are found to be higher than in the U.S. (see Dolado et al., 1996, Table 1, p. 322–323), ranging between one third and one half[17] of average earnings (see also OECD, 2008 and ETUI Policy Brief, 2009).

At the time of writing, there exists a broad consensus to raise the minimum wage as well as to make it uniform across European Union countries, at an even higher level than 50 per cent of

[17] The Kaitz index for Italy is even much higher. However, a national legislated (minimum) wage in this country does not exist, while being determined as a bargaining between unions and employers in sectoral collective agreements, and then it may greatly differ from sector to sector due to the existence of differentiated minimum wage rates.

average earnings (see, e.g., Schulten and Watt, 2007),[18] in order to protect workers against the severe recession recently experienced. Therefore, the question now is the following: would the positive effects of minimum wages on growth be confirmed if they were realistically computed as a fraction of average earnings? For a simple quantitative illustration of the theoretical predictions of the model we now refer to an example inspired by the French economy, where opponents and proponents of the existing minimum wage indexation mechanisms have recently battled (see Cahuc et al., 2008; Askenazy, 2008). Therefore, in order to answer the question above, we first assume that the minimum wage for the low-paid is computed as a percentage ($0 < z < 1$) of the weighted average wage between those of both skilled and unskilled in the case of full employment, i.e. $w_{m,t}^{U} := z \cdot \left[(1-\theta) w_{c,t}^{S} + \theta w_{c,t}^{U}\right]$. Therefore, the wage gap is $\frac{w_{c,t}^{S}}{w_{c,t}^{U}} = \frac{\alpha_2}{1-\alpha_1-\alpha_2}\frac{\theta}{1-\theta}$. Second, from the EUROSTAT (2005) data (see, e.g., Table A.2, Hipolito, 2008), we take the 90-10, 50-10 and 90-50 differentials (i.e., the ratios of the 10th, 50th and the 90th deciles) of the wage distribution, which are 3.36, 1.64 and 2.00, respectively.[19] Third, by assuming for simplicity a uniform distribution within the 50-10 and 90-50 deciles, we obtain an approximated value of the ratio between the lowest wage (i.e., the wage of 10 per cent of the low-paid) and average wage around 2.00. Since in France there is a current statutory minimum wage of about 45-50 per cent of the average wage (see OECD, 2008; ETUI Policy Brief, 2009),[20] this would approximately correspond to a wage floor that covers 10 per cent of the low-paid. Therefore, taking serious account of some recent proposals attempting to raise the minimum wage in several European countries up to 60 per

[18] In particular, "… the core proposal [of a European-wide minimum wage floor, should involve] … an undertaking by all countries to raise, within a given time scale, their minimum wage to initially 50% and subsequently 60% of the average wage." (ETUI Policy Brief, 2009, p. 5).

[19] See also Blau and Kahn (1996), Goldin and Katz (2007), Autor et al. (2008).

[20] More precisely, the ETUI Policy Brief (2009, Table 1) reports 51.4 per cent from OECD data and 48.3 per cent from ILO data for the year 2007.

cent of the average wage, we conclude that this would push the wage floor[21] up by almost 20 per cent, so that the preceding measures of wage inequality 50-10 and 90-10 deciles are in the short run compressed from 1.64 to 1.36, and from 3.36 to 2.8, respectively. What would the consequences be, in terms of economic growth, of the rise in the minimum wage from 45-50 to 60 per cent of the average wage in our context? Since the minimum wage covers 10 per cent of workers among the low-paid, we assume $\theta = 0.1$. Then we take $\alpha_1 = 0.5$ from Rodríguez and Ortega (2006, Table A.1), and calibrate $\alpha_2 = 0.473$ such that the wage gap (i.e., the ratio between the average and the lowest wages) is $\frac{w_{c,t}^{S}}{w_{c,t}^{U}} = 2$.[22] The production scale parameter ($B = 60$) is fixed to get a reasonable growth rate around 1.7 per cent per annum in the case of full employment. The subjective discount rate ($\beta = 0.11$), instead, is calibrated to obtain a propensity to save around 10 per cent (see, e.g., Jappelli and Padula, 2007, Table 1). The replacement rate γ is assumed to be 0.6.[23] Simple calculations show that if the minimum wage were raised from 50 to 60 per cent of the average wage, the wage floor of the unskilled would rise from $w_{c,t}^{U} = 14.13$ to $w_{m,t}^{U} = 16.2$ (i.e. almost 20

[21] Note that for illustrative purposes it is assumed that the wage floor only regards the first deciles of the whole population. Therefore in our model with only two categories, the share of unskilled (earning the lowest wage) is 0.1, while the remaining 0.9 is assumed to be skilled (thus earning the average wage calculated above, which is approximately double the wage received by the lowest 10 per cent of the low-paid). Note that current statutory minimum wages in Europe only cover a very low share of workers, ranging between 1 and 5 per cent in several countries, so that the conjecture that a rise in the statutory value up to 60 per cent of the average wage to cover 10 per cent of workers seems to be reasonable.

[22] It is therefore implicitly assumed that the current wage of the unskilled is the competitive one, i.e. the existing (minimum) wage floor (50 per cent of the average wage) is not binding.

[23] The amount of ARE (return to work credit) in France varies according to the wage received by the jobseeker during the reference period. Gross unemployment benefit is the higher of the following two amounts: 57.4 per cent of the SJR (reference daily salary) or 40.4 per cent of the SJR plus 11.04 euro per day. Thus in our context a replacement rate of around 55-60 per cent can be realistic.

per cent above the wage of the unskilled in the competitive-wage economy; this corresponds to $\mu = 1.14$ in our context) and the per annum growth rate (assuming each generation consists of 30 years) would increase from 1.705 per cent in the competitive wage economy to 1.712 per cent in the regulated-wage economy. The unemployment rate is reasonable at 12.5 per cent and the consumption tax rate to finance the unemployment benefit expenditure is negligible at 0.24 per cent. Interestingly, since the rise in the growth rate also pushes up the skilled wage, then not only the current unskilled but also all the future skilled will benefit from the higher minimum wage.

Our numerical examples have revealed that the minimum wage, even if realistically computed as a fraction of the average wage, and used together with a system of unemployment benefits, can stimulate growth. Of course, since our economy is highly stylised, policy conclusions should be carefully evaluated.

5.3. Capital income tax

In an OLG context taxing capital income is known to be able to stimulate growth because it causes a positive inter-generational transfer effect towards young savers (see Bertola, 1996; Uhlig and Yanagawa, 1996). In this section we briefly show that the use of capital income taxes to finance unemployment benefits, through an intergenerational transfer channel from the old dissavers to the young savers, is beneficial to economic growth.

Since with Cobb-Douglas preferences the propensity to save is independent of the interest rate (see Eq. 6.2), with capital income taxation the growth rate of the economy is still given by Eq. (18). The government budget, instead, becomes:

$$b_t u_t = \tau_{k,t} r_t k_t , \tag{39}$$

where $0 < \tau_{k,t} < 1$ is the capital income tax rate. Using Eqs. (2.2), (9.1), (10.1) and the one-period backward Eq. (13), the (constant) budget-balancing capital income tax rate is:

$$\tau_{k,t} = \tau_k(\mu) = \frac{\gamma\left(\mu^{\frac{1}{\alpha}} - 1\right)(1-\alpha)B}{\alpha B \mu - 1}. \tag{40}$$

From Eq. (40), the unemployment benefit expenditure is feasible if and only if for any $0 < \gamma < 1$, $1 < \mu < \mu_R$ holds, where μ_R is an upper feasibility bound on the wage mark-up such that $\tau_k(\mu) < 1$. Therefore, we have the following proposition.

Proposition 5. *Let $1 < \mu < \mu_R$ hold to guarantee feasibility of the unemployment benefit policy. Then, Points (1) and (2) of Proposition 1 hold.*

Proof. Since Eq. (18) holds and $1 < \mu < \mu_R$ guarantees $\tau_k(\mu) < 1$ then Proposition 5 follows. **Q.E.D.**

To illustrate Proposition 5 we take the same parameter values as in Section 3. The growth-maximising wage mark up is $\hat{\mu} = 1.272$, so that $g(\hat{\mu}) = 1.627$ and $u(\hat{\mu}) = 0.414$. The budget-balancing capital income tax rate is $\tau_k(\hat{\mu}) = 0.522$, which is higher than in the case of consumption taxes ($\tau(\hat{\mu}) = 0.26$). Moreover, the range of feasible wage mark-up to implement the unemployment benefit policy shrinks from $1 < \mu < \mu_{\mathrm{M}} = 6.568$ in the case of consumption taxation to $1 < \mu < \mu_R = 1.562$ in the case of capital income taxation.

As shown in this section, the beneficial effects of the minimum wage on growth represent a robust feature of OLG economies with both homogeneous and heterogeneous labour.

6. Conclusions

This paper takes a dynamic view of labour market rigidities. Analysis of labour market imperfections and the effects of unemployment in aggregate macroeconomic models have been widely studied in the economic literature. As regards how legislated wage minima affect economic growth, conclusions are essentially negative unless the minimum wage causes some positive externalities (for instance, on the accumulation of human capital), or when the capital and labour inputs are complementary (i.e., the elasticity of substitution between factors is fairly low).

In contrast to the previous theoretical literature, in this paper we introduced minimum wage and unemployment benefit policies in a simple double Cobb-Douglas one-sector OLG growth model à la Romer (1986), in the case of both homogeneous and heterogeneous labour. We showed that a regulated-wage economy with unemployment can grow faster than the *laissez-faire*. We also found that a minimum wage policy can be welfare-improving and the highest possible welfare level is achieved when balanced growth is maximised. In particular, we identified the conditions under which the minimum wage can make the current as well as all the infinite future generations better off, although it cannot represent a Pareto improvement.

The essential message of the present paper, therefore, is that the minimum wage can be used not only to enhance equity but also to promote economic growth and welfare even in the absence of growth-sustaining externalities which the minimum wage can generate. The present paper may be viewed as complementary to that of Irmen and Wigger (2002) where, in an overlapping generations context similar to ours, a positive relationship is established between unionised wage, unemployment and economic growth. Their results, however, hold only when capital and labour inputs are fairly complementary, and do not hold thus in the case of Cobb-Douglas technology, while our findings are confirmed (*a fortiori*) if production factors are complementary.

The present paper could be extended in several directions. For instance, utility and production functions could be generalised and an open economy framework used. Fertility choices and human capital accumulation could also be incorporated.

Acknowledgments We are indebted to Prof. Eric Maskin, Prof. Davide Fiaschi, seminar participants at: (*i*) the 19th Jerusalem School in Economic Theory "The Economics of Kenneth J.

Arrow", held on June 27–July 8, 2008 at the Institute for Advanced Studies, Hebrew University of Jerusalem, Israel, (*ii*) the XXIII AIEL National Conference of Labour Economics, held on September 11th–12th, 2008 at the University of Brescia, Italy, (*iii*) the seminar series at the Department of Economics of the University of Pisa, Italy, for stimulating discussions. Moreover, we gratefully acknowledge the editor Prof. Giacomo Corneo and three anonymous referees for valuable comments that have substantially improved the quality of the paper. The usual disclaimer applies.

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