

ON THE DECOUPLING OF FRACTURE MODES IN INTERLAMINAR FRACTURE TESTS ON BIMATERIAL SPECIMENS

P. Tsokanas¹, L.F.M. da Silva², F. Mujika³, A. Arrese³, P. Fisicaro⁴, P.S. Valvo⁴

¹ *INEGI, 4200-465 Porto, Portugal; Corresponding Author: panayiotis.tsokanas@gmail.com*

² *Faculdade de Engenharia, Universidade do Porto, 4200-465 Porto, Portugal*

³ *Faculty of Engineering of Gipuzkoa, University of the Basque Country, 20018 San Sebastián, Spain*

⁴ *Department of Civil and Industrial Engineering, University of Pisa, IT-56122 Pisa, Italy*

ABSTRACT

We start by reviewing methods for fracture mode decoupling in unconventional laboratory specimens. Then, we propose energetically orthogonal mode decoupling conditions and associated specimen design criteria to obtain pure fracture modes when bimaterial specimens are tested in asymmetric double cantilever beam and asymmetric end-notched flexure test configurations. This work hopefully sheds light on some controversial points in the relevant literature.

INTRODUCTION

We consider laboratory tests for measuring the interlaminar fracture toughness of a delamination specimen. Such tests can generally be modelled by considering the specimen as a plane (two-dimensional) cracked body, while the crack propagates in a mix of the basic fracture modes I (opening) and II (sliding). If this body is symmetrically cracked (Fig. 1a), fracture modes I and II are respectively associated to systems of symmetric and antisymmetric forces with respect to the crack plane. These forces respectively produce only normal stresses, σ_n (or crack-tip normal forces, F_n), and transverse relative displacements, δ_n , or only shear stresses, σ_t (or crack-tip shear forces, F_t), and axial relative displacements, δ_t , on the crack plane around the crack tip (Fig. 1b). These simple pure-mode conditions do not apply in general for asymmetrically cracked bodies (Fig. 1c), such as bimaterial specimens.

In this presentation, we will revisit the problem of fracture mode decoupling and again propose pure-mode conditions and derive associated specimen design criteria, aiming to resolve the controversial points in the literature. We focus on the asymmetric double cantilever beam (ADCB) (Fig. 1d) and asymmetric end-notched flexure (AENF) (Fig. 1e) test configurations, also assuming that both sub-beams are homogeneous and special orthotropic. This work assumes that pure mode I conditions occur when $\delta_t = 0$. Then, by enforcing energetical orthogonality with mode II, it follows that pure mode II conditions occur when $F_n = 0$. Using these pure-mode conditions, and by developing a simple mechanical model using laminated beam theory, Engesser–Castigliano’s theorem, and unit-load method

—this work was very recently published in Ref. [1]—, we derive the specimen design criteria for bimaterial specimens loaded using the ADCB and AENF test configurations from scratch. We will show that the two criteria to obtain pure mode I in the ADCB test and pure mode II in the AENF test coincide and are aligned with the criterion used in part of the existing literature. We hope that the present work helps resolve the confusion in the literature regarding the correctness of different mode decoupling conditions.

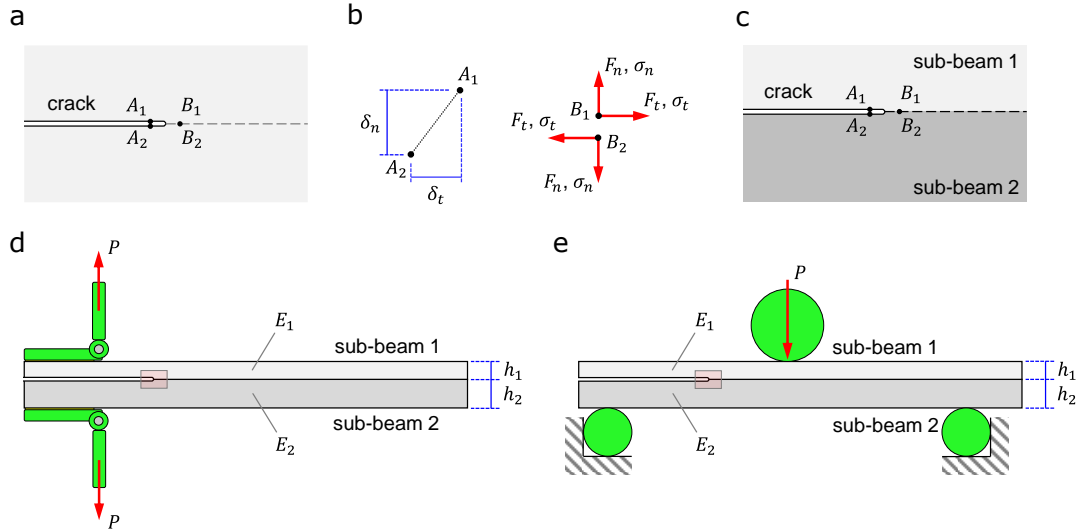


FIGURE 1 (a) Symmetric crack problem; (b) relative displacements, forces, and stresses near the crack tip; (c) asymmetric crack problem; and (d) ADCB and (e) AENF test configurations.

EXISTING PURE-MODE CONDITIONS AND SPECIMEN DESIGN CRITERIA

Energetically orthogonal pure-mode conditions

Valvo [2] demonstrated that the ‘standard’ virtual crack closure technique (VCCT)—based on the decomposition of the crack-tip nodal forces into symmetric and antisymmetric components—may be inappropriate when analysing problems with highly asymmetric cracks since negative values may be calculated for either mode I, G_I , or mode II, G_{II} , contributions to the energy release rate (ERR), G . To remedy this shortcoming, the author suggested defining pure modes based on energetically orthogonal systems of forces at the crack tip. In this way, always non-negative modal contributions to the ERR are obtained. At first, he proposed to identify pure mode I as the case when $F_t = 0$, and pure mode II as the case when $\delta_n = 0$. In a subsequent work [3], the same author reconsidered his proposal and suggested the following conditions instead: pure mode I occurs when $\delta_t = 0$, and pure mode II occurs when $F_n = 0$. The more recent proposal has a more convincing physical basis, as it assumes that mode I fracture corresponds to a pure opening of the crack faces.

Similarly to Valvo, Harvey and Wang [4, 5] developed an orthogonal pure mode methodology for partitioning the ERR into its modal contributions. They assumed that the mode I loading condition must be orthogonal to the mode II loading condition through the ERR ‘space’. Using this, any pure-mode

condition can be identified experimentally, theoretically, or numerically, and the orthogonality condition can then be used to find the other pure modes. The authors defined two sets of pure-mode conditions:

- Set 1: Pure mode I exists when $\delta_t = 0$, and pure mode II exists when $F_n = 0$
- Set 2: Pure mode I exists when $F_t = 0$, and pure mode II exists when $\delta_n = 0$

Interestingly, their first and second sets of pure modes respectively correspond to the two pure-mode sets proposed by Valvo in Refs. [3] and [2]. Harvey and Wang derived the pure modes in the contexts of Euler and Timoshenko beam theories for *rigid* interfaces and the ADCB configuration. Euler beam theory can predict negative G_I/G or G_{II}/G components over a certain number of loading conditions. Although Valvo [2] and other authors [6] state that this lacks physical interpretation, Harvey and Wang argue that there is no physical requirement on the modal contributions to the ERR, G_I and G_{II} , and the only physical requirement is that G must be non-negative definite since creating new crack surfaces requires energy.

Specimen design criteria

We now focus on the ADCB test configuration, considering both material and geometric asymmetry (Fig. 1d), as the specimen consists of two sub-beams of different longitudinal Young's moduli, E_i , and thicknesses, h_i , $i \in \{1, 2\}$. A reasonable question is whether we can appropriately design the specimen (e.g. define Young's moduli and thicknesses of both sub-beams) to achieve pure-mode I conditions (at the crack tip). Two mode decoupling conditions can be recognised in the literature:

- Mode decoupling is achieved when the differential equation of the mode I fracture is only governed by σ_n and δ_n (e.g. [7])
- Mode decoupling is achieved when the bending rigidities of the two sub-beams are equal (e.g. [8])

These conditions respectively lead to the following specimen design criteria:

$$E_1 h_1^2 = E_2 h_2^2 \quad \text{and} \quad E_1 h_1^3 = E_2 h_2^3, \quad (1)$$

which although both are used in the literature (e.g. [7, 8]), cannot both be correct. Thus, our work [1] aims to elucidate which one is ultimately correct.

THE PROPOSED SPECIMEN DESIGN CRITERIA

In this presentation, we will revisit the problem of fracture mode decoupling and discuss the various proposed definitions for pure-mode conditions, aiming to clarify which theoretical definitions for pure modes are to be preferred. We will focus on the ADCB and AENF test configurations, assuming that both sub-beams are homogeneous and special orthotropic. The work assumes that under pure mode I conditions, $\delta_t = 0$, while under pure mode II conditions, $F_n = 0$ [3–5]. Using this set of conditions and by developing a mechanical model employing laminated beam theory, Engesser–Castigliano's theorem, and unit-load method [1], we will derive the mode decoupling conditions for bimaterial specimens loaded using the ADCB and AENF test configurations from scratch. We will show that the two conditions to obtain pure mode I in the ADCB test and pure mode II in the AENF test coincide and are the same with

the first equation of Eqs. (1) that is being used in part of the literature (e.g. [7]). In addition, we will demonstrate that the analysis of the AENF test may get complicated by the contact phenomena occurring between the two sub-beams. This contact may alter the general rules for fracture mode partitioning [9].

We hope that the present work sheds light on the confusion in the literature regarding the correctness of different mode decoupling conditions and associated specimen design formulae. For more details and insights, we refer the reader to our journal publications [1]. The present work can be extended by deriving specimen design criteria for multidirectional laminated specimens [9].

ACKNOWLEDGEMENTS

PT and LFMdS acknowledge the financial support of FCT through the PTDC/EME-EME/6442/2020 project. FM and AA acknowledge the financial support of UPV/EHU to the GIU20/060 research group.

REFERENCES

1. Mujika, F.; Tsokanas, P.; Arrese, A.; Valvo, P. S.; da Silva, L. F. M. Mode decoupling in interlaminar fracture toughness tests on bimaterial specimens, *Eng. Fract. Mech.* 2023, 290, 109454.
2. Valvo, P. S. A revised virtual crack closure technique for physically consistent fracture mode partitioning, *Int. J. Fract.* 2012, 173, 1-20.
3. Valvo, P. S. A further step towards a physically consistent virtual crack closure technique, *Int. J. Fract.* 2015, 192, 235-244.
4. Wang, S.; Harvey, C. M. Mixed mode partition theories for one dimensional fracture, *Eng. Fract. Mech.* 2012, 79, 329-352.
5. Harvey, C. M.; Wang, S. Mixed-mode partition theories for one-dimensional delamination in laminated composite beams, *Eng. Fract. Mech.* 2012, 96, 737-759.
6. Maimí, P.; Gascons, N.; Ripoll, L.; Llobet, J. Mixed mode delamination of asymmetric beam-like geometries with cohesive stresses, *Int. J. Solids Struct.* 2018, 155, 36-46.
7. Ouyang, Z.; Ji, G.; Li, G. J. On Approximately Realizing and Characterizing Pure Mode-I Interface Fracture Between Bonded Dissimilar Materials, *Appl. Mech.* 2011, 78 (3), 031020.
8. Jiang, Z.; Wan, S.; Wu, Z. Calculation of energy release rate for adhesive composite/metal joints under mode-I loading considering effect of the non-uniformity, *Compos. B Eng.* 2016, 95, 374-385.
9. Garulli, T.; Catapano, A.; Fanteria, D.; Jumel, J.; Martin, E. J. Design and finite element assessment of fully uncoupled multi-directional layups for delamination tests, *Compos. Mater.* 2020, 54 (6), 773-790.