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# Prediction of Maximum Lift Coefficient of Box-Wing Aircraft through the Combination of an Analytical Adaptation of the DATCOM Method and Vortex-Lattice Simulations

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**ABSTRACT**

**The present paper concerns the introduction of a mixed analytical-numerical method for the estimation of the maximum lift coefficient of box-wing aircraft in unflapped configuration. The analytical aspect is related to the adaptation of the method included in the United States Air Force Stability and Control Data Compendium (USAF DATCOM) by means of an approach built on the characteristics of the optimal lift distribution of the box-wing and implemented through simplifying assumptions. Since the formulation depends on parameters proper of the considered aircraft, numerical simulations are performed through a Vortex-Lattice Method to complete the input dataset. The method is first presented and then validated for the case of the 2-seater amphibious PrandtlPlane from the project "IDINTOS", for which wind tunnel data are available. The method is then applied to two test cases for which CFD data are available: a 300 passengers mid-range PrandtlPlane, developed within the European research project "PARSIFAL", and a regional hybrid-electric PrandtlPlane, object of study in the Italian research project "PROSIB". Results are presented and discussed, also analyzing the links between the proposed method and relevant parameters of the box-wing design, such as taper ratios and wing loading repartition among the two wings.**

**KEYWORDS:** maximum lift coefficient; box-wing; vortex-lattice; PrandtlPlane; DATCOM.

## Acronyms

| | | |
|---|---|---|
| AVL | = | Athena Vortex Lattice |
| DoE | = | Design of Experiment |
| CG | = | Centre of Gravity |
| FEM | = | Finite Element Model |
| HLD | = | High-Lift Device |
| MDO | = | Multi-Disciplinary Optimization |
| PrP | = | PrandtlPlane |
| TLAR(s) | = | Top Lever Aircraft Requirement(s) |
| VLM | = | Vortex-Lattice Method |

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## 1. Introduction

The aviation industry is facing the challenge of reducing the environmental impact in many ways, which follows the line drawn by the European Commission (2011) and implemented through the so-called "Strategic Research and Innovation Agenda" (ACARE, 2017). Beside the evolutionary steps which allow to improve the "green" performance of today aircraft and their components, research projects aiming at developing radically new technologies can provide an important contribution to face the environmental challenge. In this context, the research here presented has been conceived, being related to two projects:

- the European research project "PARSIFAL" ("Prandtlplane ARchitecture for the Sustainable Improvement of Future AirpLanes"), carried out between 2017 and 2020 within the Horizon 2020 Program, with the goal of studying the application of the box-wing architecture to the mid-range passenger aircraft,
- the Italian research project "PROSIB" (2018-2021), dedicated to the study of regional aircraft with hybrid-electric propulsion systems, also for the case of box-wing architectures.

(a) (b)

**Fig. 1 Artistic view of the PrandtlPlanes studied in the projects (a) "PARSIFAL"; and (b) "PROSIB"**

The box-wing configuration studied in these projects, artistically represented in Fig. 1, has been called "PrandtlPlane" (or PrP), since its development comes from the studies carried out by Ludwig Prandtl in the 1920s. Prandtl (1924) indicated the box-wing architecture as the "best wing system", i.e. the lifting system capable to minimize the induced drag for given lift and wingspan.

The PARSIFAL project has been concluded showing that the box-wing architecture can be exploited to increase the payload capability and to reduce the fuel consumption per passenger-kilometer at the same time (PARSIFAL Project Deliverable D1.2, 2020) (Abu Salem, et al., 2021a), compared to tube-and-wing configurations. In fact, the box-wing allows for the avoidance of the increase of induced drag that occurs when, in a tube-and-wing configuration, the fuselage is enlarged without increasing the wingspan. Δy

Therefore, when wingspan cannot be increased arbitrarily, as it usually happens in the airports because of the size limitations due to the available apron space, the box-wing architecture allows to increase the cabin width, hence the number of seats, without reducing the span efficiency. The results of the comparison between a PrP, designed to be compliant with the ICAO Aerodrome Reference Code "C"[1], and a conventional aircraft of the same category, represented by the common reference model called CeRAS-CSR01 (CeRAS, 2021), can be summarized as follows:

- up to 22% of fuel consumption and $CO_2$ per passenger-kilometer reduction, with significant impact on both environment and market opportunities (Abu Salem, et al., 2021a) (Cipolla, et al., 2020) (Tasca, et al., 2021);

[1] wingspan between 24 and 36 m; outer main gear wheel span between 6 and 9 m.

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- up to 18% and 23% reduction of Global Warming Potential and Global Temperature change Potential, respectively (Tasca, et al., 2021).

As detailed in the following, the present paper aims to introduce a step forward in the design approach adopted for such kind of aircraft, implementing in the same optimization workflow used in both PARSIFAL and PROSIB project a new method for the preliminary estimation of the maximum lift coefficient. Since this latter usually acts as a constraint for the "high speed" optimization, the capability to predict the "low speed" performance in the early stage of the design - without affecting the computational costs significantly - allows for the aforementioned step forward in the design of box-wing aircraft.

The proposed method is based on combining an analytical adaptation of the method included in the United States Air Force Stability and Control Data Compendium (Fink, 1978), hereafter indicated as DATCOM, to the box-wing case with Vortex-Lattice simulations. More in details, the analytical part is based on the characteristics of the optimal lift distribution of the box-wing, and the implementations is performed through a set of simplifying assumptions. As shown in the following, this formulation depends on the geometrical and aerodynamic characteristics of the aircraft under study, therefore numerical simulations, here performed by means of a Vortex-Lattice code, are needed to complete the input dataset. The proposed method has been first validated using wind tunnel results from a previous research project, and then applied to two test cases obtained from the aforementioned projects PARSIFAL and PROSIB.

## 2. State of the art of maximum lift coefficient estimation methods

The most adopted preliminary method for the maximum lift coefficient of a wing is the "method 2" described in Section 4.3.1.4 of the DATCOM, which can be applied under the assumptions of subsonic conditions ($M \leq 0.2$), untwisted wings and constant airfoil section.

According to this method, the wing-to-airfoil maximum lift coefficient ratio ($CL_{max_W}/Cl_{max}$) is obtained from the DATCOM Figure 4.3.1.4-21a (Fink, 1978) , where $Cl_{max}$ refers to the airfoil defined in the free-stream direction, $\Lambda_{LE}$ is the leading edge sweep angle and $\Delta_y$ is a non-dimensional parameter related to leading edge roundness.

For airfoil with thickness-to-chord ratio above 12%, $\Delta_y$ is usually greater than 2.5 and therefore the well-known formula introduced by Torenbeek (1982) can be adopted:

$$\frac{CL_{max_W}}{Cl_{max}} = 0.9 \cdot \cos \Lambda \tag{1}$$

where $\Lambda$ is the sweep angle calculated at the wing quarter-chord line.

As the DATCOM method is derived empirically from experimental data obtained for a wide but anyway limited variety of wing shapes, it can be adopted only if the wing's aspect ratio ($AR$) fulfills the following condition:

$$AR \geq \frac{4}{(C_1 + 1)\cos \Lambda_{LE}} \tag{2}$$

were $C_1$ is a function of wing's taper ratio ($\lambda$), graphically defined in the DATCOM and for the purposes of the present papers approximated through the function defined in Eq. (3), which fits the given data with a coefficient of determination ($R^2$) higher than 0.99:

$$C_1(\lambda) = 48.58\lambda^6 - 157.81\lambda^5 + 190.96\lambda^4 - 100.87\lambda^3 + 17.72\lambda^2 + 1.45\lambda \tag{3}$$

The curves shown in the DATCOM Figure 4.3.1.4-21a (Fink, 1978) have been defined from experimental data assuming that the wing can be considered as stalled at the angle of attack at which the lift curve deviates from linear variation, hereafter indicated as $\alpha^*$. This condition can be defined as the one in which the local lift coefficient calculated for sections perpendicular to the quarter-chord line ($Cl_{\perp}$) equals the maximum lift coefficient of the perpendicular airfoil ($Cl_{max_{\perp}}$). Therefore, the problem of defining stall condition can be described as in Eq. (4):

$$\begin{cases} CL_{max_W} = \dfrac{1}{S}\displaystyle\int_{-b/2}^{b/2} Cl(y,\alpha^*)\cdot c(y)dy \\ max\big(Cl_{\perp}(y,\alpha^*)\big) = Cl_{max_{\perp}} \end{cases} \tag{4}$$

where $Cl$ is defined in the free-stream direction, c(y) is the wing chord distribution and S is defined as:

$$S = \int_{-b/2}^{b/2} c(y)dy \tag{5}$$

Eq. (4) can be modified, referring all the quantities to the lift coefficients defined in the free-stream direction, as follows:

$$\begin{cases} CL_{max_W} = \dfrac{1}{S}\displaystyle\int_{-b/2}^{b/2} Cl(y,\alpha^*)\cdot c(y)dy \\ max\big(Cl(y,\alpha^*)\big) = Cl_{max}\cdot cos^2 \Lambda \end{cases} \tag{6}$$

Concerning the box-wing architecture characteristics at stall, as summarized in Cavallaro & Demasi (2016) for past NACA/NASA studies and experimentally shown in more recent works concerning the amphibious PrP "IDINTOS" (Frediani, et al., 2015) (Cipolla, et al., 2015), a proper box-wing system design makes the front wing more critical than the rear one. As confirmed also in the present paper, this behavior depends on the constraints of longitudinal trim and stability introduced in the box-wing design. Under such constraints, in fact, the rear wing keeps generating lift when flow separation starts on the front one. Therefore, the stall condition is associated to a plateau in the $C_L - \alpha$ curve as well as to an increase of the pitch down moment. A similar tendency was already observed for the joined-wing configuration investigated by Henderson et al. (1975) and more in general by Wolkovitch (1986). More recently, the peculiarities of $C_L - \alpha$ curves have been observed experimentally for a simplified joined biplane geometry (Genco & Altman, 2009), and for several front and rear wing sweep angles combinations (Barcala, et al., 2011). In addition, experimental results (Karpovich, et al., 2020) underline the pitch down moment increase in $C_m - \alpha$ curves.

Since these peculiar characteristics are the results of the constrained design of the box-wing, the availability of specific models, even simplified, for the prediction of maximum lift coefficient in the early design stages would be of great help for the box-wing designer. Even more if these models could be somehow in relationship with the parameters influencing the box-wing longitudinal trim and stability.

On the design side, some authors have adopted the strategy of imposing an upper limit to the local lift coefficient value (Andrews & Perez, 2018) or to the difference between peak and trailing edge pressure coefficients (Kalinowski, 2017), as indicated by Valarezo et al. (1994). These strategies require several iterative aerodynamic analyses to identify the $\alpha$ (or $C_L$) value at which the stall criteria are reached and therefore are not suitable for the early design stages. Strategies as the one proposed by Phillips et al. (2007) are more suitable for the early design phases, as they rely on non-iterative low fidelity models to provide an estimation of a wing maximum lift coefficient.

The present paper aims to introduce a model that, under some assumptions but through a rigorous analytical process, can adapt the DATCOM method cited above to the case of box-wing architectures. Such main assumptions are the hypotheses that the lift distribution of each horizontal wing is close enough to the case which minimizes the induced drag, whose solution is given by the sum of a constant and an elliptic function, and that the ratio between front and rear wing lift is constant in the linear range of the $CL - \alpha$ curve. As described, the proposed method takes the main characteristics of the box-wing lift distribution into account and connects the maximum $C_L$ performance to the parameters that drive the box-wing flight mechanics characteristics.

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## 3. Method presentation

By properly modifying the DATCOM method, suitable for any cantilever, untwisted and constant-section wing in subsonic flow, the method here proposed aims to predict the maximum lift coefficient of a “clean”, i.e. unflapped, box-wing system. The strategy is based on using the Vortex-Lattice Method (VLM) results obtained for low angle of attack values, typical of cruise conditions and providing a conservative estimation of $\alpha^*$. Thus, the predicted $CL_{max}$ is not an approximation of the lift coefficient at stall, but a conservative estimation of the $C_L - \alpha$ linear range upper limit.

The reason for deriving the input data from a VLM simulation at low angle of attack lies in the strategy adopted at Pisa University for the design of box-wing aircraft, based on the studies reported by Rizzo (2009) and published more recently for the PARSIFAL project case (Abu Salem, et al., 2021b). Such strategy is implemented by means of an in-house developed tool, called AEROSTATE, whose workflow is summarized in Fig. 2.

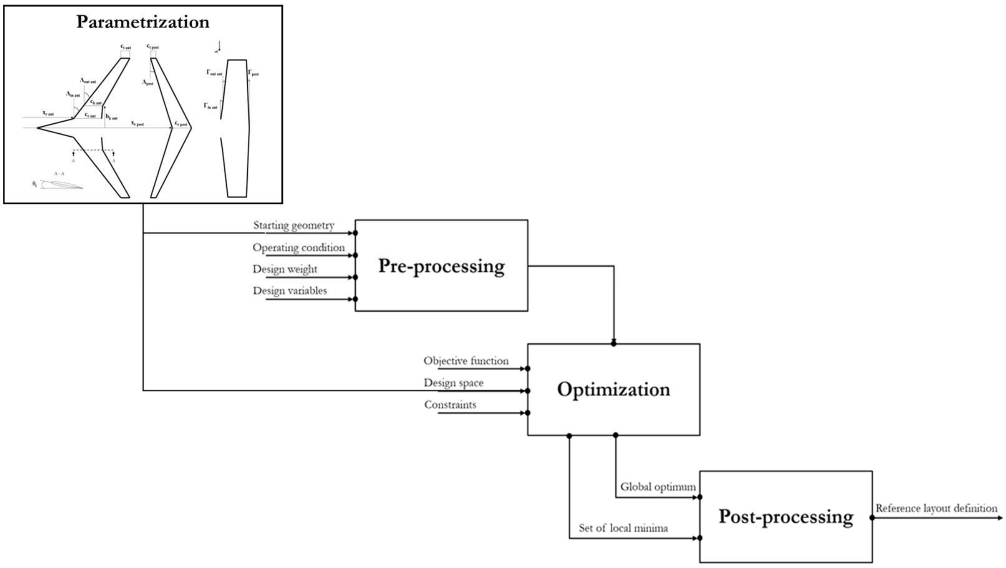


**Fig. 2 Overview of the AEROSTATE workflow**

As Fig.2 suggests, the core phase of the AEROSTATE workflow is the optimization one, in which the following problem is solved by using a multi-start local search algorithm:

$$\begin{cases} \min\left(-\dfrac{L(\bar{\boldsymbol{x}})}{D(\bar{\boldsymbol{x}})}\right)_{\text{cruise}} \\ \overline{\boldsymbol{lb}} \leq \bar{\boldsymbol{x}} \leq \overline{\boldsymbol{ub}} \\ \bar{g}(\bar{\boldsymbol{x}}) \leq 0 \end{cases} \tag{7}$$

where $\bar{\boldsymbol{x}}$ is the vector of the design variables, previously defined through the parametrization of the aircraft configuration. By minimizing the objective function indicated in Eq. (7), the algorithm looks for solutions capable to maximize the lift-to-drag ratio, for $\bar{\boldsymbol{x}}$ values within the design space, bounded by the quantities $\overline{\boldsymbol{lb}}$ and $\overline{\mathbf{ub}}$, and under the constraints introduced through the inequalities $\bar{\mathrm{g}}(\bar{\mathbf{x}}) \leq 0$. As reported by Abu Salem et al. (2021b), the objective function is evaluated in cruise condition by means of the VLM solver AVL (Drela & Youngren, 2017 ), therefore for flight conditions typically characterized by low angles of attack. In the optimization phase, AVL is run several times for an average duration of about 3 hours on a 2020 laptop computer. Therefore, adding simulations at high angles of attack with the objective of finding the conditions at which Eq. (4) is verified, means at least doubling the computational time, since each set of design variable $\bar{\boldsymbol{x}}$ should be analysed with the VLM at the cruise angle of attack and at an additional higher value. The problem of defining how much higher this latter should be is not trivial, therefore an iterative procedure should be implemented, as shown by Frediani et al., (2015).

Therefore, aiming at avoiding any significant increase in complexity and computational time, the idea behind the proposed method is to extract some characteristics of the lift distributions calculated at the cruise angle of attack, usually between the zero-lift value ($\alpha_0$) and α*, and to provide an approximate estimation of critical conditions for both front and rear wings. Once the most critical wing, i.e. the one for which α* has the lowest value, is found, the maximum lift coefficient of the box-wing within the $C_L - \alpha$ linear range ($CL_{BW}$) can be calculated.

Therefore, the main steps of the method are:

- identification of the critical condition for a generic horizontal wing composing the box-wing through the adaptation of the DATCOM method;
- evaluation of the maximum lift coefficient of the generic wing by means of VLM results at cruise condition;
- definition of the stall condition for the whole box-wing configuration.

### *3.1. Identification of the critical condition for a generic horizontal wing composing the box-wing*

According to previous (Von Kármán & Burgers, 1935) and more recent (Frediani & Montanari, 2009) studies, the solution of the problem of minimizing the induced drag of the box-wing provides optimum circulation distributions $(\Gamma(y))$ on horizontal wings which can be approximated as the sum of an elliptic and a constant component. As demonstrated by Demasi et al. (2015), this approximation is not universal since the elliptic component becomes negligible in comparison to the constant one as the heigh-to-wingspan ratio $\left(h/_b\right)$ increases. Just to give an idea, for $h/_b$ greater than 3, the elliptic part is at least one order of magnitude smaller than the constant part. Nevertheless, this approximation is acceptable for most of the practical applications of the box-wing system, in which $h/_b$ ranges between 0.1 and 0.3. In addition, Demasi et al. (2015) demonstrate that the optimal solutions are infinite, since an arbitrary constant circulation can be added to the quasi-elliptic distribution which provides the required total lift.

According to the lifting line theory, the circulation distribution can be related to the lift distribution as shown in Eq. (8), where $U_\infty$ is the freestream velocity, ρ is the air density and y is the spanwise coordinate. Therefore, the sum of an elliptic and a constant component can be also used to approximate the lift distributions over the two horizontal wings, hence the quantity $Cl(y) \cdot c(y)$.

$$\frac{1}{2} \cdot \rho \cdot U_\infty{}^2 \cdot Cl(y) \cdot c(y) = \rho \cdot U_\infty \cdot \Gamma(y) \tag{8}$$

In a generic non-optimal case, the nondimensional lift distribution on any of the two wings composing the box-wing can be described as follows:

$$Cl(y,\alpha)\frac{c(y)}{c_{\mathrm{ref}}} = Cl(b/2,\alpha)\frac{c(b/2)}{c_{\mathrm{ref}}} + \Delta Cl(y,\alpha)\frac{c(y)}{c_{\mathrm{ref}}} \tag{9}$$

where b is the wingspan, assumed the same for front wing and rear wing, $\alpha$ is the angle of attack of the whole box-wing configuration and $c_{\mathrm{ref}}$ is the chosen reference chord. For cases of practical interest, both front and rear wing provide a positive contribution to lift and thus the tip lift coefficient $Cl(b/2,\alpha)$ is assumed to be a positive quantity.

Introducing the nondimensional spanwise coordinate $\eta = \frac{2y}{b}$, Eq. (9) becomes:

$$Cl(\eta,\alpha)c(\eta) = Cl(1,\alpha)c(1) + \Delta Cl(\eta,\alpha)c(\eta) \tag{10}$$

Let us now assume that $\Delta Cl(\eta)c(\eta)$ can be described as the sum of an elliptic part, corresponding to the optimum solution, and a non-optimal component $\varepsilon(\eta)$ which is null at root and tip sections, by definition. Therefore, Eq.(10) becomes:

$$Cl(\eta,\alpha)c(\eta) = Cl(1,\alpha)c(1) + [Cl(0,\alpha)c(0) - Cl(1,\alpha)c(1)]\sqrt{1-\eta^2} + \varepsilon(\eta,\alpha) \quad (11)$$

For any wing planform, it is possible to define an equivalent simply tapered wing (ESDU-76003, 2012); hence, said $\lambda$ the tip-to-root chord ratio for simply tapered wings or the equivalent taper ratio calculated for cranked wings, the wing area is given by the following relation:

$$S = c(0)\cdot(1+\lambda)\frac{b}{2} \quad (12)$$

According to Eq. (6) and adopting Eq. (11) and (12), the critical condition can be described as follows:

$$CL_{max} = \frac{1}{S}\int_{-1}^{1} Cl(\eta,\alpha^*)\cdot c(\eta)\frac{b}{2}d\eta = \frac{1}{1+\lambda}\int_{-1}^{1}\left\{Cl(1,\alpha^*)\lambda + [Cl(0,\alpha^*) - Cl(1,\alpha^*)\lambda]\sqrt{1-\eta^2} + \frac{\varepsilon(\eta,\alpha^*)}{c(0)}\right\}d\eta \quad (13)$$

which becomes:

$$CL_{max} = \frac{1}{1+\lambda}\cdot\frac{\pi}{2}\cdot Cl(0,\alpha^*) + \frac{\lambda}{1+\lambda}\left(2-\frac{\pi}{2}\right)Cl(1,\alpha^*) + E(\alpha^*) \quad (14)$$

where:

$$E(\alpha^*) = \frac{1}{c(0)\cdot(1+\lambda)}\int_{-1}^{1}\varepsilon(\eta,\alpha^*)\,d\eta \quad (15)$$

As Eq. (6) indicates, it is necessary to establish the relation between the $CL_{max}$ estimation provided by Eq. (14) and the airfoil $Cl_{max}$. Therefore, let us introduce the first main hypothesis consisting in assuming that the function $Cl(\eta,\alpha^*)$ has its maximum value at the root section ($\eta = 0$), which means assuming the lift distribution is not too far from the optimal one, hence $\varepsilon(\eta,\alpha^*)$ is small enough. Under this assumption, it stands:

$$Cl(0,\alpha^*) = Cl_{max}\cdot cos^2\Lambda \quad (16)$$

Thus, Eq. (14) becomes:

$$CL_{max} = \frac{1}{1+\lambda}\cdot\frac{\pi}{2}\cdot Cl_{max}\cdot cos^2\Lambda + \frac{\lambda}{1+\lambda}\left(2-\frac{\pi}{2}\right)Cl(1,\alpha^*) + E(\alpha^*) \quad (17)$$

Whereas the evaluation of $Cl(1,\alpha^*)$ and $E(\alpha^*)$ would need the study of the function $Cl(\eta,\alpha)$ by means of analytical or numerical approaches, the proposed method aims at approximating the $CL_{max}$ by adopting the information provided by the DATCOM method.

For such purpose, let us now consider a cantilever wing having the same geometry, i.e. same sweep angle and airfoil, of the generic box-wing element considered previously. By observing that the lift coefficient distribution, in this case, has a null value at wing's tip, a general expression for the maximum lift coefficient can be obtained by setting $Cl(1,\alpha^*) = 0$ in Eq. (11) and introducing the non-optimal component for the cantilever wing case $\varepsilon_W(\eta)$. Adopting the same approach used to derive Eq. (17), it follows:

$$CL_{max_W} = \frac{1}{1+\lambda}\cdot\frac{\pi}{2}\cdot Cl_{max}\cdot cos^2\Lambda + E_W(\alpha_W^*) \quad (18)$$

where $\alpha_W^*$ is the critical angle of attack of the cantilever wing and $E_W$ is defined in analogy with Eq. (15).

The second and third main hypotheses to be introduced are the following:

$$\alpha_W^* = \alpha^* \tag{19}$$

$$\frac{E_W(\alpha_W^*)}{E(\alpha^*)} = 1 \tag{20}$$

which have the following meaning:

- according to Eq. (19), the critical condition of a cantilever wing having the same geometry of a box-wing horizontal element is reached at the same angle of attack, as the result of the fact that Eq. (16) is valid for both and the presence of vertical tip-wings does not affect the aerodynamic characteristics of root section;
- Eq. (20) can be written as follows

$$\frac{\int_{-1}^{1} \varepsilon_W(\eta, \alpha_W^*)\, d\eta}{\int_{-1}^{1} \varepsilon(\eta, \alpha^*)\, d\eta} = \frac{\overline{\varepsilon_W}(\alpha_W^*)}{\bar{\varepsilon}(\alpha^*)} = 1 \tag{21}$$

indicating that if the lift distribution of the box-wing horizontal element differs from the optimal one for an average value $\bar{\varepsilon}$, the cantilever wing obtained isolating that box-wing element is far from the optimal condition of a comparable average value $\overline{\varepsilon_W}$.

Given these two assumptions, Eq. (18) becomes:

$$CL_{max_W} = \frac{1}{1+\lambda} \cdot \frac{\pi}{2} \cdot Cl_{max} \cdot cos^2\, \Lambda + E(\alpha^*) \tag{22}$$

and therefore Eq. (17) can be written as:

$$CL_{max} = f(\lambda) \cdot Cl(1, \alpha^*) + CL_{max_W} \tag{23}$$

where

$$f(\lambda) = \frac{\lambda}{1+\lambda}\left(2 - \frac{\pi}{2}\right) \tag{24}$$

Said γ the tip-to-root lift coefficient ratio,

$$\gamma(\alpha) = \frac{Cl(1, \alpha)}{Cl(0, \alpha)} \tag{25}$$

considering Eq. (16), Eq. (23) becomes:

$$\frac{CL_{max}}{Cl_{max}} = f(\lambda) \cdot \gamma(\alpha^*) + \frac{CL_{max_W}}{Cl_{max}} \tag{26}$$

where the ratio ${CL_{max_W}}/{Cl_{max}}$ can be obtained by applying the DATCOM method to any box-wing horizontal element considering it as a cantilever wing with the same geometry.

Therefore Eq. (26) is the adaptation of the DATCOM "method 2" to the box-wing case, considering the peculiar lift distribution of the horizontal wings composing the box-wing system, simplified according to the strategy presented in this section.

### *3.2. Calculation of front and rear wing $CL_{max}$ for a generic horizontal wing composing the box-wing*

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Aiming to avoid overestimations of the maximum lift coefficient, the approach proposed by Torenbeek (Eq. (1)) is here adopted, hence assuming the sharpness factor ($\Delta_y$) of any considered airfoil is above the threshold value of 2.5, which is typical of airfoil with thickness-to-chord ratios above 12%. In addition, according to the DATCOM method hypotheses, it is assumed that both front wing rear wing meet the following requirements:

- untwisted wings;
- constant-section wings;
- Eq.(2) is verified.

Under such assumptions, the maximum lift coefficient achievable by front or rear wing can be obtained by implementing the following procedure:

- identification of the airfoil $Cl_{max}$ for both front and rear wing, using the best available data source;
- calculation of $\gamma$ for both front and rear wing from VLM results at an angle of attack typical of cruise condition, hence within in the linear range $[\alpha_0, \alpha^*]$;
- estimation of front and rear wing $CL_{max}$

The quantity $\gamma(\alpha^*)$ in Eq. (26) can be evaluated using the VLM results for any angle of attack within the linear range, once the following hypothesis is introduced:

$$\alpha \in [\alpha_0, \alpha^*] \Rightarrow \gamma(\alpha) = \text{constant} \tag{27}$$

Although this hypothesis introduces a significant approximation, it provides a simplification suitable for the design phase for which the proposed method is conceived. Therefore, $\gamma_{\text{FW}}$ and $\gamma_{RW}$ are calculated as follows:

- the VLM code AVL is used to estimate the aerodynamic characteristics of the box-wing aircraft modelled as shown in Fig. 3, which is related to the validation case described in Section 4. According to previous experiences (Frediani, et al., 2015) (Abu Salem, et al., 2021b), the fuselage is modelled as a lifting surface whose dimensions approximate the fuselage planform projection. Such surface is then included in the model of front wing extending this latter up to the plane of symmetry.
- the box-wing model is analyzed through AVL at low angle of attacks, typical of cruise conditions, in order to estimate quantities such as aerodynamic coefficients for the whole configuration and subcomponents, aerodynamic derivatives, trim angles and lift coefficient distributions for each lifting surface. A typical example of this output is given in Fig. 4, which is related to the configuration illustrated in Fig. 3.
- once the lift coefficient distributions are obtained, AVL results can be used to calculate $\gamma_{\text{FW}}$ and $\gamma_{RW}$ according to Eq.(25) adopting the following definitions:
- for the front wing, $Cl(0, \alpha)$ is the lift coefficient at wing root section (y=0.75 m in Fig. 4 example) and $Cl(1, \alpha)$ is taken at wing tip (y=4 m in Fig. );
- for the rear wing, $Cl(0, \alpha)$ is taken on the symmetry plane and $Cl(1, \alpha)$ is taken at wing tip as for the front wing (as shown in Fig. ).

Once $\gamma_{\text{FW}}$ and $\gamma_{RW}$ are known, Eq. (26) can be applied to calculate the lift wing-to-airfoil ratio of each wing. Since $\gamma$ values are calculated using the VLM, the results obtained take the upwash/downwash effects into account; nevertheless, it is necessary to identify the critical condition of the box-wing as a whole.

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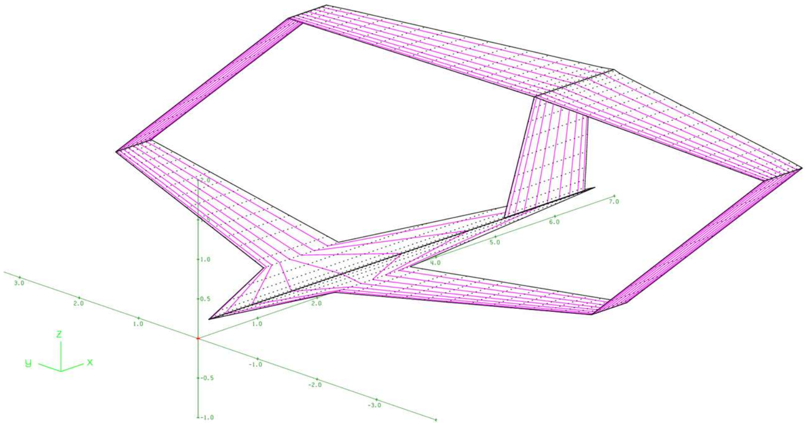

**Fig. 3 AVL model of a box-wing configuration**

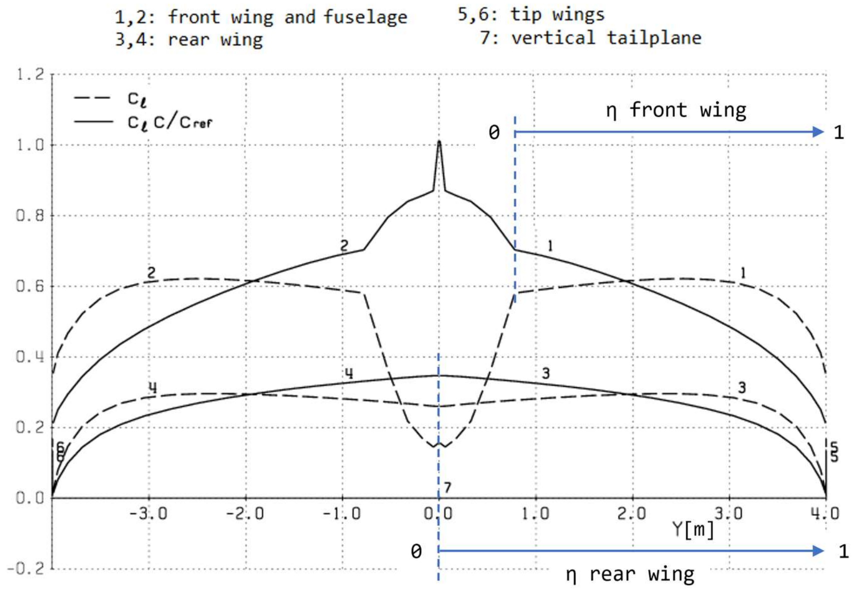


**Fig. 4 Example of lift coefficient distributions calculated on Trefftz plane for the box-wing through AVL**

## *3.3. Definition of stall condition for the box-wing configuration*

When $\alpha^*$ is reached, the lift coefficient distribution of either front or rear wing fulfils Eq. (26). If, for instance, this happens first on the front wing, we obtain:

$$\begin{cases} L_{FW}(\alpha^*) = q \cdot S_{FW} \cdot \left[ f(\lambda_{FW}) \cdot \gamma_{FW} + \left. \frac{CL_{max_W}}{Cl_{max}} \right|_{FW} \right] \cdot Cl_{max_{FW}} \\ L_{RW}(\alpha^*) = \frac{L_{FW}(\alpha^*)}{R(\alpha^*)} \end{cases} \tag{28}$$

where:

- $Cl_{max_{FW}}$ is the maximum lift coefficient of front wing airfoil, which in general differs from rear wing one,
- the quantity $\left.\frac{CL_{max_W}}{Cl_{max}}\right|_{FW}$ has to be evaluated using Torenbeek formula (Eq.(1)),
- $R$ is the ratio between front and rear wing lift:

$$R(\alpha) = \frac{L_{FW}(\alpha)}{L_{RW}(\alpha)} \tag{29}$$

Therefore, the box-wing lift at $\alpha = \alpha^*$ is:

$$L_{BW}(\alpha^*) = \left(1 + \frac{1}{R(\alpha^*)}\right) \cdot q \cdot S_{FW} \cdot \left[ f(\lambda_{FW}) \cdot \gamma_{FW} + \left.\frac{CL_{max_W}}{Cl_{max}}\right|_{FW} \right] \cdot Cl_{max_{FW}} \tag{30}$$

and the maximum lift coefficient of the box-wing system is:

$$CL_{max_{BW}} = \left[1 + \frac{1}{R(\alpha^*)}\right] \cdot \frac{S_{FW}}{S_{BW}} \cdot \left[ f(\lambda_{FW}) \cdot \gamma_{FW} + \left.\frac{CL_{max_W}}{Cl_{max}}\right|_{FW} \right] \cdot Cl_{max_{FW}} \tag{31}$$

where the reference box-wing surface ($S_{BW}$) is commonly defined as the sum of front and rear wing reference surfaces.

If instead the critical condition occurs first on rear wing, we have:

$$\begin{cases} L_{FW}(\alpha^*) = L_{RW}(\alpha^*) \cdot R(\alpha^*) \\ L_{RW}(\alpha^*) = q \cdot S_{RW} \cdot \left[ f(\lambda_{RW}) \cdot \gamma_{RW} + \left.\frac{CL_{max_W}}{Cl_{max}}\right|_{RW} \right] \cdot Cl_{max_{RW}} \end{cases} \tag{32}$$

which brings to the following maximum lift coefficient for the box-wing:

$$CL_{max_{BW}} = [1 + R(\alpha^*)] \cdot \frac{S_{RW}}{S_{BW}} \cdot \left[ f(\lambda_{RW}) \cdot \gamma_{RW} + \left.\frac{CL_{max_W}}{Cl_{max}}\right|_{RW} \right] \cdot Cl_{max_{RW}} \tag{33}$$

To define the maximum lift coefficient of the box-wing it is then necessary to evaluate the results of Eq.(31) and Eq.(33) and identify the most critical wing, i.e. the one providing the lower $CL_{max}$ value. For such purpose, the $R(\alpha^*)$ needs to be estimated and, since the scope of this approach is to provide an approximation without additional VLM computations, the following assumption is introduced:

$$R = \frac{CL_{FW}(\alpha)}{CL_{RW}(\alpha)} \frac{S_{FW}}{S_{RW}} = \text{constant for } \alpha \in [\alpha_0, \alpha^*] \tag{34}$$

As for Eq. (27), the level of approximation introduced by Eq. (34) is considered appropriate for the early design phase the proposed method is intended for. Since all the terms in Eq. (34) can be calculated by means of the VLM code, $R$ can be estimated directly.

### *3.4. Observations on the effects of wing loading repartition*

The front and rear wing loading play a crucial role in the longitudinal equilibrium and static stability of the box-wing. As described in detail by Abu Salem et al. (2021b) and here reported in Fig. 5, the rear-to-front wing loading ratio influences both the stability margin ($SM$), which needs to be positive, and the pitching moment coefficient ($C_M$), which needs to be as close as possible to zero in order to allow to trim the aircraft with as small as possible elevators deflection.

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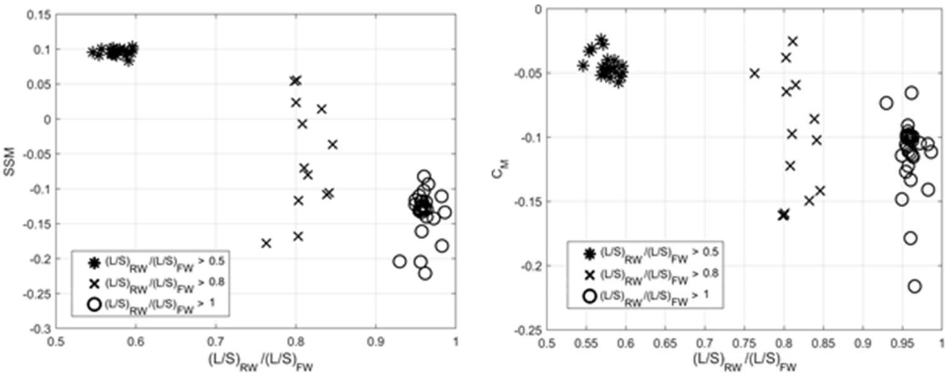

(a) (b)

**Fig. 5 Longitudinal static stability margin (a) and pitching moment (b) for different rear-to-front wing loading ratios. (Data from Abu Salem et al. 2021.)**

According to Fig. 5, obtained in the framework of the project PARSIFAL for the medium-range PrP here reported in Section 5.1, trim and stability requirements are met for values of the rear-to-front wing loading ratio in the range [0.5, 0.8].

To study the effect of wing loading ratio on also $CL_{max}$, Eq.(31) and Eq.(33) can be written as in Eq. (35), in which the first row in the brackets provides the $CL_{max}$, defined for the box-wing, achievable in the case front wing is the first one to reach the critical condition, whereas the second row concerns the case rear wing is more critical.

$$CL_{max_{BW}} = min \begin{Bmatrix} \frac{(L/S)_{FW}}{(L/S)_{BW}} \cdot \left[ f(\lambda_{FW}) \cdot \gamma_{FW} + \left. \frac{CL_{max_W}}{Cl_{max}} \right|_{FW} \right] \cdot Cl_{max_{FW}} \\ \frac{(L/S)_{RW}}{(L/S)_{BW}} \cdot \left[ f(\lambda_{RW}) \cdot \gamma_{RW} + \left. \frac{CL_{max_W}}{Cl_{max}} \right|_{RW} \right] \cdot Cl_{max_{RW}} \end{Bmatrix} \tag{35}$$

Considering the medium-range PrP test case, different wing loading repartitions can be imposed to evaluate the influence on the box-wing maximum lift coefficient, under the hypotheses of constant $\gamma_{FW}$ and $\gamma_{RW}$ values.

The results are shown in Fig. 6,where for each value of the wing loading ratio the box-wing $CL_{max}$ has been obtained selecting the minimum values of those obtained from Eq. (35). Fig. 6 underlines how, considering the range in which the wing loading ratio provides good longitudinal trim and stability characteristics, the front wing is the most critical in terms of stall occurrence.

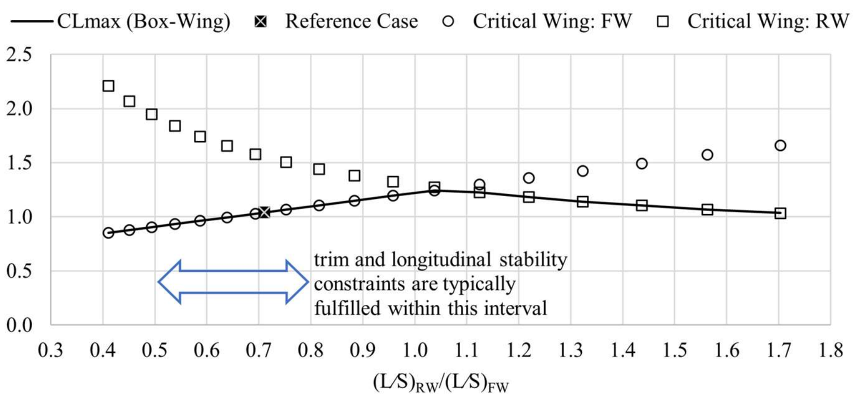


**Fig. 6 Box-wing $CL_{max}$ and critical wing identification for different rear-to-front wing loading ratios**

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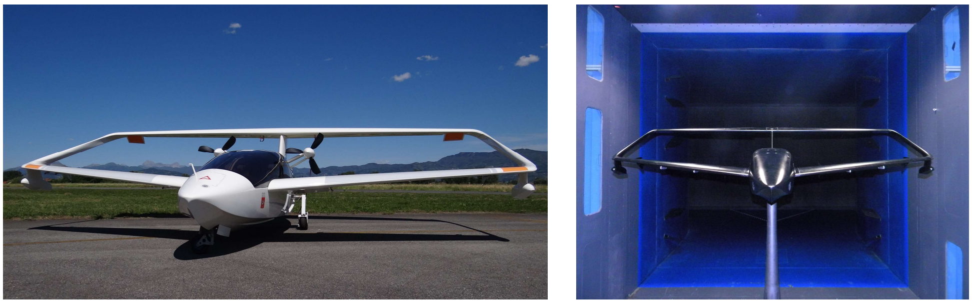
## 4. Method validation

The method has been validated considering the case of the box-wing 2-seats amphibian studied within the research project "IDINTOS" (Frediani, et al., 2015), for which a wind tunnel testing campaign has been carried out, as shown in Fig. 7 (Cipolla, et al., 2015).



(a) (b)

**Fig. 7 a): the 2-seats amphibious PrP designed within "IDINTOS" project (image courtesy of by Marco Ferracci) . b): the ¼ scaled model during a test campaign in wind tunnel (image by Vittorio Cipolla).**

As Fig. shows, the airfoil $Cl_{max}$ has been determined by using the aerodynamic solver XFoil setting the Reynolds to a reasonable value for low-speed conditions, such as those of the approach phase.

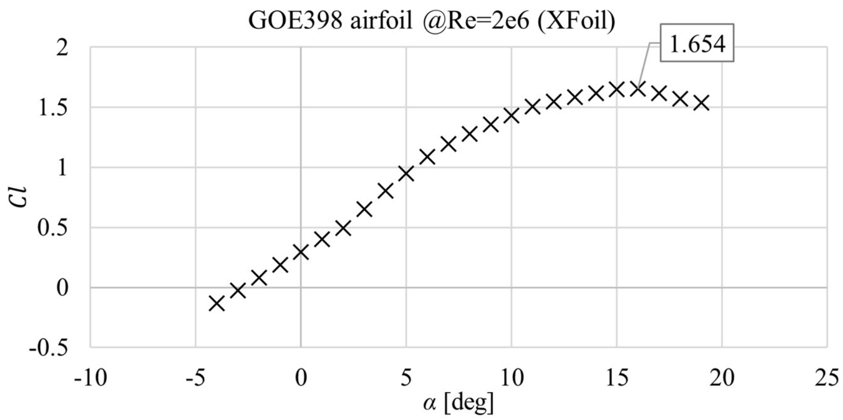



**Fig. 8 GOE398 airfoil $Cl_{max}$ calculation from XFoil data**

The box-wing maximum lift coefficient has been then evaluated using the method here presented using the input values indicated in Table 1.

The following aspects are worth to be underlined:

- for both the two wings, the airfoil sharpness factor $\Delta_y$ is above the limit value 2.5 prescribed by the Torenbeek method;
- the aspect ratio of both front and rear wings fulfils the constraint given by Eq. (2);
- the $CL_{max_{BW}}$ target value, i.e. the lift coefficient value the proposed method aims to approximate, has been extracted from wind tunnel data plotted in Fig. 9, selecting the maximum lift coefficient for which the $CL_\alpha$ derivative is not less than 2/3 of the value within the interval $CL \in [0.2, 0.8]$, which is usually in the linear range of the $CL - \alpha$ curve (see Fig. 9 for a graphical explanation);
- the VLM simulation has been performed at $\alpha$=5°, therefore the quantities $R$, $\gamma_{FW}$ and $\gamma_{RW}$ refer to such value.

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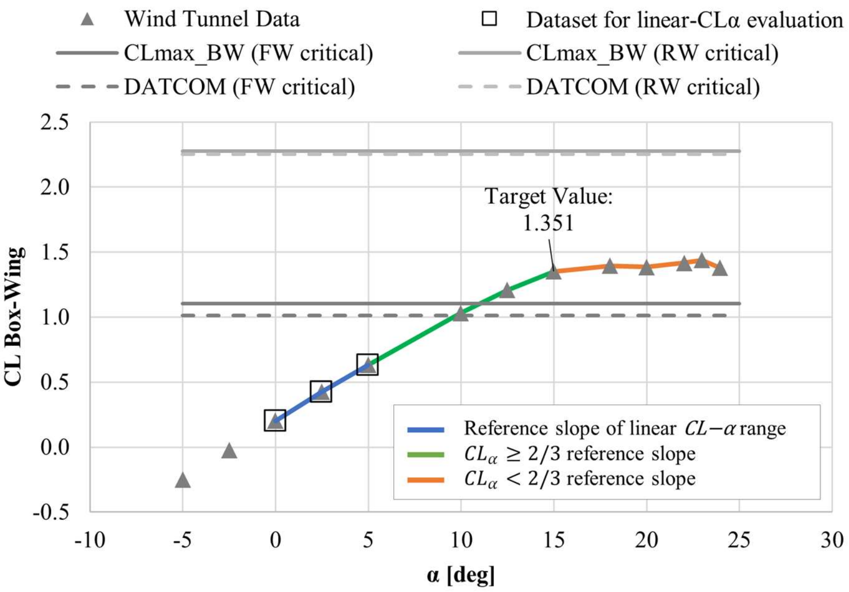


**Fig. 9 $CL_{max}$ prediction for the for the validation case "IDINTOS" with wind tunnel data (Data from Cipolla et al. 2015.)**

**Table 1. Input and output for the validation case "IDINTOS"**

| Component | Input Parameter | Value | Output Parameter | Value |
|---|---|---|---|---|
| FRONT WING | Airfoil $Cl_{max}$ (GOE398) | 1.654 | $\gamma_{FW}$ | 0.57 |
| | Airfoil $\Delta_y$ | 2.76 | $f(\lambda_{FW})$ | 0.14 |
| | $S_{FW}$ [m^2] | 6.231 | $CL_{max_{FW}}$ | 1.583 |
| | $\Lambda_{FW}$ [deg] | 12.5 | Limitation to $CL_{max_{BW}}$ (from Eq.(31)) | 1.103 |
| | $\lambda_{FW}$ | 0.49 | $CL_{max_{FW}}$ DATCOM | 1.450 |
| | $AR$ (min. value from Eq. (2)) | 6.6 (3.1) | Limitation to $CL_{max_{BW}}$ DATCOM | 1.010 |
| REAR WING | Airfoil $Cl_{max}$ (GOE398) | 1.654 | $\gamma_{RW}$ | 0.07 |
| | Airfoil $\Delta_y$ | 2.76 | $f(\lambda_{RW})$ | 0.14 |
| | $S_{RW}$ [m^2] | 7.948 | $CL_{max_{RW}}$ | 1.498 |
| | $\Lambda_{RW}$ [deg] | -4.7 | Limitation to $CL_{max_{BW}}$ (from Eq.(33)) | 2.275 |
| | $\lambda_{RW}$ | 0.49 | $CL_{max_{RW}}$ DATCOM | 1.483 |
| | $AR$ (min. value from Eq. (2)) | 8.0 (3.0) | Limitation to $CL_{max_{BW}}$ DATCOM | 2.252 |
| BOX WING | $S_{BW}$[m^2] | 14.179 | $CL_{max_{BW}}$ (from Eq. (35)) | 1.103 |
| | $R$ | 1.708 | Error vs. target value | -18% |
| | $\alpha$ for VLM simulation [deg] | 5 | $CL_{max_{BW}}$ DATCOM | 1.010 |
| | $CL_{max_{BW}}$ target value (wind tunnel) | 1.351 | Error vs. target value | -25% |

Output values related to the box-wing indicated in Table 1 and Fig. 9 show that the proposed method allows to improve the estimations provided by Eq. (1), indicated as "DATCOM" to underline the semiempirical nature of the method. The most relevant aspects can be summarized as follows:

- although the $CL_{max}$ predicted by the proposed method is higher than the DATCOM's one, it still provides a conservative estimation of the target $CL$ value with a good margin of safety;
- the differences in $CL_{max}$ estimation are not only related to the peculiar aerodynamics of the box-wing, introduced by $\gamma_{FW}$, $\gamma_{RW}$ and $R$, but depend also on the wings' taper ratio, which in the DATCOM case is taken into account only to verify the applicability of the method;
- the several approximations needed to allow the use of input from VLM simulation at cruise conditions, hence saving computational time, are acceptable given the improved estimation provided by the proposed method.

To better understand if similar results can be found for different test cases, in the following Section the method is applied to medium range and a regional PrPs, for which the $CL_{max}$ has been previously estimated by means of CFD analyses.

## 5. Test cases

### *5.1. Medium range PrP test case*

The first test case for the application of the $CL_{max}$ predictive model is related to the "PARSIFAL" project. A PrP configuration developed in the project with a significant level of detail has been selected, as a large amount of high-fidelity information regarding aerodynamics are available (PARSIFAL Project Deliverable 3.4, 2020) (PARSIFAL Project Deliverable 4.2, 2020) (Carini, et al., 2002). In particular, both the performance of the aircraft in standard operating conditions and its stall behavior at low speed have been evaluated by means of high-fidelity CFD analyses, hence providing fundamental information to assess the comparison with the model proposed in this paper.

The box-wing aircraft selected as a test case is represented in Fig. 10, while its main features are reported in the "INPUT" column of Table 2; the box-wing aircraft is an airliner operating in the medium range, with a maximum payload capacity up to 310 passengers (Cipolla, et al., 2020). The airfoil used to design the lifting system is the supercritical profile included in the CeRAS open-access database.

The main output related to the application of the $CL_{max}$ prediction model to the mid-range PrP test case are given in Table 2 and Fig. 11.

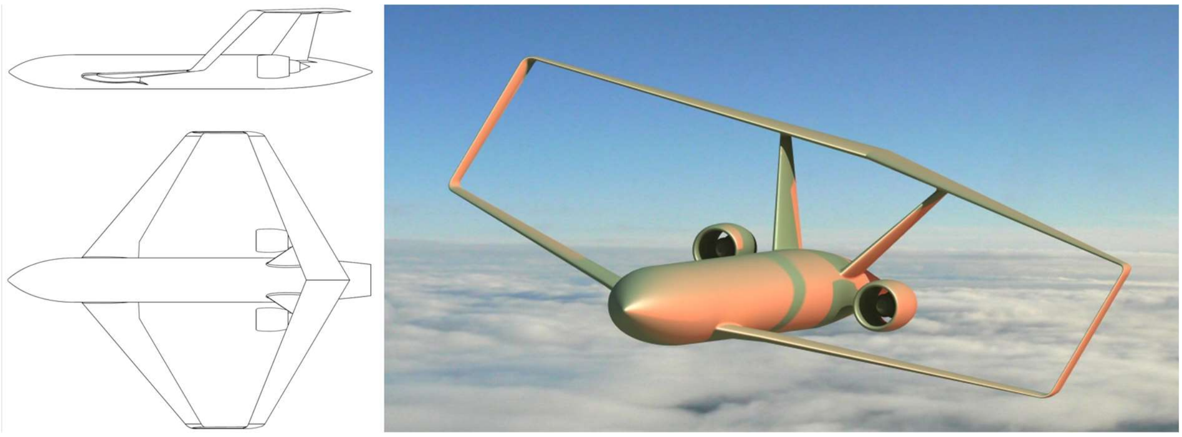

**Fig. 10 Views of the medium range PrP object of study in the project "PARSIFAL"**

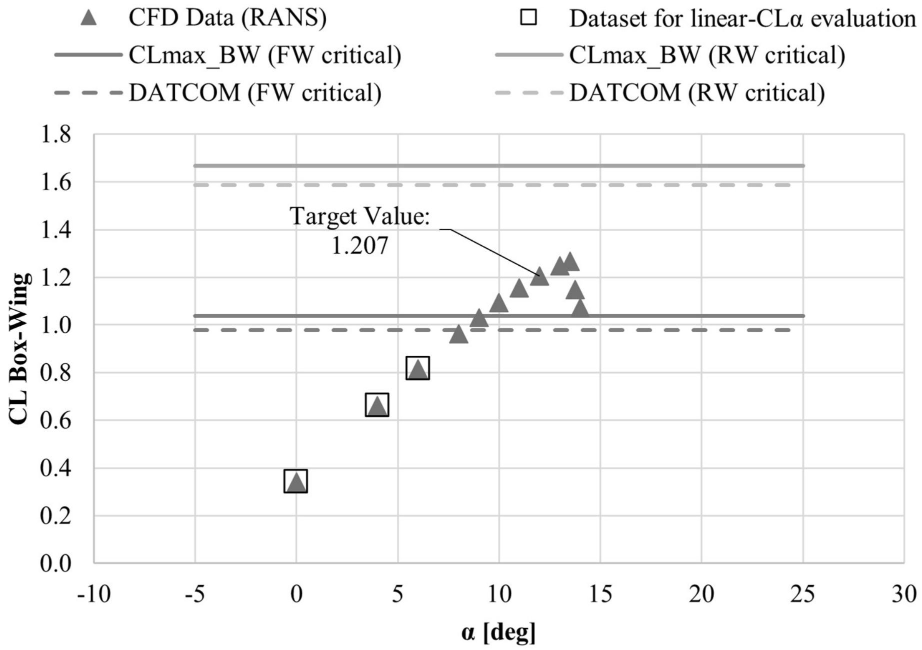


**Fig. 11 $CL_{max}$ prediction for the medium range PrP test case**

**Table 2. Input and output for the medium range PrP test case**

| Component | Input Parameter | Value | Output Parameter | Value |
|---|---|---|---|---|
| FRONT WING | Airfoil $Cl_{max}$ | 1.619 | $\gamma_{FW}$ | 0.46 |
| | Airfoil $\Delta_y$ | 1.52 | $f(\lambda_{FW})$ | 0.10 |
| | $S_{FW}$ [m^2] | 123.99 | $CL_{max_{FW}}$ | 1.221 |
| | $\Lambda_{FW}$ [deg][1] | 38.2 | Limitation to $CL_{max_{BW}}$ (from Eq.(31)) | 1.040 |
| | $\lambda_{FW}$ [1] | 0.29 | $CL_{max_{FW}}$ DATCOM | 1.148 |
| | $AR$ (min. value from Eq. (2)) | 7.3 (3.4) | Limitation to $CL_{max_{BW}}$ DATCOM | 0.978 |
| REAR WING | Airfoil $Cl_{max}$ | 1.619 | $\gamma_{RW}$ | 0.37 |
| | Airfoil $\Delta_y$ | 1.52 | $f(\lambda_{RW})$ | 0.11 |
| | $S_{RW}$ [m^2] | 129.44 | $CL_{max_{RW}}$ | 1.390 |
| | $\Lambda_{RW}$ [deg] | -24.2 | Limitation to $CL_{max_{BW}}$ (from Eq.(33)) | 1.667 |
| | $\lambda_{RW}$ | 0.36 | $CL_{max_{RW}}$ DATCOM | 1.321 |
| | $AR$ (min. value from Eq. (2)) | 10.0 (3.0) | Limitation to $CL_{max_{BW}}$ DATCOM | 1.586 |
| BOX WING | $S_{BW}$[m^2] | 253.43 | $CL_{max_{BW}}$ (from Eq. (35)) | 1.040 |
| | $R$ | 1.35 | Error vs. target value | -14% |
| | $\alpha$ for VLM simulation [deg] | 0 | $CL_{max_{BW}}$ DATCOM | 0.978 |
| | $CL_{max_{BW}}$ target value (CFD) | 1.207 | Error vs. target value | -19% |

[1] front wing is cranked; data refer to equivalent wing defined according to (ESDU-76003, 2012)

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### *5.2. Regional PrP test case*

The second test case for the application of the proposed model concerns the box-wing aircraft depicted in Fig. 12. The box-wing configuration is a hybrid-electric aircraft designed within the "PROSIB" project; the configuration has been selected after a long design process where an in-house tool called THEA-CODE has been widely used (Palaia, et al., 2021). The unflapped aircraft stall behavior has been evaluated by means of high-fidelity CFD analyses; the output of the analyses is depicted in Fig. 13. The airfoil selected for the configuration is a NACA 43018, whose $Cl_{max}$ can be conservatively assumed to be equal to 1.5 (Jacobs & Abbott, 1939). The main input and output related to the application of the $CL_{max}$ prediction model to the regional PrP test case are given in Table 3 and Fig. 13.

**Fig. 12 Views of the regional PrP object of study in the project "PROSIB"**

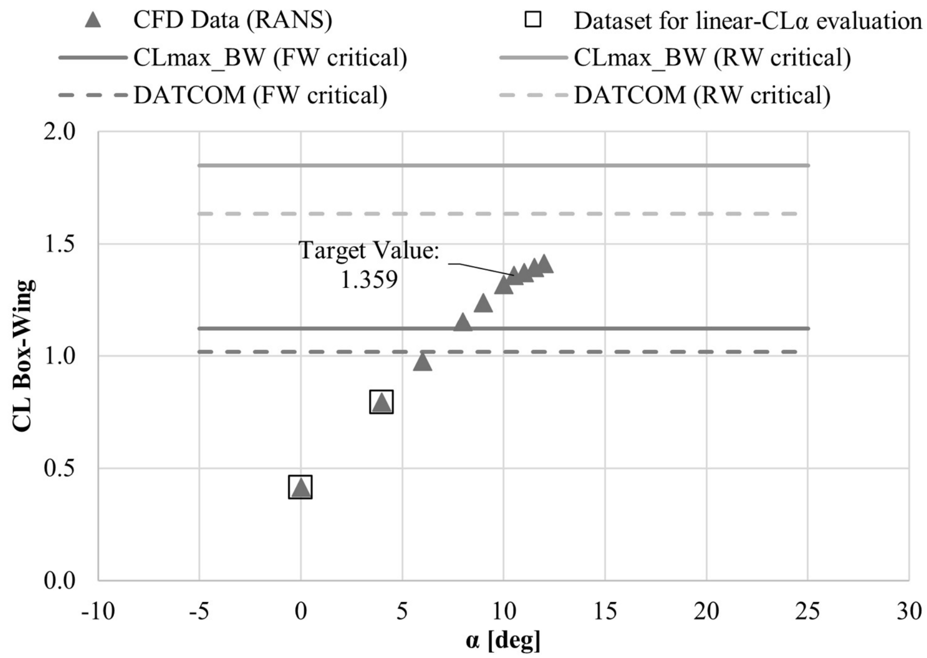


**Fig. 13 $CL_{max}$ prediction for the regional PrP test case**

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**Table 3. Input and output for the regional PrP test case**

| Component | Input Parameter | Value | Output Parameter | Value |
|---|---|---|---|---|
| FRONT WING | Airfoil $Cl_{max}$ (NACA 43018) | 1.5 | $\gamma_{FW}$ | 0.85 |
| | Airfoil $\Delta_y$ | 2.5 | $f(\lambda_{FW})$ | 0.10 |
| | $S_{FW}$ [m^2] | 34.71 | $CL_{max_{FW}}$ | 1.330 |
| | $\Lambda_{FW}$ [deg] | 26.0 | Limitation to $CL_{max_{BW}}$ (from Eq.(31)) | 1.121 |
| | $\lambda_{FW}$ | 0.29 | $CL_{max_{FW}}$ DATCOM | 1.207 |
| | $AR$ (min. value from Eq. (2)) | 10.3 (3.0) | Limitation to $CL_{max_{BW}}$ DATCOM | 1.018 |
| REAR WING | Airfoil $Cl_{max}$ (NACA 43018) | 1.5 | $\gamma_{RW}$ | 0.88 |
| | Airfoil $\Delta_y$ | 2.5 | $f(\lambda_{RW})$ | 0.13 |
| | $S_{RW}$ [m^2] | 32.53 | $CL_{max_{RW}}$ | 1.481 |
| | $\Lambda_{RW}$ [deg] | -13.5 | Limitation to $CL_{max_{BW}}$ (from Eq.(33)) | 1.848 |
| | $\lambda_{RW}$ | 0.43 | $CL_{max_{RW}}$ DATCOM | 1.309 |
| | $AR$ (min. value from Eq. (2)) | 14.9 (3.0) | Limitation to $CL_{max_{BW}}$ DATCOM | 1.634 |
| BOX WING | $S_{BW}$[m^2] | 67.24 | $CL_{max_{BW}}$ (from Eq. (35)) | 1.121 |
| | $R$ | 1.58 | Error vs. target value | -18% |
| | $\alpha$ for VLM simulation [deg] | 0 | $CL_{max_{BW}}$ DATCOM | 1.018 |
| | $CL_{max_{BW}}$ target value (CFD) | 1.359 | Error vs. target value | -25% |

## 6. Conclusions and further development

The present paper concerns the introduction of a mixed analytical-numerical method for the estimation of the maximum lift coefficient of aircraft with box-wing architecture. More in detail, the proposed method aims at estimating the $CL_{max}$ of the "clean", i.e. unflapped, configuration, which is a fundamental information to estimate the stall performance of any aircraft. The method is composed of an analytical component, which is based on the adaptation of the well-known DATCOM method, adopted in several design synthesis approaches such as the one by Torenbeek. The adaption follows an analytical strategy, built on the peculiar characteristics of the optimal lift distribution of a box-wing aircraft and implemented by means of a set of simplifying assumptions. The numerical component of the method is physics-based, since it depends on the results of Vortex-Lattice simulations carried on the considered box-wing configuration. The reason for the use of VLM simulations is given in the paper, explaining the context in which the proposed method can be applied. This is an optimization workflow, carried out using the VLM solver AVL, which aims at defining the box-wing design capable to maximize the cruise performance under a set of constraints, mostly related to flight mechanics considerations. One of the challenging aspects of the proposed method is the estimation of the $CL_{max}$ using results coming from simulations at cruise condition, hence at low angles of attack.

Therefore, the $CL_{max}$ estimation method is first presented in all its components and then validated for the case of the box-wing amphibian "IDINTOS", for which wind tunnel data are available. Given the good outcomes of the validation, the method has been applied to two test cases, the first one related to a mid-range passenger aircraft, object of study within the European research project "PARSIFAL", and the second one taken from a regional box-wing aircraft, developed in the Italian research project "PROSIB". For these test cases, the available reference data on clean stall performance are taken from CFD analyses.

Both validation and test case have shown that the $CL_{max}$ predicted by the proposed method is higher than DATCOM's one and still conservative with regards to the target $CL$ value. The improvement is in the range of 5%-7% of target value, whereas the margin of safety is not lower than 14%. As underlined in the paper, the proposed method not only allows to use the DATCOM for box-wing architectures but also introduces the direct effect of front and rear wings' taper ratio on $CL_{max}$.

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In conclusion, some remarks are worth to be made:

- the model built to estimate the box-wing $CL_{max}$ gives the possibility to associate the rear-to-front wing loading ratio, a fundamental parameter for longitudinal equilibrium and stability, also to stall performance. As shown in the paper for mid-range PrandtlPlane, this parameter undergoes contrasting requirements, since it needs to be as close to 1 to maximize the box-wing $CL_{max}$ whereas it has to be limited below 0.8 to meet longitudinal trim and stability requirements;
- in the expression used to adapt the DATCOM method to the box-wing case, the input parameters which require most of the effort for their evaluations are $\gamma_{FW}$ and $\gamma_{RW}$. As the data for the 3 analysed cases show, it is difficult to identify a typical range, small enough to provide a first estimation of the wings' $CL_{max}$ without the use of an aerodynamic solver.

A final comment concerns the introduction of High-Lift Devices effects on $CL_{max}$. The speculative idea of the authors is that the DATCOM approach of summing up $\Delta CL$ terms related to HLD extension and typology to the unflapped $CL_{max}$ may be useful also for the box-wing case. Further development may go toward this direction, using the method here proposed to estimate the unflapped $CL_{max}$ and verifying, by means of wind tunnel or CFD data, if the DATCOM approach allows for reliable and conservative preliminary estimations. Additional streams of development will concern the implementation of the proposed method, and its future versions, in the optimization workflow adopted for the design of PrandtlPlane aircraft.

## Acknowledgments

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