Economic Growth, Unemployment and Children Under Regulated Wages

Luciano Fanti* and Luca Gori**

Abstract

This paper investigates the effects of the wage regulation within a standard neoclassical growth OLG model extended with endogenous fertility. In contrast with the prevailing literature on the minimum wage, which has paid little attention to inter-temporal contexts, we conclude that under suitable conditions – say, sufficiently high both capital weight in production and unemployment benefits - a regulated wage economy may perform better than a market-wage economy, and the level of the regulated wage may also be treated as a policy parameter for controlling population growth. Finally, a further interesting result is that the correlation between unemployment and economic growth may be positive.

Keywords: Endogenous Fertility; Regulated Wage; OLG Model; Welfare

JEL Classification: E24; H24; J13; O41

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1 Introduction

Although a vast debate about the macroeconomic consequences of minimum wages\(^1\) has been developed dating from Stigler (1946), less attention has been paid as regards the long run effects of a wage regulation in a dynamical context (i.e., a simple OLG frame). The idea that the introduction of regulation of wages in a simple competitive economy brings upon an output loss due to the unemployment occurrence is a widespread belief. For instance, "it is generally recognized that minimum wage legislation induces distortions which have adverse effects on the efficiency of the economy." (Cahuc and Michel (1996, p. 1464)). Furthermore, the prevailing literature (e.g., among others, Bean and Pissarides (1993), Daveri and Tabellini (2000)) establishes a negative correlation between unemployment and growth: i.e., unemployment always deteriorates growth.

Only few papers have investigated the possibility of positive long run macroeconomic effects of the minimum wage legislation in an inter-temporal OLG context, for example Cahuc and Michel (1996) and Ravn and Sorensen (1999). However, in the two latter articles, such a possibility depended on the specific assumption of a positive relationship between the unemployment created by the minimum wage and the long-run productivity growth induced by schooling and on the-job-training, thus departing from the conventional Diamond OLG model used in this paper. In particular, none of these models is concerned with the role played by interventions on the labour market such as the regulation of wages by law accompanied with an unemployment insurance mechanism, in a context of endogenous fertility. In this paper we will try to fill the gap by developing a standard neoclassical OLG growth model (Diamond (1965)) embodying such features. In particular, we show that the regulation of wages could have a favourable impact on both economic growth\(^2\) and lifetime welfare, on the one hand, and reduce population growth on the other, concluding that under suitable conditions a regulated wage economy may perform better than a market-wage economy, and the level of the regulated wage may also be treated as a policy parameter for controlling fertility. Thus, in this paper we prove that introducing a labour market imperfection in a simple OLG frame may reverse the correlation between unemployment and economic growth.

The plan of the paper is as follows. In section 2 the main steady state results are discussed comparing both competitive and the regulated wage models (which are presented in appendix) while in section 3 a graphical illustration of the results is shown. Finally, section 4 concludes.

2 The Main Results

We characterise a basic (closed economy) dynamic general equilibrium OLG model à la Diamond (1965) with endogenous fertility and regulated wage.\(^3\)

The model together with the list of symbols used are presented in appendix. The distinctive features of the present setup are resumed as follows: 1) following a standard way to endogenise fertility in a conventional OLG framework, by assuming for simplicity that every single young adult can have children, life is divided into three periods: childhood, young adulthood, and old–age. During childhood, individuals do not make any decisions. Adult agents derive utility from (young and old age) consumption and from having children, as in Galor and Weil (1996);\(^4\) 2) rearing children requires a variable cost indexed with the young adult total income \(W_t\), that is, the cost of having a child is simply \(qW_t\) with \(q \in (0,1)\). Note that a child rearing cost linked with the workers’ total income is coherent with the microeconomic dependence of such a cost on the opportunity cost of the parents’ home time which is increasing in their working income (see Cigno (1991)). Likewise, it is natural to conjecture that also the component of the cost of child rearing due to expenditure for children-consumption is positively linked with the working income as well. Moreover, this assumption is rather usual in literature (e.g. Strulik (2004)); 3) we suppose that the unemployment subsidy (\(\gamma W\)) is a

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\(^1\) It is worth noting that in this model where, for simplicity, one type of labour only does exist, a binding minimum wage simply indicates a regulated wage fixed by law. In the case of more than one type of labour with uniformly distributed wages, this assumption would simply mean a regulated wage fixed over the average market wage.

\(^2\) In this paper the term economic growth always refers to the level (rather than to the rate of growth) of the long run income, according to the terminology of the neoclassical growth theory (e.g. Solow (1956) and Mankiw et al. (1992)). In any case, needless to say, an increase in the long run level of output, implies a transitional increase in the rate of growth as well.

\(^3\) Two reference textbooks are Azariadis (1993) and De La Croix and Michel (2002).

\(^4\) Since the scope of the paper is to isolate the relation between a regulated-wage economy and the individuals’ fertility behaviour, as a first attempt we ignore both the trade-off between child quality and quantity and the assumption that parents maximize utility of their offsprings, which has been employed to explain economic growth and stagnation by - among others - Becker et al. (1990) and Ehrlich and Lui (1991). Furthermore, note that the number of children is \(n_t \geq 1\) with \(n_t - 1\) being the population growth rate.
fraction $\gamma \in (0,1)$ of the regulated wage ($w$);\footnote{We assume $w$ to be constant over time.} \footnote{Note that, as regards the steady-state unemployment rate, which, for brevity, is not shown here, $u^*(w) \in (0,1)$ always holds.} \footnote{Table 2 and Remarks 1 and 2 straightforwardly derive from the analysis of the functions $k^*(w), y^*(w)$ and $V^*(w)$. Details are here omitted for economy of space and, of course, they are available on request.} finally, we have deliberately chosen to finance the unemployment insurance scheme with a lump-sum tax on the income of the young adult individuals. In this way the nature of unemployment benefits is purely redistributary, that is income taxed away from the young turned back to the same individuals as a benefit when unemployed. This feature is important because in OLG models, as known dating back to Bertola (1996) and Uhlig and Yanagawa (1996), taxes on the income from capital could lead to faster economic growth since all savings are performed by young agents. Then, in the present model, taxation policy does not cause any transfer from the old-age to the young adulthood (as, instead, it would have been the case with capital income taxes); thus, the positive effect on economic growth should entirely be ascribed to the regulation of wages (and, thus, to the unemployment occurrence) rather than to the intergenerational tax transfer channel.

In the following tables we resume the main analytical steady-state results of both the competitive and regulated wage economies with respect to (see appendix):\footnote{We assume $w$ to be constant over time.}

1) the per-capita stock of capital; 2) the per-capita income $y^*$; and 3) the fertility rate, $n^*$.  

**Table 1.A. Non-Competitive Wage Economy ($w \in (w_{pc}, +\infty)$).**  

<table>
<thead>
<tr>
<th>$k^*$</th>
<th>$y^*$</th>
<th>$n^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu w^\alpha$</td>
<td>$A((1-\alpha)A)^{\alpha-\gamma} \mu w^\alpha$</td>
<td>$\phi \frac{((1-\alpha)A)^{\alpha-\gamma}}{1+\rho}$</td>
</tr>
<tr>
<td>$\frac{1-\alpha}{w^\alpha} - \mu(1-\gamma)((1-\alpha)A)^{\alpha-\gamma}$</td>
<td>$\frac{1-\alpha}{w^\alpha} - \mu(1-\gamma)((1-\alpha)A)^{\alpha-\gamma}$</td>
<td>$\frac{\phi}{1+\rho}$</td>
</tr>
</tbody>
</table>

**Table 1.B. Competitive Wage Economy ($w_{pc}$).**  

<table>
<thead>
<tr>
<th>$k^*$</th>
<th>$y^*$</th>
<th>$n^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(\mu(1-\alpha)A)^{\alpha-\gamma}$</td>
<td>$A(\mu(1-\alpha)A)^{\alpha-\gamma}$</td>
<td>$\frac{\rho}{(1+\rho)q}$</td>
</tr>
</tbody>
</table>

**Table 2. Critical values of the capital weight in technology ($\alpha$), beyond which the regulated wage economy performs better than the competitive wage economy with respect to the long run capital accumulation, output and welfare.**  

<table>
<thead>
<tr>
<th>Condition for a higher capital accumulation</th>
<th>Condition for a higher output</th>
<th>Condition for a higher welfare</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha &gt; \alpha_k = 1 - \gamma$</td>
<td>$\alpha &gt; \alpha_y = \frac{1}{1+\gamma}$</td>
<td>$\alpha &gt; \alpha_v = \frac{1+\gamma(\phi+\rho)}{1+\gamma(1+\phi+\rho)}$</td>
</tr>
</tbody>
</table>

Using Tables 1.A and 1.B together with the inequalities stated in Table 2,\footnote{Table 2 and Remarks 1 and 2 straightforwardly derive from the analysis of the functions $k^*(w), y^*(w)$ and $V^*(w)$. Details are here omitted for economy of space and, of course, they are available on request.} a simple algebra implies that the following remark holds:

**Remark 1. Necessary and sufficient condition for the existence of a value of the regulated wage improving long run income and welfare, is $\alpha > 1/2$.**

In particular as regards the lifetime welfare, differentiating $\alpha_v$ with respect to $\gamma$, $\phi$ and $\rho$ we find that:

$$\frac{\partial \alpha_v}{\partial \gamma} = \frac{-1}{[1+\gamma(1+\phi+\rho)]^2} < 0,$$

(1)
\[
\frac{\partial \alpha_v}{\partial \phi} = \frac{\partial \alpha_v}{\partial \rho} = \frac{\gamma^2}{[1 + \gamma(1 + \phi + \rho)]^2} > 0.
\]  

(2)

From eqs. (1) and (2), the following remark is straightforwardly derived:

**Remark 2.** i) Any increase of the replacement ratio reduces the critical value of the capital weight in technology for which the lifetime welfare is an increasing function of the wage; ii) on the contrary, the more agents prefer to postpone consumption in future periods (\(\phi \uparrow\)), and the more parents are children-interested (\(\rho \uparrow\)), the higher the weight of capital in technology needed to have an increasing welfare function.

Therefore, provided a sufficiently high capital weight and unemployment benefit, both the long run economic growth and the lifetime welfare can be, despite the occurrence of unemployment, higher in a regulated wage economy than in the market-wage frame. Of course, we note that in the short run, as regards welfare, the introduction of regulated wages cannot be Pareto efficient (unless redistributive policies between the two generations living at the time of the introduction of minimum wages are implemented): indeed 1) the young generation may be bettered off (provided that the capital accumulation effect is sufficiently positive); however 2) even if the welfare of the young generation is increased, the old generation always incurs in a welfare loss due to the decreased interest rate. Anyway, the short run welfare analysis is beyond the scope of this paper.

Finally, the effects of the regulation of wages on the rate of fertility are summarised in the following proposition:

**Proposition 1.** Under regulated wages, the long-run population growth rate is always lower than in the market-clearing wage frame, i.e. \(n^*(w) < n^*_{pc}\).

**Proof.** The proof straightforwardly derives from

\[
\frac{\partial n^*(w)}{\partial w} = - \frac{(1-\alpha)\phi}{\alpha(1+\rho)}[(1-\alpha)A]^{1-\frac{1}{\alpha}} w^{-\frac{1}{\alpha}} < 0.
\]

Since \(n^*(w) = n^*_{pc}\) if and only if the non-competitive wage equals the market clearing wage, that is \(w = w_{pc}\), it follows directly that \(n^*(w) < n^*_{pc}\) for any \(w > w_{pc}\).

The result stated in Proposition 1 stems from the role played by the unemployment rate: indeed wages and unemployment have two opposite effects on fertility but, since the unemployment negatively depends on the wage, it occurs that the overall effect of the wage over fertility choices is negative. It is worth to note that the latter proposition shows that the level of the regulated wage may also be treated as a policy parameter for controlling population growth rate.

3 A Graphical Illustration

A simple numerical simulation, for a parametric configuration chosen only for illustrative purposes, may help us in evaluating how capital stock, income, welfare and fertility rate change along with the level of the regulated wage. Figures 1 and 2 show that long-run capital stock, income and welfare are increasing with the regulated wage, indicating that policymaker should fix, in order to maximise both the lifetime welfare of the representative individual as well as economic growth, a regulated wage as high as possible. Figure 3, instead, depicts the negative response of the fertility rate to increases of the regulated wage; due to this “modern” fertility behaviour, when the minimum wage is fixed at a too high level, population becomes stationary or it may even decrease. Overall the three figures clearly show that policymaker may choose a value of the regulated wage aiming to reach as high as possible output and welfare levels compatible with the desired population growth rate.

[Figures 1-3 about here]

4 Conclusions

In this paper we have focused on the steady state effects of the regulation of wages on economic growth, welfare and fertility within a conventional neoclassical OLG model. Our results differ markedly from the conventional wisdom which argues that the regulation of wages creates a lost in efficiency. The reason for this wisdom is that it implicitly assumes a static context where production factors are fixed, whereas in a dynamic overlapping generations frame, where capital accumulation is
affected by wages, such a wisdom could be incorrect.\textsuperscript{8} Indeed, in the paper we have showed that the introduction of regulated wages could increase both economic growth and the lifetime welfare, on the one hand, and reduce the fertility rates on the other. Therefore, we conclude that under suitable conditions a regulated wage economy may perform better than a market-wage economy, and the level of the regulated wage may also be treated as a policy parameter for controlling fertility. Moreover we note that, particularly as regards the developing countries, where low wages, low economic growth, underdeveloped welfare states and high fertility rates are current stylised facts, the findings of this work also offer some interesting policy implications. The interest of these results lies in: 1) the relevance of their messages showing a new perspective for the regulation of wages, and 2) the simplicity with which are obtained, that is within a standard dynamic general equilibrium overlapping generations model where the departures from the textbook OLG frame are simply the assumptions of endogenous fertility and wages regulated by law.
In particular, in contrast with the prevailing literature, we have shown that introducing regulated labour market in a simple OLG setup may reverse the correlation between unemployment and economic growth, and the higher the long-run unemployment rate is the higher both the long-run output and the lifetime welfare will be.

Appendix

We present here a simple (closed economy) dynamic general equilibrium OLG model with endogenous fertility and regulated wages.

The list of symbols used is the following:

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$N$</td>
<td>young adult population</td>
</tr>
<tr>
<td>$n$</td>
<td>number of children</td>
</tr>
<tr>
<td>$w$</td>
<td>regulated wage</td>
</tr>
<tr>
<td>$w_{PC}$</td>
<td>competitive wage</td>
</tr>
<tr>
<td>$W$</td>
<td>total income of the young adult agent</td>
</tr>
<tr>
<td>$r$</td>
<td>rate of return on savings</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Savings</td>
</tr>
<tr>
<td>$c_y$</td>
<td>young-age consumption</td>
</tr>
<tr>
<td>$c_o$</td>
<td>old-age consumption</td>
</tr>
<tr>
<td>$\phi$</td>
<td>consumption preference parameter</td>
</tr>
<tr>
<td>$\rho$</td>
<td>preference for children parameter</td>
</tr>
<tr>
<td>$q$</td>
<td>fraction of the young adult income for rearing one child</td>
</tr>
<tr>
<td>$K, k = K/N$</td>
<td>total and per-capital stock of capital</td>
</tr>
<tr>
<td>$L$</td>
<td>employed young population</td>
</tr>
<tr>
<td>$Y, y = Y/N$</td>
<td>total and per-capital output</td>
</tr>
<tr>
<td>$A$</td>
<td>technology index</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>capital weight in the production function</td>
</tr>
<tr>
<td>$u$</td>
<td>unemployment rate</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>replacement ratio</td>
</tr>
<tr>
<td>$\tau$</td>
<td>lump-sum tax on young adult</td>
</tr>
<tr>
<td>$\mu$</td>
<td>$\phi \eta / \rho$</td>
</tr>
</tbody>
</table>

Government. The unemployment benefit system is financed by the government who levies and adjusts over time lump-sum taxes on the young-adult income in order to balance out its budget in each period. Therefore, the time-$t$ government constraint is simply:

$$\gamma wu_t = \tau_t. \quad (A1)$$

\textsuperscript{8} In a nutshell, to better understand the economic reasons of why the introduction of a minimum wage may favour both long-run economic growth and welfare, it is sufficient to say that it acts - although only indirectly - as a reversed social security scheme: that is, in principle, it transfers resources over time from the old to the young by raising the labour income and decreasing the interest rate.
Individuals. The behaviour of individuals is usual as in the standard OLG model with endogenous fertility behaviour. The only departure is that the wage is regulated and, thus, unemployment occurs. Each young adult individuals earns a regulated wage \( \frac{W}{u} \) for the hours of work, whereas she receives an unemployment bonus for the unemployed time. The aggregate unemployment rate is \( u_t = \frac{N_t - L_t}{N_t} \), where \( L_t = (1 - u_t)N_t \) is the total number of hours worked by the young during adulthood. Note that in this model there is no uncertainty. Thus, each young-adult person will be employed for \( 1 - u_t \) hours and unemployed for \( u_t \) hours. Furthermore, we also assume that unemployed hours are without economic value. The use of the unemployment time for self-enrichment activities or for exploiting home production technologies are interesting extensions which are beyond the scope of the present paper.

Individuals of generation \( t \) are faced with the problem of maximising the following logarithmic felicity function:\(^9\)

\[
\max_{k_t, y_t, \phi, n_t} U_t \left( c_t^y, c_{t+1}^\phi, n_t \right) = \left( 1 - \phi \right) \ln \left( c_t^y \right) + \phi \ln \left( c_{t+1}^\phi \right) + \rho \ln \left( n_t \right),
\]

subject to

\[
c_t^y + c_{t+1}^\phi / (1 + r_{t+1}) = W_t (1 - qn_t) - \tau_t,
\]

where \( W_t \left( w \right) = w (1 - u_t) + \gamma w u_t \) is the total income of young-adult individuals. Thus, by using the first order conditions together with eq. (A1), the optimal number of children and the optimal savings function are given by:

\[
n_t \left( w \right) = \frac{\rho}{1 + \rho} \left[ \frac{1 - u_t}{q[1 - u_t(1 - \gamma)]} \right],
\]

\[
s_t \left( w \right) = \frac{\phi}{1 + \rho} w (1 - u_t).
\]

Firms. All the firms in the economy are identical and own a constant returns to scale Cobb-Douglas production technology \( Y_t = AK_t^a L_t^{1-a} \).\(^11\) Given the labour demand, \( L_t = (1 - u_t)N_t \), the per-capita production function is given by:

\[
y_t = A (1 - u_t) (k_t / (1 - u_t))^a.
\]

Assuming that final goods and services are traded at a unit price, profit maximisation leads to the following marginal conditions for capital and labour:\(^12\)

\[
r_t = \alpha A (k_t / (1 - u_t))^{a-1} - 1. \quad (A5)
\]

\[
w = (1 - \alpha) A (k_t / (1 - u_t))^a. \quad (A6)
\]

Since workers are paid with a higher wage than the one that clears the labour market, the representative firms hires as many workers as dictated by its perceived labour demand curve at the minimum wage level preset by law. Thus, a positive equilibrium unemployment does occur. The endogenous current unemployment rate is computed - by using eq. (A6) - as follows:

\[
u_t \left( k_t, w_t \right) = 1 - \left( \left( 1 - \alpha \right) A / w_t \right)^{1-a} \cdot k_t. \quad (A7)
\]

Note that in this case \( r \left( w \right) = \alpha A \left( (1 - \alpha)A / w \right)^{1-a} - 1 \) is a constant (that is, capital returns are independent of

\(^9\) We assume \( w_t \) to be constant over time.

\(^10\) We have chosen, as usual, the Cobb-Douglas utility specification for its analytical tractability, although it implies, as known, the restrictive assumption that the elasticity of savings with respect to the interest rate is equal to zero. Anyway, by considering a more general CIES utility function, where the interest rate affects the individual’s savings, it can be seen via numerical simulations (for economy of space not reported here) that the main findings of this paper are, a fortiori, confirmed, provided that the elasticity of savings with respect to the interest rate is not too much positive.

\(^11\) Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and, hence, it is not included here.

\(^12\) For simplicity, we assume that capital totally depreciates over time, i.e. \( \delta = 1 \). This assumption is not unrealistic in the present setup, because as noticed by De La Croix and Michel (2002), p. 338 “even is one assumes a rather low annual depreciation rate of 5%, 79% of the stock of capital is depreciated after 30 years”.

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the capital stock) and any increase in \( w \) always pushes down the real interest rate below the competitive level.

The Long-Run Equilibrium. The market-clearing condition is given by the equality between savings and investments, that is \( n_kr+1 = s \). Substituting out for \( n \) and \( s \), according to (A2) and (A3) respectively, and using eq. (A7), capital stock evolves according to:

\[
k_{r+1} = \mu y w + \mu(1-\gamma)((1-\alpha)A)^{1/w} \frac{1}{\gamma w} k^*(w),
\]

(A8)

where \( \mu := \phi q / \rho \).

From eq. (A8) steady state results are straightforwardly derived and presented in table 1.A. It is worth to note that the equilibrium is globally stable for whatever regulated wage level. The proof straightforwardly derives by the inspection of eq. (A8). For the sake of brevity we do not report here the complete proof which is of course available on request.

After having discussed the economic growth outcomes of the model we turn to the welfare analysis, which has been carried out in terms of comparing steady state paths of the lifetime welfare of the representative generation, following, among many others, Samuelson (1975).

The steady-state young-adult and old-age consumption are the following:

\[
c^y(w) = \frac{1-\phi}{1+\rho}(1-\alpha)A^\frac{1}{\omega} \frac{1}{\omega} k^*(w), \tag{1}
\]

and

\[
c^o(w) = \frac{\phi}{1+\rho}(1+r(w))(1-\alpha)A^\frac{1}{\omega} \frac{1}{\omega} k^*(w). \tag{2}
\]

The government’s objective is to maximise the following representative generation indirect utility function:

\[
V^*(w) = (1-\phi)\ln(c^y(w)) + \phi \ln(c^o(w)) + \rho \ln(n^*(w)). \tag{3}
\]

We may then proceed to derive explicitly a parametric condition for the OLG model under consideration to analyse the relationship among regulated wage and welfare.

The first order conditions are given by:

\[
\frac{\partial V^*(w)}{\partial w} = 0 \iff \frac{1-\phi}{c^y(w)} \frac{\partial c^y(w)}{\partial w} + \frac{\phi}{c^o(w)} \frac{\partial c^o(w)}{\partial w} + \frac{\rho}{n^*(w)} \frac{\partial n^*(w)}{\partial w} = 0. \tag{4}
\]

From eq. (4), a critical value of the capital weight ensuring that regulated wages are always welfare-preferred, is derived and shown in Table 2.

Finally, when the market-clearing wage prevails and thus the unemployment rate is zero, the standard steady state results are shown in table 1.B. The building of the Diamond OLG model with competitive wage and endogenous fertility as well as the derivation of its steady state outcomes in terms of capital stock, output, fertility rate and utility (i.e. welfare) are rather conventional and for the sake of brevity here omitted. Of course they are available on request.

References


13 It is worth to note that in this paper we only perform a positive rather than a normative analysis.
Figure 1. The long-run stock of capital and the long-run income in both the market-wage (\(w_{pc}\)) and regulated-wage (\(w\)) economies. \(y^*(w, \gamma)\) is scaled 1:10. The starting point of the horizontal axis is the market-clearing wage, that is \(w_{pc} = 27.83\). Parameter set: \(A = 100\), \(\alpha = 0.55\), \(\phi = 0.03\), \(\gamma = 0.95\), \(\rho = 0.10\) and \(q = 0.05\).

\[\text{In order to better clarify the meaning of the coefficient } \alpha \text{ (the capital weight in technology), it is worth noting that a possible interpretation is that the capital stock may be thought in its broad concept, including physical and human components and that the labour input only includes non-specialised labour. In fact, as argued by Mankiw et al. (1992), p. 417, the minimum wage may be thought to be a proxy of the return to labour without human capital; they suggest that since the minimum wage has averaged about 30 to 50 percent of the average wage in manufacturing, then 50 to 70 percent of total labour income represents the return to human capital, so that if the physical capital’s share of income is expected to be about 1/3, the human capital’s share of income should be between 1/3 and one half. In sum, with the broad view of capital the coefficient } \alpha \text{ may be fairly about 0.6 and 0.8. Indeed, for instance, Barro and Sala-i-Martin (2003), p. 110), used } \alpha = 0.75 \text{ saying that: “Values in the neighbourhood of 0.75 accord better with the empirical evidence, and these high values of } \alpha \text{ are reasonable if we take a broad view of capital to include human components.”}\]
Figure 2. The long-run lifetime welfare in both the market-wage ($w_{pc}$) and regulated-wage ($w$) economies. The starting point of the horizontal axis is the market-clearing wage, that is $w_{pc} = 27.83$. Parameter set: $A = 100$, $\alpha = 0.55$, $\phi = 0.03$, $\gamma = 0.95$, $\rho = 0.10$ and $q = 0.05$.

Figure 3. The long-run fertility rate in both the market-wage ($w_{pc}$) and regulated-wage ($w$) economies. The starting point of the horizontal axis is the market-clearing wage, that is $w_{pc} = 27.83$. Parameter set: $A = 100$, $\alpha = 0.55$, $\phi = 0.03$, $\gamma = 0.95$, $\rho = 0.10$ and $q = 0.05$. 