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Magnetic properties of the strongly interacting matter

C. BONATI(1), M. D'ELIA(1), M. MARITI(1)(*), F. NEGRO(1) and F. SANFILIPPO(2)

(1) *Dipartimento di Fisica dell'Università di Pisa and INFN, Sezione di Pisa, Pisa, Italy*
(2) *School of Physics and Astronomy, University of Southampton, Southampton, UK*

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Summary. — We study the magnetic properties of the strongly interacting matter using Lattice QCD simulations. The QCD medium shows a paramagnetic behavior in the range of temperatures 100–400 MeV, with a sharp increase of the magnetic susceptibility above the deconfinement temperature. We expect a significant magnetic contribution to the pressure of the system in non-central heavy-ion collisions.

PACS `12.38.Aw` – General properties of QCD (dynamics, confinement, etc.).
PACS `11.15.Ha` – Lattice gauge theory.
PACS `12.38.Gc` – Lattice QCD calculations.
PACS `12.38.Mh` – Quark-gluon plasma.

1. – Introduction

QCD is the theory which describes the behavior of the strongly interacting matter, made of quarks and gluons. Since quarks are also subject to the electromagnetic interactions, one expects that intense magnetic fields ($e|\mathbf{B}| \simeq m_\pi^2$) can influence the dynamics of the QCD matter. This scenario is expected to be relevant in many phenomenological contexts, like in a class of neutron stars, the magnetars ($e|\mathbf{B}| \simeq 10^{10}$ tesla) [1], in the early stage of the Universe ($e|\mathbf{B}| \simeq 10^{16}$ tesla) [2] and in off-central heavy-ion collisions ($e|\mathbf{B}| \simeq 10^{15}$ tesla) [3]. The first questions we can rise in this context are: how does the strongly interacting matter react to these fields? Is it a paramagnet or a diamagnet?

2. – The method

We use lattice QCD calculations to evaluate the dependence of the free energy density to the external magnetic fields [4]. We simulate $N_f = 2 + 1$ quarks at the physical point, using state-of-the-art discretization of the Lagrangian. We introduce the

(*) Poster author.

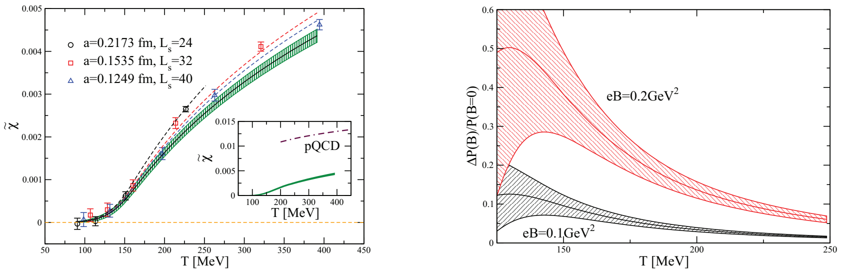

Fig. 1. – Left: Magnetic susceptibility for the different values of the lattice spacing a and of the temperature T. Right: Magnetic contribution to the pressure relative to the $B = 0$ case.

external magnetic fields in the discretized covariant derivative (like for the continuum formulation). We work in a finite space-time lattice of volume $a^4(L_s^3 \times L_t)$ (L_s, L_t are, respectively, the spatial and the temporal lattice extents, a is the lattice spacing), where we impose periodic boundary conditions to reduce spurious finite size effects. These conditions lead to a quantization rule for the magnetic field: $e|\mathbf{B}| = \frac{2\pi b}{q(L_s a)^2}$, where $b \in \mathbb{Z}$, and q is the smallest charge in the system. Because of this quantization, one can not access directly to derivatives of the free energy density with respect to the magnetic field, $\frac{\partial^n f(T,eB)}{\partial(eB)^n}$. However, one can overcome this problem by measuring free energy differences:

$$\Delta f(b,T) - \Delta f(b-1,T) = \int_{b-1}^{b} M(\bar{b},T)\,\mathrm{d}\bar{b} \quad \text{with} \quad M(\bar{b},T) = \frac{\partial f(\bar{b},T)}{\partial \bar{b}}, \tag{1}$$

where we extend $f(b,T)$ to $b \notin \mathbb{Z}$ to compute $M(b,T)$. In the linear regime approximation, $\Delta f(b,T) = f(b,T) - f(0,T) \approx \frac{1}{2}c_2(T)b^2$, we can extract $c_2(T)$ from: $\Delta f(b,T) - \Delta f(b-1,T) = \frac{1}{2}c_2(T)(2b-1)$. Then the magnetic susceptibility $\tilde{\chi}$ (in SI units) can be obtained using $\Delta f(B,T) = -\frac{\tilde{\chi}(T)}{2\mu_0}B^2$, which gives $\tilde{\chi}(T) = -\frac{e^2\mu_0 c}{18\hbar\pi^2}L_s^4 c_2(T)$.

3. – Results

We find that the QCD medium is weakly paramagnetic in the confined phase and strongly paramagnetic in the deconfined phase, with a linear behavior up to $e|\mathbf{B}| \approx 0.2\,\mathrm{GeV}^2$. However, within our current precision, we cannot exclude a diamagnetic behavior of the medium for $T < 100\,\mathrm{MeV}$, where the pseudoscalar mesonic degrees of freedom are expected to dominate and to give rise to a diamagnetic response. Our data are well described by the functions

$$\tilde{\chi}(T) = A\exp\left(-\frac{M}{T}\right), \quad \text{for} \quad T \le T_0 \quad \text{or} \quad \tilde{\chi}(T) = A'\log\left(\frac{T}{M'}\right), \quad \text{for} \quad T > T_0, \tag{2}$$

with fit values $M \simeq 900\,\mathrm{MeV}$ and $T_0 \simeq 160\,\mathrm{MeV}$. Near the transition temperature, the magnetic contribution to the pressure is of the order 10% for $e|\mathbf{B}| = 0.1\,\mathrm{GeV}^2$, 50% for $e|\mathbf{B}| = 0.2\,\mathrm{GeV}^2$.

REFERENCES

[1] Duncan R. C. and Thompson C., *Astrophys. J.*, **392** (1992) L9.
[2] Vachaspati T., *Phys. Lett. B*, **265** (1991) 258.
[3] Skokov V., Illarionov A. Y. and Toneev V., *Int. J. Mod. Phys. A*, **24** (2009) 5925.
[4] Bonati C., D'Elia M., Mariti M., Negro F. and Sanfilippo F., *Phys. Rev. Lett.*, **111** (2013) 182001; *Phys. Rev. D*, **89** (2014) 054506.