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Reclamation of a resource extraction site: A differential game approach

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Abstract

We study an extraction site reclamation problem in a two-player differential game setting over a finite time horizon. Environmental regulation requires each firm to engage in reclamation efforts during the entire lifespan of the extraction site and to pay an abandonment reclamation fee at the end of its lease term for the unclaimed pollution caused by firms' activities. Firms determine their reclamation efforts in order to minimize their reclamation cost. We analyze and compare individual firms' choices and the pollution stock in the noncooperative and the cooperative cases by distinguishing between situations in which firms are homogeneous and heterogeneous. We study the case in which firms have different lease durations and different degrees of environmental liability. We show that the dynamics of the reclamation efforts may be substantially different under noncooperation and cooperation, and in both cases, it is mainly determined by how the rate of time preference and the growth rate of firms' liabilities compare. Moreover, in all scenarios, the reclamation efforts generally rise with the degree of liability and fall with the lease duration, suggesting that in order to promote better environmental outcomes, the regulators should

carefully determine the lease conditions by introducing intra-term reclamation fees along with stringent environmental accountability.

KEYWORDS

degree of liability, heterogeneity, lease duration, reclamation, resource extraction, site cleanup

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1 | INTRODUCTION

Extraction of natural resources deteriorates the environment both on and near the extraction field. The side effects of resource extraction on land are mostly severe and include the construction of access roads, seismic lines, well-sites, tailing ponds, worker camps, storage areas and facilities, processing plants, compressor and pumping stations, as well as other miscellaneous infrastructure of various sizes (e.g., pipeline corridors in the case of oil and gas industry). However, the degree and the size of the deterioration associated with these activities varies considerably from case to case and tends to be more severe when the extraction field consists of surface mines and unconventional resources. In general, the environmental disturbance is larger or, at least more salient, when extraction occurs through surface mining also known as open-pit, which involves extracting rock or minerals from the earth by their removal from an open pit or borrow, in contrast to other extractive methods that require tunneling into the earth.[1] Aside from the specific extraction site, in many cases, land is also disturbed due to hazardous tailings and waste generated from the varied processes of extraction and production. For example, oil sands extraction results in the accumulation of large amounts of residual waste, which contains a mixture of water, clay, unrecovered bitumen and solvent, and dissolved chemicals including organic toxic compounds, captured by the tailing ponds.[2] Moreover, abandoned oil and gas wells may provide pathways for subsurface fluid migration, which can lead to groundwater contamination and gas emissions into the atmosphere (Kang et al., 2015). The environmental deterioration is particularly severe in the case of exploitation of unconventional resources, while recently production from these resources has grown much faster than expected due to the improvements and developments in the extraction technologies which have drastically reduced the unit production cost (Mistré et al., 2018). Such a technological improvement has had a significant impact on the energy portfolios of different countries especially in North America over the past decade;

[1]For example, just to give a sense of the size of the problem, in Canada about 4800 square kilometers of land could be impacted by the mining method of extracting oil sands in Alberta, mostly through surface mining (Canadian Association of Petroleum Producers, 2018).

[2]For example, in 2017 in Alberta, the oil sand's tailing ponds cover more than 220 square kilometers and hold an estimated 1.2 trillion liters of contaminated water (Herald, 2017). Even if the water released from the ponds can be recycled and reused in oil sand processing, the majority remains as mud almost indefinitely (Natural Resource Canada, 2018).

nevertheless, the reclamation technologies have not improved at a same pace and are still in their infancy. For example, in Canada firms are required to have specific plans in place to ensure reclamation; however, the effective reclamation of extraction sites is predicted to be very difficult without significant improvements in tailings management techniques (Natural Resource Canada, 2018). Moreover, that recent downturn in energy prices exacerbates the issue of cleaning up inactive oil and gas wells further, as the number of unclaimed and abandoned wells (i.e., orphaned wells) has increased dramatically, from less than 100 in 2012 to more than 3000 in 2017, only in Alberta (Dachis et al., 2017). In addition to the limits in the reclamation technologies, the current regulations may not be effective either, as confirmed by the US example, where extraction firms are required to post a bond to ensure environmental reclamation of abandoned sites; however, the bonding requirements mostly cover only a small fraction of the reclamation costs even though they vary across states (Ho et al., 2018; Mitchell & Casman, 2011). The scenario is not more comforting in developing countries either. For example, China has introduced a bonding system in 1998 but only by 2013 all its provinces have become compliant with it; moreover, such a bonding system suffers from critical deficiencies (including the absence of a national administrative authority to oversee reclamation standards and bonding, the absence of any national standard for the types of financial instruments that can be used for bonding) resulting in few companies reclaiming the disturbed land, or receiving any bond refund since the bond paid turns out to be lower than the reclamation costs (Cheng & Skousen, 2017; Yan et al., 2012).

Given the size and the spread of the environmental deterioration caused by resource extraction industries in general and the fast development of extraction of unconventional resources in particular, reclamation of these sites are crucial in order to reduce the negative environmental consequences imposed on future generations. Site reclamation, also known as rehabilitation or revitalization, is the process of restoring the land that has been mined or drilled to a natural or economically usable state. In some cases, the process of site reclamation occurs once extraction is completed, but proper reclamation planning activities are needed and often occur prior to even beginning the site development. For example, different Canadian provinces have recently introduced diverse regulations to ensure that reclamation effectively takes place. Alberta's Environmental Protection and Enhancement Act (EPEA) of 2014 requires firms to prepare for reclamation early in the extraction phase in coal or oil sands mines, and in particular, companies must submit several plans and reports to guide progressive reclamation through the life of a mine in order to be able to complete reclamation within one year of the surface disturbance, as encouraged by the EPEA (Alberta Energy Regulator, 2018). In Ontario even before a mine opens for production, plans need to be in place for the future reclamation of the land; in particular, every advanced exploration and mining project needs to include a closure plan to restore the land to a natural state upon completion of exploration and mining activities, and such rehabilitation plans may include razing buildings, planting trees and natural grasses, and restoring wildlife habitats (Ontario Mining Association, 2018). Similar regulations are also present in the US, where the Federal Land Policy and Management Act of 1976 demands to take any action to prevent "unnecessary or undue degradation" of the land. To do so, the Bureau of Land Management requires oil and gas operators to reclaim the land they disturb and to post a bond as a warranty; these bonds are meant to ensure the lease terms are fulfilled, including covering reclamation costs in the event that the operator abandons the well. However, the bonding requirements have not been updated since the 60s with the consequence that the bureau currently faces potential reclamation liabilities that exceed the value of the bonds it holds, possibly by a considerable amount (United States Government Accountability Office, 2010; Center for Western Priorities, 2018).

The policy relevance of site reclamation and the increasing number of extraction sites all over the world have given rise to a growing interest in reclamation issues, especially over the last decade. Several papers analyze the consequences of reclamation liabilities on firms' investment decisions and the effectiveness of current reclamation policies in specific contexts and specific case studies (e.g. Andersen & Coupal, 2009; Bishop, 2013; Dachis et al., 2017; Davis, 2015; Espinoza & Morris, 2017; Gerard, 2000; Mitchell & Casman, 2011; Sullivan & Amacher, 2009). Most of these works focus on reclamation bonds, discussing their strong limitations (Andersen & Coupal, 2009; Gerard, 2000; Mitchell & Casman, 2011) and thus the need for a reform for the regulatory system (Bishop, 2013; Dachis et al., 2017).[3] Some compare the effectiveness of different policy instruments on firms' behavior showing their strong impact on the viability of alternative project opportunities (Espinoza & Morris, 2017), showing either that bonds may reduce but not eliminate the wedge between private and socially optimal reclamation choices (Sullivan & Amacher, 2009), or that despite their limitations bonds still represent the best option available (Davis, 2015). More limited are the papers analyzing the reclamation policies from a theoretical point of view, which clearly requires to rely upon dynamic settings. In a dynamic hazardous wastes cleanup model, Caputo and Wilen (1995) analyze the intertemporal tradeoffs imposed by regulations to assess the environmental damage and resource cost of cleaning up the wastes. Lappi (2018) studies a social planner's problem deciding upon cleaning up multiple polluted sites by choosing the time and order of cleanups, showing that because of the natural decay of hazardous wastes it may be optimal to delay cleanup activities, potentially explaining why at the reclamation date in reality firms rarely deliver. By considering the implications of reclamation on the extraction decisions, even if in different setups, both Yang and Davis (2018) and Lappi (2020) analyze how different policy instruments can be used to decentralize the social optimum, eventually underlying the problems associated with bonding requirements (Yang & Davis, 2018).

To the best of our knowledge, no single study thus far has tried to analyze the issues associated with the design of reclamation policy and how firms' reclamation efforts are affected by the presence of multiple firms operating close to each other. Indeed, extraction activities of different firms often take place in narrow areas in which firms pollute the surrounding environment (e.g., any possible spills and leakages that contaminate the nearby air and water resources) and also end up sharing a variety of facilities (including, but not limited to, access roads, infrastructures, landfill sites, lakes or other water bodies, pipeline corridors). Such interconnections between firms gives rise to the possibility of free riding since it is hard to distinguish who should be held responsible for the environmental damages generated by the side effects of extraction activities. The implications of such shared facilities have been mostly ignored in both theoretical literature and policy settings while their relevance for environmental outcomes may be very significant, especially in cases like western Canada's oil sand industry where firms operate in close proximity of each other and do share main development facilities including, but not limited to, access roads, landfill sites, and pipelines. Our paper tries to make a first theoretical contribution in this context complementing extant reclamation literature by focusing on strategic interactions between firms whose extraction activities result in polluting shared nearby environment and may require to set up shared facilities

[3]Mitchell and Casman (2011) show that the oil and gas bonding requirements in the United States, which generally determine the minimum bonding values either on a blanket basis or for individual wells (and which may be met even without transferring money), do not incentivize reclamation since their levels are typically set too low; in particular, due to the expected growth of extraction activities which will lead several operators to drill thousands of wells, the blanket bond is particularly inadequate to cover the reclamation costs.

polluting the environment. Without a doubt, the shared nature of the environment and, therefore, the strategic interaction of the involved parties introduce additional layers of complexity to the reclamation problem, allowing us to assess the effectiveness of actual reclamation policies from a broader perspective.

Specifically, we develop a theoretical model of site reclamation from the point of view of two neighboring firms whose extraction generates emissions jointly polluting a common environment. We assume that environmental regulation requires firms to engage in reclamation and restoration efforts during the entire lifespan of the site, and also it makes firms liable to pay an abandonment reclamation fee at the end of their extraction lease for the amount of pollution they fail to reclaim by then. Therefore, each firm plans for its dynamic reclamation activities strategically in an attempt to minimize its costs, which is the discounted sum of the instantaneous losses associated with reclamation activities and the end-of-lease reclamation fee which depends on the unclaimed pollution stock at the abandonment date. First, we study the case in which the two firms are homogeneous and then we depart from this baseline setup by introducing heterogeneity, including different lease terms (i.e., different duration and degrees of liability). While it is possible to derive all of our results analytically in the former case, we need to rely on numerical simulations to illustrate some of our results in the latter due to the complexity that heterogeneities introduce into the model. In both cases, we analyze firms' decisions under two different scenarios, namely a situation in which they do not cooperate on their reclamation strategies and a situation in which they do. Our results show that, if firms are homogeneous, the time trajectory of the reclamation effort in the noncooperative case may be nonmonotonic, while in the cooperative case, it is always monotonic and dependent on how the rate of time preference and the gross growth rate of pollution compare. Moreover, for any given level of pollution, the cooperative rule prescribes a higher level of reclamation effort than the noncooperative one for both firms. If firms are heterogeneous they generally choose different reclamation efforts under noncooperation, while under cooperation this occurs only if the efficiency of their environmental maintenance is different. Under noncooperation, the reclamation efforts of the two firms can show opposite dynamics, while this does not occur under cooperation, where instead there is a discontinuity in the reclamation effort of the firm with longer lease duration at the other firm's site dismissal date. Independently of the fact that firms are homogeneous or heterogeneous, in all scenarios the reclamation efforts generally rise with the degree of liability and fall with the lease duration, suggesting that in order to promote better environmental outcomes, the regulators should carefully determine the lease conditions by introducing intra-term reclamation fees along with stringent environmental accountability. Also, by determining the lease terms such to compel firms to continually involve in reclamation activities proportional to the pollution level, they can effectively address individual firms' reclamation efforts along the optimal cooperative level.

Different from extant literature which focuses on the reclamation problem from the point of view of the regulator by either determining the socially optimal reclamation efforts (Caputo & Wilen, 1995; Lappi, 2018) or how decentralizing the first best (Lappi, 2020; Yang & Davis, 2018), our paper analyzes the issue from the site operators' perspective by discussing the implications of strategic interaction between firms. Our focus at the firm-level allows us to understand how different lease terms (i.e., different durations and degrees of liability) impact the reclamation choices of the firms operating in and sharing a common extraction site not only at the site dismissal date but also during the entire production phase, showing that the introduction of intra-term reclamation fees along with stringent environmental accountability may result in better environmental outcomes. Previous studies have abstracted from the analysis of the role of lease

conditions in driving firms' decisions (since focusing on the regulator's problems), and from the possibility to engage in reclamation activities during the production phase (since assuming the existence of a separate reclamation phase, completely independent from extraction). We believe that our approach is more consistent with the current regulations in place that prescribe reclamation since early in the production process (e.g., in Canada) and provides simpler to implement suggestions to support reclamation policymaking.

This paper proceeds as follows. Section 2 presents our model, which consists of a reclamation problem in a differential game setting in which firms seek to minimize their reclamation cost, given by the discounted sum of the instantaneous losses associated with environmental maintenance activities and the abandonment reclamation fee. In Section 3, we focus on the simplest case in which the firms are homogeneous, by analytically deriving and comparing the noncooperative and cooperative outcomes. In Section 4, we extend the analysis to a framework with heterogeneous firms by comparing the noncooperative and cooperative outcomes through numerical simulations which allow us to discuss how different lease durations and degrees of liability affect individual reclamation efforts. Section 5 presents concluding remarks and highlights directions for future research. All mathematical technicalities are presented in the Appendix 6.

2 | THE MODEL

We consider a two-firm (indexed by z, $z \in \{i,j\}$) differential game of site reclamation in which the extraction leases have finite duration. At the site dismissal time, each firm is liable to pay a reclamation fee proportional to the level of unclaimed environmental degradation (i.e., the pollution stock p_t) caused by their joint production activities. At each moment in time, each firm z engages in resource extraction and reclamation activities, $\tau_t^z > 0$, which we refer to as "reclamation effort" for simplicity. For the sake of simplicity, we assume that firms extract the resource at an exogenous rate γ^z, where $\gamma^z > 0$ is a technological parameter, while emissions due to extraction are not constant but increasing in the pollution stock due to, for example, the decreasing returns of the extraction technologies. In other words, emissions, e_t^z, are given by $e_t^z = \epsilon^z \gamma^z p_t$, where $\epsilon^z > 0$ is the environmental inefficiency of extraction activity. Comparing to the standard pollution dynamic models, this assumption is indeed a move toward increasing realism: in the case of minerals, for example, the higher-grade ores are extracted first, followed by an increasing reliance on lower-grade ones, resulting in the marginal emissions to increase with the cumulative amount extracted.[4] The reclamation effort defines the ability of each firm to curb the pollution and reclaim the environment via purposive abatement activities $a_t^z = \alpha^z \tau_t^z$, where a_t^z denotes abatement or environmental maintenance and $\alpha^z > 0$ the efficiency of environmental reclamation activity.[5] Pollution accumulates according to the difference between the emissions generated by the two firms i and j, net of pollution decay and the two firms environmental maintenance: $\dot{p}_t = (\epsilon^i \gamma^i + \epsilon^j \gamma^j - \delta) p_t - \alpha^i \tau_t^i - \alpha^j \tau_t^j$, where $\delta > 0$ is the natural pollution decay rate. We assume

[4]Note also that a similar dynamic equation has recently been employed in a single-player pollution control problem frameworks by Saltari and Travaglini (2016) and La Torre et al. (2017, 2021).

[5]We may assume that firms' extraction rates and their environmental inefficiencies are known to the regulator, through tax and technology information (income taxes provide details regarding sales and thus extraction, and extraction does depend on the technology available by firms).

that the gross growth rate of pollution (i.e., the growth rate of pollution in the absence of abatement) is positive, that is, $\epsilon^i\gamma^i + \epsilon^j\gamma^j > \delta$, meaning that without purposive reclamation activities pollution will increase over time leading the firms to face a substantial reclamation fee on their lease termination date.

We assume that by environmental regulations each firm is liable to cleanup and reclaim the environment by the end of its lease duration (i.e., on site dismissal). If a firm fails to thoroughly reclaim the environment by then it has to pay a fee to cover the cost of the environmental damage it has left behind, which we shall refer to as "abandonment reclamation fee." Different from current bonding requirements, we assume that existing regulations require firms to post a large enough bond to ensure they will effectively cover these fees. Firm z's cost function, denoted by $\mathscr{C}^z$, is the sum of two different terms: the discounted ($\rho > 0$ is the rate of time preference) sum of instantaneous losses generated by the firm's reclamation effort (i.e., abatement cost), and the discounted abandonment reclamation fee associated with the unclaimed damage at the end of firm z's (finite and known) lease duration, T^z.[6] We assume that abatement activities are subject to decreasing returns-to-scale such that the abatement cost function, $\ell(\tau_t^z)$, is increasing and convex in the reclamation effort. In other words, removing pollution and reclaiming the environment becomes more and more difficult as the removed stock of pollution increases (see, among others, Barrett, 1994; Finus, 2008; Nordhaus, 2010). For the sake of simplicity, we assume the abatment cost function to be quadratic as follows: $\ell(\tau_t^z) = \frac{(\tau_t^z)^2}{2}$. The environmental damage is assumed to be increasing and convex in the pollution, and to take a quadratic form, thus the abandonment reclamation fee, $f(p_{T^z})$, is assumed to be $f(p_{T^z}) = \phi^z \frac{p_{T^z}^2}{2}$, where $\phi^z \geq 0$ is determined by the regulator and quantifies the extent to which firm z is effectively liable for the damage caused by pollution at site dismissal.[7] Thus, each firm chooses the reclamation effort τ_t^z to devote to reclamation activities in order to minimize its cost, by taking into account the evolution of pollution, its given initial level and, of course, the choice made by the other firm. The initial level of pollution, p_0, represents the extent of environmental degradation before extraction effectively takes place, due to, for example, pre-extraction activities like preparing infrastructures and seismic lines. In other words, the time zero in our model, represents the beginning of the extraction phase for the firms, while, p_0 represents the stock of pollution that during the pre-development and site preparation has taken place. Naturally, firms are responsible for this initial stock of pollution and should be held accountable if failed to reclaim it.

The representative firm z's problem can be summarized as follows:

$$\min_{\tau_t^z} \mathscr{C}^z = \int_0^{T^z} \frac{(\tau_t^z)^2}{2} e^{-\rho t} dt + \phi^z \frac{p_{T^z}^2}{2} e^{-\rho T^z}, \tag{1}$$

$$s.t. \dot{p}_t = (\epsilon^i\gamma^i + \epsilon^j\gamma^j - \delta)p_t - \alpha^i\tau_t^i - \alpha^j\tau_t^j, \tag{2}$$

$$p_0 > 0 \text{ given.} \tag{3}$$

[6]A similar two-terms objective function has been used in different one-player environmental and macroeconomic contexts by La Torre et al. (2017) and La Torre and Marsiglio (2020), respectively.

[7]A lower bound for each firm's liability may be borrowed from Caputo and Wilen's (1995) and Lappi's (2018) works. By studying the optimal timing and amount of final cleanup of abandoned mines from the social planner's perspective, their minimum cost provides an estimate of the minimum firm's liability in our setup.

As explained above, the second component in (1) represents the discounted value of the lump-sum penalty that firm z should pay to the regulator if it fails to reclaim the environment by the end of its lease. If $\phi^z = 0$ then the firm is not liable for reclamation and thus only the sum of instantaneous losses will determine the firm's choices (and in this case the cost minimizing reclamation effort will clearly be null). For any $\phi^z > 0$ (and finite), the firm needs to account for both its instantaneous losses and abandonment reclamation fee to determine its rehabilitation efforts. We are particularly interested in quantifying how firms' reclamation decisions depend on the value of this parameter, which we shall refer to as the "degree of liability," in order to provide a first assessment of how reclamation regulations may impact firms' behaviors and environmental outcomes.

Before proceeding to the solution of our model, it may be convenient to mention some comments on our modeling approach and assumptions. Similar to extant reclamation literature, we assume that the damages are reversible (Caputo & Wilen, 1995; Lappi, 2018, 2020; Yang & Davis, 2018), and we assume that reclamation may occur only during the site lifespan (Caputo & Wilen, 1995; Yang & Davis, 2018). Different from Yang and Davis (2018), in order to preserve tractability due to the presence of strategic interaction between firms, we abstract from modeling the resource stock dynamics, and different from Caputo and Wilen (1995) in our setting firms pays a reclamation fee at the site dismissal date related to the amount of unclaimed damages, along the lines of the historical policy scenario in Yang and Davis (2018). The introduction of a game between firms is the main aspect that distinguish our paper from all previous studies and makes comparison of results difficult. For the sake of simplicity, we have assumed that only two firms operate on the site sharing the common environment, but in reality it may well be possible that the number of strategically interacting firms is larger than two. Introducing multiple firms in our setting will not modify our conclusions from a qualitative point of view, thus it seems convenient to present the model in the simplest possible form. In addition to the game aspect, another aspect that this work contributes to the extant literature is suggesting important departures in the model from previous works on the optimal clean up of polluting sites. Given the specific focus on reclamation activities, the works most closely related to ours are Lappi (2018, 2020) and Yang and Davis (2018). The polluted sites in Lappi's (2018) framework can be interpreted as the abandoned or orphan mine sites that the operators have failed to reclaim in ours, and thus our paper is complementing Lappi's (2018). As a matter of fact, we consider the firms' abatement and reclamation activities before the site closure and abandonment, therefore, the reclamation cost estimated in Lappi's (2018) social planner framework may be considered as a lower bound for the abandonment reclamation fee determined by the regulator in our setup. Unlike Yang and Davis (2018), we consider environmental costs that are nonlinear in the pollution stock, which is a more realistic assumption (Dockner et al., 2000), and thus while some of our special case results are consistent with their findings we find that the time path of abatement (and thus the marginal abatement cost) and pollution are dependent on the difference between firms' time preference and pollution accumulation (and thus future liabilities accumulation) rates. Similar to Lappi (2018), we do consider the natural revitalization of the environment into our model, which instead Yang and Davis (2018) abstract from. Nevertheless, different from both Yang and Davis (2018) and Lappi (2018), our focus is on the firms' choices and their continuous abatement and reclamation activities by taking into account possible fines and punishments they may face for leaving pollution behind. This gives us the possibility to study and discuss the impact of different aspects of lease terms (such as duration and liabilities) on the results in a rather realistic way, since these terms are normally predetermined and, for example, firms cannot simply extend their lifespan forever to avoid facing reclamation fees. With that purpose, we also include the possible strategic interactions

between neighbor firms which share a variety of facilities and as such share the environmental degradation liabilities. It is also worth mentioning that another aspect that our research add to the considerably limited theoretical literature in environmental reclamation is that extant works focus on open-loop solutions based on an optimal control method while in our setup we rely on dynamic programming and thus focus on the closed-loop solutions.

In what follows, we first study our model under the assumption that firms are homogeneous under noncooperative and cooperative assumptions, and then we depart from the homogeneity assumption to present a rather general model.

3 | THE HOMOGENEOUS CASE

We first assume that the two firms are exactly identical. This simplification allows us to explicitly characterize and compare the noncooperative and cooperative outcomes and identify the sources of distortions underlying the inefficient noncooperative behavior.

3.1 | The noncooperative solution

In a business as usual scenario, since firms do not cooperate on their reclamation strategies, they will only be concerned about their own individual cost and make their choices of reclamation effort accordingly. When the two firms are perfectly identical ($T^i = T^j = T$, $\phi^i = \phi^j = \phi$, $\epsilon^i = \epsilon^j = \epsilon$, $\gamma^i = \gamma^j = \gamma$ and $\alpha^i = \alpha^j = \alpha$), the above minimization problem reads as follows:

$$\min_{\tau_t^z} \mathscr{C}^z = \int_0^T \frac{(\tau_t^z)^2}{2} e^{-\rho t} dt + \phi \frac{p_T^2}{2} e^{-\rho T}, \tag{4}$$

$$s.t. \dot{p}_t = (2\epsilon\gamma - \delta) p_t - \alpha(\tau_t^i + \tau_t^j). \tag{5}$$

We are interested in finding the closed-loop feedback solution for the above problem. Thus, solving it requires to find an explicit expression for the value function satisfying its associated Hamilton–Jacobi–Bellman equation. After some algebra, it is possible to claim the following (the proofs of all the following propositions are presented in the Appendix 6).

Proposition 1 *The Cournot–Nash rule for the reclamation effort of the representative firm $z \in \{i, j\}$, $(\tau_t^z)^N$, and the Cournot–Nash time path of pollution, p_t^N, are, respectively, given by:*

$$(\tau_t^z)^N = \frac{\lambda \phi \alpha e^{\lambda t}}{(\lambda + 3\phi\alpha^2) e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}} p_t^N, \tag{6}$$

$$p_t^N = p_0 \left[\frac{(\lambda + 3\phi\alpha^2) e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}}{(\lambda + 3\phi\alpha^2) e^{\lambda T} - 3\phi\alpha^2} \right]^{\frac{2}{3}} e^{(2\epsilon\gamma - \delta)t}, \tag{7}$$

where $\lambda \equiv \rho - 2(2\epsilon\gamma - \delta)$.

Proposition 1 states that the Cournot–Nash reclamation effort (which defines the effective environmental maintenance rule, $\alpha(\tau_t^z)^N$) is proportional to pollution. The reclamation

effort turns out to be well defined (i.e., positive) independently of the sign of λ. Note that λ equals the time preference, ρ, net of twice the gross growth rate of pollution, $2(2\epsilon\gamma - \delta)$,[8] where the latter will eventually determine the abandonment reclamation fee. Thus, a positive (negative) λ represents a situation in which firms discount future faster (slower) than they are concerned with the rate at which their reclamation liabilities in the distant future increase in the absence of reclamation efforts. As we are going to see later, the model's outcome is going to be crucially different according to which of these two alternative situations holds true.

By plugging (7) into (6), the Cournot–Nash reclamation effort time path is given by the following time-varying expression:

$$\tau_t^N = \frac{p_0 \lambda \phi \alpha e^{[\rho-(2\epsilon\gamma-\delta)]t}}{[(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}]^{\frac{1}{3}}[(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2]^{\frac{2}{3}}}, \tag{8}$$

suggesting that the reclamation effort increases with the pollution due to pre-extraction activities, which is thus effectively accounted for by the single firm in the determination of its abatement level. Straightforward algebra allows us to show that the time path of the Cournot–Nash reclamation effort may not be monotonic, since the sign of the following derivative is ambiguous and depends on a number of factors:

$$\frac{\partial \tau_t^N}{\partial t} = \frac{p_0 \alpha \lambda \phi e^{[\rho-(2\epsilon\gamma-\delta)]t} \left\{[\rho-(2\epsilon\gamma-\delta)](\lambda+3\phi\alpha^2)e^{\lambda T} - [2\rho-(2\epsilon\gamma-\delta)]\phi\alpha^2 e^{\lambda t}\right\}}{[(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}]^{4/3}[(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2]^{2/3}}.$$

However, provided that the rate of time preference is smaller than the gross growth rate of pollution, that is $\rho < (2\epsilon\gamma - \delta)$ (which can be verified only whenever $\lambda < 0$), the Cournot–Nash reclamation effort will monotonically fall over time if λ is small enough in absolute value. When this is the case, the reclamation effort is initially high, then decreases over time, otherwise it might be initially low and then gradually increase over time. The intuitive explanation behind this behavior of reclamation efforts is that when firms discount future at a rate relatively lower than the growth of their future liabilities (i.e., when the discount rate is mildly less than the gross growth rate of pollution, $\rho < (2\epsilon\gamma - \delta)$) but $|\lambda|$ remains small, they act more aggressively to reduce and control pollution by implementing higher levels of abatement and environmental maintenance early on and viceversa. However, if $\lambda < 0$ and large in absolute value then as we will show later on in our numerical examples the reclamation effort might show a non-monotonic behavior. Such non-monotonic behavior can be understood by relating the size of the natural pollution decay which ultimately determines whether it may be convenient for the individual firm to intervene in environmental maintenance as soon as possible or to postpone it to some future date. If the natural decay is low, there is no point in postponing maintenance activities and thus the reclamation effort will be high initially to then decrease gradually over time. However, if this is high then the natural dynamics tend to reduce pollution providing a clear incentive for firms to postpone intervention: in this case, the reclamation effort will be high initially (the net growth rate of pollution is higher than the discount factor) and tend to fall for some time (exploiting the natural environmental cleaning capacity) to then radically increase

[8]Due to the quadratic forms of the instantaneous losses and end-of-lease reclamation fee, firms attach to the net extraction activities twice the value they attach to time.

closer to the dismissal date in order to minimize the unclaimed damage and thus the abandonment reclamation fee.[9]

Aside from the temporal evolution of the reclamation effort, from Proposition 1, we can also infer the implications of policy parameters (i.e., the lease duration and the degree of liability) which can eventually be determined by the regulator when setting the lease conditions of single individual firms. Indeed, the reclamation effort will monotonically fall with the lease duration whenever $\lambda > 0$ or $\lambda < 0$ but small enough in absolute value:

$$\frac{\partial \tau_t^N}{\partial T} = -\frac{p_0 \alpha \lambda^2 \phi(\lambda + 3\phi\alpha^2)\left[(\lambda + 3\phi\alpha^2)e^{\lambda T} - 2\phi\alpha^2 e^{\lambda t} - \phi\alpha^2\right] e^{[\rho-(2\epsilon\gamma-\delta)]t} e^{\lambda T}}{[(\lambda + 3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2]^{5/3}[(\lambda + 3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}]^{4/3}}, \tag{9}$$

and will rise with the degree of liability:

$$\frac{\partial \tau_t^N}{\partial \phi} = \frac{p_0 \alpha \lambda^2 \left[(\lambda + 3\phi\alpha^2)e^{\lambda T} - 2\phi\alpha^2 e^{\lambda t} - \phi\alpha^2\right] e^{[\rho-(2\epsilon\gamma-\delta)]t} e^{\lambda T}}{[(\lambda + 3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2]^{5/3}[(\lambda + 3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}]^{4/3}} > 0. \tag{10}$$

These last two results are intuitive by considering that a longer lease duration tends to reduce reclamation efforts at any moment in time by postponing the payment of the abandonment reclamation fee associated with pollution to a later date in the future. Similarly, a lower degree of liability by reducing the size of the reclamation fee tends to lower abatement at any moment in time.

Note also that Proposition 1 implies that the abandonment reclamation fee to be paid by firm z is given by:

$$f(p_T^z)^N = \frac{\phi p_0}{2}\left[\frac{\lambda e^{\lambda T}}{(\lambda + 3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2}\right]^{\frac{2}{3}} e^{(2\epsilon\gamma-\delta)T}, \tag{11}$$

which monotonically increases (decreases) with the degree of liability if $\lambda > 0$ (if $\lambda < 0$), while it monotonically increases (decreases) with the lease duration if λ is large enough (if λ is small enough). Both these results are intuitive by recalling that λ measures the how fast firms discount the future with respect to the pollution gross growth rate. When $\lambda > 0$ firms have weak incentives to engage in reclamation efforts and this will lead to a large stock of pollution in the future and thus to a large abandonment reclamation fee, and the higher λ the higher the abandonment fee. Since such effects compound over the time, when $\lambda > 0$ and possible high, the shorter the lease duration the lower the end-of-lease fee.

To complete our discussion about the impact of different parameters on the reclamation efforts and the pollution dynamics, we present some numerical examples. The parameter values are arbitrarily chosen in order to make sure that our numerical solution leads to economically meaningful values for all our variables and also to visualize the implications of different parameters in the most effective way; moreover, changing the parametrization allows us to conclude that our results along with our sensitivity analysis are robust.[10] Specifically, in our benchmark

[9]This explanation is similar to Lappi's (2018) argument, even if derived in a completely different setting with jump reclamation activities. Different from Lappi (2018) who abstracts completely from emissions, in our setting emissions are exogenously deteriorating the environment at a constant rate (equal to $2\epsilon\gamma$ is our current homogeneous case), thus what really drives firms' decision is the rate of natural decay net of total emissions (i.e., $\delta - 2\epsilon\gamma$).

[10]The practice of choosing parameter values to ensure economically meaningful values for all model variables is common in the literature using dynamic and differential games to study environmental economics (see, e.g., Breton et al., 2010; Masoudi & Zaccour, 2013, 2018).

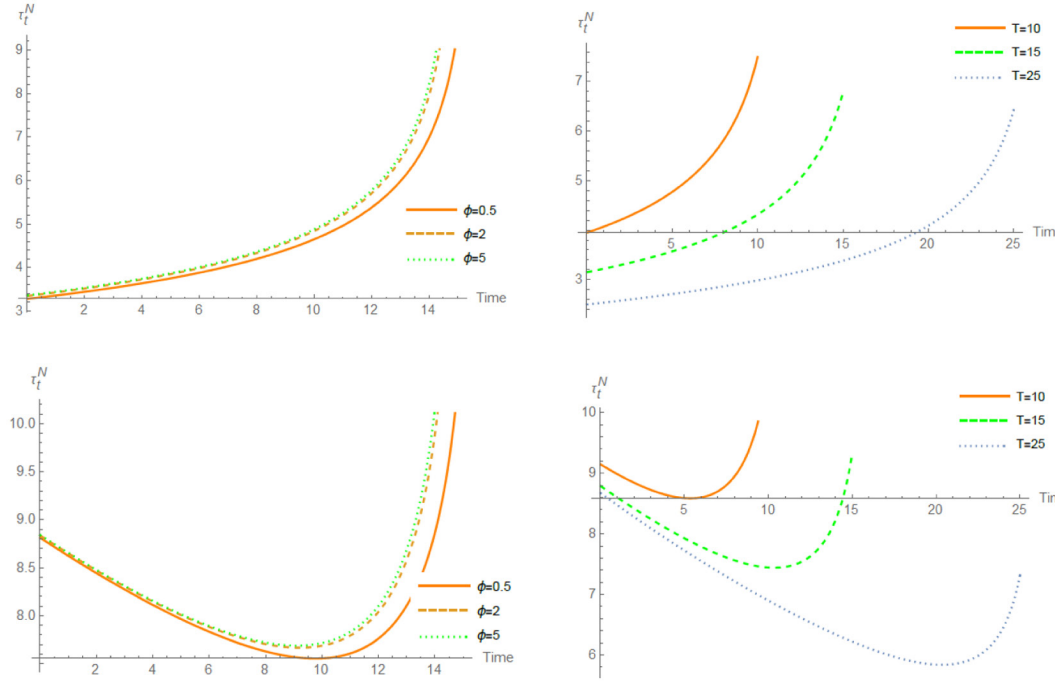

FIGURE 1 Impact of ϕ (left) and T (right) on τ_t^N in the $\lambda > 0$ (top) and $\lambda < 0$ (bottom) cases [Colour figure can be viewed at wileyonlinelibrary.com]

example, we set $p_0 = 100$, $\alpha = 1$, $\rho = 0.04$, $\delta = 0.05$, $\epsilon = 0.1$, $\gamma = 1$, $\phi = 0.05$, $T = 15$ and either $\gamma = 1$ (such that $\lambda > 0$) or $\gamma = 0.2$ (such that $\lambda < 0$), and let ϕ and T eventually change. Figure 1 presents the results of our numerical simulations, showing the impact of changes in the degree of liability (left panels) and in the lease duration (right panels) on the time paths of the reclamation effort for the $\lambda > 0$ (top panels) and $\lambda < 0$ (bottom panels) cases. We can observe that, while in the $\lambda > 0$ case the reclamation effort is monotonically increasing, in the $\lambda < 0$ it is nonmonotonic—decreasing in earlier times and increasing closer to the lease end date. Regardless of the sign of λ, consistent with what discussed above, the higher the lease duration or the lower the degree of liability, the lower the reclamation efforts and so the higher the pollution stock at each moment in time. Therefore, intuitively, the environment will benefit from lower lease duration and higher degrees of liability, suggesting that in order to ensure better environmental outcomes, the regulators should carefully determine the lease conditions, by introducing intra-term reclamation fees along with stringent environmental accountability. For example, this may be implemented by anticipating part of the penalty for the past unclaimed pollution at some earlier phase following intermediate evaluation and assessment of the effectiveness of firms' reclamation activities.

3.2 | The cooperative solution

The above case may present how firms interact with each other in a noncooperative business-as-usual scenario but this is clearly not optimal since firms fail to internalize the externality that their activities impose on each other. In order to determine the optimal equilibrium, we now

focus on the case in which the two firms cooperate and coordinate their reclamation efforts, and as such, they minimize their joint reclamation cost as follows:

$$\min_{\tau_t^i,\tau_t^j} \mathscr{C} = \mathscr{C}^i + \mathscr{C}^j = \int_0^T \left[\frac{(\tau_t^i)^2}{2} + \frac{(\tau_t^j)^2}{2} \right] e^{-\rho t} dt + \phi p_T^2 e^{-\rho T}, \tag{12}$$

$$s.t. \dot{p}_t = (2\epsilon\gamma - \delta) p_t - \alpha(\tau_t^i + \tau_t^j). \tag{13}$$

In this case, it is possible to derive the following result.

Proposition 2 *The cooperative rule for the reclamation effort for firm $z \in \{i,j\}$, $(\tau_t^z)^C$, and the cooperative time path of pollution, p_t^C, are, respectively, given by:*

$$(\tau_t^z)^C = \frac{2\lambda\phi\alpha e^{\lambda t}}{(\lambda + 4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2 e^{\lambda t}} p_t^C, \tag{14}$$

$$p_t^C = p_0 \left[\frac{(\lambda + 4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2 e^{\lambda t}}{(\lambda + 4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2} \right] e^{(2\epsilon\gamma - \delta)t}, \tag{15}$$

where $\lambda \equiv \rho - 2(2\epsilon\gamma - \delta)$.

Proposition 2 states that the cooperative reclamation effort is proportional to pollution. Exactly as in the noncooperative case, the optimal reclamation effort turns out to be well defined (i.e., positive) independently of the sign of λ; however, its behavior is dependent on the (sign and magnitude) of λ. Similar to the noncooperative case, in the very specific case in which $\lambda = 0$, the cooperative reclamation effort is null.

By substituting (15) into (14), the cooperative time path of the reclamation effort is given by the following expression:

$$\tau_t^C = \frac{2p_0\lambda\phi\alpha e^{[\rho - (2\epsilon\gamma - \delta)]t}}{(\lambda + 4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2}. \tag{16}$$

Unlike the previous case, it is straightforward to see that the time evolution of the optimal reclamation effort is monotonic since the sign of the following derivative only depends on the sign of $\rho - (2\epsilon\gamma - \delta)$:

$$\frac{\partial \tau_t^C}{\partial t} = \frac{2p_0\lambda\phi\alpha[\rho - (2\epsilon\gamma - \delta)]e^{[\rho - (2\epsilon\gamma - \delta)]t}}{(\lambda + 4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2}. \tag{17}$$

Specifically, the cooperative reclamation effort will rise (fall) over time if the discount factor is larger (smaller) than the gross growth rate of pollution, which can occur only when $\lambda < 0$. Moreover, exactly as in the noncooperative case, the optimal reclamation effort will monotonically fall with the length of the lease duration, T, whenever $\lambda > 0$ or $\lambda < 0$ but small enough in absolute value since:

$$\frac{\partial \tau_t^C}{\partial T} = -\frac{2p_0\lambda^2\phi\alpha(\lambda + 4\phi\alpha^2)e^{\lambda T}e^{[\rho - (2\epsilon\gamma - \delta)]t}}{[(\lambda + 4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2]^2}, \tag{18}$$

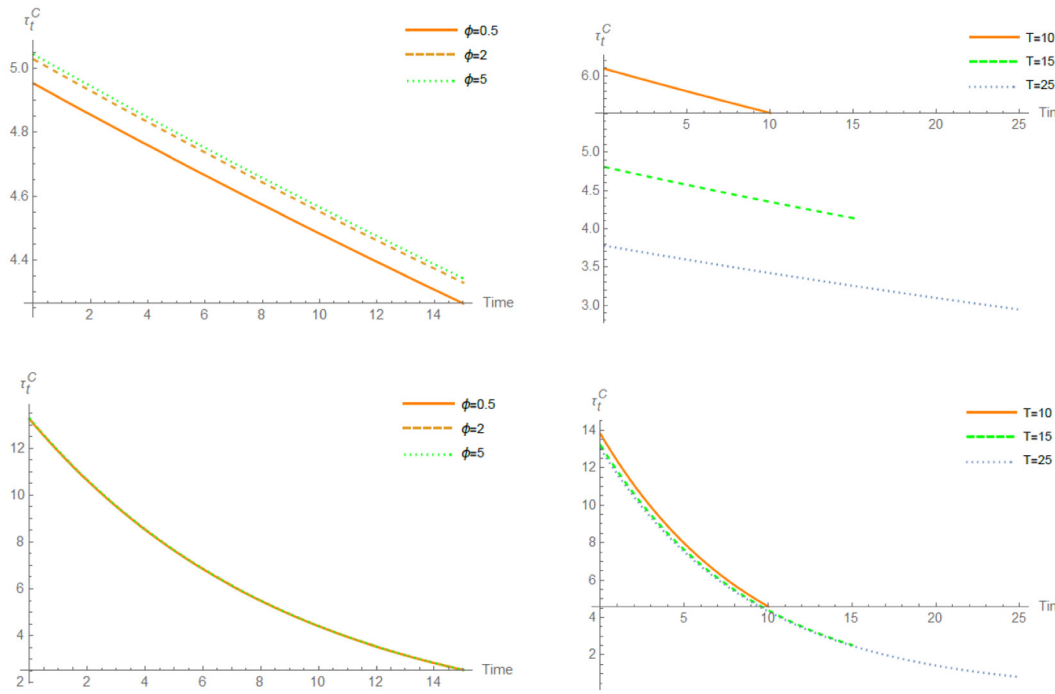

FIGURE 2 Impact of ϕ (left) and T (right) on τ_t^C in the $\lambda > 0$ (top) and $\lambda < 0$ (bottom) cases [Colour figure can be viewed at wileyonlinelibrary.com]

and the cooperative reclamation effort will rise with the degree of liability since we have:

$$\frac{\partial \tau_t^C}{\partial \phi} = \frac{2p_0\lambda^2\alpha e^{\lambda T}e^{[\rho-(2\epsilon\gamma-\delta)]t}}{[(\lambda+4\phi\alpha^2)e^{\lambda T}-4\phi\alpha^2]^2} > 0.$$

The same arguments discussed earlier to explain the effect of changes in the lease duration and in the degree of liability also apply to the cooperative case. The main difference between the reclamation efforts in the noncooperative and the cooperative scenarios is related to the fact that the time trajectory of the reclamation effort will always be monotonic in the latter case while it may be non-monotonic in the former.

Note also that Proposition 2 implies that the abandonment reclamation fee to be paid by firm z is given by:

$$f(p_T^z) = \frac{\phi^z p_0}{2}\left[\frac{\lambda e^{\lambda T}}{(\lambda+4\phi\alpha^2)e^{\lambda T}-4\phi\alpha^2}\right]e^{(2\epsilon\gamma-\delta)T}. \tag{19}$$

which monotonically increases (decreases) with the degree of liability if $\lambda > 0$ (if $\lambda < 0$), while it monotonically increases (decreases) with the lease duration if λ is large enough (if λ is small enough). These results can be explained exactly as in the previous noncooperative case, and exactly the same comments apply.

By relying on the same parameter values employed earlier in the noncooperative case, Figure 2 presents some numerical examples to illustrate the above discussed comparative dynamics in the $\lambda > 0$ and $\lambda < 0$ cases. We can note that, different from the noncooperative case, the time paths of the cooperative reclamation efforts are always monotonically decreasing, while the effects of

changes in the lease duration and the degree of liability on the reclamation efforts mimic those presented in the noncooperative case.

3.3 | Comparison

After presenting the noncooperative and cooperative solutions, now we analyze the difference between these two scenarios by quantifying the size of the distortion imposed by firms' strategic behavior and examining the impact of the model parameters on this distortion. By comparing the cooperative and noncooperative reclamation efforts, given by (16) and (8), respectively, we obtain the following expression whose sign is ambiguous since it depends on how specific parameter values compare:

$$\tau_t^C - \tau_t^N = \frac{p_0 \lambda \phi \alpha e^{[\rho-(2\epsilon\gamma-\delta)]t} \left\{ 2[(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}]^{\frac{1}{3}} [(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2]^{\frac{2}{3}} - [(\lambda+4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2] \right\}}{[(\lambda+4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2][(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}]^{\frac{1}{3}} [(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2]^{\frac{2}{3}}}. \tag{20}$$

However, except under extremely special circumstances that require $2[(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}]^{\frac{1}{3}} [(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2]^{\frac{2}{3}} = [(\lambda+4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2]$ (or $\lambda\phi\alpha = 0$, which may occur when the discount factor is equal to twice the gross growth rate of pollution, $\lambda = 0$, when there is no penalty for leaving the environment unclaimed, $\phi = 0$, or when reclamation activities are not effective at all, $\alpha = 0$; in each of these cases there will be no reclamation under both scenarios), the gap between the cooperative and noncooperative reclamation efforts will not be null, meaning that in general the noncooperative solution distorts the reclamation effort away from its socially optimal value, as expected. However, we cannot derive a general conclusion about the sign of the distortion in (20), as this depends non-monotonically on the different model's parameters as well as time. Nevertheless, some additional conclusion can be obtained by directly comparing the reclamation effort coefficients. Indeed, from (6) and (14), we get:

$$\frac{\partial \tau_t^C}{\partial p_t^C} - \frac{\partial \tau_t^N}{\partial p_t^N} = \frac{\lambda \phi \alpha e^{\lambda t} \left[\left(3\lambda + 10\alpha^2\phi\right) e^{\lambda T} - 10\phi\alpha^2 e^{\lambda t} \right]}{\left[(\lambda+4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2 e^{\lambda t}\right] \left[(\lambda+3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}\right]} > 0, \tag{21}$$

which implies that, for any given level of pollution, the cooperative rule will prescribe a higher level of reclamation effort than the noncooperative one. As a result, beginning from the same initial value, the pollution trajectory resulting from cooperation will be located underneath the noncooperative one. Therefore, it is possible that at some point in time the cooperative reclamation effort becomes lower than the noncooperative effort. This may occur whenever the pollution stock becomes considerably smaller in the cooperative than in the noncooperative case, such that this difference more than compensates the difference in the coefficients given by (21).

Figures 3 and 4 illustrate the size of the difference between the cooperative and noncooperative reclamation efforts and pollution, respectively, in both the $\lambda > 0$ and $\lambda < 0$ cases. We can observe that, as discussed above, beginning from the same initial value of pollution, cooperation implies more reclamation efforts than noncooperation (such that the size of the difference between the two reclamation efforts is positive; see Figure 3) which leads into a widening of the distortion between the cooperative and noncooperative pollution (see Figure 4); however, when pollution becomes large enough the noncooperative reclamation effort becomes larger than the cooperative effort (the difference between the two reclamation efforts becomes negative).

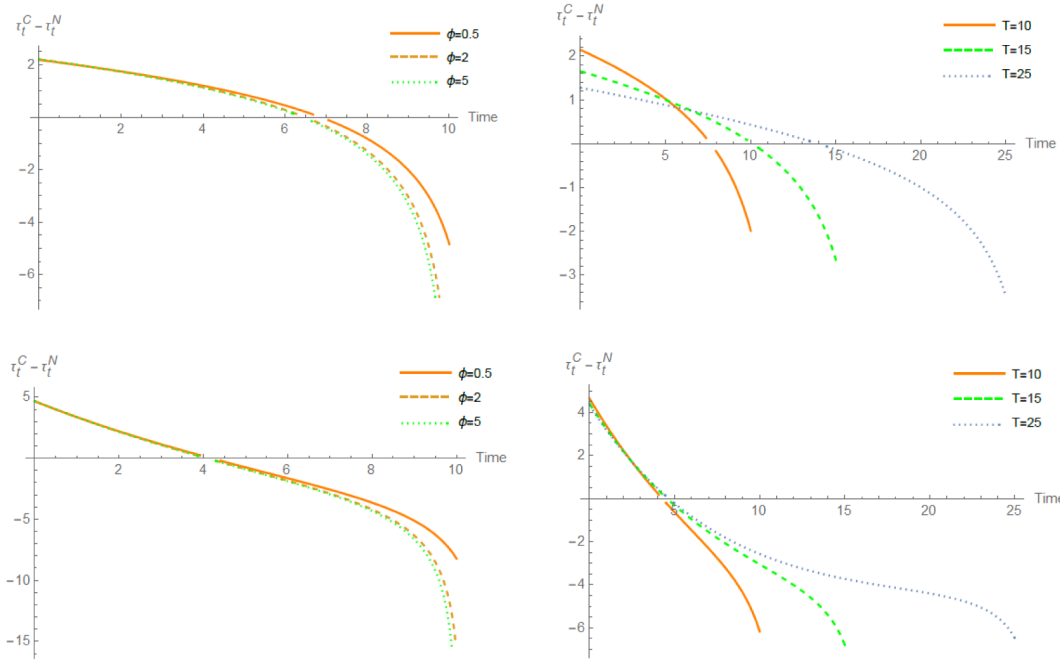

FIGURE 3 Impact of ϕ (left) and T (right) on $\tau_t^C - \tau_t^N$ in the $\lambda > 0$ (top) and $\lambda < 0$ (bottom) cases [Colour figure can be viewed at wileyonlinelibrary.com]

Another noteworthy observation is that, as we get closer to the end of the lease duration, the noncooperative reclamation effort increases faster than the cooperative one, giving rise eventually to a decrease in the gap between the cooperative and noncooperative pollution. Moreover, the larger the firm's degree of liability, the smaller this gap. The figures also show that, as expected, the impact of the lease duration and the degree of liability on the difference between the cooperative and noncooperative reclamation efforts and pollution is similar to what has been discussed in relation to Figures 1 and 2.

Assuming that the cooperative solution is the optimal solution, we may wonder whether the regulator can adjust the policy parameter ϕ in order to decentralize the cooperative solution under noncooperation. In other words, if the business as usual assumption is that firms do not cooperate, the regulator must adjust its policy accordingly. Given the complex and dynamic setting of our model, it is not possible to adjust ϕ to a new value, ϕ', to equalize the noncooperative and cooperative reclamation efforts, but the regulator can still adjust this parameters to ensure that the pollution left at site dismissal under noncooperation coincides with the cooperative level, that is $p_T^C(\phi) = p_T^N(\phi')$, where $\phi' = g\phi$ and g is the adjustment parameter. Under the same parametrization earlier employed, Table 1 determines the value of the adjustment parameter as a function of the policy parameters, ϕ and T, by considering one of the them fixed and varying the other. We can see that the adjustment parameter increases with the lease duration and with the degree of liability: since both the lease duration and the degree of liability determine the firms' incentive to engage in reclamation efforts, these parameters affect the size of the gap between the cooperative and noncooperative pollution and so the size of the adjustment parameter to decentralize the cooperative pollution outcome.

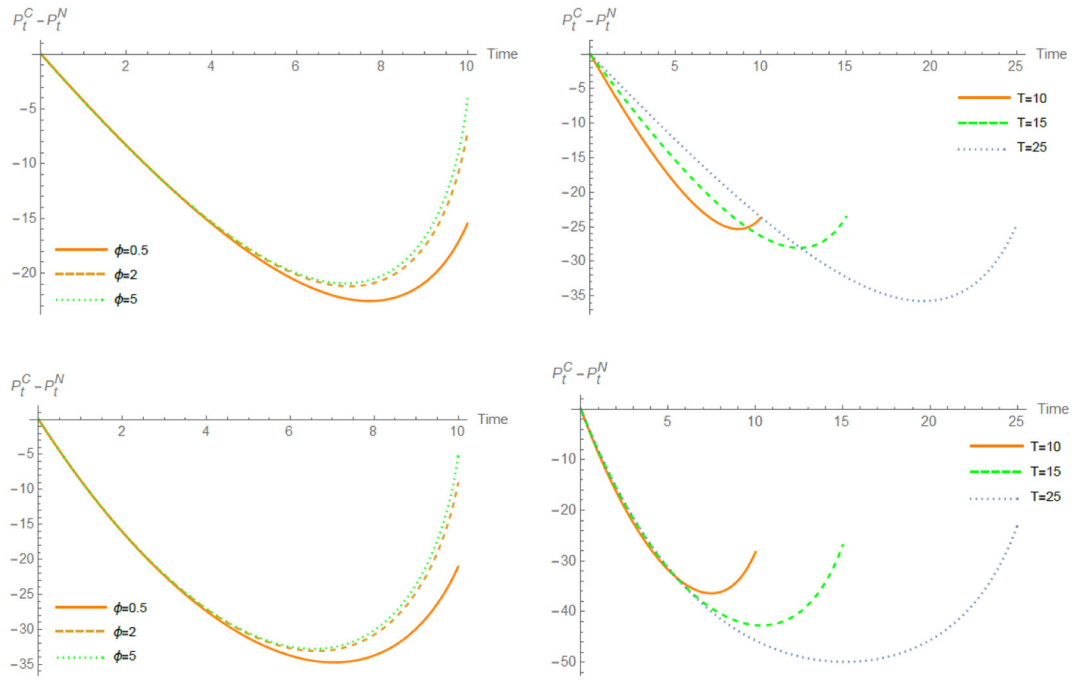

FIGURE 4 Impact of ϕ (left) and T (right) on $p_t^C - p_t^N$ in the $\lambda > 0$ (top) and $\lambda < 0$ (bottom) cases [Colour figure can be viewed at wileyonlinelibrary.com]

TABLE 1 Decentralization of the cooperative pollution level at site dismissal

Fixed ϕ ($\phi = 0.5$)	Adjustment (g)	Fixed T ($T = 15$)	Adjustment (g)
$\lambda > 0$		$\lambda > 0$	
$T = 10$	3.59992	$\phi = 0.5$	3.67156
$T = 15$	3.67156	$\phi = 2$	7.32724
$T = 25$	3.69604	$\phi = 5$	11.5795
$\lambda < 0$		$\lambda < 0$	
$T = 10$	7.59878	$\phi = 0.5$	9.94695
$T = 15$	9.94695	$\phi = 2$	18.9374
$T = 25$	15.15240	$\phi = 5$	29.6266

4 | THE HETEROGENEOUS CASE

We now assume that the two firms have different characteristics and thus we restore all sources of heterogeneity. Specifically, we allow the degree of liability and the lease duration to be different between the two firms to represent a situation in which firms differ in their bargaining power with the public authority and in their extraction lease terms. The introduction of heterogeneity substantially complicates the model, thus we have to rely on numerical simulations to characterize and compare the noncooperative and cooperative solutions.

4.1 | The non-cooperative solution

When the two firms face different lease durations, we need to specify which time horizon is longer and what happens beyond the shortest time horizon. Without loss of generality, we assume that $T^i > T^j$, thus after T^j (and before T^i) only firm i makes decisions since firm j has already dismissed the site, and by assumption, at time T^j has paid the abandonment reclamation fee to the public authority. Since after T^j only firm i needs to determine its reclamation efforts, we use backward induction to find the solution. Specifically, we first solve firm i's problem for the period from T^j to T^i assuming that pollution at time T^j is given. Then by using the discounted value function found in this step as the continuation value in firm i's problem, we can solve the game between the two firms from time 0 to T^j. The former problem reads as follows:

$$\min_{\tau_t^i} \mathscr{C}^i = \int_{T^j}^{T^i} \frac{(\tau_t^i)^2}{2} e^{-\rho t} dt + \phi^i \frac{p_{T^i}^2}{2} e^{-\rho T^i}, \tag{22}$$

$$s.t. \dot{p}_t = (\epsilon^i \gamma^i - \delta) p_t - \alpha^i \tau_t^i. \tag{23}$$

while for the latter problem for firm i we have:

$$\min_{\tau_t^i} \mathscr{C}^i = \int_0^{T^j} \frac{(\tau_t^i)^2}{2} e^{-\rho t} dt + \overline{\mathscr{F}}^i(T^j, p_{T^j}) e^{-\rho T^j}, \tag{24}$$

$$s.t. \dot{p}_t = (\epsilon^i \gamma^i + \epsilon^j \gamma^j - \delta) p_t - \alpha^i \tau_t^i - \alpha^j \tau_t^j, \tag{25}$$

where $\overline{\mathscr{F}}^i(T^j, p_{T^j})$ is the value function associated with the problem in (22) and (23). For firm j, the problem is similar to the one faced by firm i between 0 and T^j, aside from the fact that the second component in (24) is replaced by firm j's discounted abandonment reclamation fee, that is $\frac{\phi^j}{2} p_{T^j}^2 e^{-\rho T^j}$. The noncooperative solution of these problems is characterized below.

Proposition 3 *From time 0 to T^j, the Cournot–Nash reclamation effort for firm* i *and* j, $(\tau_t^i)^N$ *and* $(\tau_t^j)^N$, *and the time path of pollution are, respectively, given by:*

$$(\tau_t^i)^N = \alpha^i p_t^N A_t^i, \tag{26}$$

$$(\tau_t^j)^N = \alpha^j p_t^N A_t^j, \tag{27}$$

$$\dot{p}_t^N = [\epsilon^i \gamma^i + \epsilon^j \gamma^j - \delta - (\alpha^i)^2 A_t^i - (\alpha^j)^2 A_t^j] p_t^N, \tag{28}$$

where A_t^i and A_t^j are the solutions of the following system of differential equations:

$$\dot{A}_t^i = (\alpha^i)^2 (A_t^i)^2 + 2(\alpha^j)^2 A_t^i A_t^j + [\rho - 2(\epsilon^i \gamma^i + \epsilon^j \gamma^j - \delta)] A_t^i, \tag{29}$$

$$\dot{A}_t^j = (\alpha^j)^2 (A_t^j)^2 + 2(\alpha^i)^2 A_t^i A_t^j + [\rho - 2(\epsilon^i \gamma^i + \epsilon^j \gamma^j - \delta)] A_t^j, \tag{30}$$

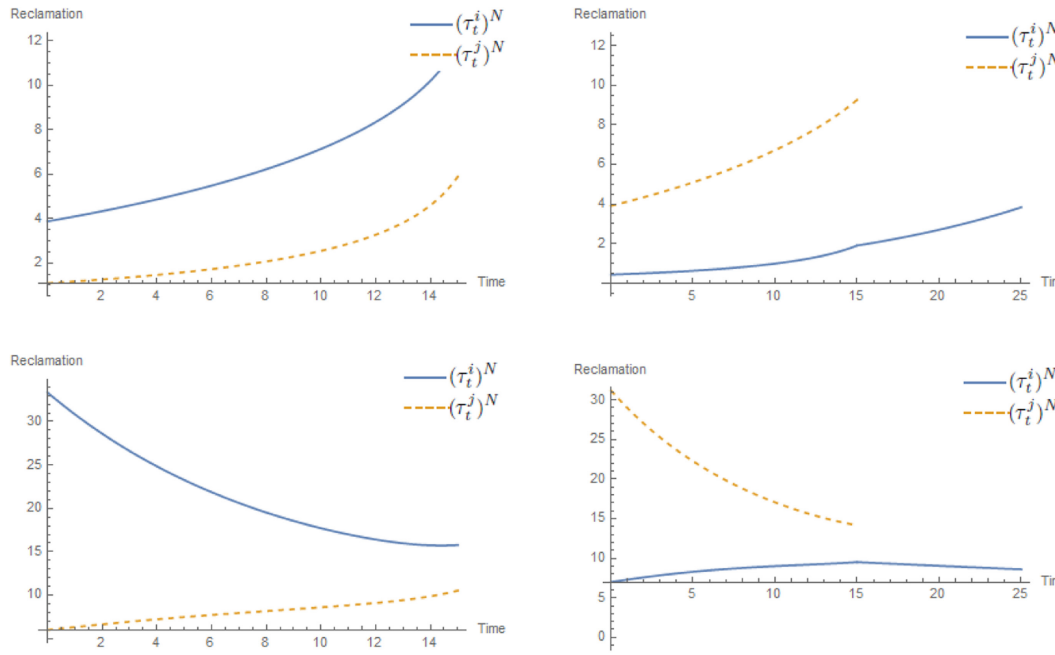

FIGURE 5 Impact of ϕ (left, $\phi^i = 0.75$, $\phi^j = 0.5$) and T (right, $T^i = 25$, $T^j = 15$) on τ_t^N in the $\lambda > 0$ (top, $\gamma = 1$) and $\lambda < 0$ (bottom, $\gamma = 5$) cases [Colour figure can be viewed at wileyonlinelibrary.com]

with terminal conditions $A^i_{T^j} = \frac{\bar{\lambda}\phi^i e^{\bar{\lambda}T^j}}{[\bar{\lambda}+\phi^i(\alpha^i)^2]e^{\bar{\lambda}T^i} - \phi^i(\alpha^i)^2 e^{\bar{\lambda}T^j}} \geq 0$ *and* $A^j_{T^j} = \phi^j \geq 0$.

Between time T^j *and* T^i *firm* i*'s Cournot–Nash rule for the reclamation effort,* $(\tau_t^i)^N$*, and the time path of pollution are, respectively, given by:*

$$(\tau_t^i)^N = \frac{\alpha^i \bar{\lambda}\phi^i e^{\bar{\lambda}t}}{[\bar{\lambda}+\phi^i(\alpha^i)^2]e^{\bar{\lambda}T^i} - \phi^i(\alpha^i)^2 e^{\bar{\lambda}t}} p_t^N, \tag{31}$$

$$p_t^N = p_{T^j}^N \left[\frac{[\bar{\lambda}+\phi^i(\alpha^i)^2]e^{\bar{\lambda}T^i} - \phi^i(\alpha^i)^2 e^{\bar{\lambda}t}}{[\bar{\lambda}+\phi^i(\alpha^i)^2]e^{\bar{\lambda}T^i} - \phi^i(\alpha^i)^2 e^{\bar{\lambda}T^j}} \right] e^{(\epsilon^i\gamma^i-\delta)(t-T^j)}, \tag{32}$$

where $\bar{\lambda} \equiv \rho - 2(\epsilon^i\gamma^i - \delta)$.

Proposition 3 characterizes the rule for the reclamation effort for both firms and the pollution dynamics under noncooperation in the case of heterogeneity. Unlike what is discussed in the homogeneous case, it is not possible to derive an explicit expression for the reclamation strategies of the two firms since the simultaneous system of differential Equations (29) and (30) cannot be solved analytically, and thus we cannot explicitly derive all the results we have presented earlier in the homogeneous case. Therefore, we will need to rely on numerical simulations to illustrate the results embedded in the above proposition between time 0 and T^j. Nevertheless, from Proposition 3 we can conclude that, while we expect that heterogeneities lead to a divergence between the two firms' choices of reclamation efforts, heterogeneity in only the rate of emission (i.e., $\epsilon^z\gamma^z$) does not.

Figure 5 presents numerical examples to shed light on the impact of heterogeneity on the firms' reclamation effort time path in the $\lambda > 0$ and $\lambda < 0$ cases. To show the impact of the main

parameters, we remove all sources of heterogeneity apart from one at once, either in ϕ or T. Our benchmark parameter values are the following: $p_0 = 100$, $\rho = 0.04$, $\delta = 0.05$, $\alpha^i = \alpha^j = 0.5$, $\epsilon^i = \epsilon^j = 0.02$, $\phi^i = \phi^j = 0.5$, $T^i = T^j = 15$ and either $\gamma = 1$ (such that $\lambda > 0$) or $\gamma = 5$ (such that $\lambda < 0$), with ϕ and T eventually changing. We can observe that the evolution of the reclamation efforts shows a larger degree of variability with respect to what is discussed in the homogeneous case. While the firm with shorter lease duration provides higher reclamation effort, the dynamics of the reclamation efforts depends on the sign of λ: in the $\lambda > 0$ case both firms' reclamation efforts monotonically increase, in the $\lambda < 0$ case they can show opposite dynamics (increasing in one firm and decreasing in the other, with the firm with shorter lease duration showing high reclamation effort initially which then gradually decreases). As expected, our numerical results suggest that the firm with the higher degree of liability chooses higher reclamation efforts.

4.2 | The cooperative solution

Similar to the noncooperative case, since after T^j only firm i is active, from T^j to T^i the cooperative solution coincides with the noncooperative one summarized by (31) and (32). The value function associated with firm i's problem between T^j to T^i, evaluated at time T^j, allows to determine the effective degree of liability of firm i associated with the remaining level of pollution after T^j. Thus, in order to characterize the cooperative solution we need to solve the joint minimization problem between 0 and T^j as follows:

$$\min_{\tau_t^i,\tau_t^j} \mathscr{C} = \mathscr{C}^i + \mathscr{C}^j = \int_0^{T^j} \left[\frac{(\tau_t^i)^2}{2} + \frac{(\tau_t^j)^2}{2} \right] e^{-\rho t} dt + \frac{\overline{\phi}^i + \phi^j}{2} p_{T^i}^2 e^{-\rho T^i}, \tag{33}$$

$$s.t. \dot{p}_t = (\epsilon^i \gamma^i + \epsilon^j \gamma^j - \delta) p_t - \alpha^i \tau_t^i - \alpha^j \tau_t^j, \tag{34}$$

where $\overline{\phi}^i = \frac{\overline{\lambda}\phi^i e^{(\overline{\lambda})T^j}}{[\overline{\lambda} + \phi^i(\alpha^i)^2]e^{\overline{\lambda}T^i} - \phi^i(\alpha^i)^2 e^{\overline{\lambda}T^j}}$. Proposition 4 characterizes the solution.

Proposition 4 *Between time 0 and T^j, the cooperative reclamation effort rule for firm $z = \{i, j\}$, denoted by $(\tau_t^z)^C$ and the cooperative time path of pollution are, respectively, given by:*

$$(\tau_t^z)^C = \frac{\lambda(\overline{\phi}^i + \phi^j)\alpha^z e^{\lambda t}}{\{\lambda + (\overline{\phi}^i + \phi^j)[(\alpha^i)^2 + (\alpha^j)^2]\} e^{\lambda T^j} - (\overline{\phi}^i + \phi^j)[(\alpha^i)^2 + (\alpha^j)^2] e^{\lambda t}} p_t^C, \tag{35}$$

$$p_t^C = p_0 \left[\frac{\{\lambda + (\overline{\phi}^i + \phi^j)[(\alpha^i)^2 + (\alpha^j)^2]\} e^{\lambda T^j} - (\overline{\phi}^i + \phi^j)[(\alpha^i)^2 + (\alpha^j)^2] e^{\lambda t}}{\{\lambda + (\overline{\phi}^i + \phi^j)[(\alpha^i)^2 + (\alpha^j)^2]\} e^{\lambda T^j} - (\overline{\phi}^i + \phi^j)[(\alpha^i)^2 + (\alpha^j)^2]} \right] e^{(\epsilon^i \gamma^i + \epsilon^j \gamma^j - \delta)t}. \tag{36}$$

Between time T^j and T^i firm i's cooperative reclamation effort, $(\tau_t^i)^C$, and the cooperative dynamic path of pollution are, respectively, given by:

$$(\tau_t^i)^C = \frac{\alpha^i \bar{\lambda} \phi^i e^{\bar{\lambda} t}}{[\bar{\lambda} + \phi^i (\alpha^i)^2] e^{\bar{\lambda} T^i} - \phi^i (\alpha^i)^2 e^{\bar{\lambda} t}} p_t^C, \tag{37}$$

$$p_t^C = p_{T^j}^C \left[\frac{[\bar{\lambda} + \phi^i (\alpha^i)^2] e^{\bar{\lambda} T^i} - \phi^i (\alpha^i)^2 e^{\bar{\lambda} t}}{[\bar{\lambda} + \phi^i (\alpha^i)^2] e^{\bar{\lambda} T^i} - \phi^i (\alpha^i)^2 e^{\bar{\lambda} T^j}} \right] e^{(\epsilon^i \gamma^i - \delta)(t - T^j)}, \tag{38}$$

where $p_{T^j}^C$ *is the pollution stock (36) evaluated at time* T^j*, given by the following expression:* $p_{T^j}^C = p_0 \left[\frac{\lambda e^{\left[\rho - (\epsilon^i \gamma^i + \epsilon^j \gamma^j - \delta)\right] T^j}}{\{\lambda + (\bar{\phi}^i + \phi^j)[(\alpha^i)^2 + (\alpha^j)^2]\} e^{\lambda T^j} - (\bar{\phi}^i + \phi^j)[(\alpha^i)^2 + (\alpha^j)^2]} \right]$.

Different from what is discussed earlier for the noncooperative case, Proposition 4 explicitly derives the rule for the reclamation effort for both firms and the dynamics of pollution under heterogeneity in the case of cooperation. Consistent with what is shown in the homogeneous case, the cooperative reclamation effort is proportional to pollution. However, the presence of heterogeneity in their environmental maintenance efficiency (i.e., α^i and α^j) drives a wedge between the reclamation efforts of the two firms up to T^j such that the firm with higher efficiency is required to make a higher reclamation effort. Nevertheless, other sources of heterogeneity will not lead to heterogeneity in the cooperative reclamation efforts. The following corollary summarizes these results.

Corollary 1 *Between time 0 and* T^j*, firm* i *and* j *will be required to provide different reclamation efforts only if the efficiency of their maintenance activities (i.e.,* α^i *and* α^j*) is different, and in particular reclamation effort will be higher for the firm with higher efficiency.*

From Proposition 4, we can note that the reclamation effort for firm i is discontinuous at T^j since the difference between (37) and (35) evaluated at time T^j is strictly positive as given by the following expression:

$$(\tau_t^i)^C|_{T^j} - (\tau_t^i)^C|_{(T^j)^+} = \frac{\left\{ e^{T^i \bar{\lambda}} \left[\bar{\lambda} + (\alpha^i)^2 \phi^i \right] (\bar{\phi}^i + \phi^j) - e^{T^j \bar{\lambda}} \phi^i \left[\bar{\lambda} + (\alpha^i)^2 (\bar{\phi}^i + \phi^j) \right] \right\} p_0 \alpha^i \lambda e^{(\lambda - \delta + \gamma_i \epsilon_i + \gamma_j \epsilon_j) T^j}}{\left[\bar{\lambda} e^{T^i \bar{\lambda}} + e^{T^j \bar{\lambda}} (\alpha^i)^2 \phi^i (e^{T^i \bar{\lambda}} - 1) \right] \left\{ \lambda e^{T^j \lambda} + \left[(\alpha^i)^2 + (\alpha^j)^2 \right] (\bar{\phi}^i + \phi^j)(e^{T^j \lambda} - 1) \right\}} > 0 \tag{39}$$

This suggests that the reclamation effort of firm i will drop after the end of firm j's lease duration. This is due to the fact that the cooperation mechanism requires both active firms to internalize the negative impact of their emissions on each other—captured by the sum of the two abandonment fee parameters, $\bar{\phi}^i + \phi^j$ —and thus tend to devote a larger amount of resources to preserve the environment. This leads to a lower level of pollution at time T^j than in the noncooperative case. However, by assumption, firm j will not be active after T^j and it is only firm i that continues to extract the resource and reclaim the environment which means that during this period cooperative and noncooperative behaviors coincide. This will lead to a discontinuous drop in the reclamation effort of firm i at T^j. This result is summarized in the next proposition.

Proposition 5 *The cooperative reclamation effort of firm* i *is discontinuous at* T^j.

By plugging (36) into (35), the cooperative time path of the reclamation effort, $(\tau_t^z)^C$ for $z = \{i, j\}$, between time 0 to T^j, is given by the following expression:

$$(\tau_t^z)^C = \frac{p_0\lambda(\overline{\phi}^i + \phi^j)\alpha^z e^{[\rho-(\epsilon^i\gamma^i+\epsilon^j\gamma^j-\delta)]t}}{\{\lambda+(\overline{\phi}^i+\phi^j)[(\alpha^i)^2+(\alpha^j)^2]\}e^{\lambda T^j}-(\overline{\phi}^i+\phi^j)[(\alpha^i)^2+(\alpha^j)^2]}, \tag{40}$$

from which it follows that the effects of different parameters are qualitatively identical to what is discussed earlier in the homogeneous case. Indeed, the time dynamics of the cooperative reclamation effort for both firms is monotonic and dependent upon how the time preference rate and gross growth rate of pollution compare (i.e., the sign of $\rho - (\epsilon^i\gamma^i + \epsilon^j\gamma^j - \delta)$, yielding a monotonically increasing dynamic if positive or a monotonically decreasing dynamic if negative). Moreover, the cooperative reclamation efforts for both firms are increasing in their own and their cross degree of liability since we have:

$$\frac{\partial(\tau_t^z)^C}{\partial\overline{\phi}^i} = \frac{\partial(\tau_t^z)^C}{\partial\phi^j}\frac{p_0\lambda^2\alpha^i e^{\lambda T^j+(\rho-(\epsilon^i\gamma^i+\epsilon^j\gamma^j-\delta))t}}{\left[\left\{\lambda+(\overline{\phi}^i+\phi^j)[(\alpha^i)^2+(\alpha^j)^2]e^{\lambda T^j}\right\}-(\overline{\phi}^i+\phi^j)\left((\alpha^i)^2+(\alpha^j)^2\right)\right]^2} > 0, \tag{41}$$

for $z = \{i, j\}$, and thus $\frac{\partial(\tau_t^z)^C}{\partial\phi^i} > 0$ as well. Moreover, the impact of the length of firm j's lease duration on the cooperative reclamation efforts of firm $z = \{i, j\}$, is negative regardless of whether $\lambda > 0$ or $\lambda < 0$, as long as it is small enough in absolute value:

$$\frac{\partial(\tau_t^z)^C}{\partial T^j} = -\frac{p_0\lambda^2\alpha^i(\overline{\phi}^i+\phi^j)\left\{\lambda+(\overline{\phi}^i+\phi^j)[(\alpha^i)^2+(\alpha^j)^2]\right\}e^{\lambda T^j+[\rho-(\epsilon^i\gamma^i+\epsilon^j\gamma^j-\delta)]t}}{\left[\left\{\lambda+(\overline{\phi}^i+\phi^j)[(\alpha^i)^2+(\alpha^j)^2]e^{\lambda T^j}\right\}-(\overline{\phi}^i+\phi^j)\left((\alpha^i)^2+(\alpha^j)^2\right)\right]^2}. \tag{42}$$

This result is in contrast with what we have observed in the noncooperative case for firm i, where its reclamation effort falls with the length of firm j's lease duration due to the strategic reaction of this firm to j's relatively higher level of reclamation. Under cooperation, both firms increase their reclamation in response to a shorter lease duration for either of them in order to minimize their joint cost.

Figure 6 shows that the two firms will be required to provide the same level of reclamation efforts when the source of heterogeneity is either the degree of liability or the lease duration, as suggested by Corollary 1. We can observe that the firms' reclamation effort is increasing (decreasing) when $\overline{\lambda} > 0$ (when $\overline{\lambda} < 0$). Moreover, different from what happens in the noncooperative case, the heterogeneity in the lease duration yields a discontinuity at T^j, exactly as discussed in Proposition 5.

4.3 | Comparison

The absence of an explicit expression for the reclamation effort in the noncooperative case makes it impossible to perform any explicit comparison between the cooperative and the noncooperative

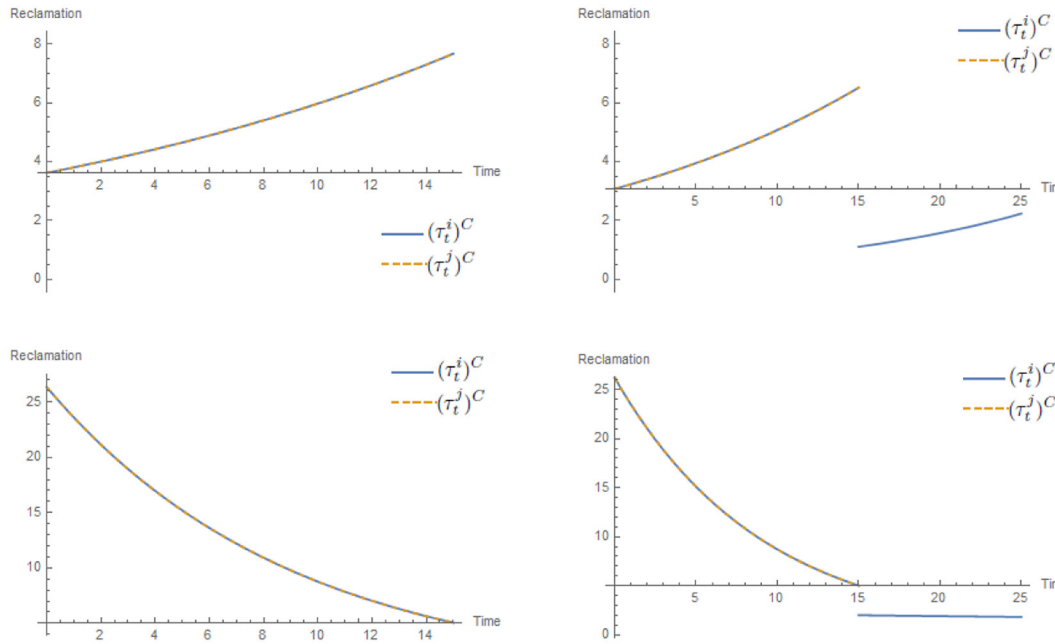

FIGURE 6 Impact of ϕ (left, $\phi^i = 0.75, \phi^j = 0.5$) and T (right, $T^i = 25, T^j = 15$) on τ_t^C in the $\overline{\lambda} > 0$ (top) and $\overline{\lambda} < 0$ (bottom) cases [Colour figure can be viewed at wileyonlinelibrary.com]

outcomes. We will, therefore, rely on numerical simulations to shed some light on how they effectively compare. This is illustrated in Figures 7 and 8, for the reclamation effort and pollution difference, respectively, for the cases in which $\overline{\lambda} > 0$ and $\overline{\lambda} < 0$.

The numerical simulations suggest that for the firm with either higher degree of liability or shorter lease duration the cooperative rule will prescribe a lower level of reclamation effort than the noncooperative rule (see Figure 7). Even though this result is unusual, it is very intuitive. Under noncooperation the firm with either a lower degree of liability or longer lease duration strategically chooses to contribute lower reclamation efforts knowing that the other firm will contribute more in order to avoid a high abandonment reclamation fee, which leads to a relatively high effort by the latter firm. However, under cooperation which requires both active firms to internalize the negative impact of their extraction activities on each other and given the increasing marginal cost of the reclamation efforts, the former firm is required to contribute much more than in the noncooperative case. Thus the reclamation effort required by the firm with a shorter lease duration or larger degree of liability becomes smaller than under noncooperation. Nevertheless, the increase in the reclamation efforts of the other firm more than compensates the decrease in the efforts of this firm, which consequently leads to a lower pollution trajectory under cooperation than under noncooperation (see Figure 8). Similar to the symmetric case, the difference between the cooperative and noncooperative pollution initially widens, but as firms get closer to the dismissal date, this gap tends to decrease.

In terms of policy implications, our results suggest that in order to address individual firms' reclamation efforts along the optimal cooperative level, the environmental regulators should determine the lease terms such to compel firms to continually involve in reclamation activities proportional to the pollution level. According to Corollary 1, such an approach does not need to be firm-specific as far as firms do not differ in the efficiency of their maintenance activities, and if

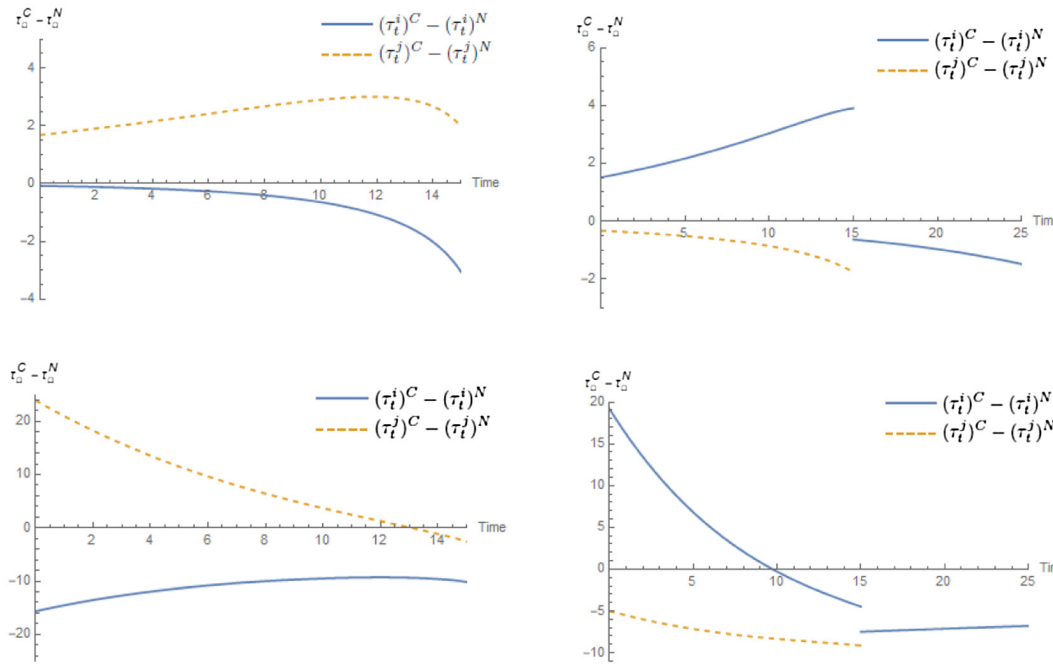

FIGURE 7 Impact of ϕ (left, $\phi^i = 0.75, \phi^j = 0.5$) and T (right, $T^i = 25, T^j = 15$) on $\tau_t^C - \tau_t^N$ in the $\overline{\lambda} > 0$ (top) and $\overline{\lambda} < 0$ (bottom) cases [Colour figure can be viewed at wileyonlinelibrary.com]

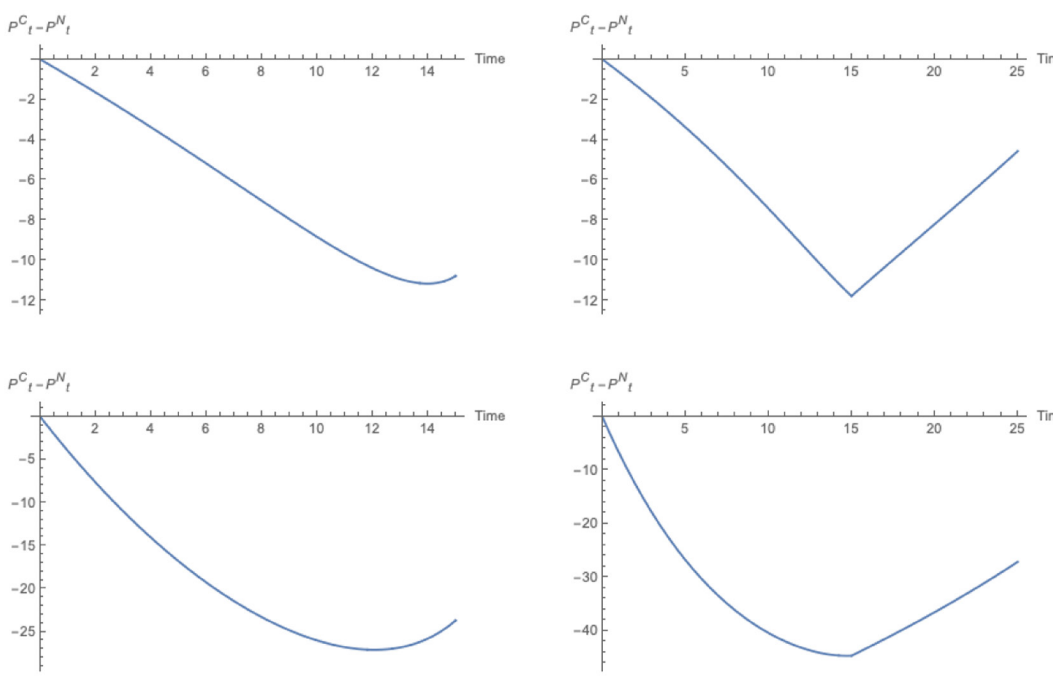

FIGURE 8 Impact of ϕ (left, $\phi^i = 0.75, \phi^j = 0.5$) and T (right, $T^i = 25, T^j = 15$) on $p_t^C - p_t^N$ in the $\lambda > 0$ (top) and $\lambda < 0$ (bottom) cases [Colour figure can be viewed at wileyonlinelibrary.com]

this is the case the effective implementation of such an approach to reclamation regulation may not be particularly demanding.

5 | CONCLUSIONS AND POLICY IMPLICATIONS

Extraction activities severely deteriorate the environment on and near the extraction field in several ways. Consequently, reclamation and rehabilitation of extraction sites are crucial in order to ensure that the environment remains viable for the future generations. The policy relevance of reclamation and the increasing number of extraction sites all over the world have given rise to a growing interest in reclamation issues. Thus far, from a theoretical perspective, all papers have focused on how a single firm is affected by reclamation policy, and none has considered how results may change with the presence of multiple firms in a single extraction area because of their strategic interactions. Our article tries to make the first theoretical contribution in this context in order to understand how individual firms' decisions and environmental outcomes are affected by the existence of a penalty at site dismissal for the unclaimed damages jointly caused by firms' extraction activities. We develop a theoretical model of reclamation of an extraction site from the point of view of two neighboring firms which plan for their dynamic reclamation activities strategically. We assume that each firm is liable to pay an abandonment fee at the end of its extraction lease duration for the part of pollution that it fails to reclaim by then, and such a fee depends on the extent of the environmental damage associated with the two firms' extraction activities. We analyze and compare how individual firms' behaviors and environmental outcomes change under noncooperation and cooperation scenarios, by distinguishing between situations in which firms are homogeneous and heterogeneous.

Our results show that, if firms are homogeneous, the time trajectory of the reclamation effort in the noncooperative case may be nonmonotonic, while in the cooperative case it is always monotonic and dependent on how the rate of time preference and the gross growth rate of pollution compare. Intuitively, when firms discount the future at a rate relatively lower than the growth of their future liabilities (i.e., when the discount rate is mildly less than the gross growth rate of pollution) they act more aggressively to reduce and control pollution by implementing higher levels of abatement and environmental maintenance early on and viceversa. However, if the discount rate is considerably lower than the rate of pollution accumulation we may observe that in the noncooperative case the reclamation effort might show a non-monotonic behavior. Such a non-monotonic behavior can be understood by relating the size of the natural decay of pollution which ultimately determines strategically whether it may be convenient for the individual firm to intervene in environmental maintenance as soon as possible or to postpone it to some future date. In the contrary, in the cooperative case, the internalization of the externalities assures that the reclamation trajectory remains monotone. Nevertheless, for any given pollution level, the cooperative rule prescribes a higher level of reclamation effort than the noncooperative one for both firms. However, if firms are heterogeneous, under noncooperation they choose different reclamation efforts while, under cooperation, this occurs only if the efficiency of their maintenance activities is different.

We also observe that under noncooperation, due to strategic reactions of firms to each other's choices, the reclamation efforts of the two firms can show opposite dynamics as long as the firm with the shorter lease duration is active and thus will contribute more to avoid high penalty at its site dismissal, while this does not occur under cooperation. As in the homogeneous case, the internalization of externalities leads to equal reclamation efforts for the two firms unless the

efficiency of their maintenance activities is different. Instead in the cooperative case there is a discontinuity in the reclamation efforts of the firm with longer lease duration at the other firm's site dismissal date. Independently of the fact that firms are homogeneous or heterogeneous, in all scenarios the reclamation efforts generally rise with the degree of liability and fall with the lease duration, suggesting that in order to promote better environmental outcomes, the regulators should carefully determine the lease conditions by introducing intra-term reclamation fees along with stringent environmental accountability. Also, by determining the lease terms, to compel firms to continually involve in reclamation activities proportional to the pollution stock, they can effectively address individual firms' reclamation efforts along the optimal cooperative level. From a policy perspective, we know that the current bonding that is usually used as a warranty to ensure the lease terms are fulfilled has been failing since the posted bonds have been far short to cover the reclamation fees. Our results instead suggest that the regulator must put policies in place to motivate continuous reclamation efforts (posted bonds) that are proportional to pollution during the entire life of the project. This will ensure that by the time the operators abandon their sites they either have reclaimed the environment or have posted enough bonds to cover the costs.

To the best of our knowledge, this is the first theoretical article analyzing the implications of reclamation activities on firms' behavior and environmental outcomes in a game setting, and thus for the sake of clarity of arguments we have maintained our framework as simple as possible. This simplification has precluded us from endogenizing extraction choices, thus, an extension of the model to analyze the firms' joint determination of extraction and reclamation efforts would be of special interest. Also, allowing for the presence of multiple (i.e., more than two) heterogenous firms sharing the site and of a regulator optimally determining the lease terms would be particularly interesting from a policy perspective. Another complementary line of research consists of collecting information from actual extraction sites in order to calibrate and apply our model to assess the effectiveness of reclamation activities in real world situations. These additional tasks are left for future research.

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REFERENCES

Alberta Energy Regulator. (2018). *Environmental Protection and Enhancement Act*. Retrieved from https://www.aer.ca/project-closure/reclamation/mine-reclamation-requirements

Andersen, M., & Coupal, R. (2009). Economic issues and policies affecting reclamation in Wyoming's oil and gas industry. *Proceedings America Society of Mining and Reclamation*, 1–17.

Barrett, S. (1994). Self-enforcing international environmental agreements. *Oxford Economic Papers*, *46*, 878–894.

Bishop, R. E. (2013). Historical analysis of oil and gas well plugging in New York: Is the regulatory system working? *New Solutions*, *23*, 103–116.

Breton, M., Sbragia, L., & Zaccour, G. (2010). A dynamic model for international environmental agreements. *Environmental and Resource Economics*, *45*, 25–48.

Calgary, H. (2017). *Tailings ponds a critical part of Alberta's oilsands legacy*. Retrieved from https://calgaryherald.com/business/energy/tailings-ponds-a-critical-part-of-albertas-oilsands-legacy

Canadian Association of Petroleum Producers. (2018). *Canada's oil sands industry is working to reduce the size of its footprint in order to maintain the biodiversity of the region and to support the function of natural ecosystems*. Retrieved from https://www.canadasoilsands.ca/en/environmental-innovation/land

Caputo, M. R., & Wilen, J. E. (1995). Optimal cleanup of hazardous wastes. *International Economic Review*, *36*, 217–243.

Center for Western Priorities. (2018). *Reclaiming oil and gas wells on federal lands: Estimate of costs*. Retrieved from http://westernpriorities.org/wp-content/2018/02/Bonding-Report.pdf

Cheng, L., & Skousen, J. G. (2017). Comparison of international mine reclamation bonding systems with recommendations for China. *International Journal of Coal Science & Technology*, *4*, 67–79.

Dachis, B., Shaffer, B., & Thivierge, V. (2017). *All's well that ends well: Addressing end-of-life liabilities for oil and gas wells*. Retrieved from https://www.cdhowe.org/research_papers/mixed/Commentary492.pdf

Davis, L. W. (2015). Policy monitor–Bonding requirements for U.S. natural gas producers. *Review of Environmental Economics and Policy*, *9*, 128–144.

Dockner, E. J., Sorger, G., Jorgensen, S., & Long, N. V. (2000). *Differential games in economics and management science*. Cambridge University Press.

Espinoza, R. D., & Morris, J. W. F. (2017). Towards sustainable mining (part II): Accounting for mine reclamation and post reclamation care liabilities. *Resources Policy*, *52*, 29–38.

Finus, M. (2008). Game theoretic research on the design of international environmental agreements: Insights, critical remarks, and future challenges. *International Review of Environmental and Resource Economics*, *2*, 29–67.

Gerard, D. (2000). The law and economics of reclamation bonds. *Resources Policy*, *26*, 189–119.

Ho, J. S., Shih, J.-S., Muehlenbachs, L. A., Munnings, C., & Krupnick, A. J. (2018). Managing environmental liability: An evaluation of bonding requirements for oil and gas wells in the United States. *Environmental Science & Technology*, *52*, 3908–3916.

Kang, M., Baik, E., Miller, A. R., Bandilla, K. W., & Celia, M. A. (2015). Effective permeabilities of abandoned oil and gas wells: Analysis of data from Pennsylvania. *Environmental Science & Technology*, *49*, 4757–4764.

Lappi, P. (2018). Optimal clean-up of polluted sites. *Resource and Energy Economics*, *54*, 53–68.

Lappi, P. (2020). A model of optimal extraction and site reclamation. *Resource and Energy Economics*, *29*, 101126.

La Torre, D., Liuzzi, D., & Marsiglio, S. (2017). Pollution control under uncertainty and sustainability concern. *Environmental and Resource Economics*, *67*, 885–903.

La Torre, D., Liuzzi, D., & Marsiglio, S. (2021). Transboundary pollution externalities: Think globally, act locally? *Journal of Mathematical Economics*, in press.

La Torre, D., & Marsiglio, S. (2020). A note on optimal debt reduction policies. *Macroeconomic Dynamics*, *24*(7), 1850–1860.

Masoudi, N., & Zaccour, G. (2013). A differential game of international pollution control with evolving environmental costs. *Environment and Development Economics*, *18*(6), 680–700.

Masoudi, N., & Zaccour, G. (2018). Adaptation and international environmental agreements. *Environmental and Resource Economics*, *71*, 1–21.

Mistré, M., Crénes, M., & Hafner, M. (2018). Shale gas production costs: Historical developments and outlook. *Energy Strategy Reviews*, *20*, 20–25.

Mitchell, A. L., & Casman, E. A. (2011). Economic incentives and regulatory framework for shale gas well site reclamation in Pennsylvania. *Environmental Science & Technology*, *45*, 9506–9514.

Natural Resource Canada. (2018). *Environmental challenges*. Retrieved from https://www.nrcan.gc.ca/energy/oil-sands/5855

Nordhaus, W. D. (2010). *DICE-2010 model*. Retrieved from http://nordhaus.econ.yale.edu/RICEmodels.htm

Ontario Mining Association. (2018). *Closure and land rehabilitation*. Retrieved from https://www.oma.on.ca/en/ontariomining/ClosureandRehabilitation.asp

Saltari, E., & Travaglini, G. (2016). Pollution control under emission constraints: Switching between regimes. *Energy Economics*, *53*, 212–219.

Sullivan, J., & Amacher, G. S. (2009). The social costs of mineland restoration. *Land Economics*, *85*, 712–726.

United States Government Accountability Office. (2010). *Oil and gas bonds: Bonding requirements and BLM expenditures to reclaim orphaned wells*. GAO. Retrieved from https://www.gao.gov/assets/310/300218.pdf

Yan, S., Shen, W., & Zou, C. (2012). Theory and practice of deposit system for restoration and control of mining environment. *Environmental Science & Technology*, *35*, 423–427.

Yang, P., & Davis, G. (2018). Non-renewable resource extraction under financial incentives to reduce and reverse stock pollution. *Journal of Environmental Economics and Management*, *92*, 282–299.

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APPENDIX A

TECHNICAL APPENDIX

Homogeneous case: Non-cooperative solution

By denoting with $\mathcal{J}^z(t,p_t)$ the value function associated with problem (4) and (5) for firm $z \in \{i,j\}$, the Hamilton–Jacobi–Bellman equation reads as:

$$-\frac{\partial \mathcal{J}^z(t,p_t)}{\partial t} = \min_{\tau_t^z} \left\{ \frac{(\tau_t^z)^2}{2} e^{-\rho t} + [(2\epsilon\gamma - \delta)p_t - \alpha(\tau_t^i + \tau_t^j)] \frac{\partial \mathcal{J}^z(t,p_t)}{\partial p_t} \right\}, \tag{A1}$$

while the corresponding terminal condition:

$$\mathcal{J}^z(T,p_T) = \frac{\phi}{2} p_T^2 e^{-\rho T}, \tag{A2}$$

The first-order conditions for firm i and j yield, respectively:

$$\tau_t^i = \alpha e^{\rho t} \frac{\partial \mathcal{J}^i(t,p_t)}{\partial p_t}, \tag{A3}$$

$$\tau_t^j = \alpha e^{\rho t} \frac{\partial \mathcal{J}^j(t,p_t)}{\partial p_t}. \tag{A4}$$

Since the two firms are identical, we guess that the value functions for each firm's problem is the same. Specifically, our guess takes the following form:

$$\mathcal{J}(t,p_t) = \mathcal{J}^i(t,p_t) = \mathcal{J}^j(t,p_t) = \frac{1}{2} p_t^2 A_t e^{-\rho t}, \tag{A5}$$

where A_t is a variable to be determined. By computing the derivatives $\frac{\partial \mathcal{J}}{\partial t}$ and $\frac{\partial \mathcal{J}}{\partial p_t}$, and plugging these into (A3), (A4) and (A1), we obtain:

$$\tau_t = \tau_t^i = \tau_t^j = \alpha p_t A_t, \tag{A6}$$

$$\dot{A}_t = 3\alpha^2 A_t^2 + [\rho - 2(2\epsilon\gamma - \delta)]A_t. \tag{A7}$$

At the terminal time, by comparing (A2) and (A5) evaluated at T, it follows that $A_T = \phi \geq 0$. Note that (A7) is a Bernoulli differential equation with a terminal condition, and its exact solution is given by:

$$A_t = \frac{\lambda \phi e^{\lambda t}}{(\lambda + 3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}}, \tag{A8}$$

where $\lambda \equiv \rho - 2(2\epsilon\gamma - \delta)$. By plugging (A8) into (A5), we obtain the following expression for the value function:

$$\mathcal{J}(t, p_t) = \mathcal{J}^i(t, p_t) = \mathcal{J}^j(t, p_t) = \frac{1}{2}\frac{\lambda \phi e^{(\lambda-\rho)t}}{(\lambda + 3\phi\alpha^2)e^{\lambda T} - 3\phi\alpha^2 e^{\lambda t}} p_t^2. \tag{A9}$$

By plugging (A8) into (A6) and (5) we get the closed-loop feedback Cournot–Nash policy rule for the reclamation efforts and the pollution dynamics given in (6) and (7), respectively.

Homogeneous case: Cooperative solution

By denoting with $\mathcal{J}(t, p_t)$ the value function associated with problem (12) and (13), the Hamilton–Jacobi–Bellman equation reads as:

$$-\frac{\partial \mathcal{J}(t, p_t)}{\partial t} = \min_{\tau_t^i, \tau_t^j} \left\{ \frac{(\tau_t^i)^2}{2} e^{-\rho t} + \frac{(\tau_t^j)^2}{2} e^{-\rho t} + [(2\epsilon\gamma - \delta)p_t - \alpha(\tau_t^i + \tau_t^j)] \frac{\partial \mathcal{J}(t, p_t)}{\partial p_t} \right\}, \tag{A10}$$

while the corresponding terminal condition is:

$$\mathcal{J}(T, p_T) = \phi p_T^2 e^{-\rho T}. \tag{A11}$$

The first-order conditions for firm i and j yield, respectively:

$$\tau_t^i = \alpha e^{\rho t} \frac{\partial \mathcal{J}(t, p_t)}{\partial p_t}, \tag{A12}$$

$$\tau_t^j = \alpha e^{\rho t} \frac{\partial \mathcal{J}(t, p_t)}{\partial p_t}. \tag{A13}$$

We guess the following functional form for the value function:

$$\mathcal{J}(t, p_t) = \frac{1}{2} p_t^2 A_t e^{-\rho t}, \tag{A14}$$

where A_t is a variable to be determined. By computing its derivatives $\frac{\partial \mathcal{J}}{\partial t}$ and $\frac{\partial \mathcal{J}}{\partial p_t}$, and plugging these into (A12), (A13) and (A10), we obtain:

$$\tau_t = \tau_t^i = \tau_t^j = \alpha p_t A_t, \tag{A15}$$

$$\dot{A}_t = 2\alpha^2 A_t^2 + [\rho - 2(2\epsilon\gamma - \delta)]A_t. \tag{A16}$$

At the terminal time, by comparing (A11) and (A14) evaluated at T, it follows that $A_T = 2\phi \geq 0$. Note that (A16) is a Bernoulli differential equation with a terminal condition, and its exact solution is given by:

$$A_t = \frac{2\lambda\phi e^{\lambda t}}{(\lambda + 4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2 e^{\lambda t}}, \tag{A17}$$

where $\lambda \equiv \rho - 2(2\epsilon\gamma - \delta)$. By plugging (A17) into (A14) we obtain the following expression for the value function:

$$\mathcal{J}(t, p_t) = \frac{1}{2}\frac{2\lambda\phi e^{(\lambda-\rho)t}}{(\lambda + 4\phi\alpha^2)e^{\lambda T} - 4\phi\alpha^2 e^{\lambda t}} p_t^2. \tag{A18}$$

By plugging (A17) into (A15) and (13) we get the closed-loop cooperative policy rule for the reclamation efforts and the time path of the pollution given in (14) and (15), respectively.

Heterogeneous case: Noncooperative solution

By denoting with $\overline{\mathcal{J}}^i(t, p_t)$ the value function associated with problem (22) and (23) for firm i, the Hamilton–Jacobi–Bellman equation reads as:

$$-\frac{\partial \overline{\mathcal{J}}^i(t, p_t)}{\partial t} = \min_{\tau_t^i} \left\{ \frac{(\tau_t^i)^2}{2} e^{-\rho t} + [(\epsilon^i\gamma^i - \delta)p_t - \alpha^i \tau_t^i] \frac{\partial \overline{\mathcal{J}}^i(t, p_t)}{\partial p_t} \right\}, \tag{A19}$$

while the corresponding terminal condition:

$$\overline{\mathcal{J}}^i(T^i, p_{T^i}) = \frac{\phi^i}{2} p_{T^i}^2 e^{-\rho T^i}. \tag{A20}$$

The first-order condition yields:

$$\tau_t^i = \alpha^i e^{\rho t} \frac{\partial \overline{\mathcal{J}}^i(t, p_t)}{\partial p_t}. \tag{A21}$$

We guess the following functional form for the value function:

$$\overline{\mathcal{J}}^i(t, p_t) = \frac{1}{2} p_t^2 \overline{A}_t^i e^{-\rho t}, \tag{A22}$$

where $\overline{A}_t^i$ is a variable to be determined. By computing its derivatives $\frac{\partial \overline{\mathcal{J}}}{\partial t}$ and $\frac{\partial \overline{\mathcal{J}}}{\partial p_t}$, and plugging these into (A21) and (A19), we obtain:

$$\tau_t^i = = \alpha^i p_t \overline{A}_t^i, \tag{A23}$$

$$\dot{\overline{A}}_t^i = (\alpha^i)^2 (\overline{A}_t^i)^2 + \{\rho - 2[\epsilon^i\gamma^i - \delta]\}\overline{A}_t^i. \tag{A24}$$

At the terminal time, by comparing (A20) and (A22) evaluated at T^i, it follows that $\overline{A}_T^i = \phi^i \geq 0$. Note that (A24) is a Bernoulli differential equation with a terminal condition, and its exact solution is given by:

$$\overline{A}_t^i = \frac{\overline{\lambda}\phi^i e^{\lambda t}}{[\overline{\lambda} + \phi^i(\alpha^i)^2]e^{\overline{\lambda}T^i} - \phi^i(\alpha^i)^2 e^{\overline{\lambda}t}}, \tag{A25}$$

where $\overline{\lambda} \equiv \rho - 2(\epsilon^i\gamma^i - \delta)$. By plugging (A25) into (A22), we obtain the following expression for the value function:

$$\overline{\mathcal{F}}^i(t, p_t) = \frac{1}{2}\frac{\overline{\lambda}\phi^i e^{(\overline{\lambda}-\rho)t}}{[\overline{\lambda} + \phi^i(\alpha^i)^2]e^{\overline{\lambda}T^i} - \phi^i(\alpha^i)^2 e^{\overline{\lambda}t}} p_t^2. \tag{A26}$$

By plugging (A25) into (A23) and (23) we get the firm i's closed-loop feedback Cournot–Nash rule for the environmental effort and the pollution dynamics between T^j and T^i, given in (31) and (32), respectively.

By denoting with $\mathcal{J}^i(t, p_t)$ the value function associated with problem (24) and (25) for firm i, the Hamilton–Jacobi–Bellman equation reads as:

$$-\frac{\partial \mathcal{J}^i(t, p_t)}{\partial t} = \min_{\tau_t^i} \left\{ \frac{(\tau_t^i)^2}{2} e^{-\rho t} + [(\epsilon^i\gamma^i + \epsilon^j\gamma^j - \delta)p_t - \alpha^i\tau_t^i - \alpha^j\tau_t^j]\frac{\partial \mathcal{J}^i(t, p_t)}{\partial p_t} \right\}, \tag{A27}$$

while the corresponding terminal condition is:

$$\mathcal{J}^i(T^j, p_{T^j}) = \frac{1}{2}\overline{\phi}^i p_{T^j}^2 e^{-\rho T^j}, \tag{A28}$$

where $\overline{\phi}^i = \frac{\overline{\lambda}\phi^i e^{(\overline{\lambda}-\rho)T^j}}{[\overline{\lambda} + \phi^i(\alpha^i)^2]e^{\overline{\lambda}T^i} - \phi^i(\alpha^i)^2 e^{\overline{\lambda}T^j}}$. Similarly, denoting with $\mathcal{J}^j(t, p_t)$ the value function associated with problem for firm j, the Hamilton–Jacobi–Bellman equation reads as:

$$-\frac{\partial \mathcal{J}^j(t, p_t)}{\partial t} = \min_{\tau_t^j} \left\{ \frac{(\tau_t^j)^2}{2} e^{-\rho t} + [(\epsilon^i\gamma^i + \epsilon^j\gamma^j - \delta)p_t - \alpha^i\tau_t^i - \alpha^j\tau_t^j]\frac{\partial \mathcal{J}^i(t, p_t)}{\partial p_t} \right\}, \tag{A29}$$

while the corresponding terminal condition is:

$$\mathcal{J}^i(T^j, p_{T^j}) = \frac{\phi^j}{2} p_{T^j}^2 e^{-\rho T^j}. \tag{A30}$$

The first-order conditions for firm i and j yield, respectively:

$$\tau_t^i = \alpha^i e^{\rho t}\frac{\partial \mathcal{J}^i(t, p_t)}{\partial p_t}, \tag{A31}$$

$$\tau_t^i = \alpha^j e^{\rho t}\frac{\partial \mathcal{J}^j(t, p_t)}{\partial p_t}. \tag{A32}$$

For each firm, we guess the following functional forms for the value functions:

$$\mathcal{J}^i(t,p_t)=\frac{1}{2}p_t^2A_t^ie^{-\rho t}, \tag{A33}$$

$$\mathcal{J}^j(t,p_t)=\frac{1}{2}p_t^2A_t^je^{-\rho t}, \tag{A34}$$

where A_t^i and A_t^j are some variables to be determined. By computing the derivatives $\frac{\partial\mathcal{J}}{\partial t}$ and $\frac{\partial\mathcal{J}}{\partial p_t}$, and plugging these into (A31), (A32), (A27), and (A29), we obtain, respectively:

$$\tau_t^i==\alpha^ip_tA_t^i, \tag{A35}$$

$$\tau_t^j==\alpha^jp_tA_t^j, \tag{A36}$$

$$\dot{A}_t^i=(\alpha^i)^2(A_t^i)^2+2(\alpha^j)^2A_t^iA_t^j+[\rho-2(\epsilon^i\gamma^i+\epsilon^j\gamma^j-\delta)]A_t^i, \tag{A37}$$

$$\dot{A}_t^j=(\alpha^j)^2(A_t^j)^2+2(\alpha^i)^2A_t^iA_t^j+[\rho-2(\epsilon^i\gamma^i+\epsilon^j\gamma^j-\delta)]A_t^j. \tag{A38}$$

At the terminal time, by comparing (A28) and (A33) and (A30) and (A34) evaluated at T^j, it follows that $A_{T^j}^i=\overline{\phi}^i\geq 0$ and $A_{T^j}^j=\phi^j\geq 0$. Since (A37) and (A38) form a simultaneous system of differential equations, an exact solution for A_t^i and A_t^j cannot be determined. The above equations determine the closed-loop feedback Cournot–Nash reclamation efforts for countries i and j, while the pollution dynamics between 0 and T^j is determined by plugging these reclamation efforts into (28).

Heterogeneous case: Cooperative solution

By denoting with $\mathcal{J}(t,p_t)$ the value function associated with problem (33) and (34), the Hamilton–Jacobi–Bellman equation reads as:

$$-\frac{\partial\mathcal{J}(t,p_t)}{\partial t}=\min_{\tau_t^i,\tau_t^j}\left\{\frac{(\tau_t^i)^2}{2}e^{-\rho t}+\frac{(\tau_t^j)^2}{2}e^{-\rho t}+[(\epsilon^i\gamma^i+\epsilon^j\gamma^j-\delta)p_t-\alpha^i\tau_t^i-\alpha^j\tau_t^j]\frac{\partial\mathcal{J}(t,p_t)}{\partial p_t}\right\}, \tag{A39}$$

while the corresponding terminal condition is:

$$\mathcal{J}^i(T^j,p_{T^j})=\frac{\overline{\phi}^i+\phi^j}{2}p_{T^j}^2e^{-\rho T^j}. \tag{A40}$$

The first-order conditions for firm i and j yield, respectively:

$$\tau_t^i=\alpha^ie^{\rho t}\frac{\partial\mathcal{J}(t,p_t)}{\partial p_t}, \tag{A41}$$

$$\tau_t^j=\alpha^je^{\rho t}\frac{\partial\mathcal{J}(t,p_t)}{\partial p_t}. \tag{A42}$$

We guess the following functional form for the value function:

$$\mathcal{J}(t,p_t)=\frac{1}{2}p_t^2A_te^{-\rho t}, \tag{A43}$$

where A_t is a variable to be determined. By computing its derivatives $\frac{\partial \mathcal{J}}{\partial t}$ and $\frac{\partial \mathcal{J}}{\partial p_t}$, and plugging these into (A41), (A42) and (A39), we obtain, respectively:

$$\tau_t^i=\alpha^i p_tA_t, \tag{A44}$$

$$\tau_t^j=\alpha^j p_tA_t, \tag{A45}$$

$$\dot{A}_t=[(\alpha^i)^2+(\alpha^j)^2]A_t^2+[\rho-2(\epsilon^i\gamma^i+\epsilon^j\gamma^j-\delta)]A_t. \tag{A46}$$

At the terminal time, by comparing (A40) and (A43) evaluated at T^j, it follows that $A_T=\overline{\phi}^i+\phi^j\geq 0$. Note that (A46) is a Bernoulli differential equation with a terminal condition, and its exact solution is given by:

$$A_t=\frac{\lambda(\overline{\phi}^i+\phi^j)e^{\lambda t}}{\{\lambda+(\overline{\phi}^i+\phi^j)[(\alpha^i)^2+(\alpha^j)^2]\}e^{\lambda T^j}-(\overline{\phi}^i+\phi^j)[(\alpha^i)^2+(\alpha^j)^2]e^{\lambda t}}. \tag{A47}$$

By plugging (A47) into (A43), we obtain the following expression for the value function:

$$\mathcal{J}(t,p_t)=\frac{1}{2}\frac{\lambda(\overline{\phi}^i+\phi^j)e^{(\lambda-\rho)t}}{\{\lambda+(\overline{\phi}^i+\phi^j)[(\alpha^i)^2+(\alpha^j)^2]\}e^{\lambda T^j}-(\overline{\phi}^i+\phi^j)[(\alpha^i)^2+(\alpha^j)^2]e^{\lambda t}}p_t^2. \tag{A48}$$

By plugging (A47) into (A45), (A44) and (34) we get the closed-loop cooperative reclamation efforts and the pollution dynamics given in (35) and (36), respectively.