PUBLIC EDUCATION, FERTILITY INCENTIVES, NEOCLASSICAL ECONOMIC GROWTH AND WELFARE

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ABSTRACT

Using a simple overlapping generations model of neoclassical growth we analyse the effects of both child allowances and the system of public education on the rate of fertility, the per capita income and the individual lifetime welfare, with particular attention to the long-run interaction between education and child benefit policies. The essential message of the present paper is that developed countries plagued by below-replacement fertility and income stagnation may raise per capita income and the rate of fertility at the same time by increasing the public education expenditure rather than resorting to child allowances. The latter in fact are found to be harmful for the long-run neoclassical economic growth and, in contrast with the common belief, for the rate of population growth as well. Moreover, the welfare analysis has shown the existence of a Pareto-efficient welfare-maximising educational contribution rate.

Keywords: public education, child policy, OLG model

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I. INTRODUCTION

In several developed countries plagued by below-replacement fertility (e.g., Germany, Italy, Japan and Spain) policymakers aimed at increasing the rate of population growth (and, hence, the ratio of economically active workers to total population) by resorting essentially to child benefit policies[1] – in particular, to avoid dramatic consequences to the viability of the generous and widely adopted pay-as-you-go (PAYG) pension budgets.

The most relevant pillars of public intervention (welfare state) in developed countries – especially in Europe – may be considered the social security system, the education system and the child benefit system.[2] The economic literature has developed models embodying both the first and the third (see, amongst others, Peters, 1995; van Groezen et al. 2003; Fenge and Meier, 2005, 2009; van Groezen and Meijdam, 2008) or the first two ones (e.g., Pogue and Sgontz, 1977; Becker and Murphy, 1988; Pecchenino and Pollard, 2002; Bommier et al., 2004; Boldrin and Montes, 2005; Žamac, 2007). However, to the best of our knowledge, scarce attention has been paid to models which instead take into account the existence of an interaction between the public education system and the child benefit system. In this paper, therefore, we will try to fill the gap by developing a standard general equilibrium overlapping generations (OLG) model of neoclassical growth embodying such features to investigate whether and how the child benefit policy and the system of public education affect individual fertility, the level of per capita income and the individual lifetime welfare in the long run.

[1] For instance, in Italy a 1,000 euro child grant for each new born was introduced in the year 2005; moreover, the current Italian Government has even suggested to provide a 2,500 euro benefit per year for each child from zero up to three years of life with a progressive extension up to 18 years of life. In Poland every woman will benefit from a one-off 258 euro payment for each child, and women from poorer families will receive double the previous amount.

[2] It is worth noting that another pillar of the welfare state is the health care system: the interaction between such a system and the other sectors of the economy is the object of a growing body of economic literature.

Recently Fanti and Gori (2007), building on a textbook OLG model of neoclassical growth without schooling expenditure, have shown that the public provision of child allowances, especially for economies experiencing a relatively high capital share in production and/or a relatively low degree of individual thriftiness, may be dramatically fertility-reducing. In the present paper, we test for the robustness of this result in a context in which a child benefit policy and a system of public education do coexist. It will be proved that when the government finances a certain amount of schooling expenditure, the introduction of child allowances – although it is expected to be detrimental for the long-run per capita income – always depresses individual fertility. By contrast, a policymaker may increase the rate of population growth and the neoclassical economic growth at the same time by implementing a public education system financed at balanced budget with labour income taxes.

Moreover, the welfare analysis shows that depending on the value of the distributive capital share in production, the maximisation of the steady-state lifetime indirect utility index of the representative generation is Pareto efficient, while the introduction of child allowances (taxes) is always Pareto-worsening.

The policy implications are interesting: providing an increasing amount of resources to the education system at the expense of the child benefit system may enhance the (neoclassical) economic growth and it may also promote both population growth and welfare. As a consequence, developed countries plagued by below-replacement fertility and income stagnation may achieve the goal of both higher fertility and higher per capita income in the long run by expanding the schooling expenditure rather than resorting to child benefit policies. Moreover, we have shown that the public provision of child allowances is detrimental not only for economic growth but, rather interestingly, even for the rate of population growth.

Our findings also represent a policy warning, especially for a country such as Italy, which – if compared with the other developed countries – is one of the worst plagued by low fertility and income stagnation, and where scarce public expenditure on education and generous child allowance systems do coexist. Interestingly, we also picked up the long-run fertility-maximising and the long-run saving-maximising values of the educational contribution rate, and both are equal to the share of the human capital in the production of final goods and services. However, the maximisation of both the fertility rate and the saving rate is always inefficient because the welfare of the representative individual can be still increased by raising the educational contribution rate.

The remainder of the paper is organised as follows. In Section II we develop the model. In Section III the main steady-state results are analysed and discussed. In Section IV we study the welfare effects of the public expenditure on education. Section V concludes.

II. THE MODEL

II.1. Individuals

Agents have identical preferences and live for three periods (childhood, young adulthood and old-age) in an OLG economy. During childhood individuals do not make economic decisions. Adult individuals belonging to generation t (N_t) have a homothetic and separable lifetime utility function (U_t) defined over $c_{1,t}$, $c_{2,t+1}$ and n_t,[3] that is, working period consumption, retirement period consumption and the number of children, respectively.[4] Only young-adult individuals – endowed with h_t units of human capital in every period – join the workforce and supply labour inelastically on the labour market, while receiving wage income at the competitive rate w_t (paid for one efficiency unit of labour supplied at time t). This income is used to consume, to bear children, to pay taxes and to save. We assume that raising children requires a fixed cost $m > 0$ per child

[3] Note that the variable n represents the number of children with $n-1$ being the population growth rate.

[4] The point of view arguing that parents derive utility directly from having children but not from the utility of their children – namely a weak form of altruism – has been suggested early by Eckstein and Wolpin (1985) and it has been largely assumed recently (e.g., amongst many others, Galor and Weil, 1996 and van Groezen et al., 2003).

measured in units of market goods. Moreover, the government provides a per child monetary transfer, $\beta > 0$, to support child-rearing of households.

When old individuals are retired and consume on the basis of their own past savings (s_t) plus the accrued interest at the rate r_{t+1}.

Therefore, the representative individual entering the working period at time t faces the following problem:

$$\max_{\{c_{1,t}, c_{2,t+1}, n_t\}} U_t(c_{1,t}, c_{2,t+1}, n_t) = (1-\phi)\ln(c_{1,t}) + \rho \ln(c_{2,t+1}) + \phi \ln(n_t), \tag{P}$$

subject to

$$\begin{aligned} c_{1,t} + s_t &= w_t h_t (1 - \theta - \tau_t) - (m - \beta) n_t \\ c_{2,t+1} &= (1 + r_{t+1}) s_t \end{aligned},$$

where $0 < \tau_t < 1$ and $0 < \theta < 1$ represent the labour income taxes used to finance the child allowance system and the public education system, respectively, $0 < \rho < 1$ is the subjective discount factor and $0 < \phi < 1$ captures the importance in the welfare function of raising children relative to material consumption when young, i.e., the higher ρ is the more agents want to smooth consumption over time, and the higher ϕ is the more parents are children-interested.

The necessary and sufficient conditions for an interior solution are given by:

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1-\phi}{\rho} = 1 + r_{t+1}, \tag{1}$$

$$\frac{c_{1,t}}{n_t} \cdot \frac{\phi}{1-\phi} = m - \beta. \tag{2}$$

Eq. (1) equates the marginal rate of substitution between working period and retirement period consumption to their relative prices, whereas Eq. (2) equates the marginal rate of substitution between material consumption when young and the number of children to the involved marginal costs of raising an extra child. Note that a necessary and sufficient condition for the existence of a finite positive solution for n_t is $m-\beta>0$, that is, the net marginal cost of raising children must be strictly positive.

Exploiting (1), (2) and the lifetime budget constraint, the demand for children and the saving function are respectively given by:

$$n_t = \frac{\phi}{1+\rho} \cdot \frac{w_t h_t (1-\theta-\tau_t)}{m-\beta}, \tag{3}$$

$$s_t = \frac{\rho}{1+\rho} \cdot w_t h_t (1-\theta-\tau_t). \tag{4}$$

II.2. Government

Following van Groezen et al. (2003) and Žamac (2007), we assume that the government runs two distinct balanced budget policies in every period, and then uses the revenues so collected to finance separately both the child benefit and the schooling expenditures.

The child subsidy is assumed to be entirely financed by levying and adjusting over time a labour income tax. Therefore, the time-t child policy budget reads as:

$$\beta n_t = \tau_t w_t h_t, \tag{5}$$

where the left-hand side represents the per capita childcare expenditure and the right-hand side the per capita tax receipts.

As regards the public education plan, we assume that the education expenditure (g_t) is adjusted and balanced by the government according to

$$g_t n_t = \theta w_t h_t, \tag{6}$$

where the left-hand side represents the per capita schooling expenditure and the right-hand side the per capita tax receipts.

It is worth noting that the wage tax rate to finance child benefits depends on the time period while the wage tax rate to finance the schooling expenditure is taken to be constant, that is, the government fixes a contribution rate to finance the educational expenditure, while adjusting the latter from period to period to balance out the budget. This because we want to capture the effect of the per child schooling expenditure on the accumulation of human capital in equilibrium, given the value of the policy instrument θ. The per child benefit expenditure (β), instead, has been kept constant because we are interested in analysing the long-run effects of child allowances on both fertility and income in the long run.[5]

Now, inserting (5) into (3) to eliminate τ_t and rearranging terms yields:

$$n_t = \frac{\phi w_t h_t (1-\theta)}{(1+\rho)(m-\beta)+\phi\beta}. \tag{7}$$

Then, combining Eqs. (4), (5) and (7), the saving function becomes:

$$s_t = \frac{\rho(m-\beta) w_t h_t (1-\theta)}{(1+\rho)(m-\beta)+\phi\beta}. \tag{8}$$

II.3. Human capital

The public expenditure on education creates a positive externality on the accumulation of human capital by increasing the future productivity of children. In particular, we assume that the human capital evolves according to the following Cobb-Douglas learning technology, where the skills produced in period $t+1$ depend on the public education expenditure as well as on the existing level of human capital, that is (following an usual formulation, e.g., Glomm and Ravikumar, 1997; Žamac, 2007):

$$h_{t+1} = B h_t^{\mu} g_t^{1-\mu}, \tag{9}$$

where $B > 0$ is a scale parameter and $0 < \mu < 1$.[6]

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[5] Analysis of different combinations of policy parameters, e.g., assuming either the child allowance or the educational contribution rate to be time-dependent, and thus adjusted by the government to balance the budget in every period, could be an interesting extension of the present paper.

[6] Human capital is embodied in the workers and thus it is natural to assume that it is completely depreciated when workers are retired.

II.4. Firms

As regards the production of final goods and services, firms are assumed to be identical and to act competitively on the market. The (aggregate) constant returns to scale Cobb-Douglas technology is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t h_t$ are output, capital and the time-t labour input (in efficiency units), respectively, $A > 0$ represents a scale parameter and $0 < \alpha < 1$ is the share of physical capital in production. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as per capita capital stock and output respectively, the intensive form production function may be written as:

$$y_t = Ak_t^{\alpha} h_t^{1-\alpha} . \tag{10}$$

Assuming that physical capital totally depreciates at the end of each period and knowing that the price of final output is normalised to unity, profit maximisation leads to the following marginal conditions for capital and labour in efficiency units, respectively:

$$r_t = \alpha A \left(\frac{k_t}{h_t} \right)^{\alpha - 1} - 1 , \tag{11}$$

$$w_t = (1 - \alpha) A \left(\frac{k_t}{h_t} \right)^{\alpha} . \tag{12}$$

II.5. Equilibrium

Exploiting Eqs. (5) and (6), and knowing that population evolves according to $N_{t+1} = n_t N_t$, the market-clearing condition in goods as well as in capital markets is expressed by the equality $n_t k_{t+1} = s_t$, that is, the per capita stock of capital installed in period $t+1$ equals the amount of resources saved in period t discounted by the number of individuals in the same period. Substituting out for n and s from Eqs. (7) and (8) respectively, the equilibrium condition boils down to the following per capita stock of capital in the long run

$$k_{t+1} = k^* = \frac{\rho}{\phi}(m - \beta). \tag{13}$$

As can readily be seen from Eq. (13), the per capita stock of capital approaches the steady-state after one period only. Moreover, a rise in the child allowance always reduces the capital stock in the long run, that is, $\frac{\partial k^*}{\partial \beta} = -\frac{\rho}{\phi} < 0$ (i.e., the higher the child grant, the lower the saving rate and – owing to a reduced cost of child rearing – the higher the demand for children in the short run), while being unaffected by the educational contribution rate.

Exploiting Eqs. (6) and (7), the public expenditure on education in equilibrium is:

$$g^* = \frac{\theta}{1-\theta} \cdot \frac{(1+\rho)(m-\beta) + \phi\beta}{\phi}. \tag{14}$$

From Eq. (14) the following remark holds:

Remark 1 A rise in β *(*θ*) always reduces (increases)* g^*.

Therefore, from Eqs. (9) and (14) the steady-state level of human capital, which is therefore negatively (positively) related with the child allowance (the schooling expenditure), is determined, after one period only, by:

$$h^* = Z \cdot g^*, \tag{15}$$

where $Z := B^{\frac{1}{1-\mu}}$.

III. FERTILITY, SAVINGS AND INCOME IN THE LONG RUN

III.1. Fertility in the long run

We now investigate how the child subsidy and the system of public education affect the long-run individual fertility.

Given Eq. (7), we can express the long-run fertility rate as a generic function of β and θ as

$$n^* = n^*\{\beta, \theta, w^*[k^*(\beta), h^*(\beta,\theta)], h^*(\beta,\theta)\}. \tag{16}$$

Totally differentiating Eq. (16) with respect to β yields:[7]

$$\frac{dn^*}{d\beta} = \overbrace{\frac{\partial n^*}{\partial \beta}}^{+} + \underbrace{\overbrace{\frac{\partial n^*}{\partial h^*}}^{+} \cdot \overbrace{\frac{\partial h^*}{\partial \beta}}^{-}}_{-} + \underbrace{\overbrace{\frac{\partial n^*}{\partial w^*}}^{+} \cdot \left(\underbrace{\overbrace{\frac{\partial w^*}{\partial k^*}}^{+} \cdot \overbrace{\frac{\partial k^*}{\partial \beta}}^{-}}_{-} + \underbrace{\overbrace{\frac{\partial w^*}{\partial h^*}}^{-} \cdot \overbrace{\frac{\partial h^*}{\partial \beta}}^{-}}_{+} \right)}_{+/-}. \tag{17}$$

Eq. (17) reveals that the final effect of a rise in the child allowance on the long-run rate of fertility depends on three counterbalancing forces, and thus appears to be ambiguous: (*i*) a positive (direct) effect which tends to increase fertility by decreasing the cost of children;[8] (*ii*) a negative (indirect) general equilibrium effect which acts negatively on fertility due to the reduced human capital accumulation; (*iii*) an ambiguous (indirect) general equilibrium feedback effect which acts on fertility through (*iii*.1) a reduced wage rate owing to a lower stock of physical capital and (*iii*.2) an increased wage rate because of the reduced human capital accumulation. Given the positive relationship between fertility and wages, if the physical (human) capital effect prevails, then the lower (higher) the wage rate, the lower (higher) the demand for children.

As regards the effects of a rise in the educational contribution rate, the total derivative of Eq. (16) with respect to the θ gives:

$$\frac{dn^*}{d\theta} = \overbrace{\frac{\partial n^*}{\partial \theta}}^{-} + \underbrace{\overbrace{\frac{\partial n^*}{\partial h^*}}^{+} \cdot \overbrace{\frac{\partial h^*}{\partial \theta}}^{+}}_{+} + \underbrace{\overbrace{\frac{\partial n^*}{\partial w^*}}^{+} \cdot \overbrace{\frac{\partial w^*}{\partial h^*}}^{-} \cdot \overbrace{\frac{\partial h^*}{\partial \theta}}^{+}}_{-}. \tag{18}$$

From Eq. (18) we can see that that the final effect of a rise in θ on the long-run fertility rate is ambiguous and depends on three counterbalancing forces: (*i*) a negative (direct) effect which,

[7] Details of Eqs. (17) and (18) are given in the Appendix.

[8] Given the assumption of logarithmic preferences, each child is evaluated as a normal good by parents and thus the substitution effect prevails on the income effect as a consequence of the reduction in the price of children owing to the public provision of child allowances.

through the reduced disposable working income of the young people, tends to decrease fertility, (*ii*) a positive (indirect) general equilibrium feedback effect which promotes fertility owing to a higher human capital accumulation, and, finally, (*iii*) an indirect general equilibrium feedback effect which reduces the demand for children given the lower wage rate earned by the young following a rise in the accumulation of human capital determined by the higher public educational expenditure.

To analyse ultimately which of the forces dominates when either the child allowance or the educational contribution rate changes, we now combine Eqs. (7) and (12)-(15) to obtain:

$$n^{*}(\beta,\theta)=(1-\alpha)AZ^{1-\alpha}\,\theta^{1-\alpha}(1-\theta)^{\alpha}\left[\frac{\rho(m-\beta)}{(1+\rho)(m-\beta)+\phi\beta}\right]^{\alpha}, \qquad (19)$$

From Eq. (19) we may derive the following propositions:

Proposition 1 (Effects of the child benefit expenditure on the rate of fertility in the long run). Let $0<\theta<1$ *hold. Then the long-run rate of fertility is always lower than whether the child allowance is not introduced at all.*

Proof. The proof straightforwardly derives from:

$$\frac{\partial n^{*}(\beta,\theta)}{\partial\beta}=\frac{-n^{*}(\beta,\theta)\phi\alpha m}{(m-\beta)[(1+\rho)(m-\beta)+\phi\beta]}<0,$$

for any $0\leq\beta<m$ and $0<\theta<1$. Q.E.D.

Proposition 2 (Effects of the public expenditure on education on the rate of fertility in the long run). Let $0\leq\beta<m$ *hold. Then the long-run fertility rate is maximised at* $\theta=\overline{\theta}$ *.*

Proof. The proof uses the following derivative:

$$\frac{\partial n^{*}(\beta,\theta)}{\partial\theta}=\frac{n^{*}(\beta,\theta)(1-\alpha-\theta)}{\theta(1-\theta)}.$$

Therefore,

$$\frac{\partial n^*(\beta,\theta)}{\partial\theta}\begin{matrix}>\\ <\end{matrix}0 \Leftrightarrow \theta \begin{matrix}<\\ >\end{matrix}\bar{\theta},$$

for any $0 \leq \beta < m$, with $\bar{\theta} := 1-\alpha$ being an interior global maximum. Q.E.D.

Proposition 1 shows that the rate of fertility is always lower than whether the child allowance is not introduced at all. This occurs because of the general equilibrium effect which reduces the human capital accumulation and thus discourages fertility: this negative effect always rules over both the direct positive effect of the child subsidy (which reduces the cost of children) and the ambiguous general equilibrium effect, independently of whether the latter is positive or negative.

Proposition 2 reveals that the rate of fertility is non-monotonic in θ. In particular, there exists a fertility-maximising value of the educational contribution rate which corresponds exactly to the weight of the human capital in technology. The higher the relative importance of human capital in production is, the higher the effectiveness of the schooling expenditure as an instrument for the actual rise of the population growth rate in the long run. In particular, when the educational contribution rate is low enough, the positive effect of the human capital accumulation dominates and thus the demand for children rises up to the point in which a further increase in θ leads to lower fertility, because of (*i*) the negative direct effect of the educational contribution rate in the short run (which reduces the disposable income of the working-age generation), and (*ii*) the negative general equilibrium wage effect of the increased human capital accumulation in the long run. Note that the value of the educational contribution rate that marks the start of the fertility-declining region (where both the positive and the negative effects of θ on the demand for children match each other exactly) is simply the weight of human capital in the production function.

Therefore, governments aiming at increasing the population growth rate (especially in countries plagued by below-replacement fertility) should rise the public expenditure on education rather then resort to the more traditional child allowance instrument; the latter in fact, although it is expected to reduce per capita income in the long run, is always dramatically fertility-reducing when an interaction with the education system exists and individuals accumulate human capital through a learning technology.

III.2. Savings in the long run

As regards the effects on the long-run individual saving function

$$s^*(\beta,\theta)=\frac{\rho}{\phi}(m-\beta)\cdot n^*(\beta,\theta), \tag{20}$$

of both the child benefit system and the public education system, the following proposition holds:

Proposition 3 (Effects of the child benefit expenditure on the saving rate in the long run). Let $0<\theta<1$ *hold. Then the long-run saving rate is always lower than whether the child allowance is not introduced at all.*

Proof. The proof uses the following derivative:

$$\frac{\partial s^*(\beta,\theta)}{\partial \beta}=\frac{-s^*(\beta,\theta)[(1+\rho)(m-\beta)+\phi(\beta+\alpha m)]}{(m-\beta)[(1+\rho)(m-\beta)+\phi\beta]}<0,$$

for any $0\leq\beta<m$ and $0<\theta<1$. Q.E.D.

Proposition 4 (Effects of the public expenditure on education on the saving rate in the long run). Let $0\leq\beta<m$ *hold. Then the long-run saving rate is maximised at* $\theta=\overline{\theta}$ *.*

Proof. The proof straightforwardly derives from:

$$\frac{\partial s^*(\beta,\theta)}{\partial \theta}=\frac{s^*(\beta,\theta)(1-\alpha-\theta)}{\theta(1-\theta)}.$$

Therefore,

$$\frac{\partial s^{*}(\beta,\theta)}{\partial\theta} \gtrless 0 \Leftrightarrow \theta \lessgtr \overline{\theta},$$

for any $0 \le \beta < m$, with $\overline{\theta}$ being an interior global maximum. Q.E.D.

Propositions 2 and 4 reveal that the effects of the educational contribution rate on both fertility and savings in the long run are exactly the same. In particular, even the fertility-maximising and the saving-maximising values of the educational contribution rate both are the same.[9] Therefore, since the equilibrium stock of capital is obtained as the ratio between the saving rate and the number of persons, the effect of θ on k^{*} is ruled out.

III.3. Income in the long run

As regards the relationship between per capita income and both the child allowance system and the public education system in the long run, the following propositions hold:

Proposition 5 (Effects of the child benefit expenditure on the long-run per capita income). Let $0 < \theta < 1$ *hold. Then the public provision of child allowances monotonically reduces the long-run per capita income.*

Proof. The proof uses the following derivative:

[9] It is worth noting that this result, showing the maximising effect of the equivalence between a tax rate and the share of human capital as a productive input, seems to be a reminiscence of the Barro's (1990) result, who analysed the effects of a productive government expenditure in an endogenous growth context, rather than studying the role of schooling expenditure in a model of neoclassical growth. Roughly speaking, while in Barro (1990) the endogenous growth rate is maximised by equalising the income tax to the share attributed to the public input in production, in our model the government should set the wage tax (corresponding to the percentage ratio between per capita schooling expenditure – gn – and per capita income – wh) to equal the share attributed to the human capital in the production function, in order to maximise both the fertility rate and the saving rate.

$$\frac{\partial y^*(\beta,\theta)}{\partial \beta} = \frac{-y^*(\beta,\theta)[(1+\rho-\phi)(m-\beta)+\alpha\phi m]}{(m-\beta)[(1+\rho)(m-\beta)+\phi\beta]} < 0,$$

for any $0 \le \beta < m$ and $0 < \theta < 1$. Q.E.D.

Therefore the introduction of a child allowance, used by the policymaker with pro-natalist purposes, has an expected negative effect on the long-run per capita income as well as an unexpected negative effect on the long-run individuals' fertility.

Proposition 6 (Effects of the public expenditure on education on the long-run per capita income). Let $0 \le \beta < m$ hold. Then the public expenditure on education monotonically increases the long-run per capita income.

Proof. The proof uses the following derivative:

$$\frac{\partial y^*(\beta,\theta)}{\partial \theta} = \frac{y^*(\beta,\theta)(1-\alpha)}{\theta(1-\theta)} > 0,$$

for any $0 \le \beta < m$ and $0 < \theta < 1$. Q.E.D.

Although the capital stock is not affected by the educational contribution rate θ, the other component of the production function, the human capital input, is a positive monotonic function of the schooling expenditure (see Eqs. 14 and 15). This is the reason why the long-run output rises monotonically along with the educational contribution rate.

In contrast with the child allowance policy, a public education system may promote both per capita income and the population growth rate in the long run. In particular, while per capita income is monotonically increasing with the public schooling expenditure (see Proposition 6), the positive effect of the educational contribution rate on the population growth rate holds up to the point in which the latter is equal to the share of human capital in production; beyond such a level the demand for children shrinks (see Proposition 2). Therefore, the following proposition holds:

Proposition 7 Let $0 \leq \beta < m$ *hold. The government can increase the level of per capita income and the population growth rate at the same time with a public education policy such that* $0 < \theta \leq \overline{\theta}$.

Proof. The proof is obvious given the results stated in Propositions 2 and 6. Q.E.D.

Therefore, the negative relationship between per capita income and population growth – which is a tenet of the standard neoclassical growth theory – can be dramatically reconsidered, since the government can let both the population growth rate and the (neoclassical) economic growth rate rise by implementing a public education system financed at balanced budget with labour income taxes.

However, it is worth noting that from an economic point of view, increasing either the fertility rate or the saving rate cannot be an objective in itself. If there is no reason for the government to maximise n or s per se, then the appropriate objective for a benevolent government could be the maximisation of the utility attained by the representative individual in a market setting. In particular, we assume a benevolent government who seeks to maximise (choosing appropriately the policy parameters) the steady-state lifetime indirect utility index of the representative individual, taking as given the individual choices about working period consumption, retirement period consumption and the number of children raised.

IV. WELFARE

In this section[10] we analyse the welfare effects of child allowances and the system of public education by assuming a benevolent government whose appropriate objective is to maximise the steady-state lifetime utility of the representative individual.[11]

Although the issue of optimality has been largely treated in literature (e.g., van Groezen et al., 2003; Abio et al., 2004; Fenge and Meier, 2005, 2009; van Groezen and Meijdam, 2008), the policies adopted by governments for achieving their welfare objectives have been in particular: (*i*) traditional PAYG pension systems and child allowances (van Groezen et al., 2003, 2008); (*ii*) partially fertility-related PAYG pension systems[12] and child allowances (Fenge and Meier, 2005, 2008); (iii) fully fertility-related PAYG pension systems (Abio et al., 2004). In this section we will describe another channel (i.e., the effects of both the public financing of education and the child allowance/tax policy), so far, to the best of our knowledge, not enough scrutinised, through which the steady-state lifetime welfare of the representative individual can be maximised.

Below we show that if all generations are treated symmetrically by the government, the maximisation of the steady-state lifetime indirect utility index with respect to the educational contribution rate is Pareto-efficient, because it is impossible to make all generations better off by either increasing or decreasing the wage tax used to finance the public schooling expenditure.[13] It is worth noting that the analysis is conducted by setting $\beta = 0$ without loss of generality.

Now, define

[10] We thank an anonymous referee for suggesting the welfare analysis.

[11] This amounts to say, as usual, that the government acts as a Stackelberg leader with respect to individuals and firms (Stackelberg followers).

[12] Note that the fertility-related PAYG system is a mechanism that relates (partially or totally) the pension benefit to the number of children raised by individuals.

[13] Moreover, it is also possible to show that the introduction of either a child allowance or a child tax can never represent a Pareto improvement, since either the current generation incurs a welfare loss (if a child tax is introduced) or all future generations lose (if a child allowance is introduced), as compared with the *laissez-faire* economy. Of course, the proof is available on request.

$$\overline{\alpha} := \frac{\rho}{1+2\rho}, \tag{21}$$

as a threshold value of the distributive capital share in the production function. Then the following proposition holds:

Proposition 8 Assume that all generations are weighted symmetrically. Then

(1) if $\alpha > \overline{\alpha}$*, the maximisation of the steady-state indirect lifetime welfare function of the representative individual with respect to the educational contribution rate is Pareto efficient. In particular, the Pareto efficient allocation is obtained when* $\theta = \theta^*$ *where*

$$\theta^* := \frac{(1-\alpha)(1+2\rho)}{(1+\rho)} < 1; \tag{22}$$

(2) if $\alpha \leq \overline{\alpha}$*, then the steady-state individual lifetime welfare is a positive monotonic function of the educational contribution rate and the highest possible welfare level is obtained as long as* $\theta \to 1$.

Proof. The proof uses the following line of reasoning. Assume that the public expenditure on education is already in place backward from present, so that the accumulation of human capital at time t depends on the educational contribution rate, θ. Since: (*i*) the government expenditure on education is financed at balanced budget with labour income taxes in every period, (*ii*) the per capita stock of physical capital approaches the steady-state after one period only[14] and (*iii*) the human capital accumulation is constant at every date, then assuming that all generations are weighted symmetrically (i.e., the social discount factor is equal to one), the lifetime welfare of the

[14] It is worth noting that in this class of models, in which fertility is an endogenous economic variable (where parents derive utility directly from having children) and the cost of children is fixed, the transitional dynamics is such that the per capita stock of capital is constant, i.e., it approaches the steady-state after one period only.

generation entering the working period at time t, as measured by the indirect lifetime utility index $V_t(\theta)$, as well as the lifetime welfare of all generations entering the working period subsequently, $V_{t+1}(\theta)$ for any t, are the same and, in particular, they are measured by the lifetime indirect utility index of the steady-state generation, that is, $V^*(\theta)$.

Therefore, (given Eq. P and using the properties of logarithms) we find that:

$$V^*(\theta) = \ln\left(\left(c_1^*\right)^{1-\phi} \cdot \left(c_2^*\right)^{\rho} \cdot \left(n^*\right)^{\phi}\right), \tag{23}$$

where

$$c_1^* = \frac{1-\phi}{\phi} m(1-\alpha) A Z^{1-\alpha} \left(\frac{\rho}{1+\rho}\right)^{\alpha} \theta^{1-\alpha} (1-\theta)^{\alpha}, \tag{24}$$

$$c_2^* = \frac{m}{\phi} \alpha (1-\alpha) A^2 Z^{2(1-\alpha)} \left(\frac{\rho}{1+\rho}\right)^{2\alpha} (1+\rho) \theta^{2(1-\alpha)} (1-\theta)^{2\alpha-1}, \tag{25}$$

$$n^* = (1-\alpha) A Z^{1-\alpha} \left(\frac{\rho}{1+\rho}\right)^{\alpha} \theta^{1-\alpha} (1-\theta)^{\alpha}, \tag{26}$$

Using (24)-(26), Eq. (23) can be written equivalently as:

$$V^*(\theta) = \ln\left(\Lambda \cdot \theta^{(1-\alpha)(1+2\rho)} \cdot (1-\theta)^{\alpha(1+2\rho)-\rho}\right), \tag{27}$$

where

$$\Lambda := (1-\phi)^{1-\phi} \left(\frac{m}{\phi}\right)^{1-\phi+\rho} \alpha^{\rho} (1-\alpha)^{1+\rho} A^{1+2\rho} Z^{(1-\alpha)(1+2\rho)} \left(\frac{\rho}{1+\rho}\right)^{\alpha(1+2\rho)} (1+\rho)^{\rho}.$$

Differentiating Eq. (27) with respect to the educational contribution rate, θ, gives:

$$\frac{\partial V^*(\theta)}{\partial \theta} = \frac{(1-\alpha)(1+2\rho) - \theta(1+\rho)}{\theta(1-\theta)}. \tag{28}$$

Let $\alpha > \overline{\alpha}$ hold. Then $\frac{\partial V^*(\theta)}{\partial \theta} \underset{<}{\overset{>}{}} 0$ if and only if $\theta \underset{>}{\overset{<}{}} \theta^*$ with $\theta = \theta^*$ (defined by Eq. 22) being an interior global maximum, and $\lim_{\theta \to 1} V^*(\theta) = -\infty$. This proves point (1).

Let $\alpha \leq \overline{\alpha}$ hold. Then $\dfrac{\partial V^*(\theta)}{\partial \theta} > 0$ for any $0 < \theta < 1$ and $\lim_{\theta \to 1} V^*(\theta) = +\infty$. This proves point (2). Q.E.D.

Proposition 8 shows that when the distributive capital share is relatively high, there exists a value of the educational contribution rate which maximises the steady-state lifetime indirect utility index of the representative individual. Since, after one period there is no transitional dynamics, then all generations have the same lifetime welfare function, and thus a Pareto-efficient allocation is achieved, i.e., it is impossible to make all generations better off by either increasing or decreasing the educational contribution rate if $\theta = \theta^*$.

However, when the distributive capital share is relatively low, the lifetime welfare is a monotonic increasing function of the educational contribution rate and thus the highest possible welfare can be achieved by increasing the public schooling expenditure at the highest possible level.

Moreover, comparison of the welfare-maximising policy with both the fertility-maximising and the saving-maximising policies (which we recall they are represented by the same educational contribution rate, see Propositions 2 and 4), gives the following proposition:

Proposition 9 Let $\alpha > \overline{\alpha}$ hold. Then the maximisation of both the fertility rate and the saving rate can never be Pareto optimal. In particular, both the fertility-maximising and the saving-maximising educational contribution rate, $\theta = \overline{\theta}$, is always lower than welfare-maximising educational contribution rate, $\theta = \theta^*$.

Proof. The proof can easily be derived showing that $\theta^* > \overline{\theta}$ always holds. Since,

$$\theta^* = \overline{\theta} \cdot \frac{1 + 2\rho}{1 + \rho}, \tag{29}$$

and $\frac{1+2\rho}{1+\rho}>1$ holds for any $0<\rho<1$, then Eq. (34) implies $\theta^{*}>\overline{\theta}$ is always fulfilled. Q.E.D.

Proposition 9 shows that the maximisation of both individual fertility and the saving rate is always inefficient, because the lifetime welfare of the representative generation can be still raised by increasing further the educational contribution rate.

V. CONCLUSIONS

In this paper we explored the effects of both the child benefit system and the public education system on the rate of population growth, the per capita income and the lifetime welfare in a conventional general equilibrium OLG model of neoclassical growth (Diamond, 1965) extended to account for endogenous fertility, with particular attention to the long-run interaction between education and child benefit policies.

We showed that the introduction of child allowances – often invoked by policymakers for a pro-natalist purpose – always has an unexpected negative effect on individual fertility and, moreover, it also reduces, as expected owing to the reduced human capital accumulation, the long-run per capita income. By contrast, a policy aimed at increasing the public schooling expenditure promotes both population growth and the (neoclassical) economic growth. In particular, the former increases up to the point in which the contribution rate used to finance the educational expenditure equals the share of human capital in the production function. The essential message of the present paper therefore is that developed countries plagued by below-replacement fertility and income stagnation may achieve a twofold goal by increasing the expenditure on public education: a higher per capita income and a higher population growth.

As for the welfare analysis, by assuming a benevolent government, we showed interestingly that when the share of physical capital in the production function is relatively high, the maximisation of the steady-state lifetime welfare function of the representative individual with respect to the educational contribution rate is Pareto efficient, while the introduction of child allowances (taxes) can never represent a Pareto improvement, because the welfare of the steady-state generation (current generation) is reduced by the introduction of the child policy.

APPENDIX

In this appendix we present details of Eqs. (17) and (18) in the main text, showing the effects of a rise in both the child allowance and the educational contribution rate on the rate of fertility and the human capital accumulation in the long run.

$$\frac{\partial n^*}{\partial \beta} = \frac{(1+\rho-\phi)n^*}{(1+\rho)(m-\beta)+\phi\beta} > 0, \tag{A1}$$

$$\frac{\partial h^*}{\partial \beta} = \frac{-Z\theta(1+\rho-\phi)}{(1-\theta)\phi} < 0, \tag{A2}$$

$$\frac{\partial n^*}{\partial \theta} = \frac{-n^*}{1-\theta} < 0, \tag{A3}$$

$$\frac{\partial h^*}{\partial \theta} = \frac{Z[(1+\rho)(m-\beta)+\phi\beta]}{\phi(1-\theta)^2} > 0. \tag{A4}$$

Moreover,

$$\frac{\partial n^*}{\partial h^*} = \frac{\phi w^*(1-\theta)}{(1+\rho)(m-\beta)+\phi\beta} > 0, \tag{A5}$$

$$\frac{\partial n^*}{\partial w^*} = \frac{\phi h^*(1-\theta)}{(1+\rho)(m-\beta)+\phi\beta} > 0, \tag{A6}$$

$$\frac{\partial w^*}{\partial k^*} = \alpha(1-\alpha)A(k^*)^{\alpha-1}(h^*)^{-\alpha} > 0, \tag{A7}$$

$$\frac{\partial w^*}{\partial h^*} = -\alpha(1-\alpha)A(k^*)^{\alpha}(h^*)^{-\alpha-1} < 0. \tag{A8}$$

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