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Research article

MODELLING OF DEPLOYABLE CABLE NETS FOR ACTIVE SPACE DEBRIS REMOVAL

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ABSTRACT

Space debris represents a true risk for current and future activities in the circumterrestrial space, and remediation activities must be set out to guarantee the access to space in the future. For active debris removal, the development of an effective capturing mechanism remains an open issue. Among several proposals, cable nets are light, easily packable, scalable, and versatile. Nonetheless, guidance, navigation, and control aspects are especially critical in both the capture and post-capture phases. We present a finite element model of a deployable cable net. We consider a lumped mass/cable net system taking into account non-linearities arising both from large displacements and deformations, and from the different response of cables when subject to tension and compression. The problem is stated by using the nodal coordinates as Lagrangian coordinates. Lastly, the nonlinear governing equations of the system are obtained in a form ready for numerical integration.

KEYWORDS: space debris, cable net, finite element model

1. Introduction

Missions in the circumterrestrial space produce debris that represent a threat for current and future space activities. Once the main national space agencies became aware of the problem, they have developed and adopted mitigation guidelines to reduce the debris production rate from new missions [1] [2]. However, the overall number of space debris is steadily increasing because of increasing space activities. As a consequence, remediation activities must be set out to guarantee the access to space for future generations. In particular, the disposal of massive objects abandoned around the Earth would result effective to secure the most valuable orbital regions [3] [4].

For active debris removal (ADR), the development of an effective capturing mechanism is still a problematic aspect of the mission architecture. Two main alternatives have been considered: using robotic arms or tethered nets. In the last twenty years both Space Shuttle and International Space Station have been equipped with robotic arms. Among several uses, robotic arms have been used effectively for capture of cooperative and attitude-stabilized spacecrafts, both under human control and by automated procedures [5]. Anyhow, their employment for capture of non-cooperative debris has still to be proven. In fact, the complexity of approaching manoeuvres can be guessed if we consider a tumbling target with large appendages, form which a distance between 1 and 3 meters has to be reached. For comparison, cable nets can be thrown from distances of 20 meters. Also, they are light, easily packable, scalable, and versatile. Nonetheless, guidance, navigation, and control (GNC) aspects are especially critical for nets in the capture and post-capture phases [6].

In a typical ADR mission, the chaser will first rendezvous with the target and then throw a tethered net to capture it. The deployment of the net can be achieved by ejecting a number of bullet masses placed on its border. When cables connected to the bullets start tensioning, they pull along neighbouring portions of the net.

Several theoretical models have been proposed to describe the deployment and capture processes. Benvenuto *et al*. [7] and Botta *et al*. [8] modelled the net as a system of concentrated masses connected to each other by linear spring-dampers. In their models, springs react only in tension and infinitesimal strains are considered. Shan *et al*. [9] compared the simple lumped mass-spring model with a more refined model based on the absolute nodal coordinate formulation (ANCF) proposed by Shabana [10]. They used a third-order cable element [11] and considered finite strains through the Green-Lagrange strain tensor. Their cable element admits compressive stresses, which may produce buckling. The lumped mass-spring and ANCF models gave similar results in terms of the overall behaviour of the net, but the ANCF model was much more computationally expensive.

We propose a finite element (FE) model of the cable net with lumped nodal masses and first-order cable elements. We assume the nodal positions as the main unknowns of the problem. Large displacements and finite deformations are considered through the Green-Lagrange strain tensor [12]. Cable elements are assumed to react only in tension with a linear relationship between the axial strain and the corresponding component of the work-conjugate second Piola-Kirchhoff stress tensor [13]. Global damping is introduced into the model according to Rayleigh's hypothesis [14]. Lastly, the governing equations of the nonlinear dynamic problem are obtained by using the standard assembly procedure of the finite element method. Hence, the dynamic response of the cable net can be determined by applying a suitable numerical integration scheme.

2. Finite element formulation

2.1. Kinematics

In the framework of the finite element method, the cable net is modelled as a discrete system consisting of m elements of finite size, connected to each other at n points called nodes (Fig. 1). Mass, damping, elastic properties, as well as applied loads and restraints are modelled as lumped nodal entities in the FE model. Nodes are located at the intersections between cables, while elements correspond to the portions of cables included between them.

Current and reference configurations are referred to a fixed Cartesian reference system, $O\, x_1x_2x_3$. Denoting by P_i the point corresponding to the i^{th} node ($i = 1, \ldots, n$), its position vector is $\boldsymbol{x_i} = P_i - O \in \mathbb{R}^3$, and its displacement vector is $\boldsymbol{u}_i = P_i - \overline{P}_i = \boldsymbol{x}_i - \overline{\boldsymbol{x}}_i$, where a bar is used here and in the following to distinguish the reference configuration, $\overline{\Omega}$, from the current one, Ω. The reference configuration is by definition undeformed. We collect the position vectors of all the nodes of the system into a single vector $\boldsymbol{x} \in \mathbb{R}^{3n}$, which we assume as the main unknown of the problem.

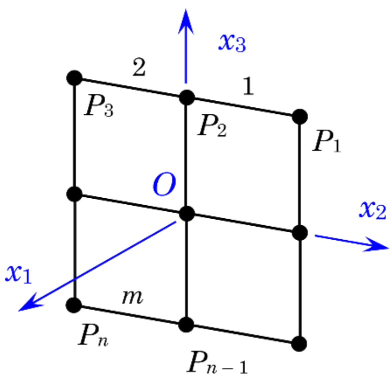

Figure 1. Finite element model of a generic cable net

2.2. Cable element

Let us consider the e^{th} cable element ($e = 1, \ldots, m$) connecting the i^{th} and j^{th} nodes. We denote the nodal position vector of the element in the current configuration as $\boldsymbol{x}_e = [\boldsymbol{x}_i;\ \boldsymbol{x}_j] \in \mathbb{R}^6$. The element length, L_e, as the distance between its nodes, can be calculated from the following formula:

$$L_e^2 = \boldsymbol{x}_e^{\mathrm{T}} \boldsymbol{\Delta}\, \boldsymbol{x}_e\,, \tag{1}$$

where

$$\boldsymbol{\Delta} = \begin{bmatrix} \mathbf{I} & -\mathbf{I} \\ -\mathbf{I} & \mathbf{I} \end{bmatrix} \tag{2}$$

is a constant matrix, here introduced to automate the subtraction between the nodal displacements, and $\mathbf{I} \in \mathbb{R}^{3\times 3}$ is the identity matrix. The component of the Green-Lagrange strain tensor in the element axial direction is evaluated with respect to the reference configuration as:

$$E_{11} = \frac{1}{2}\frac{L_e^2 - \bar{L}_e^2}{\bar{L}_e^2}\,. \tag{3}$$

The axial stress in the cable element is obtained by a conventional constitutive law that accounts for the inability of cables to sustain relevant compressive stresses. When the distance between the element nodes is less than its reference length, the element is considered slack, and the axial stress is assumed null. Otherwise, a linearly elastic relationship is assumed between the Green-Lagrange strain and second Piola-Kirchhoff stress [13]:

$$S_{11} = \begin{cases} E E_{11}, & \text{if } E_{11} \geq 0; \\ 0, & \text{otherwise}; \end{cases} \tag{4}$$

where E is the Young's modulus of the material.

By assuming linear shape functions for the element, the secant elastic stiffness matrix can be expressed as follows [12]:

$$\mathbf{S}_e(\boldsymbol{x}_e) = S_{11}\frac{\bar{A}_e}{\bar{L}_e}\boldsymbol{\Delta}\,, \tag{5}$$

where $\bar{A}_e$ is the cross-section area of the element, evaluated in the reference configuration. Besides, the tangent elastic stiffness matrix is

$$\mathbf{T}_e(\boldsymbol{x}_e) = \mathbf{S}_e(\boldsymbol{x}_e) + \frac{\partial S_{11}}{\partial E_{11}}\frac{\bar{A}_e}{\bar{L}_e^3}\boldsymbol{\Delta}\,\boldsymbol{x}_e\,\boldsymbol{x}_e^{\mathrm{T}}\boldsymbol{\Delta}\,. \tag{6}$$

A simple lumped mass matrix is considered for each element [14]:

$$\mathbf{M}_e = \frac{1}{2}\bar{\rho}_e\bar{L}_e\bar{A}_e\begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}, \tag{7}$$

where $\bar{\rho}_e$ is the mass density in the reference configuration, and $\mathbf{0}$ denotes the null matrix in $\mathbb{R}^{3\times 3}$.

For the clarity of presentation, finally we introduce the assembly matrix of the element, $\mathbf{A}_e \in \mathbb{R}^{6\times 3n}$. It is defined as a null matrix, except for the entries corresponding to columns from $3i - 2$

to $3i$ into the first 3 rows, and for the entries corresponding to columns from $3j-2$ to $3j$ into the rows from 4 to 6, where identity matrices are placed. As a result, the nodal position vector of the element can also be expressed as $\boldsymbol{x}_e = \mathbf{A}_e\,\boldsymbol{x}$.

2.3. Cable net

The mass, secant stiffness, and tangent stiffness matrices – belonging to $\mathbb{R}^{3n\times 3n}$ – of the cable net are obtained by assembling the corresponding matrices of the elements:

$$\mathbf{M}=\sum_{e=1}^{m}\mathbf{A}_e^{\mathrm{T}}\,\mathbf{M}_e\,\mathbf{A}_e,\quad \mathbf{S}(\boldsymbol{x})=\sum_{e=1}^{m}\mathbf{A}_e^{\mathrm{T}}\,\mathbf{S}_e(\boldsymbol{x}_e)\mathbf{A}_e,\quad \text{and}\quad \mathbf{T}(\boldsymbol{x})=\sum_{e=1}^{m}\mathbf{A}_e^{\mathrm{T}}\,\mathbf{T}_e(\boldsymbol{x}_e)\mathbf{A}_e, \tag{8}$$

where dependence on nodal positions is highlighted.

Finally, global damping is introduced into the model according to Rayleigh’s hypothesis [14]. The damping matrix is obtained as the sum of two contributions proportional to the mass and tangent stiffness matrices of the system, respectively, evaluated in the reference configuration:

$$\mathbf{D}=\alpha\,\mathbf{M}+\beta\,\mathbf{T}(\bar{\boldsymbol{x}}), \tag{9}$$

where α and β are suitable combination coefficients.

3. Governing equations

The nonlinear dynamic problem for the deployable cable net is governed by the following differential equation set:

$$\mathbf{M}\,\ddot{\boldsymbol{x}}+\mathbf{D}\,\dot{\boldsymbol{x}}+\mathbf{S}(\boldsymbol{x})\,\boldsymbol{x}=\boldsymbol{p}(t), \tag{10}$$

where both the nodal positions, $\boldsymbol{x}$, and the nodal loads, $\boldsymbol{p}$, depend on time t. The upper dot denotes differentiation with respect to time.

The governing equations can be integrated numerically, and this will be part of the prosecution of our work. Besides, the order of magnitude of the main external forces shall to be evaluated, and the most significant ones among them shall be include into the theoretical model.

4. Conclusions

Deployable cable nets are promising capture systems for the active removal of space debris, but the development of accurate and effective simulation tools is crucial for allowing their application.

We have proposed a FE model of a cable net with lumped nodal masses and first-order cable elements. In line with the ANCF, nodal positions have been adopted as the main unknowns of the problem. Large displacements and finite deformations have been considered through the Green-Lagrange strain tensor. Cable elements have been assumed to react only in tension with a linear relationship between the axial strain and the corresponding component of the work-conjugate second Piola-Kirchhoff stress tensor. Global damping has been introduced into the model according to Rayleigh's hypothesis. The governing equations have been stated.

The presented theoretical model is only a first step towards the effective simulation of a deployable cable net. In future works we are going to integrate numerically the governing equations starting from suitable initial conditions. Then, we will enhance the model by accounting for the contact phenomena arising after the impact of the net with the target.

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