Public health spending and unfunded public pensions in an OLG model of neoclassical growth: some new results about the poverty trap problem

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Abstract This paper introduces unfunded pay-as-you-go public pensions in a two-period overlapping generations model with endogenous lifetime à la Chakraborty (2004). We study the transitional dynamics and steady states outcomes showing that the public provision of health services may have not only the well-known beneficial effect to help to escape from poverty, but it can also produce unexpected negative consequences for per capita GDP in both low-income–high-mortality and high-income–low-mortality economies. Moreover, under myopic expectations, the government health expenditure can generate non-monotonic fluctuations in low mortality societies.

Keywords Health; OLG model; Public pensions

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1. Introduction

The importance of longevity as a determinant of macroeconomic outcomes over the very long run is recognised to be preeminent and has received increasing attention in recent years, especially in overlapping generations (OLG) growth models with either endogenous or exogenous fertility. In this regard, two seminal papers are Blackburn and Cipriani (2002) and Chakraborty (2004). The former introduces endogenous longevity in a three-period OLG model of growth with endogenous fertility and individuals that accumulate human capital through education. Their model is able to explain the so-called Demographic Transition since multiple regimes of development are possible. The latter, instead, assumes individual lifetime as a variable determined by public investments on health into an otherwise genuine model of the Diamond's (1965) variety with exogenous fertility, and study the connections between pervasive ill-health and economic growth. He finds that an economy can permanently be stuck into poverty if health investments are scarce.[1]

More recently, Bhattacharya and Qiao (2007) endogenise adult mortality through a non-linear function of both the private and public expenditure on health in an OLG growth model with exogenous fertility.[2] They find that endogenous fluctuations and deterministic chaos can appear

[1] In fact, the "Interest in health economics has soared over the past three decades, stimulated by intellectual innovations, greater availability of data, and, most importantly, a surge in health care spending." (Fuchs, 1996, p. 1). See also Fuchs (1987).

[2] In this paper we abstract from considering private health insurance. However, it is important to note that collecting taxes to finance a public system of health services may reduce the private insurance coverage (as the recent debate as regards the US health reform has also witnessed). For instance, Cutler and Gruber (1996a) have pointed out that a noteworthy crowding out effect of an expansion in the tax-financed public health expenditure exists on private decisions to spend on health, and estimated, in particular, for US pregnant women over the period 1987–1992 that "approximately 50 percent of the increase in Medicaid coverage was associated with a reduction in private insurance coverage. This occurred largely because employees took up employer-based insurance less frequently" Therefore, through this channel, there may be a negative feedback effect on longevity that, however, we did not gather in our model. For empirical

under two rather stringent hypotheses: (*i*) the public health expenditure is a convex function of the government's tax collections, and (*ii*) the public and private inputs in the longevity function are fairly complementary.

In addition to the public health system, another pillar of the welfare state, that have characterised several developed countries after the World War II, is represented by unfunded pensions. There exists a vast literature that discusses the effects of public pensions in macroeconomic models,[3] also as regards, more specifically, the way of reforming the social security system to make pensions sustainable over time, because of the dramatic fall in birth rates and the increase in longevity, namely population ageing, that developed countries are facing with (see, e.g., Boeri et al., 2001, 2002; Blinder and Krueger, 2004; Feldstein, 2005).

Population ageing is one of the most important stylised facts in the world, which can produce serious economic consequences especially for the financing of both public health services and pensions to the steadily rising number of mature individuals. Theses issues are briefly sketch below.

Longevity and the health system. The weight of government health expenditure on total health expenditure is essentially homogeneous in several European countries and Japan (almost 80 per cent), while it is rather low in the United States and in some developing countries (see Table 1). Moreover, at the first sight (a thoughtful econometric analysis is beyond the scope of the present paper), from the Table 1 we also notice that: (*i*) countries with long-lived individuals (e.g., Italy and Japan) are even those that historically have had and have an almost entirely public financed health system (even more that 80 per cent of the whole amount), as well as a high health expenditure to GDP ratio (9-10 per cent). In contrast, as regards some the most populous countries in the world (e.g., India, Indonesia, Nigeria, Pakistan), Table 1 reveals that government health expenditure is

analyses of public and private health insurances, see, e.g., Currie and Gruber (1996a, 1996b) and Cutler and Gruber (1996b). We thank an anonymous referee for suggesting to clarify this point.

[3] See, amongst many others, Cigno (1995), Cutler and Johnson (2004), Boldrin et al. (2005), Cigno and Werding (2007), Caucutt et al. (2007) and Galasso et al. (2009).

scarce and both the government health expenditure and the government health expenditure to GDP ratio are really low. The United States represent a different case: this because the public expenditure as a percentage of total health expenditure is relatively low, but the whole amount of health spending relative to GDP is high (almost double the amount of Italy). To sum up, the data presented Table 1 seem to reveal a remarkable role the government can play through the financing of health spending as a means of lower adult mortality. Moreover, as recently evidenced by some empirical articles, while Thornton (2002) finds that the effect of total health spending on mortality is quite small (using U.S. data from 1990), Lichtenberg (2004) concludes, with respect to US longevity over the period 1960-2001, that the public health expenditure has a positive effect on the individual lifetime. Moreover, Self and Grabowsky (2003) show that health public spending is less significant in developed countries, while being effective in developing economy as a means of healthy life. Finally, Aísa et al. (2010, p. 1), based their analysis on OECD countries for the period 1960-2000, finding that the "public health expenditure plays a significant role in increasing longevity. Moreover, the influence of government expenditure is non-linear and […] the biggest effect on longevity is when public expenditure is around 7% of GDP". In the overall, the financing of health systems by governments seems to has positive effects on longevity, especially in developing countries. Indeed, following one again Aísa et al. (2010), it should be advised that an expansion of the public sector at the expense of the private sector in those countries where the private/public ratio is high can positively affect longevity (see the developing countries in Table 1).[4] For an analysis of projection methods to assess the effects of health spending on growth and longevity, see Dormont et al. (2008).

Longevity and the pension system. Social security will be under strain in the near future, essentially because of both the steadily rising longevity and steadily reducing birth rates. The ratio

[4] In fact, Aísa et al. (2010, p. 15) conclude that "for countries in which the private-public ratio is higher than 0.5, a policy recommendation could be that an expansion of the public sector would be desirable in terms of longevity if it is accompanied by a reduction, or at least a stagnation, of the private sector."

of active people in total population is currently shrinking and the old-age dependency ratio (as given by the number of retirees in total population) is then increasing (see CESifo Forum, 2007, Figures 5 and 6, p. 10, where, for a hundred years 1950-2050 period, it is evident that the old-age dependency ratio has already doubled since 1950 and it is even increasing as regards the future). The current situation in Japan and Italy seem to be the worst in this regards, with dramatic implications for the PAYG pension system at least in the forecast until 2050. In this respect, CESifo Forum (2007, p. 9) concludes that "Either we double the contribution rate if we want to keep the pensions in line with wages, or we halve the pensions relative to wages. Or we might choose a mixture of these policies." For these reasons, also the theoretical analysis of the macroeconomic effects of longevity deserves an increasing attention."

Table 1. Health expenditure and demographic statistics in some developed, developing and underdeveloped countries.

	Government expenditure on health as % of total expenditure on health		Total expenditure on health as % of GDP		Life expectancy at birth (years, both sexes)			Adult morality (probability of dying between 15 and 60 years per 100 population)		
	2000	2007	2000	2007	1990	2000	2008	1990	2000	2008
France	79.4	79.0	10.1	11.0	77	79	81	11.5	10.0	8.7
Germany	79.7	76.9	10.3	10.4	75	78	80	11.8	9.4	7.8
Ireland	73.5	80.7	6.3	7.6	75	76	80	10.8	9.6	7.3
Italy	72.5	76.5	8.1	8.7	77	79	82	9.5	7.6	6.1
Japan	81.3	81.3	7.7	8.0	79	81	83	8.1	7.3	6.5
United States	43.2	45.5	13.4	15.7	75	77	78	13.2	11.4	10.7
Brazil	40.0	41.6	7.2	8.4	67	70	73	21.2	18.3	15.8
Indonesia	36.6	54.4	2.0	2.2	61	65	67	27.5	23.2	20.6
India	24.5	26.2	4.4	4.1	58	61	64	27.4	25.6	21.3
Nigeria	33.5	25.6	4.6	6.6	46	47	49	40.5	41.9	41.1
Pakistan	21.3	30.0	3.0	2.7	58	61	63	25.1	21.8	20.4
Chad	42.5	56.3	6.3	4.8	49	47	46	37.5	42.0	44.6

In the economic literature, however, scarce attention has been paid, to the best of our knowledge, to the interaction between the provision of public pensions and the health care system in a dynamic macroeconomic model with overlapping generations and production. This paper, therefore, wants to fill this gap by incorporating public pensions in an OLG model à la Chakraborty (2004) while also assuming: (*i*) the existence of a positive natural or biological rate of longevity, and (*ii*) a S-shaped relationship between longevity and health expenditure,[5] i.e. it is postulated the existence of thresholds in the accumulation of knowledge required for new medical advances and discoveries in the treatment of diseases (e.g. vaccines) to be effective.[6]

Our main finding is that the introduction of either a small or large enough public health system is harmful for the GPD per capita, while within a range of intermediate values it may favour the birth of a high equilibrium where mortality is low as well as the death of the poverty trap. However, raising health expenditure too much may have the unpleasant effect to bring the GDP in the unique regime of development (coming after the disappearance of the poverty trap) to a level even lower than that would be obtained in the poor equilibrium in the absence of health spending. This means that the effectiveness of the public health system as an instrument for both improving economic growth and development is limited to a narrow range of government's tax collections.

We also find that the public provision of health care services in an economy with public pensions may generate (under myopic expectations) endogenous fluctuations in rich–low mortality societies. Our results also represent a policy warning about the unexpected growth-reducing and destabilising effects of financing the health system.

The rest of the paper is organised as follows. Section 2 presents the model. Section 3 studies the steady-state and dynamical features of the economy and shows (numerically) the existence of non-monotonic behaviours. Section 4 concludes.

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[5] Such a functional form (different from Chakraborty, 2004, who instead assumes a concave function) is widely adopted to represent the relationships between longevity and human capital and/or longevity and health spending (see, e.g., Blackburn and Cipriani, 2002; Blackburn and Issa, 2002; de la Croix and Ponthiere, 2010).

[6] This means that the health expenditure to finance new research projects may be apparently high scarcely effective until a certain degree of knowledge is achieved. Beyond such a threshold, however, a "sudden" effect exists to allow to trigger and bring to light the beneficial effects of the new medical discoveries, to make them efficient, usable and operative across population and eventually transformed into higher longevity. As regards the effectiveness of the new medical knowledge on adult mortality reduction in historical epochs, see, e.g., de la Croix and Sommacal (2009).

2. The model

Consider a general equilibrium OLG closed economy populated by a continuum of identical agents of measure one, identical firms and a government that separately finances health services and pay-as-you-go (PAYG) pensions. Population is stationary.

The lifetime of the typical agent is divided into youth (working period) and old-age (retirement period), as in Diamond (1965). Individuals have preferences towards young-age and old-age consumptions. When young, individuals of each generation are endowed with one unit of labour inelastically supplied to firms. The probability of surviving from youth to old age of an individual born at t, π_t, is assumed to be endogenously determined by each person's health level which is, in turn, augmented by the public provision of health services, (see Chakraborty, 2004). We assume that π_t is determined by the following non-decreasing bounded function of health investments:[7]

$$\pi_t = \pi(h_t) = \pi_0 + \frac{\pi_1 \Delta h_t^{\delta}}{1 + \Delta h_t^{\delta}}, \tag{1}$$

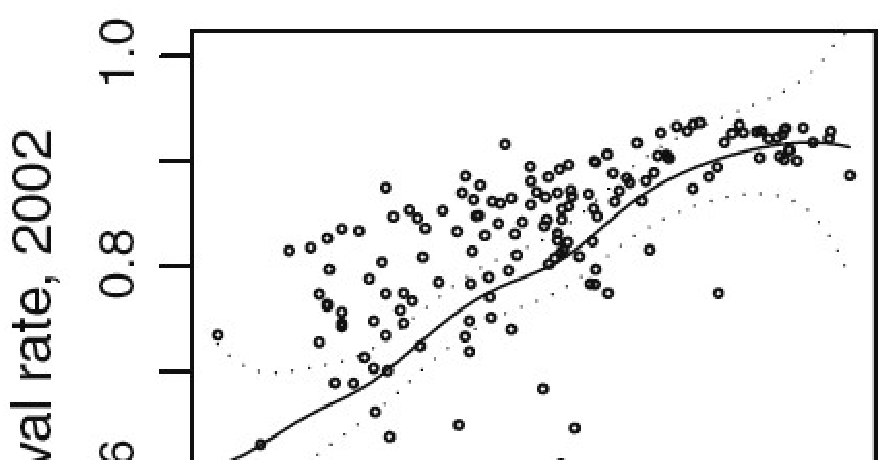
where $\delta, \Delta > 0$, $0 < \pi_0 < 1$, $0 < \pi_1 < 1$, $0 < \pi_0 + \pi_1 \leq 1$, $\pi(0) = \pi_0 > 0$, $\lim_{h \to \infty} \pi(h) = \pi_0 + \pi_1 \leq 1$,

$\pi'_h(h) = \dfrac{\pi_1 \delta \Delta h^{\delta-1}}{\left(1 + \Delta h^{\delta}\right)^2} > 0$, $\pi''_{hh}(h) < 0$ if $\delta \leq 1$ and $\pi''_{hh}(h) \gtrless 0$ for any $h \lessgtr h_T := \left[\dfrac{\delta - 1}{(1+\delta)\Delta}\right]^{\frac{1}{\delta}}$ if $\delta > 1$.

Now, some clarification on Eq. (1) are in order: first, π_0 is defined as being the natural or biological rate of longevity of people (see, e.g., Ehrlich, 2000; Leung and Wang, 2010). It represents a measure of the individual life span regardless of the provision of health services; second, π_1 is a measure of the intensity of the efficiency of health expenditure on longevity; third, we may think, realistically, that health investments have a more intense effect in reducing adult mortality when a certain threshold is achieved, while becoming scarcely effective when longevity is close to its saturating value (e.g., the functional relationship between public health expenditure and

[7] See Blackburn and Cipriani (2002) and de la Croix and Ponthiere (2010) for similar longevity functions.

longevity may be S-shaped). The parameters δ and Δ allow to capture such an idea and determine both the turning point of $\pi_h'(h)$ and speed of convergence from the biological length of life π_0 to the saturating value π_1. If $\delta > 1$ ($\delta \le 1$), threshold effects of health investment of longevity does (not) exist. Eq. (1), therefore, encompasses the "saturating" function used in the numerical examples by Chakraborty (2004) when $\delta = \Delta = 1$ and $\pi_0 = 0$ as well as the S-shaped function when $\delta > 1$. At the empirical level, there is a burgeoning literature that deals with the relationship between income and longevity, and even tries to ascertain the causal effects between them. Some of the evidence presents a rather significant non-linearity in the relationship between longevity rates and either per capita income (which may be a proxy of the public health spending) or the per capita health spending. For instance, Martikainen et al. (2009, Figure 1, p. 152) found a logistic-like shape relationship of hazard ratios of mortality by different measures of income for Finnish men and women aged 30-64 years, while Fioroni (2010, Figures 1 and 2, p. 357 – reported below for comparison purposes) shows that a non-monotonic relationship between per capita GDP and longevity, and per capita health spending and longevity can exist (from an historical point of view, also some evidence reported in Fogel, 2004, supports this non-linearity).



Source: Fioroni (2010, Figure 1, p. 357).

The government separately finances health care services and PAYG pensions to the survived old at balanced budget.[8] As regards the former, we assume that the per capita health expenditure at t (h_t) is constrained by

$$h_t = \tau w_t, \tag{2.1}$$

where w_t is the unitary wage and $0 < \tau < 1$ the (constant) labour income tax rate. As regards the latter, per capita pension expenditure at t ($\pi_{t-1} p_t$) is determined as

$$\pi_{t-1} p_t = \theta w_t. \tag{2.2}$$

where $0 < \theta < 1$ is the (constant) contribution rate to the PAYG system.[9]

Therefore, the budget constraint faced by the young at t reads as:

$$c_{1,t} + s_t = w_t(1 - \theta - \tau), \tag{3.1}$$

i.e. wage income – net of the contributions paid to finance both health and pension expenditures, where $\theta + \tau < 1$ – is divided into consumption, $c_{1,t}$ and saving, s_t.

When old, individuals retire and live on the proceeds of savings plus expected interests ($r^e{}_{t+1}$), and the expected pension benefit ($p^e{}_{t+1}$). The existence of a perfect annuities market (where savings

[8] The modelling of balanced-budget policies in a separate way is standard in the economic literature, see, e.g, van Groezen et al. (2003) and van Groezen and Meijdam (2008), which consider two separate budgets as regards the financing of PAYG pensions and child allowances, and Žamac (2007), which, instead, analyses the case of a government that collects taxes to finance PAYG pensions and schooling separately. For instance, Žamac (2007, p. 623) notes that "The assumption regarding two separate systems is mainly based on the fact that the existing social security programs have a very weak connection with the education system, if any." This line of reasoning can therefore be used to justify the funding of pensions and health services through *ad hoc* taxation. For instance, as regards the Italian case, it is well known that pension budgets are separately managed by INPS (*Istituto Nazionale di Previdenza Sociale*).

[9] We note that it should also be considered that the reduction in the disposable income of workers caused by the payment of contributions to finance pension payouts may negatively affect the length of the life span of people. In fact, it has been argued that the rise in fiscal burden for the financing of both pension and heath spending can significantly affect adult mortality (see, e.g., Viscusi, 1996).

are intermediated through mutual funds) implies that old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased. Hence, the budget constraint of the old both at t can be expressed as

$$c_{2,t+1} = \frac{1 + r^e{}_{t+1}}{\pi_t} s_t + p^e{}_{t+1} . \tag{3.2}$$

where $c_{2,t+1}$ is old-aged consumption.

By taking the government budgets Eqs. (2.1) and (2.2), and both current and future factor prices as given, the representative individual of generation t chooses how much to save out of her disposable income to maximise the lifetime utility index

$$U_t = \ln(c_{1,t}) + \pi_t \beta \ln(c_{2,t+1}), \tag{4}$$

subject to Eqs. (3.1) and (3.2), where $0 < \beta < 1$ is the subjective discount factor. The saving rate, therefore, is

$$s_t = \frac{\pi_t \beta w_t (1 - \theta - \tau)}{1 + \pi_t \beta} - \frac{\pi_t p^e{}_{t+1}}{(1 + \pi_t \beta)(1 + r^e{}_{t+1})} . \tag{5}$$

Firms are identical and markets are competitive. Aggregate production at t (Y_t) takes place by combining capital (K_t) and labour (L_t) according to the constant returns to scale Cobb-Douglas technology $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where $A > 0$ and $0 < \alpha < 1$. Assuming that capital fully depreciates at the end of each period and a unit price of output, profit maximisation implies that factor inputs are paid their marginal products, that is:

$$r_t = \alpha A k_t^{\alpha - 1} - 1, \tag{6.1}$$

$$w_t = (1 - \alpha) A k_t^{\alpha}, \tag{6.2}$$

where k_t is the capital-labour ratio.

3. Steady state analysis

Using Eqs. (1), (2.1) and (6.2) to substitute out for π_t into Eq. (5) as well as the one period forward pension accounting rule Eq. (2.2), the saving rate can be re-arranged as:

$$s_t = \frac{\pi_t(k_t)\beta w_t(1-\theta-\tau)}{1+\pi_t(k_t)\beta} - \frac{\theta}{1+\pi_t(k_t)\beta}\frac{w^e{}_{t+1}}{1+r^e{}_{t+1}}. \tag{7}$$

Since the expected values of factor prices are included in Eq. (7), it is necessary to specify the type of expectations formation of individuals with respect to the future values of both the wage and interest rate. The two extreme cases generally used to study the dynamics of a general equilibrium deterministic economy are: (*i*) myopic foresight, and (*ii*) perfect foresight (see, e.g., de la Croix and Michel, 2002). Since the assumption of myopic expectations is rather usual in literature (see Michel and de la Croix, 2000), and a certain level of myopia seems to be inherent with the life-cycle context and PAYG pensions in existence (see Pecchenino and Pollard, 2005),[10] below we study the case of myopic expectations for the interesting dynamical features that the model generates in such a case.[11]

With myopic foresight, individuals expect the future values of the interest and wage rates to depend on the current value of the capital stock, that is:

$$\begin{cases} 1+r^e{}_{t+1} = \alpha A k_t^{\alpha-1} \\ w^e{}_{t+1} = (1-\alpha) A k_t^{\alpha} \end{cases}. \tag{8}$$

Given Eqs. (2.1) and (2.2), the equilibrium condition is given by:

$$k_{t+1} = s_t, \tag{9}$$

[10] For instance, according to Samuelson (1975), a reason for justifying the introduction of a social security system might be based on an "intrinsic" myopia of agents when young. In fact, he stated that "Many social security systems, like the New Deal U.S. system, may be deemed most valuable precisely because the myopia ignored by the present models does in fact prevail. People live miserably in old age because they do not realize when young what are the consequences of their private saving habits. So by democratic fiat, they paternalistically impose on themselves a within-life pattern of consumption that favors old age at the expense of young. Precisely because of the myopia that makes paternalism optimal…" (p. 543).

[11] For the sake of completeness, however, we analyse the case of perfect foresight in the Appendix.

which is combined with Eqs. (6.2), (7), (8) and (9), to obtain:

$$k_{t+1} = \frac{\pi_t(k_t)\beta(1-\theta-\tau)(1-\alpha)Ak_t^{\alpha}}{1+\pi_t(k_t)\beta} - \frac{\theta}{1+\pi_t(k_t)\beta}\frac{1-\alpha}{\alpha}k_t, \tag{10}$$

or, alternatively

$$k_{t+1} = \frac{1}{1+\pi_0\beta+[1+(\pi_0+\pi_1)\beta]Bk_t^{\alpha\delta}}\left\{\beta D(\tau,\theta)k_t^{\alpha}\left[\pi_0+(\pi_0+\pi_1)B(\tau)k_t^{\alpha\delta}\right]-\theta\frac{1-\alpha}{\alpha}\left(1+Bk_t^{\alpha\delta}\right)k_t\right\}, \tag{11}$$

where $B(\tau) := \Delta[\tau(1-\alpha)A]^{\delta}$ and $D(\tau,\theta) := (1-\theta-\tau)(1-\alpha)A$ are two positive constants used to simplify notation. The system described by Eq. (11) completely characterises the dynamics of our economy.

We note that Eq. (11), in the special case $\theta = 0$, $\pi_0 = 0$ and $\delta = \Delta = 1$ replicates Chakraborty (2004, Proposition 1 (i), p. 126), that is there exist both low and high regimes of development and the poverty trap is represented by the attractive zero equilibrium point.

The study of Eq. (11) gives rise to interesting dynamical features, especially as regards the effects of health taxes in an economy with public pensions, that must be carefully examined. We note first the main effect of the existence of PAYG pensions: while a rise in θ reduces the disposable income of the young and thus savings and capital accumulation shrink, it also makes the crowding out effect of public pensions on capital accumulation larger – i.e., the higher PAYG pensions, the lower the need to save to sustain consumption of mature individuals – and this, in turn, implies that the shape of the saving function may becomes "backward bending" when individuals are myopic foresighted.[12]

It should be noted that the interaction between the pension and health systems is crucial because of the general equilibrium feedback effects that PAYG pensions play on health spending and

[12] Note that, as a consequence, an economy described the dynamic system Eq. (11) is prone to be characterised by non-monotonic dynamics. Moreover, cyclical instability can also be observed when θ varies. However in this paper we limit us to only consider sufficiently small size of pensions to preserve stability of the unique positive equilibrium in the absence of public health investments.

eventually on longevity. Thus, the use labour income taxes is important for the results. In fact, the use of consumption taxes would be neutral because it would not distort saving decisions in our model, as preferences are of the Cobb-Douglas type. In other words, with consumption taxes, capital accumulation would still be described by Eq. (11), while a generalised income tax would only have the effect of reducing further disposable income but not that of stimulating capital accumulation through a rise in longevity (as instead would be the case with the health wage income tax rate).

As regards both the dynamical features and long-run outcomes of the economy, from Eq. (11) the following results can be established.

Result 1. *(Dynamics). Let* $0 < \pi_0 < 1$ *hold. Then, depending on the mutual relationship between the output elasticity of capital,* α*, and the intensity of the threshold effect of the accumulated health capital as an inducement to higher longevity,* δ*, the dynamic system described by Eq. (11) admits either two steady states* $\{0, \bar{k}\}$*, the former being locally unstable and the latter may be either locally stable or unstable, or four steady states* $\{0, \bar{k}_1, \bar{k}_2, \bar{k}_3\}$*, the first and the third being locally unstable, while the second and the forth may be either locally stable or unstable.*

Result 2. *(Policy effects). Let* $0 < \pi_0 < 1$ *hold. Then, the introduction of either a fairly small or large health expenditure is harmful to per capita GPD, while within a range of intermediate values it may favour the appearance of a second high steady state (thus allowing for the possibility of escaping from poverty), which can in turn become unique (i.e., the low equilibrium disappears and, hence, the poverty trap is definitely eliminated), if health spending is further increased. However, although when only a unique high equilibrium exists, a large enough increase in the health tax rate may have the paradoxical effect of reducing capital accumulation and per capita GDP up to a level even lower than that when the poverty trap existed.*

Since the non-linear dynamic system Eq. (11) cannot be handled in a neat analytical form, we now resort to numerical experiments to illustrate Results 1 and 2 (see Figure 1, which displays the whole process of birth and death of the four equilibria when τ rises). The parameter set (chosen only for illustrative purposes) is the following: $A = 16$, $\alpha = 0.3$ (see Gollin, 2002; Kehoe and Perri, 2002), $\beta = 0.6$ (see Žamac, 2007), $\pi_0 = 0.2$, $\pi_1 = 0.75$, $\Delta = 1$, $\delta = 80$[13] and $\theta = 0.28$ (see, e.g., Liikanen, 2007, p. 4, argued that the "pension contributions in Europe would rise from their present level of around 16% of aggregate wages to around 28% by the year 2040).

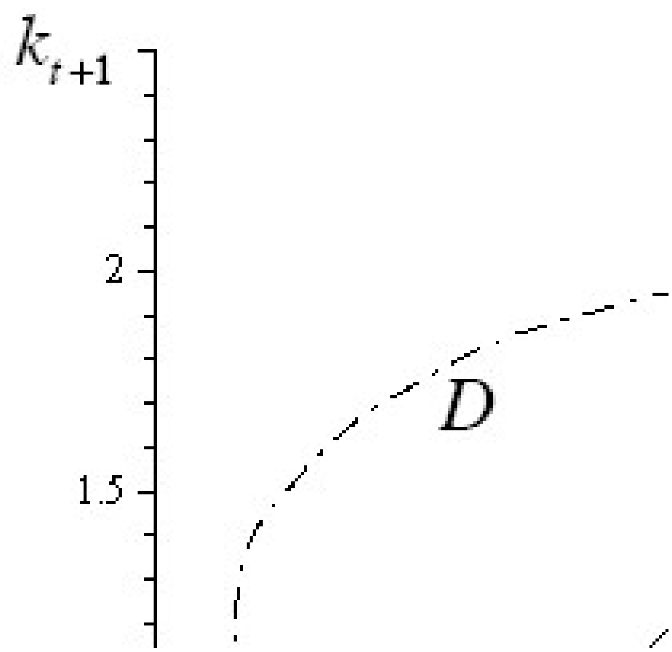


Figure 1. Evolution of the steady states when τ varies.

[13] Note that, although in the numerical examples we used a rather high value of δ, in order to stress the sigmoid shape of the longevity function, the analysis is unaltered also if we assume lower values of δ, such as, for instance, those used by Blackburn and Cipriani (2002) and de la Croix and Ponthiere (2010) ($\delta = 4$ and $\delta = 10$, respectively).

Starting from the benchmark case $\tau = 0$ (the bold solid line A), where a unique positive (poor) steady state exists $\bar{k}_1 = \bar{k} = 0.4209$ (since $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t=\bar{k}} = -0.1083$, $\bar{k}$ is stable), Figure 1 clearly shows that a slight increase in the health tax from 0 to 0.05 (the normal solid line B) significantly reduces the unique steady state in the poor country ($\bar{k} = 0.3797$) and, hence, the GDP per capita shrinks (since $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t=\bar{k}} = -0.1083$, the equilibrium is still stable). Interestingly, it should be noted that within the range $0 < \tau < 0.072$, the per capita GDP is reduced as the health tax becomes larger. However, when $\tau \cong 0.072$ a tangent bifurcation emerges (not reported in the figure), and, hence, increasing health tax rate further, e.g. $\tau = 0.08$, causes a dramatic change. In fact, as (the dotted) line C shows, the economy shift from a unique non-trivial equilibrium configuration to a multiple threefold positive steady states): indeed, a high steady state appears ($\bar{k}_3 = 2.3851$) and the low one shrinks further on ($\bar{k}_1 = 0.3557$). However, both steady states are stable since $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t=\bar{k}_1} = -0.1083$ and $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t=\bar{k}_3} = 0.0088$. Therefore, when $\tau = 0.08$ an economy may start converging towards either the low-income–high-mortality equilibrium or high-income–low-mortality regime depending on initial conditions. A further increase in the tax rate from 0.08 to 0.15 (see the dashed-dotted line D), causes the disappearance of the low (poor) equilibrium and all the trajectories will converge towards the unique high (rich) stable steady state ($\bar{k}_3 = 2.0214$ with $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t=\bar{k}_3} = 0.0087$), i.e. the poverty trap is definitely eliminated. We observe, however, that the per capita GDP is now lower than the corresponding value in the high equilibrium when the poverty trap existed. Finally, if the tax rate is increased to $\tau = 0.58$ (see the dashed line E), we obtain the paradoxical effect of reducing the steady state in the high regime of development to a level even lower than that would be obtained in the benchmark case $\tau = 0$, where only a unique (poor) equilibrium existed (i.e.,

$\bar{k}_3 = 0.272$ if $\tau = 0.58$ versus $\bar{k}_1 = 0.4209$ if $\tau = 0$). This, in turn, thus cause a reduction in the GDP below the corresponding value when $\tau = 0$.

The effect of raising health taxes in an economy with public pensions can therefore, rather surprisingly, be dramatic as regards the per capita GDP. Indeed, raising the health tax rate in an economy with public pensions causes a twofold effect on capital accumulation. First, it directly reduces the disposable income of the young and thus savings and capital accumulation shrinks through this channel. Second, it raises longevity and, hence, as an indirect effect, savings and capital accumulation increase because individuals expect to live longer. If the health tax is fairly low (high), the positive (negative) indirect (direct) effect dominates and capital accumulation raises (reduces). However, the existence of public pensions makes negative effect of health taxes larger. In fact, since $B(\tau) > 0$ for any $0 < \tau < 1$, then the financing of health care services increases the weight of the crowding out effect of public pensions on capital accumulation, as can easily be ascertained by looking at the last addendum of right-hand side of Eq. (11) within braces. If this additional effect is strong enough, both the poor and rich economies may suffer when health expenditure is raised.

3.1. A numerical exercise for the Italian economy

In this section we perform a simulation for Italy to show the sensitivity of our model to some realistic economic parameters. The chosen parameter values are, therefore, the following: $\alpha = 0.5$ (see Rodriguez and Ortega, 2006, Table A.1), $\beta = 0.6$, which corresponds to a propensity to save around 12 per cent if each generation consists of 25 years, $\pi_0 = 0.2$ and $\pi_1 = 0.85$ (see World Health Statistics (2010), Part II. Global Health Indicators). Then we choose $\Delta = 1$, $\delta = 10$ and calibrate $A = 23$, $\tau = 0.17$ and $\theta = 0.27$ to obtain a government health expenditure to per capita GDP ratio of 8.5 per cent (see World Health Statistics, 2010, for Italy). This case is represented by line A in Figure 2, where a stable unique (high) regime of development exists ($\bar{k} = 4.56$). Figure 2 shows that a reduction in the health tax rate from 0.17 to 0.08 increases capital accumulation and, hence, the steady state in the high regime raises ($\bar{k}_3 = 6.14$), which corresponds to the same adult life spam as individuals already live close to the saturating value in the longevity function even if the health spending-GDP ratio shrinks. The reduction in the health expenditure, however, causes the appearance of a stable low equilibrium where an economy can be entrapped to due to initial conditions, where capital accumulation ($\bar{k}_1 = 0.742$) and longevity ($\bar{\pi}_1 = 0.26$) are really low and close to values corresponding to those of developing countries (see line B in Figure 2).

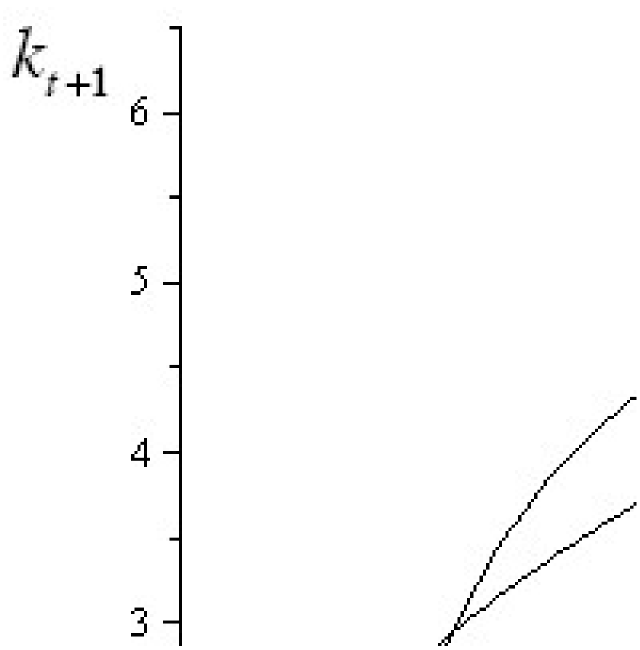


Figure 2. Evolution of the steady states when τ varies (the case of Italy).

3.2. Deterministic business cycles under myopic foresight

Result 3. *(Stability). Under myopic expectations, the public provision of health services in an economy with public pensions may be a source of cyclical instability and persistent business cycles in low mortality societies.*

In order to study the stability properties of the high equilibrium and show the emergence of cyclical instability using τ as the bifurcation parameter, we resort to numerical simulations by assuming $\alpha = 0.13$ and $\theta = 0.39$ and then keeping the other parameters unchanged.

Starting from $\tau = 0.076$ (corresponding to which four steady states exist), it can easily be seen that the high equilibrium $\bar{k}_3 = 1.0915$ is stable, (i.e. $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = \bar{k}_3} = -0.5842$). Assuming now that the government wishes to slightly increase the health expenditure by raising the tax rate to $\tau = 0.08$, we get $\bar{k}_3 = 1.1027$ with $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = \bar{k}_3} = -1.0395$, i.e. the stock of capital is increased but the equilibrium is now unstable. When $\tau = 0.10$ we observe a strong reduction in capital accumulation and per capita GDP. Moreover, the steady state $\bar{k}_3 = 1.0542$ becomes unstable (i.e., $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = \bar{k}_3} = -1.0464$). We now investigate the local behaviour of $\bar{k}_3$ in the neighbourhood of the flip bifurcation point, i.e. $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t = \bar{k}_3} = -1$, using τ as the bifurcation parameter. The bifurcation diagram for τ (Figure 3) shows the emergence of a stable 2-period cycle (stable flip bifurcation). The vertical axis shows the limit points of the high equilibrium sequence of capital, $\bar{k}_3$. While when the health tax rate is too low the high equilibrium does not exist, it appears beyond a value of τ around 0.0767. When the health tax rate further increases up to a value slightly higher than 0.0778, a period doubling bifurcation occurs and a 2-period cycles emerges. This is an example of an endogenous business cycle determined by the public health system[14] in an economy with PAYG pensions.

[14] Although it is well known that in OLG growth models deterministic business cycles can emerge (e.g., Grandmont, 1985; Farmer, 1986; Reichlin, 1986; Medio 1992), in this paper it is shown that a cycle may be triggered by an excessive public health system. However, different from Bhattacharya and Qiao (2007), non-monotonic dynamics exist even without the rather stringent assumption that longevity is a convex function of the government's tax collections, as well as the existence of a private (complement to the public) input in the longevity function.

Therefore, economies that combine public pensions and health care services may not be able to sustain high levels of capital accumulation and per capita GDP, and low adult mortality, because of the endogenous persistent economic fluctuations and even chaotic motions.[15]

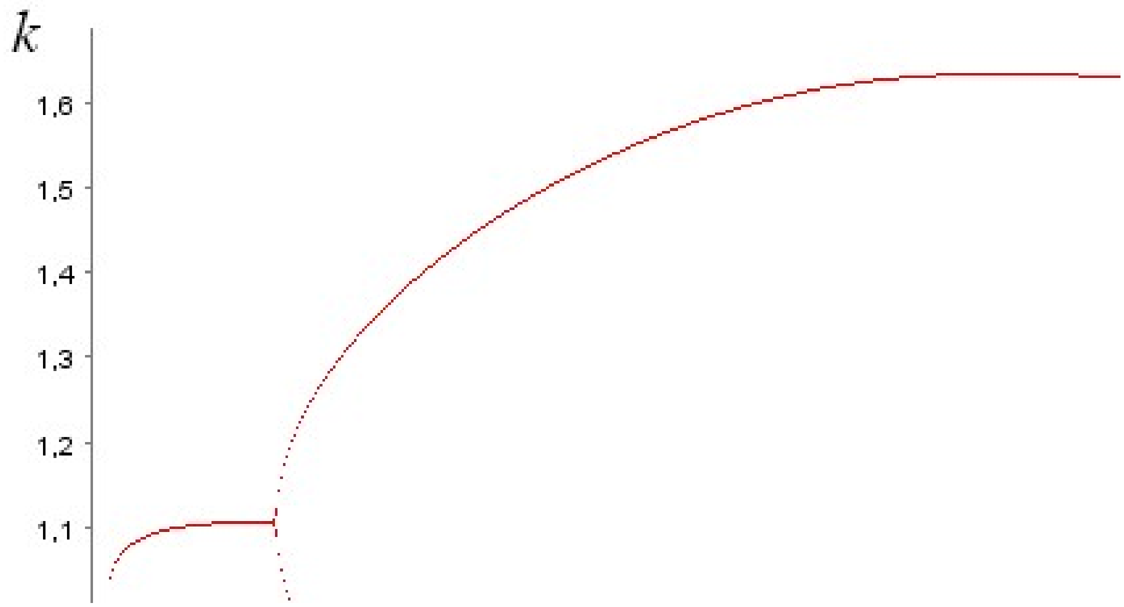


Figure 3. Bifurcation diagram for τ ($k_0 = 1$): enlarged view of $0.4 < k < 1.7$ and $0.075 < \tau < 0.11$.

4. Conclusions

In modern economies the provision of public pensions and health services co-exist. The goal of the former program is to secure old-age consumption, while the latter is implemented mainly to improve human health conditions, to promote healthy lifestyles and, hence, the quality of life. In this paper we present a simple growth model with overlapping generations and endogenous lifetime à la Chakraborty (2004), extended with public pensions. While Chakraborty (2004) shows that scarce health investments can explain the existence of poverty traps, in this paper we show that a

[15] Note that extensive numerical simulations revealed, for other plausible parameter values, that possible chaotic behaviours can also occur.

rise in health spending may give rise to unexpected reductions in per capita GPD either for small or large enough health programmes.[16] Moreover, it is shown that (*i*) the possibility to escape from poverty and ill-health is limited to a rather narrow (intermediate) range of government's tax collections, and (*ii*) a rise in health spending may expose low mortality societies to endogenous fluctuations and even chaotic behaviours. Therefore, our results are at odds with those an economy without public pensions (see Chakraborty, 2004).

Appendix

We briefly present here the dynamics of capital when individuals are perfectly foresighted. In this case, individuals expect the future values of both the interest and wage rates to depend on the future value of the stock of capital, that is

$$\begin{cases} 1 + r^e{}_{t+1} = \alpha A k_{t+1}{}^{\alpha-1} \\ w^e{}_{t+1} = (1-\alpha) A k_{t+1}{}^{\alpha} \end{cases} . \tag{A.1}$$

Combining Eqs. (6.2), (7), (A.1) and (9), the dynamic path of capital accumulation is now:

$$k_{t+1} = \frac{\pi_t(k_t)\beta(1-\theta-\tau)\alpha(1-\alpha)A}{[1+\pi_t(k_t)\beta]\alpha + \theta(1-\alpha)} k_t{}^{\alpha} , \tag{A.2}$$

The steady states emerging from the dynamic system Eq. (A.2) are the same as those in the case of myopic expectations (see Eq. 11 in the main text).[17] The steady state analysis presented in the main text, therefore, holds irrespective of the type of expectation formations about the future factor prices by individuals.

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[16] Indeed, the use of either a too small or large health expenditure may bring the per capita GPD in the high equilibrium at a level lower than that would be obtained without the health provision.

[17] Note that with perfect foresight, the shape of the phase map Eq. (A.2) and the qualitative behaviour of the trajectories approaching the same steady states are of course different than the case with myopic foresight, but deterministic business cycles cannot emerge in such a case (see Galor and Ryder, 1989).

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