The R&D investment decision game with product differentiation

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Abstract This article extends the cost-reducing R&D model with spillovers by d'Aspremont and Jacquemin (1988, 1990) to allow quantity-setting firms (Cournot rivalry) to play the non-cooperative R&D investment decision game with horizontal product differentiation. Unlike Bacchiega et al. (2010), who identify a parametric region (defined by the extent of technological spillovers and the efficiency of R&D activity), in which the game is a prisoner's dilemma (self-interest and mutual benefit of cost-reducing innovation conflict), this work shows that product differentiation changes the game into a deadlock (self-interest and mutual benefit do not conflict), regardless of the parameter scale (i.e., also *in the absence of spill-over effects*). Then investing in R&D challenges the improvement of interventions aimed at favouring product differentiation. This is because social welfare when firms invest in cost-reducing R&D is greater than when firms do not invest in R&D. Alternatively, R&D subsidies can be used as a social welfare maximising tool also in the absence of R&D spillovers. These results also hold for price-setting firms (Bertrand rivalry).

Keywords Process innovation; Nash equilibrium; Social welfare

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Declarations of interest None

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1. Introduction

This article revisits the influential model by d'Aspremont and Jacquemin (AJ; 1988, 1990) of cost-reducing R&D with spillovers. It considers a non-cooperative three-stage game in which firms endogenously choose whether to invest in R&D – as in Bacchiega et al. (2010) – and consumers have preferences characterising horizontally differentiated products (Singh and Vives, 1984).

In their pioneering contributions, AJ study a two-stage game in which two identical firms invest in R&D in the first stage. In the second stage, they act as Cournot competitors in a market for homogeneous goods. The authors consider three cases under which firms (i) non-cooperatively compete at both stages (a two-stage non-cooperative game), (ii) cooperate at both stages (a two-stage cooperative solution), and (iii) collude at the R&D stage and compete at the market stage (a two-stage mixed game). The process innovation R&D investment flows and reduces the investing firms' marginal cost; however, it also exogenously spills over, thereby reducing the rival's marginal cost. Specifically, AJ's article aims at comparing the magnitude of cost-reducing technical advance achieved when firms conduct R&D either competitively or cooperatively, finding that cooperative R&D leads to greater technological advance than competitive R&D for sufficiently large spill-over effects.

Later, Henriques (1990) and Suzumura (1992) extend AJ in two distinct directions. First, Henriques (1990) augments AJ's findings by providing stability conditions showing that the main results obtained by AJ's quantity-setting duopoly are meaningful only when the solution of the non-cooperative game is stable, in the sense of Seade (1980). This implies the existence of thresholds, in the parameter space defined by the extent of technological spillovers and the efficiency of R&D activity (as also clarified by Bacchiega et al., 2010), such that the reaction functions in the R&D space should cross adequately. In particular, at any level of technological spillovers, the efficiency of the R&D technology should be low enough to avoid over-investment in R&D that would then increase the degree of competition (by reducing the average variable costs of production, increasing output, and lowering market price), thereby eroding profits in both cases of strategic substitutability (little technological spillovers) and complementarity (large technological spillovers) of R&D investments.[1] Second, Suzumura (1992) applies AJ's idea to a general class of oligopoly models, in the cases of both cooperative and non-cooperative R&D with spillovers.

Subsequent extensions by Kamien et al. (1992), Ziss (1994), De Bondt (1996), Amir (2000), Amir et al. (2003), and Lambertini and Rossini (2009), amongst others, have typically shown that cooperative R&D decisions by firms competing in the product market are socially beneficial. This literature portrays spillovers as exogenous; i.e., a fixed fraction of a firm's R&D investment exogenously flows to competitors so that each firm has no direct control over the extent of disclosure.

However, another branch of the literature assumes that firms can endogenously control spillovers aiming at investigating whether owners decide about information sharing. The works belonging to this literature can be divided into two groups. One group (Poyago-Theotoky, 1999; Atallah, 2004; Lambertini et al., 2004) studies the case in which firms decide about information sharing after they invested in R&D (i.e., spillovers do not affect the extent of R&D investments). The main result of this branch of literature is that firms choose to keep their R&D knowledge secret, and thus R&D spillovers are absent (non-disclosure). The second group (Gersbach and Schmutzler, 2003; Gil-Moltó et al., 2005; Piga and Poyago-Theotoky, 2005; Milliou, 2009) considers the possibility that firms choose whether to share R&D outcomes before they invest in R&D (i.e., spillovers do affect the

[1] In the case of the strategic substitutability (resp. complementarity) of the R&D activity, the intensity of the R&D externality is small (resp. large), and the amounts of R&D investment x_i of firm i and x_j and of firm j ($i = \{1,2\}$; $i \neq j$) are negatively (resp. positively) related in the R&D space, i.e., the R&D reaction curves are downward-sloping (resp. upward-sloping). The stability conditions require that $\left|\frac{dx_i}{dx_j}\right| < 1$ in both cases. Poyago-Theotoky (1999) interpreted strategic substitutability (resp. complementarity) as resembling the case in which R&D-related information disclosure is small (resp. large), and firms follow similar (resp. distinct) research paths.

extent of R&D investments). Protecting or sharing knowledge depends on price or quantity competition (the first paper) on location (second and third papers), and if the extent of R&D spillovers is not too strong, firms can let R&D knowledge flow to competitors (the last paper). The main result of this branch of literature is that firms choose to disclose R&D knowledge with the rivals.[2]

The AJ framework is widely used in the industrial organization literature as a basic textbook model for process innovation in oligopolistic contexts; however, the analysis of the properties and solutions of the (three-stage) R&D investment decision game played by non-cooperative and selfish firms is scant. Indeed, while AJ and all of the subsequent aforementioned works just analyse exogenously given contexts with R&D investments (irrespective of whether the focus is on the analysis of cooperation versus competition in the R&D phase or the endogeneity or exogeneity of spillovers), the underlying game played by firms choosing whether to invest in R&D *in the absence of spill-over effects* is a prisoner's dilemma (i.e., conflict exists between self-interest and mutual benefit to undertake cost-reducing innovation). In other words, R&D investments result in profit reduction.

Although showing this result in the simplified AJ's two-stage model seems to be straightforward, it has never been explicitly noted. As an exception, Bacchiega et al. (2010) add the investment-decision stage to the two-stage AJ's game. *By assuming that R&D spill-over effects occur,* those authors pinpoint the existence of: (1) a parameter region, defined by the interplay between the extent of technological spillovers and the efficiency of the R&D activity, in which the three-stage R&D game (played by the firm that non-cooperatively chooses both R&D investment at the R&D stage and output at the market stage) is a prisoner's dilemma (low values of R&D spillovers or no spillovers); (2) conversely and more importantly, a parameter region in which the prisoner's dilemma is solved and the game turns to be an anti-prisoner's dilemma (larger values of R&D spillovers). Under this condition, no conflict between self-interest and mutual benefit to undertake cost-reducing innovation exists. However, Bacchiega et al. (2010) choose to restrict the analysis to the case of homogeneous products and quantity-setting firms. Moreover, they also neglect to consider the R&D cost conditions related to the symmetric subgame I/I and the asymmetric subgame I/NI needed to define the parametric space in which the non-cooperative version of the R&D game is feasible.[3]

By assuming both Cournot and Bertrand rivalries, the present work aims at generalising on the variety of products, allowing capture of the case of (horizontally) differentiated goods, and augmenting the analysis to consider all the relevant constraints of the R&D game. Definitively, the article proposes to question whether the unpleasant prisoner's dilemma (on the firm side) is a robust feature of the R&D duopoly game framed in the AJ setup. It explicitly shows that horizontal product differentiation solves the dilemma obtained by Bacchiega et al. (2010) and lets the game become a deadlock, *irrespective of the extent of technological spillovers*. It also represents an attempt to provide a thoughtful analysis of the non-cooperative version of the R&D model developed by AJ – surprisingly missing in the IO literature – by clarifying (a) the role played by the relevant thresholds (the stability conditions and the R&D cost conditions) so that the feasibility conditions of the model are accurately disentangled to ensure the non-violation of several bounds (e.g., the non-negativity of the R&D costs), and (b) the parameter configurations under which the game is a prisoner's or an anti-prisoner's dilemma. As is known, the prisoner's dilemma of the R&D game in the AJ setting reveals the sub-optimality of the non-cooperative solution (alternatively, cooperation is Pareto improving).

[2] There exists another relevant branch of the literature augmenting the AJ model to the analysis of R&D subsidies, i.e., Hinloopen (1997, 2000) and Amir et al. (2019).

[3] There are some very recent works (Naskar and Pal, 2020; Lee and Muminov, 2021; Shrivastav, 2021; Heywood et al., 2022) aiming at augmenting the basic AJ framework with additional ingredients. The first and third ones introduce network externalities, product compatibility and product differentiation with the aim at analysing their effects on equilibrium outcomes of process innovation in Cournot and Bertrand duopolies. The second one studies the incentives for R&D information sharing (disclosure) in a mixed duopoly (i.e., with private and public firms). The fourth and more recent compares R&D competition and cooperation rivalry in a model of endogenous network compatibility. The authors show that cooperation in R&D should be important in network industries with endogenous compatibility. None of them, however, concentrate on the endogenous emergence of Nash equilibria comparing the strategic incentive of quantity-setting or price-setting firms to invest or not to invest in R&D with and without spillovers.

Thus, AJ's basic model always shows (regardless of the spill-over effects) the unpleasant result that non-cooperation is harmful. This also explains why much of the subsequent literature has focused on the cooperative solution. Unlike the existing literature, this article shows that an appropriate generalisation with product differentiation allows the non-cooperative and unilateral R&D investment behaviour to be better for players.

Our results differ from those of Bacchiega et al. (2010) in one crucial respect: in their work, the prisoner's dilemma vanishes if and only if the extent of technological spillovers is sufficiently high, which would then require that firms disclose (or, equivalently, they are unable to keep undisclosed) the information on the results of their R&D investment. However, the non-disclosure (i.e., keeping secret) of the R&D-related result in the AJ setting is in the unilateral interest of each non-cooperative firm. *Under product differentiation, the prisoner's dilemma can vanish also in the absence of R&D spillovers in both cases of quantity-setting and price-setting duopolies.* Moreover, the welfare analysis also reveals that, at the non-cooperative Pareto efficient Nash equilibrium with product differentiation in which both firms invest in cost-reducing R&D, firms and consumers are better off than without R&D.

Our findings suggest that investing in R&D challenges the improvement of interventions aimed at fostering product differentiation. Though this policy might seem detrimental for consumers, as it strengthens firms' market power, it also encourages R&D investments (at least starting with a relatively high degree of differentiation), thereby allowing firms to increase their market share at the expense of the rival's share in the market for the product of their variety (and thus consumers' surplus increases in aggregate terms as total supply increases, due to the stimulus of the higher variety). This eventually contributes to increasing profits and thus social welfare, compared to the scenario in which products are perceived as homogeneous.

When products are perfect substitutes, an increase in the degree of R&D spillovers always induces the investing firm to reduce its investment effort (to prevent the rival from taking advantage by free-riding on its investment activity). An increase in product heterogeneity resulting from consumer preferences tends to promote R&D strategic complementarity as the degree of competition in the product market reduces and profits increases. Therefore, there is room for the joint use of resources devoted to R&D (i.e., the R&D externality of one firm favours the R&D investment of the rival and vice versa) if the efficiency of R&D activity is high enough. Finally, product complementarity promotes cooperative behaviour in the product market by letting firms act as if they were maximising joint profits, and this eventually leads each of them to benefit from R&D complementarity.

As implementing policies aiming at favouring product differentiation trough ad hoc advertising investments can be a complex exercise (the policy maker should have information on the characteristics of consumer preferences and the cost structure of firms), the present work shows that R&D subsidies in a model with product differentiation can be used as a welfare-maximising tool also in the absence of R&D spillovers, which represents a robust outcome of the disclosure decision game according to a game theoretic approach (Buccella et al., 2021).

Definitively, the present article clarifies some already existing key points of the AJ literature and focuses on the fifth of the points below.

- First, as was shown by Buccella et al. (2021), to which we refer for details, the robust outcome according to a correct game-theoretic approach based on the AJ setting is the *absence* of R&D spillovers (perfect patenting). This is because the unique sub-game perfect Nash equilibrium (SPNE) of the non-cooperative *disclosure decision game* with exogenous spillovers is a *prisoner's dilemma* in which firms choose not to disclose information about R&D investments.
- Second, the existence of a *positive* extent of technological spillovers is certainly correct from a modelling perspective, as it helps describing cases in which perfect patenting is not allowed for reasons beyond the firm's control, but R&D disclosure does not emerge (at any positive

degree) as a SPNE of the disclosure decision game with R&D investing firms in the absence of public intervention.

- Third, the SPNE of the non-cooperative *investment decision game* à la AJ with homogeneous products in the absence of R&D spillovers (including the modelling framework developed by Bacchiega et al., 2010) is a *prisoner's dilemma* in which investing in R&D is a dominant strategy.
- Fourth, investing in R&D when products are homogeneous is therefore not Pareto improving from a societal perspective. This is because R&D investments increase the consumers' surplus but reduce firms' profits.
- Fifth, horizontal product differentiation eliminates the dilemma also in the *absence* of R&D spillovers (perfect patenting). The SPNE of the R&D investment decision game becomes a *deadlock*, i.e., there is no conflict between self-interest and mutual benefit to undertake cost-reduction actions through R&D, and investing in R&D is Pareto improving, as both consumers and firms are better off.

The remainder of the article is organised as follows. Section 2 outlines the model and discusses the main ingredients of the R&D investment decision game with product differentiation. Section 3 concentrates on quantity-setting firms.[4] Section 4 introduces the R&D subsidy as a welfare maximising tool in an environment with product differentiation. Section 5 concludes the article. The online Appendix provides mathematical details, the proofs of the propositions and the geometrical projections of the main analytical results.

2. The model

The starting point of our analysis can be spelled out by following the pioneering idea of cost-reducing innovation developed by AJ, augmented almost two decades later by Bacchiega et al. (2010), who consider the non-cooperative R&D investment decision game played by two quantity-setting firms producing homogeneous products.

Unlike Bacchiega et al. (2010), this section aims at developing the investment decision game in an AJ-like setting (including the stability conditions as in Henriques, 1990) by considering horizontal product differentiation à la Singh and Vives (1984).

Consider an industry in which two quantity-setting firms, i and j ($i = \{1,2\}$; $i \neq j$), face the perspective of investing in R&D and then sell goods of variety i and j, respectively, in the product market. Following Dixit (1979) and Singh and Vives (1984), the linear (inverse) demand for product of variety i is given by $p_i = a - b(q_i + dq_j)$, where p_i denotes the price of product of variety i (representing the marginal willingness to pay of consumers towards products of firm i), q_i, and q_j are the quantities of product of varieties i and j produced by firm i and firm j, respectively, $a > 0$ represents the market size, $b > 0$ measures the slope of the market demand being part of its elasticity, and $-1 \leq d \leq 1$ is the degree of product differentiation as perceived by customers (Singh and Vives, 1984). Positive (resp. negative) values of d refer to product substitutability (resp. complementarity). When $d = 0$, goods are totally differentiated; i.e., each firm acts as a monopolist. $d = 1$ refers to homogeneous goods and resembles the model developed by Bacchiega et al. (2010). For reasons of analytical tractability (and without loss of generality), we set $a = b = 1$ henceforth. Therefore, the indirect demand for product of variety i under horizontal differentiation is:

$$p_i = 1 - q_i - dq_j,\ i,j = \{1,2\},\ i \neq j. \tag{1}$$

The total cost of production and the cost of R&D effort of firm i are respectively given by the functions $C_i(q_i, x_i, x_j)$ and $X_i(x_i)$, in which x_i and x_j represent the R&D effort (investment) firm i

[4] For space constraints, the analysis of Bertrand competition in the AJ setting is available upon request. In this regard, we pinpoint that the results of the quantity competition model qualitatively hold for price-setting firms (with quantitative differences, especially about the thresholds identified by the stability conditions and the R&D cost conditions).

and firm j exert, respectively. Following AJ, these functions can be specified using the standard expressions:

$$C_i(q_i, x_i, x_j) = (w - x_i - \beta x_j)q_i,\ i,j = \{1,2\}, i \neq j, \tag{2}$$

and

$$X_i(x_i) = \frac{g}{2}x_i^2,\ i,j = \{1,2\}, i \neq j, \tag{3}$$

where $g > 0$ is a parameter measuring R&D efficiency that scales up/down R&D investment total costs. It represents an exogenous index of technological progress, measuring for example the appearance of a new, cost-effective technology, weighting the degree at which the available technology for process innovation affects investment decisions and firm's profits. A reduction in g can be interpreted as a technological advance so that investing in R&D becomes cheaper (i.e., the efficiency of R&D investment increases). In addition, $\beta \in [0,1]$ captures the extent of technological spillovers (externality) resulting from the R&D investment activity of firm j exogenously flowing as a cost-reducing device towards firm i (i.e., the amount of information that firm j exogenously discloses). We assume that both firms symmetrically share this characteristic of the extent of technological spillovers. This scenario represents the standard case of exogenous spillovers according to which a fixed fraction of a firm's R&D process innovation exogenously flows to competitors so that each firm has no direct control over the extent of disclosure for, e.g., technological reasons. This case directly follows AJ and the subsequent contributions by Henriques (1990), Suzumura (1992), Kamien et al. (1992), De Bondt (1996) and Bacchiega et al. (2010). When $\beta = 0$, there are no R&D externalities, resembling the case of non-disclosure of R&D information. When $\beta = 1$, R&D information is fully shared, so that R&D disclosure is at its (exogenous) highest intensity. As was discussed in the introduction, though assuming $\beta > 0$ is reasonable, it cannot emerge as a SPNE of the disclosure decision game in the absence of R&D subsidies (see Buccella et al., 2021, for details).

The expression representing the firm's technology in Eq. (2) implies that the unitary cost of production should be positive so that $w - x_i - \beta x_j > 0$ should always hold, where $0 < w < 1$ measures the unitary technology of production cost irrespective of R&D investments. Moreover, the expression representing the cost of R&D effort in Eq. (3) reveals diminishing returns in the R&D technology exerted by firm i. Therefore, each firm sustains the cost of R&D effort with technology displaying decreasing returns to scale to achieve the benefit of reducing the total unit costs of production with constant returns to scale.

Definitively, selfish firms are engaged in a three-stage non-cooperative *R&D investment decision game with horizontal product differentiation* and complete information in which they choose whether to invest in R&D activities at stage one (*the investment-decision stage*). At stage two (*the R&D stage*), firms choose the extent of process innovation R&D investment (if they invest) or, alternatively, they do not invest in R&D. At stage three (*the market stage*), firms compete on quantity (Section 3), or they set the price in the Bertrand scenario, available upon request. As usual, the game is solved by adopting the backward induction logic.

3. The R&D investment decision game with product differentiation: Cournot competition

This section initiates the analysis of the non-cooperative R&D investment decision game with horizontal product differentiation in a quantity-setting (Cournot rivalry) game. To avoid lengthening the article too much, the analysis and the discussion of the investment configurations that are not part of a Nash equilibrium, i.e., the symmetric subgame in which firms do not invest in R&D (NI/NI) and the asymmetric subgame in which only one firm invests in R&D (I/NI) can be found in the online Appendix.

3.1. The symmetric subgame in which both firms invest in R&D (I/I)

Consider now the possibility that firms symmetrically choose to invest in R&D, that is $x_i > 0$ and $X_i(x_i) > 0$ ($i = \{1,2\}, i \neq j$). Therefore, by using Eqs. (1), (2), and (3), the profit function of firm i becomes:

$$\Pi_i^{I/I} = p_i q_i - C_i(q_i, x_i, x_j) - X_i(x_i) = (1 - q_i - dq_j)q_i - (w - x_i - \beta x_j)q_i - \frac{g}{2}x_i^2, \quad (4)$$

where the upper script I/I stands for positive R&D investments of both firms. At the market stage of the game, each firm chooses the amount of output to maximise profits. Maximisation of (4) with respect to q_i leads to the following downward-sloping reaction function of firm i in the (q_i, q_j) space as a function also of R&D efforts x_i and x_j, that is:

$$\frac{\partial \Pi_i^{I/I}}{\partial q_i} = 0 \Leftrightarrow \overline{q}_i^{I/I}(q_j, x_i, x_j) = \frac{1 - w - dq_j + x_i + \beta x_j}{2}. \quad (5)$$

Eq. (5) reveals that an increase in the R&D investment of firm i shifts outwards its own reaction function, thereby contributing to increase production. Conversely, an increase in the R&D investment of the rival (firm j) shifts outward firm i's reaction function with a lower intensity, as x_j can be beneficial for q_i only when there exists positive R&D externalities ($\beta > 0$), so that R&D-related information can flow to competitors partially if $0 < \beta < 1$ or totally if $\beta = 1$ generating benefits without payments to the rival. In the last case, the x_j-related externality contributes (at its highest intensity) exactly as x_i as a device fostering the production of firm i. Using Eq. (5) together with the symmetric counterpart for firm j provides the system of reaction functions in the (q_i, q_j) space as a function of the R&D effort. The solution of the system of output reaction functions $\overline{q}_i^{I/I}(q_j, x_i, x_j)$ ($i = \{1,2\}$, $i \neq j$) allows us to get the following equilibrium output obtained at the third stage of the symmetric subgame I/I:

$$\overline{\overline{q}}_i^{I/I}(x_i, x_j) = \frac{(1-w)(2-d) + (2 - d\beta)x_i + (2\beta - d)x_j}{(2-d)(2+d)}. \quad (6)$$

Eq. (6) shows that production of firm i depends on its own R&D investments (due to a twofold reason) as well as on R&D investments conducted by firm j (due to R&D the externality). On one hand, the R&D investment undertaken by firm i allows for a direct increase in the amount of its production (whose intensity is weighted by the term $\frac{2}{(2-d)(2+d)}$) due to the strategic interaction with the rival. Therefore, firm i increases production through this channel. On the other hand, if products are substitutes ($d > 0$), there exists a mitigation effect of x_i on q_i because of the amount of R&D-related information externalities flowing from firm i to firm j (whose intensity is weighted by the term is weighted by the term $\frac{d\beta}{(2-d)(2+d)}$). Therefore, firm i reduces production through this channel. However, the strength of the latter effect can never counterbalance the strength of the former, including under the most extreme conditions, i.e., when the products are perfect substitutes ($d = 1$) and disclosure is at its maximum intensity ($\beta = 1$). This is because $\frac{d\beta}{(2-d)(2+d)} < \frac{2}{(2-d)(2+d)}$ always holds. Definitively, an increase in x_i always causes an increase in q_i if products are substitutes. Product complementarity ($d < 0$), instead, sharply modifies the incentives on firm i's flowing through R&D towards firm j. Specifically, the mitigation effect becomes a strengthening one so that there exists a positive effect on firm i's output of its own R&D externality flowing towards the rival. It is as though firm i has the advantage of agreeing with firm j to disclose R&D information. This effect reaches its maximal effectiveness when products are perfect complements ($d = -1$) and disclosure is at its maximum intensity ($\beta = 1$). Additionally, the R&D disclosure of both firms allows for a further increase in the output production of firm i through the R&D investments of firm j if and only if the extent of technological spillovers is sufficiently large ($\beta > \frac{d}{2}$, i.e., x_i and x_j are strategic complements). Otherwise, the case turns to a reduction if the extent of technological spillovers is sufficiently small ($\beta < \frac{d}{2}$, i.e., x_i and x_j are strategic substitutes). This condition holds when products are substitutes and the degree of product substitutability tends to counterbalance the positive feedback effect of the spillovers, reaching its maximal intensity under product substitutability ($d = 1$). This implies that the higher the degree of product substitutability, the higher the need for firms to disclose R&D information to increase their own output in the product market. Conversely, product complementarity moves exactly in the opposite direction, thereby letting output production always be positively correlated with the amount of R&D investment of the rival.

Substituting Eq. (6) together with its counterpart for firm j in Eq. (4) allows to obtain firm i's profits as a function of R&D efforts x_i and x_j, i.e., $\Pi_i^{I/I}(x_i, x_j)$. Following the tradition initiated by AJ in the non-cooperative version of the process investment R&D duopoly, firms maximise profits non-cooperatively at the second (R&D) stage of the game by choosing the amount of cost-reducing investment. Formally, this implies that:

$$\frac{\partial \Pi_i^{I/I}(x_i, x_j)}{\partial x_i} = 0 \Leftrightarrow \overline{x}_i^{I/I} = \frac{2(2-d\beta)\left[(2-d)(1-w)+(2\beta-d)x_j\right]}{g[16-d^2(8-d^2)]+2d\beta(4-d\beta)-8}. \tag{7}$$

Using Eq. (7) together with the corresponding counterpart for firm j allows us to obtain the system of reaction functions in the R&D space, that is (x_i, x_j). Solving the system of the R&D reaction functions allows us to obtain the amount of equilibrium investment (denoted as usual with an asterisk) following the process innovation effort of firm i at the second stage of the game (and consequently the symmetrical firm j's response), that is:

$$x_i^{*I/I} = \frac{2(1-w)(2-d\beta)}{g(2-d)(2+d)^2-2(1+\beta)(2-d\beta)}. \tag{8}$$

From Eq. (8), $x_i^{*I/I} > 0$ if and only if the denominator is positive, that is $g > g_{SC}^{\beta_{high}}(\beta, d)$, which is defined in Eq. (12) and the subscript SC denotes "Stability Condition".

The second-order condition for a maximum (concavity) requires that $\left.\frac{\partial^2 \Pi_i^{I/I}(x_i, x_j)}{\partial x_i^2}\right|_{x_i = x_i^{*I/I}} < 0$. This implies that the inequality

$$g > \frac{2(2-d\beta)^2}{(2-d)^2(2+d)^2} := g_{SOC}(\beta, d) \text{ (second-order condition),} \tag{9}$$

must hold to guarantee that the solution to the profit maximisation problem is economically meaningful, where the subscript SOC denotes "Second Order Condition". This condition boils down to $g > g_{SOC}(\beta, 1) := \frac{2}{9}(2-\beta)^2$ if $d = 1$, which replicates exactly the AJ's result in our normalised setup. The R&D equilibrium characterised by the expression in (8) is stable (in the sense of Seade, 1980) if and only if the reaction functions defined in the R&D space should adequately cross (Henriques, 1990). Indeed, Henriques (1990) and Bacchiega et al. (2010) find that the R&D reaction curves can be downward sloping or upward sloping depending on the relative size of β (the R&D externality). If they are downward sloping (resp. upward-sloping), x_i and x_j are strategic substitutes (resp. complements). This holds when the R&D externality is small (resp. large). The stability conditions require that $\left|\frac{dx_i}{dx_j}\right| < 1$ in both cases of strategic substitutability and complementarity and thus leads to a relationship only between g and β in the case of perfect substitutability, i.e., $d = 1$ (Bacchiega et al., 2010). Differently, the R&D reaction curves in the case of product differentiation can be downward sloping or upward sloping depending on the relative size of β and d and consequently the stability conditions include three parameters (g, β and d). By computing the derivative

$$\frac{dx_i}{dx_j} = \frac{2(2\beta-d)(2-d\beta)}{g(2-d)^2(2+d)^2-2(2-d\beta)^2}, \tag{10}$$

the denominator is positive if and only if $g > g_{SOC}(\beta, d)$, which should always be fulfilled for concavity. Therefore, $\frac{dx_i}{dx_j} < 0$ if and only if $\beta < \frac{d}{2}$ (x_i and x_j are strategic substitutes) and $\frac{dx_i}{dx_j} > 0$ if and only if $\beta > \frac{d}{2}$ (x_i and x_j are strategic complements). These conditions boil down to those found by Bacchiega et al. (2010) under the assumption of perfect substitutability ($d = 1$). The stability conditions $\left|\frac{dx_i}{dx_j}\right| < 1$ in the R&D model with product differentiation require that one should impose (see Hinloopen, 2015):[5]

$$g > \frac{2(1-\beta)(2-d\beta)}{(2-d)^2(2+d)} \coloneqq g_{SC}^{\beta_{low}}(\beta, d) \text{ if } 0 \leq \beta < \frac{d}{2}, (x_i \text{ and } x_j \text{ are strategic substitutes}), \quad (11)$$

and

$$g > \frac{2(1+\beta)(2-d\beta)}{(2-d)(2+d)^2} \coloneqq g_{SC}^{\beta_{high}}(\beta, d) \text{ if } \frac{d}{2} < \beta \leq 1, (x_i \text{ and } x_j \text{ are strategic complements}), \quad (12)$$

where $g_{SC}^{\beta_{low}}(\beta, d) > g_{SC}^{\beta_{high}}(\beta, d)$ for any $0 \leq \beta \leq \frac{d}{2}$ and $g_{SC}^{\beta_{low}}(\beta, d) < g_{SC}^{\beta_{high}}(\beta, d)$ for any $\frac{d}{2} \leq \beta \leq 1$, that is $g_{SC}^{\beta_{low}}(\beta, d)$ is binding for any $0 \leq \beta < \frac{d}{2}$, and $g_{SC}^{\beta_{high}}(\beta, d)$ is binding for any $\frac{d}{2} < \beta \leq 1$ in the (β, g) space. Therefore, the condition that guarantees positive R&D investments ($g > g_{SC}^{\beta_{high}}(\beta, d)$) from Eq. (8) and the second-order condition ($g > g_{SOC}(\beta, d)$) are fulfilled for any $0 \leq \beta \leq 1$ if the stability conditions are satisfied. We recall that the second-order condition and the stability conditions suggest that the efficiency of R&D activity should not be too high; i.e., parameter g should not be too low to avoid excessive R&D investments that would contribute to greatly reduce marginal and average production costs and increase output pushing down the market price of products of both varieties. This condition would reduce the market power of firms, the extent of which depends on the price elasticity of demand. Indeed, if $g < g_{SOC}(\beta, d)$, the percentage reduction in market price is high enough to let profits become negative (as will be clear from Eq. (17)). For a geometrical representation of the shape and position of the second-order condition, the stability conditions and the R&D cost condition for the cases of homogeneous and heterogeneous products see Figures A.1, A.2 and A.3 in the online Appendix. Figure A.1 refers to the case of *homogeneous products* ($d = 1$) showing that $g > g_{SOC}(\beta, d)$ holds for every g satisfying the stability conditions irrespective of the value of β for which (given d) x_i and x_j are strategic substitutes and strategic complements. If q_i and q_j are complements (negative values of d), the relevant stability condition is always given by the inequality in (12), i.e., x_i and x_j are strategic complements for any couple (β, g) in that case. An increase in the degree of product differentiation ($d \downarrow$) changes the shape of the stability conditions in the (β, g) space. In particular, the β-threshold separating the region of strategic substitutability from the region of strategic complementarity, i.e., $\frac{d}{2}$, shifts leftward in the (β, g) space, thus favouring strategic complementarity in the R&D effort that works therefore out as a device enforcing the R&D externality. Figure A.2 contrasts Figure A.1 and shows this result for the case *heterogeneous products*. This can also be ascertained analytically by studying how $x_i^{*I/I}$ reacts to a change in β by comparing the cases of homogeneous and heterogeneous products. In fact,

$$\frac{\partial x_i^{*I/I}}{\partial \beta} = \frac{-2(1-w)\left[gd(2-d)(2+d)^2 - 2(2-d\beta)^2\right]}{[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)]^2}. \quad (13)$$

From Eq. (13), one can see that $\frac{\partial x_i^{*I/I}}{\partial \beta} < 0$ for any β and g when products are perfect substitutes ($d = 1$) as $g > g_{SOC}(\beta, 1)$ must hold. As the externality of R&D investment becomes larger, each firm has an incentive to reduce its own amount of cost-reducing R&D investment, as it can benefit from the externality resulting from the rival's investment. Instead, when products are differentiated ($d < 1$) the effect on $x_i^{*I/I}$ of a change in β depends on whether products are substitutes ($d > 0$) or complements ($d < 0$). In the former case, $\frac{\partial x_i^{*I/I}}{\partial \beta} > 0$ if $g < \frac{g_{SOC}(\beta,d)}{d}$ and $\frac{\partial x_i^{*I/I}}{\partial \beta} < 0$ if $g > \frac{g_{SOC}(\beta,d)}{d}$,

[5] The definition of strategic substitutability and complementarity between the R&D investments x_i and x_j depends on the relative size of β, which – in a model with horizontally differentiated goods – is affected by the degree of product differentiation (d). If goods are substitutes ($d > 0$), the relevant constraints to characterise strategic substitutability and complementarity between x_i and x_j are those defined in the expressions (11) and (12). If goods are complements ($d < 0$), x_i and x_j are always strategic complements, as the inequality in (12) holds for any $0 \leq \beta \leq 1$.

where $\frac{g_{SOC}(\beta,d)}{d} > g_{SOC}(\beta,d)$ for any $0 < d < 1$, and the difference between the two thresholds increases as the degree of product differentiation increases. An increase in product heterogeneity resulting from consumer preferences tends to promote R&D strategic complementarity as the degree of competition in the product market reduces and profits increases. Therefore, there is room for the joint use of resources devoted to R&D (i.e., the R&D externality of one firm favours the R&D investment of the rival and vice versa) if the efficiency of R&D activity is high enough (low values of g). In the latter case, instead, $\frac{\partial x_i^{*I/I}}{\partial \beta} > 0$ for any β and g as product complementarity promotes cooperative behaviour in the product market by letting firms act as if they were maximising joint profits, and this eventually leads each of them to benefit from R&D complementarity.

The analysis so far should be augmented with additional constraints on the side of the costs of production. Indeed, as we know from Eq. (2), the unitary production cost $w - x_i - \beta x_j$ as part of the total costs of production $C_i(q_i, x_i, x_j)$ must always be positive. Therefore, by using Eq. (8), the inequality $w - x_i - \beta x_j > 0$ is fulfilled if and only if:

$$g > \frac{2(1+\beta)(2-d\beta)}{w(2-d)(2+d)^2} \coloneqq g_T^{I/I}(\beta,d,w), \text{ (R\&D cost condition)}, \tag{14}$$

where the subscript T stands for "Threshold".

The inequality in (14) must hold as an additional threshold in determining meaningful Nash equilibrium outcomes of the game, as will be clear from the analysis presented in Section 3.2. However, Figure A.3 in the online Appendix, plotted in the (β, g) space for four different values of w depicted in Panels A-D, clarifies the behaviour of the threshold $g_T^{I/I}(\beta,d,w)$ for the subgame I/I (the blue line in the figure) by overlapping it with the stability conditions in (11) and (12), as depicted in Figure A.1 for the case of homogeneous products (the second-order condition was not drawn as it is always fulfilled once the stability conditions are satisfied). As can be seen from (14), the shape of the threshold $g_T^{I/I}(\beta,d,w)$ also depends on w. Therefore, it is important to study the conditions under which the threshold $g_T^{I/I}(\beta,d,w)$ is binding compared to the stability conditions *for the subgame I/I*.

Comparison of (12) and (14) clearly reveals that $g_T^{I/I}(\beta,d,w) > g_{SC}^{\beta_{high}}(\beta,d)$ for any $w < 1$ and $g_T^{I/I}(\beta,d,1) \to g_{SC}^{\beta_{high}}(\beta,d)$ from above for $w \to 1$. In contrast, comparison of (11) and (14) reveals that $g_T^{I/I}(\beta,d,w)$ can be higher or lower than $g_{SC}^{\beta_{low}}(\beta,d)$, depending on the relative size of β, d and w. Proposition 1 deepens this result and complements Figure A.3 in the online Appendix by showing that $g_T^{I/I}(\beta,d,w)$ can be binding in the (β, g) space, depending on some conditions on the main parameters of the problem. Let us first define $\beta_T^{I/I} \coloneqq \frac{-(2-d)+w(2+d)}{2-d+w(2+d)}$ as a threshold value of the intensity of the R&D externality such that $g_T^{I/I}(\beta,d,w) = g_{SC}^{\beta_{low}}(\beta,d)$ in the (β, g) space. Then, $\beta_T^{I/I} \to \frac{d}{2}$ if $w \to 1$ and $\beta_T^{I/I} < \frac{d}{2}$ for any d and $w < 1$. In addition, $\beta_T^{I/I} < 0$ if $w < \frac{2-d}{2+d} \coloneqq w_T^{I/I}$ and $\beta_T^{I/I} > 0$ if $w > w_T^{I/I}$, where $w_T^{I/I} = \frac{1}{3}$ if $d = 1$, $w_T^{I/I} = 1$ if $d = 0$ and $w_T^{I/I} > 1$ if $d < 0$. Then, the following proposition holds.

Proposition 1 [Constraints of the subgame I/I]. 1) If products are complements ($d < 0$), then $g_T^{I/I}(\beta,d,w)$ is binding in the (β, g) space for any $0 \le \beta \le 1$ and $0 < w < 1$ for the subgame I/I. 2.1) If products are substitutes ($d > 0$) and $w < w_T^{I/I}$ then $g_T^{I/I}(\beta,d,w)$ is binding in the (β, g) space for any β for the subgame I/I. 2.2) If products are substitutes ($d > 0$) and $w > w_T^{I/I}$, then 2.2.1) $g_{SC}^{\beta_{low}}(\beta,d)$ is binding in the (β, g) space for any $\beta < \beta_T^{I/I}$ for the subgame I/I, and 2.2.2) $g_T^{I/I}(\beta,d,w)$ is binding in the (β, g) space for any $\beta > \beta_T^{I/I}$ for the subgame I/I. 3) If products are substitutes ($d > 0$) and $w \to 1$, then $g_{SC}^{\beta_{low}}(\beta, d)$ is binding for any $0 \leq \beta < \frac{d}{2}$ and $g_{SC}^{\beta_{high}}(\beta, d)$ is binding for any $\frac{d}{2} < \beta \leq 1$ for the subgame I/I.

Proof. 1) If $d < 0$, then $w_T^{I/I} > 1$ and $\beta_T^{I/I} < 0$ for any $0 < w < 1$. Therefore, $g_T^{I/I}(\beta, d, w) > g_{SC}^{\beta_{low}}(\beta, d)$ for any $0 \leq \beta \leq 1$ and $0 < w < 1$. 2) If $d > 0$, then $w_T^{I/I} < 1$, and $\beta_T^{I/I} < 0$ for any $w < w_T^{I/I}$ and $\beta_T^{I/I} > 0$ for any $w > w_T^{I/I}$. Therefore, 2.1) if $w < w_T^{I/I}$ then $g_T^{I/I}(\beta, d, w) > g_{SC}^{\beta_{low}}(\beta, d)$ for any β, and 2.2) if $w > w_T^{I/I}$ then $g_T^{I/I}(\beta, d, w) < g_{SC}^{\beta_{low}}(\beta, d)$ for any $\beta < \beta_T^{I/I}$ and $g_T^{I/I}(\beta, d, w) > g_{SC}^{\beta_{low}}(\beta, d)$ for any $\beta > \beta_T^{I/I}$. 3) If $d > 0$ and $w \to 1$ then $\beta_T^{I/I} \to \frac{d}{2}$ and $g_T^{I/I}(\beta, d, 1) \to g_{SC}^{\beta_{high}}(\beta, d)$ from above for any $0 \leq \beta \leq 1$. Therefore, $g_{SC}^{\beta_{low}}(\beta, d) > g_{SC}^{\beta_{high}}(\beta, d)$ for any $0 \leq \beta \leq \frac{d}{2}$ and $g_{SC}^{\beta_{low}}(\beta, d) < g_{SC}^{\beta_{high}}(\beta, d)$ for any $\frac{d}{2} \leq \beta \leq 1$. **Q.E.D.**

Proposition 1 clarifies the role of the unitary (and marginal) costs of production in an R&D environment within the subgame I/I. Basically, it tells us that $g_T^{I/I}(\beta, d, w)$ tends to be binding for the subgame I/I when 1) products are complements, as this kind of consumer's tastes favour R&D strategic complementarity by increasing the R&D investment of each firm; 2) products are substitutes, but the unitary cost resulting from the technology of production of final goods is sufficiently low; and 3) product are substitutes, and the extent of technological spillovers is sufficiently high. In each of these cases, in fact, there are reasons for each firm to reduce production costs. A huge reduction in the unitary (and marginal) production costs through R&D effort implies a dramatic increase in output (as shown by the output reaction function Eq. (5)) and a corresponding reduction in the market price that erodes the market power of firms, leading them to produce eventually quantities corresponding to the downward-sloping branch of their own total revenue function, in which the price elasticity of demand is smaller than one, so that firms should reduce instead of increase production to maximise profits in that case. An increase in the degree of product differentiation ($d \downarrow$) has an ambiguous effect on $g_T^{I/I}(\beta, d, w)$ depending on the extent of the R&D externality (β). This can be ascertained by computing $\frac{\partial g_T^{I/I}(\beta,d,w)}{\partial d}$, whose sign is the same (and depends on the same thresholds) as the sign of $\frac{\partial x_i^{*I/I}}{\partial d}$, as shown in Proposition 2.

Proposition 2 [Effects of product differentiation on the R&D cost condition in the subgame I/I]. The derivative $\frac{\partial g_T^{I/I}(\beta,d,w)}{\partial d}$ is (*i*) positive (resp. negative) for any $\frac{2}{3} < d < 1$ (resp. $-1 < d < \frac{2}{3}$) if $\beta = 0$, (*ii*) positive (resp. negative) for any $\frac{3+\beta-\sqrt{(1-\beta)(9+7\beta)}}{2\beta} < d < 1$ (resp. $-1 < d < \frac{3+\beta-\sqrt{(1-\beta)(9+7\beta)}}{2\beta}$) if $0 < \beta \leq \frac{1}{2}$, where $\frac{3+\beta-\sqrt{(1-\beta)(9+7\beta)}}{2\beta} > \frac{2}{3}$ and $\frac{3+\beta-\sqrt{(1-\beta)(9+7\beta)}}{2\beta} = 1$ if $\beta = \frac{1}{2}$, and (*iii*) negative for any $-1 < d < 1$ if $\frac{1}{2} < \beta \leq 1$.

Proof. The proof follows from $sgn\left\{\frac{\partial g_T^{I/I}(\beta,d,w)}{\partial d}\right\} = sgn\left\{\frac{\partial x_i^{*I/I}}{\partial d}\right\} = sgn\{-\beta d^2 + (3+\beta)d - 2(1+\beta)\}$. **Q.E.D.**

Therefore, an increase in the degree of product heterogeneity ($d \downarrow$) broadens the feasible parameter space bounded by the R&D cost condition ($g_T^{I/I}(\beta, d, w) \downarrow$) if the degree of product substitutability is sufficiently small (ranging from the cases of no externality up to half the strength of the externality of R&D information). If products are perceived as poorly differentiated, a reduction in d allows each firm to reduce R&D investment at the optimum (as product differentiation per se increases output and then reduces the need to invest in R&D), thus widening the parameter region in the (g, β) space in which the R&D cost condition is fulfilled. However, the larger the externality of the cost-reducing R&D activity (β), the smaller this effect.

Differently, product heterogeneity narrows the feasible parameter space bounded by the R&D cost condition ($g_T^{I/I}(\beta, d, w) \uparrow$) if the degree of product substitutability is sufficiently large. Unlike the previous case, in fact, when products of variety i and j are perceived as highly differentiated, a further reduction in d marks the start for an increase in R&D investment at the optimum to capture unilaterally the benefits of higher product differentiation, and the larger the externality of the cost-reducing R&D activity (β), the stronger this effect. Finally, if the extent of R&D externality is high enough ($\beta > \frac{1}{2}$), product differentiation always favours strategic complementarity of R&D effort and thus contributes to let the R&D cost condition become more binding in the (g, β) space for the subgame I/I.

We now move forward by continuing the equilibrium analysis of the subgame I/I. By using the symmetrical equilibrium R&D expression in (8) and substituting out for $x_i^{*I/I}$ in the equilibrium output obtained at the third stage of the game, one gets the amount of output produced by firm i ($i = \{1,2\}$, $i \neq j$) at equilibrium under I/I, that is:

$$q_i^{*I/I} = \frac{g(1-w)(2-d)(2+d)}{g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)}. \tag{15}$$

Eq. (15) reveals that $g > g_{SC}^{\beta_{high}}(\beta, d)$ is sufficient to guarantee a positive output production for both firms. In addition, from the expressions in (8) and (15), one can easily get the equilibrium values of the market price of product of variety i and profits of firm i, which are respectively given by the following equations:

$$p_i^{*I/I} = \frac{g(2-d)(2+d)[1+w(1+d)] - 2(1+\beta)(2-d\beta)}{g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)}, \tag{16}$$

where the denominator is positive if $g > g_{SC}^{\beta_{high}}(\beta, d)$, and

$$\Pi_i^{*I/I} = \frac{g(1-w)^2\left[g(2-d)^2(2+d)^2 - 2(2-d\beta)^2\right]}{[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)]^2}. \tag{17}$$

The expressions of the equilibrium price and equilibrium profits in (16) and (17) reveal that $p_i^{*I/I} > 0$ if $g > \frac{2(1+\beta)(2-d\beta)}{(2-d)(2+d)[1+w(1+d)]} \coloneqq g_p^{I/I}(\beta, d, w)$ and $\Pi_i^{*I/I} > 0$ if $g > g_{SOC}(\beta, d)$. We note that $g_p^{I/I}(\beta, d, w) < g_{SC}^{\beta_{high}}(\beta, d)$ for any $0 < w < 1$ and $g_p^{I/I}(\beta, d, 1) \to g_{SC}^{\beta_{high}}(\beta, d)$ if $w \to 1$. Therefore, both thresholds are satisfied if either $g > g_T^{I/I}(\beta, d, w)$ or $g > g_{SC}^{\beta_{low}}(\beta, d)$ holds.

The equilibrium values of consumers' surplus ($CS^{*I/I}$)[6] and producers' surplus ($PS^{*I/I}$) that can be obtained in the I/I subgame are summarised as follows:

$$CS^{*I/I} = \frac{1}{2}\left[\left(q_i^{*I/I}\right)^2 + \left(q_j^{*I/I}\right)^2 + 2d q_i^{*I/I} q_j^{*I/I}\right] = \frac{g^2(1-w)^2(1+d)(2-d)^2(2+d)^2}{[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)]^2}. \tag{18}$$

and

$$PS^{*I/I} = \Pi_i^{*I/I} + \Pi_j^{*I/I} = \frac{2g(1-w)^2\left[g(2-d)^2(2+d)^2 - 2(2-d\beta)^2\right]}{[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)]^2}. \tag{19}$$

Therefore, social welfare under I/I, $W^{*I/I}$, is given by:

$$W^{*I/I} = CS^{*I/I} + PS^{*I/I} = \frac{g(1-w)^2\left[g(3+d)(2-d)^2(2+d)^2 - 4(2-d\beta)^2\right]}{[g(2-d)(2+d)^2 - 2(1+\beta)(2-d\beta)]^2}. \tag{20}$$

Comparison of Eqs. (20) and (21) – see the online Appendix for analytical derivations of the relevant variables of the subgame NI/NI – allows to get Proposition 3.

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[6] In this model with linear demands, the consumers' surplus is given by the area bounded by the difference between the price that consumers are willing to pay, and the price actually paid for the quantity consumed in the market for product i and for product j, which in equilibrium is given by the area of the triangle $CS^* = \frac{1}{2}(1 - p_i^*)q_i^* + \frac{1}{2}(1 - p_j^*)q_j^* = \frac{1}{2}[(q_i^*)^2 + (q_j^*)^2 + 2dq_i^*q_j^*]$.

Proposition 3 [Social welfare]. Social welfare under I/I is larger than social welfare under NI/NI.

Proof. By computing the difference $W^{*I/I} - W^{*NI/NI}$ and solving for g allows to get the threshold $g_W(\beta, d) \coloneqq \frac{(3+d)(2-d\beta)(1+\beta)^2}{(2+d)^2[4-d-d^2+6\beta-d^2\beta]} > 0$, which is smaller than $g_{SC}^{\beta_{high}}(\beta, d)$ for any $0 \leq \beta \leq 1$, $-1 < d < 1$. Therefore, $W^{*I/I} > W^{*NI/NI}$ either for any $g > g_{SC}^{\beta_{low}}(\beta, d)$ or for any $g > g_T^{I/I}(\beta, d, w)$. **Q.E.D.**

The result behind Proposition 3 has a simple economic intuition. On one hand, the R&D investment causes an increase in production and this, in turn, implies an increase in consumers' surplus as customers can consume more than in the absence of R&D. Therefore, social welfare increases through this channel. On the other hand, the increased production can only be sold through a reduction in the price consumers are willing to pay, thus generating an ambiguous effect on profits that can therefore be larger or smaller than in the absence of R&D depending on the relative size of the main parameters of the problem, i.e., the efficiency of the R&D technology and the extent of technological spillovers. Therefore, social welfare can increase or reduce through this channel. However, irrespective of whether profits under I/I are larger or smaller than under NI/NI, the positive CS effect dominates so that social welfare with R&D is larger than without R&D. In what follows, we will analyse the effects of product differentiation on profits (and social welfare) showing that although an increase in the degree of product differentiation increases the market power of firms and thus tends to reduce the consumers' surplus, the benefit to firms' profits is high enough to offset the reduction in the consumers' surplus and thus generate a Pareto efficient outcome for firms and society.

3.2. The investment-decision stage under quantity competition: Nash equilibria and discussion

This section examines the first stage of the game, in which firms (owners) choose whether to invest in R&D in a non-cooperative quantity-setting environment à la d'Aspremont and Jacquemin (1988, 1990), by also clarifying the role of the constraints (the stability conditions and the R&D cost conditions) of the R&D investment decisions game and stressing the differences between homogeneous and heterogeneous products.[7]

Making use of the profit equation (17) of firm i for the symmetric subgame I/I, the profit equation of firm i for the symmetric subgame NI/NI

$$\Pi_i^{*NI/NI} = \left(\frac{1-w}{2+d}\right)^2, \tag{21}$$

and the profit equations of firm i and firm j for the asymmetric subgame I/NI,

$$\Pi_i^{*I/NI} = \frac{g(1-w)^2(2-d)^2}{g(2-d)^2(2+d)^2-2(2-d\beta)^2}, \tag{22}$$

and

$$\Pi_j^{*I/NI} = \frac{(1-w)^2\left[g(2-d)^2(2+d)-2(1-\beta)(2-d\beta)\right]^2}{[g(2-d)^2(2+d)^2-2(2-d\beta)^2]^2}, \tag{23}$$

we can build on the payoff matrix summarised in Table 1 regarding the Cournot R&D game with product differentiation (following the analysis of Bacchiega et al., 2010).

[7] We pinpoint that, amongst other things, the online Appendix provides 1) a detailed analyses of the symmetric subgame NI/NI and the asymmetric subgame I/NI, and 2) a discussion on the role of the relevant additional constraint, i.e., the R&D cost condition of the investing firm, that contributes – together with the stability conditions and the R&D cost condition emerging in the symmetric subgame I/I – to identify the feasible region of the R&D investment decision game.

Table 1. The investment-decision game (payoff matrix). Cournot competition.

Firm 2 $\rightarrow$ Firm 1 $\downarrow$	I	NI
I	$\Pi_1^{*I/I}, \Pi_2^{*I/I}$	$\Pi_1^{*I/NI}, \Pi_2^{*I/NI}$
NI	$\Pi_1^{*NI/I}, \Pi_2^{*NI/I}$	$\Pi_1^{*NI/NI}, \Pi_2^{*NI/NI}$

To satisfy the technical restrictions and have well-defined equilibria in pure strategies for every strategic profile (one for each player), the analysis is restricted to the feasibility constraints discussed in Proposition A.1 in the online Appendix (which henceforth are assumed to be always satisfied), whose role is clarified in Remark 1.

Remark 1. Proposition A.1 allows us to make clear the constraints emerging in the non-cooperative version of the cost-reducing R&D model by d'Aspremont and Jacquemin (1988, 1990) with spillovers with homogeneous or heterogeneous products and let the R&D investment decision game be meaningful. These constraints represent the stability condition emerging when x_i and x_j are strategic substitutes or strategic complements, the R&D cost condition emerging in the symmetric subgame I/I, and the R&D cost condition of the investing firm emerging in the asymmetric subgame I/NI. The proposition tells us that $g_T^{I/I}(\beta, d, w)$ is always binding for the R&D investment decision game when products are complements (irrespective of the other parameters of the model). This is because this kind of consumer's tastes favour R&D strategic complementarity by incentivising both firms to invest in R&D to increase their own profits. Differently, product substitutability increases the complexity and let spillovers and the unitary cost become relevant in determining the feasibility conditions.

To derive all of the possible equilibria of the game, one must study the sign of the profit differentials $\Delta\Pi_A = \Pi_i^{*I/NI} - \Pi_i^{*NI/NI}$, $\Delta\Pi_B = \Pi_i^{*NI/I} - \Pi_i^{*I/I}$ and $\Delta\Pi_C = \Pi_i^{*NI/NI} - \Pi_i^{*I/I}$ for $i = \{1,2\}, i \neq j$. The formulations and the analytical details of the profit differentials are in the online Appendix. The analysis of the profit differentials reveal that $\Delta\Pi_A > 0$ and $\Delta\Pi_B < 0$, irrespective of the parameter scale, whereas $\Delta\Pi_C$ can be positive or negative depending on the relative values of β, g and d. In this regard, let

$$g_C(\beta, d) \coloneqq \frac{2(1+\beta)^2(2-d\beta)}{(2+d)^2[2(1-d)+\beta(4-d)]}, \tag{24}$$

be the threshold value of g as a function of the intensity of the spillovers effect and the degree of product differentiation such that $\Delta\Pi_C = 0$. If $g < g_C(\beta, d)$ then $\Delta\Pi_C > 0$, and profits for both firms under the strategic profile NI/NI are higher than profits under the strategic profile I/I. If $g > g_C(\beta, d)$, then $\Delta\Pi_C < 0$, and profits both firms under the strategic profile I/I are higher than profits under the strategic profile NI/NI.

Eq. (24) highlights the most significant difference between the R&D investment decision game with homogeneous products (Bacchiega et al., 2010) and the R&D investment decision game with heterogeneous products, as was already pinpointed in the introduction. Indeed, our results differ from those of Bacchiega et al. (2010) in one crucial respect: in their work, the prisoner's dilemma vanishes if and only if the extent of technological spillovers is positive and sufficiently high. This would require that firms disclose (or equivalently be unable to keep undisclosed) the information on the results of their own R&D investment at a certain degree (perfect patenting). However, the non-disclosure of R&D-related result in the AJ setting is in the unilateral interest of each non-cooperative firm. *If the degree of product differentiation is sufficiently high, the prisoner's dilemma can vanish also in the absence of R&D spillovers, in turn, representing the robust outcome according to a game-*

theoretic approach (Buccella et al., 2021). When $d = 1$, Eq. (24) boils down to $g_C(\beta, 1) \coloneqq \frac{2(1+\beta)^2(2-\beta)}{27\beta}$, so that $g_C(0,1) \to +\infty$ if $\beta \to 0$.

Therefore, g can be higher or lower than $g_C(\beta, 1)$ if and only if $0 < \beta \leq 1$ if products are homogeneous ($d = 1$). This implies that, in the case of no disclosure or perfect patenting ($\beta = 0$), it is not possible to find a finite value of g to solve the prisoner's dilemma of the R&D game if products are homogeneous or perfect substitutes.

Differently, if products are heterogeneous (imperfect substitutes or complements), then $g_C(0, d) \coloneqq \frac{2}{(1-d)(2+d)^2} > 0$ if $\beta = 0$ (no disclosure). Therefore, g can be higher or lower than $g_C(\beta, d)$ for any $0 \leq \beta \leq 1$ if products are heterogeneous ($-1 \leq d < 1$). This means that there exists a finite value of g that solves the prisoner's dilemma of the R&D game, regardless of the extent of R&D spillovers, including the case of no disclosure, which is in the unilateral interest of each firm in the AJ setting. This result is clarified in Proposition 4, which concentrates on the case of product heterogeneity ($d < 1$). This case avoids facing the case of perfect substitutability addressed by Bacchiega et al. (2010), according to which the prisoner's dilemma can vanish only when firms are disclosing. This case is reported in Corollary 1 following Proposition 4.

Define now some thresholds useful to disentangle the results of the points in which Proposition 4 is divided depending on the relative value of w. All the cases of Proposition 4 (product heterogeneity) and Corollary 1 (product homogeneity) following this classification are detailed in Proposition A.2 and Corollary A.1 (online Appendix), respectively. Proposition 4 and Corollary 1 are written for the case $\frac{2}{3} \leq w < 1$ for reason of clarity of the presentation.

Now, let

$$\bar{\beta}_C(d, w) \coloneqq \frac{(8+d^2)(1-w)+d(10w-6)-\sqrt{(1-w)(2-d)^2[16+(1-w)d^2+(16w-8)d]}}{8w}, \tag{25}$$

be a threshold value of the intensity of the R&D externality such that $g_T^{I/NI}(\beta, d, w) = g_C(\beta, d)$ in the (β, g) space prevailing when $0 < w \leq \frac{5}{8}$, where $\bar{\beta}_C(d, w) \geq 0$ for any $\bar{d}_C(w) \leq d \leq 1$ and $\bar{\beta}_C(d, w) < 0$ for any $-1 \leq d < \bar{d}_C(w)$, and

$$\bar{d}_C(w) \coloneqq \frac{3(1-w)-\sqrt{(1-w)(1+3w)}}{2-3w} > 0, \tag{26}$$

for any $0 < w \leq \frac{5}{8}$ represents a threshold value of the degree of product differentiation such that $\bar{\beta}_C(d, w) = 0$. Let

$$\bar{\bar{\beta}}_C(d) \coloneqq \frac{3d-2}{6-d}, \tag{27}$$

be a threshold value of the intensity of the R&D externality such that $g_{SC}^{\beta_{low}}(\beta, d) = g_C(\beta, d)$ in the (β, g) space prevailing when $\frac{5}{8} < w < \frac{2}{3}$, where $\bar{\bar{\beta}}_C(d) \geq 0$ for any $\bar{\bar{d}}_C \leq d \leq 1$ and $\bar{\bar{\beta}}_C(d) < 0$ for any $-1 \leq d < \bar{\bar{d}}_C$, and

$$\bar{\bar{d}}_C = \frac{2}{3} \tag{28}$$

represents a threshold value of the degree of product differentiation such that $\bar{\bar{\beta}}_C(d) = 0$. Let

$$d_{CC}(w) \coloneqq 6 - 8w, \tag{29}$$

be a threshold value of the degree of product differentiation such that $\bar{\beta}_C(d, w) = \bar{\bar{\beta}}_C(d)$, where $d_{CC}(w) > 0$ for any $0 < w < \frac{3}{4}$ (so that it is certainly positive for any $0 < w < \frac{2}{3}$), $d_{CC}(w) \leq 1$ for any $\frac{5}{8} \leq w < 1$ and $d_{CC}(w) > 1$ for any $0 < w < \frac{5}{8}$. Therefore, if $0 < w < \frac{5}{8}$, then $\bar{\beta}_C(d, w) < \bar{\bar{\beta}}_C(d)$ for any $-1 \leq d \leq 1$; if $\frac{5}{8} \leq w < \frac{2}{3}$, then $\bar{\beta}_C(d, w) \geq \bar{\bar{\beta}}_C(d)$ for any $d_{CC}(w) \leq d \leq 1$ and $\bar{\beta}_C(d, w) < \bar{\bar{\beta}}_C(d)$ for any $-1 \leq d < d_{CC}(w)$. In addition, $d_{CC}(w) > d_T^{I/NI}(w) > \bar{d}_C(w) > \bar{\bar{d}}_C$ for any $0 < w < \frac{2}{3}$, $d_{CC}(w) < d_T^{I/NI}(w) < \bar{d}_C(w) < \bar{\bar{d}}_C$ for any $\frac{2}{3} < w < 1$, $\beta_{T_2}^{I/NI}(d, w) < \bar{\beta}_C(d, w)$ for any

$d_T^{I/NI}(w) \leq d \leq 1$ and $\beta_{T_2}^{I/NI}(d,w) < 0$ for any $d < d_T^{I/NI}(w)$ if $\frac{1}{2} \leq w < \frac{5}{8}$, and $\bar{\bar{\beta}}_C(d) \leq \beta_{T_2}^{I/NI}(d,w)$ for any $d_{CC}(w) \leq d \leq 1$ and $\bar{\bar{\beta}}_C(d) > \beta_{T_2}^{I/NI}(d,w)$ for any $d < d_{CC}(w)$ if $\frac{5}{8} \leq w < \frac{2}{3}$.

Proposition 4 and Corollary 1 (resp. Proposition A.2 and Corollary A.1 in the online Appendix) summarise the Nash equilibrium outcomes of the R&D investment decision game with heterogeneous and homogeneous products, respectively, under quantity competition for $\frac{2}{3} \leq w < 1$ (resp. for all the other values of w, including the case $w \to 1$). We chose to present the analytical details of the cases of product complementarity and product substitutability (divided according to several thresholds of w), along with the corresponding geometrical projections (from Figure A.4 to Figure A.7) directly in the online Appendix, as they are cumbersome and do not add innovations in the paradigms emerging endogenously in the R&D investment decision game with product differentiation, which remain the prisoner's dilemma and the anti-prisoner's dilemma (deadlock).

Proposition 4 [Nash equilibrium outcomes when products are substitutes]. The outcomes of the R&D investment decision game with quantity competition and product differentiation (substitutability) when $\frac{2}{3} \leq w < 1$ are the following.

[1] Products are substitutes ($d > 0$). Let $\frac{2}{3} \leq w < 1$ hold. If $\bar{\bar{d}}_C = \frac{2}{3} \leq d < 1$ then [1.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g_{SC}^{\beta_{low}}(\beta,d) < g < g_C(\beta,d)$, and [1.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g > g_C(\beta,d)$, for any $\bar{\bar{\beta}}_C(d) < \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

[2] Products are substitutes ($d > 0$). Let $\frac{2}{3} \leq w < 1$ hold. If $0 < d < \frac{2}{3} = \bar{\bar{d}}_C$ then (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$.

Proof. [1] If products are substitutes ($d > 0$), $\frac{2}{3} \leq w < 1$ and $\frac{2}{3} \leq d < 1$ then $g_{SC}^{\beta_{low}}(\beta,d)$ is binding for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$, $g_T^{I/NI}(\beta,d,w)$ is binding for any $\beta_{T_2}^{I/NI}(d,w) < \beta < \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ in the (β,g) space (from Proposition A.1 in the online Appendix). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \leq \beta \leq 1$ and $\frac{2}{3} \leq w < 1$. Then, $g_C(\beta,d) > g_{SC}^{\beta_{low}}(\beta,d)$ for any $0 \leq \beta < \bar{\bar{\beta}}_C(d)$, $g_C(\beta,d) < g_{SC}^{\beta_{low}}(\beta,d)$ for any $\bar{\bar{\beta}}_C(d) \leq \beta < \beta_{T_2}^{I/NI}(d,w)$, $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ for any $\beta_{T_2}^{I/NI}(d,w) \leq \beta < \beta_{T_1}^{I/NI}(d,w)$, $g_C(\beta,d) < g_T^{I/I}(\beta,d,w)$ for any $\beta_{T_1}^{I/NI}(d,w) \leq \beta \leq 1$, where $\bar{\bar{\beta}}_C(d) > 0$ and $\bar{\bar{\beta}}_C(d) < \beta_{T_2}^{I/NI}(d,w) < \beta_{T_1}^{I/NI}(d,w)$ for any $\frac{2}{3} \leq w < 1$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C > 0$ for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g_{SC}^{\beta_{low}}(\beta,d) < g < g_C(\beta,d)$, and $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(d)$ and $g > g_C(\beta,d)$, for any $\bar{\bar{\beta}}_C(d) \leq \beta < \beta_{T_2}^{I/NI}(d,w)$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) \leq \beta < \beta_{T_1}^{I/NI}(d,w)$, $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) \leq \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$. [2] If products are substitutes ($d > 0$), $\frac{2}{3} \leq w < 1$ and $0 < d < \frac{2}{3}$ then $g_{SC}^{\beta_{low}}(\beta,d)$ is binding for any $0 \leq \beta \leq$

$\beta_{T_2}^{I/NI}(d,w)$, $g_T^{I/NI}(\beta,d,w)$ is binding for any $\beta_{T_2}^{I/NI}(d,w) < \beta < \beta_{T_1}^{I/NI}(d,w)$ and $g_T^{I/I}(\beta,d,w)$ is binding $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ in the (β,g) space (from Proposition A.1 in the online Appendix). In addition, $\frac{\partial g_C(\beta,d)}{\partial \beta} < 0$ for any $0 \leq \beta \leq 1$ and $\frac{2}{3} \leq w < 1$. Then, $\bar{\bar{\beta}}_C(d) < 0$ and $g_C(\beta,d) < g_{SC}^{\beta_{low}}(\beta,d)$ for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$, $g_C(\beta,d) < g_T^{I/NI}(\beta,d,w)$ and $g_C(\beta,d) < g_T^{I/I}(\beta,d,w)$ for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$. Therefore, $\Delta\Pi_A > 0$, $\Delta\Pi_B < 0$ and $\Delta\Pi_C < 0$ for any $0 \leq \beta \leq \beta_{T_2}^{I/NI}(d,w)$ and $g > g_{SC}^{\beta_{low}}(\beta,d)$, for any $\beta_{T_2}^{I/NI}(d,w) < \beta \leq \beta_{T_1}^{I/NI}(d,w)$ and $g > g_T^{I/NI}(\beta,d,w)$, and for any $\beta_{T_1}^{I/NI}(d,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,d,w)$. **Q.E.D.**

Corollary 1 [Nash equilibrium outcomes when products are homogeneous]. Products are perfect substitutes ($d = 1$). Let $\frac{2}{3} \leq w < 1$ hold. Then [1.1] (I,I) is the unique Pareto inefficient Nash equilibrium and the R&D investment decision game with product differentiation is a prisoner's dilemma for any $0 \leq \beta \leq \bar{\bar{\beta}}_C(1)$ and $g_{SC}^{\beta_{low}}(\beta,1) < g < g_C(\beta,1)$, and [1.2] (I,I) is the unique Pareto efficient Nash equilibrium and the R&D investment decision game with product differentiation is a deadlock for any $0 < \beta \leq \bar{\bar{\beta}}_C(1)$ and $g > g_C(\beta,1)$, for any $\bar{\bar{\beta}}_C(1) < \beta \leq \beta_{T_2}^{I/NI}(1,w)$ and $g > g_{SC}^{\beta_{low}}(\beta,1)$, for any $\beta_{T_2}^{I/NI}(1,w) < \beta \leq \beta_{T_1}^{I/NI}(1,w)$ and $g > g_T^{I/NI}(\beta,1,w)$, and for any $\beta_{T_1}^{I/NI}(1,w) < \beta \leq 1$ and $g > g_T^{I/I}(\beta,1,w)$.

Proof. The proof follows the same line of reasoning and uses the same arguments as the proof Proposition 4 by assuming $d = 1$ and knowing that $g_C(\beta,1) \coloneqq \frac{2(1+\beta)^2(2-\beta)}{27\beta}$, so that $g_C(0,1) \to +\infty$ if $\beta \to 0$. This implies that it is not possible to solve the prisoner's dilemma in the absence of R&D spillovers if products are perfect substitutes. **Q.E.D.**

The shape of $g_C(\beta,d)$ (dotted line) and the shapes of the other relevant constraints of the model are depicted in Figure 1A and Figure 1B, which are plotted in the parameter space (β,g) for $w = 0.8$ ($\frac{2}{3} \leq w < 1$). Panel A (resp. B) shows the case of homogeneous (heterogeneous) products, i.e., $d = 1$ (resp. $d = 0.66667$, i.e., product substitutability). These figures aim to help the reader understand the narrative of the analytical results. The sand-coloured region represents the unfeasible parameter space. This region is bounded by the constraints discussed in Proposition A.1 in the online Appendix and tells us that, for any given value of the spillovers, the efficiency of R&D activity should not be too high, i.e., parameter g should not be too low to avoid excessive R&D investments that would contribute to greatly reduce marginal and average production costs and increase output pushing down the market price of products of both varieties at too low a level. However, product differentiation favours technological progress by pushing downwards the stability conditions and the R&D cost conditions in the (β,g) space.

Propositions A.1 (online Appendix) identifies the parametric conditions under which the stability conditions and R&D cost conditions are binding for the R&D investment decision game. Proposition 4 and Corollary 1 show that a sufficiently high degree of product differentiation can solve the prisoner's dilemma. This allows us (i) to let the R&D investment decisions game become a deadlock, irrespective of the intensity of the R&D spillovers, *including the case of no disclosure* (Proposition 4), and (ii) to pinpoint under perfect substitutability that the prisoner's dilemma can be solved if and only if *firms are disclosing on their R&D activity* (Corollary 1). However, this outcome cannot emerge as a Nash equilibrium of a non-cooperative game with complete information where investing firms choose whether to disclose R&D-related information (Buccella et al., 2021), whose subgame perfect Nash equilibrium in the absence of public intervention is perfect patenting.

Though there are several parametric conditions in which either a stability condition or an R&D cost condition is binding, the outcomes of the R&D investment decision game belong to two standard paradigms: the prisoner's dilemma (when the degree of product differentiation is sufficiently low, $d \uparrow$) and the anti-prisoner's dilemma or deadlock (when the degree of product differentiation is sufficiently high, $d \downarrow$). In the former case, investing in R&D represents a dominant strategy and the Nash equilibrium (I,I) is Pareto inefficient, and thus there exists a conflict between self-interest and mutual benefit to undertake cost-reducing innovation. In the latter case, investing in R&D represents a dominant strategy, the Nash equilibrium (I,I) is Pareto efficient, and thus there exists no conflict between self-interest and mutual benefit to undertake cost-reducing R&D.

In the basic AJ model with homogeneous products (Bacchiega et al., 2010) the extent of the R&D spillovers and the efficiency of the R&D technology are the parameters responsible for the occurrence of the prisoner's dilemma or the anti-prisoner's dilemma (see also Burr et al. 2013). On one hand, high spillovers are always expected to reduce the firm's incentives for R&D due to the free-riding effect of the public goods, and thus also tend to reduce the prisoner's dilemma loss of profitability. Additionally, higher costs of R&D effort always reduce the engagement in R&D investments, and thus they indirectly reduce the prisoner's dilemma loss of profitability. On the other hand, when firms engage in a non-cooperative R&D and are entrapped in a profit-worsening dilemma, consumers are better off; moreover, these benefits more than offset the negative impact on the firm's profits, and thus social welfare increases. The prisoner's dilemma (following R&D investments, see Proposition 3) enhances social welfare as a whole; however, it is Pareto-inferior because it benefits consumers but, on the other hand, it hurts firms. The dilemma emerging in the scenario developed by Bacchiega et al. (2010) – homogeneous products or low degree of product differentiation and low or no spillovers – follows a situation in which both players have a dominant strategy, i.e., each player aims at obtaining the best outcome regardless of his opponent's choice. Indeed, no one is unilaterally interested to do not investing in R&D if the rival does not invest in R&D. This is because each player unilaterally prefers to be the unique firm to invest in R&D and thereby increase output and profits rather than forego those advantages by not investing. Moreover, no one is interested to do not investing even when the rival invests in R&D in order not to be the only one to forego the benefits of the investment. Therefore, irrespective of the rival's decision, each player will invest in R&D. However, if both players had decided to cooperate in order not to (jointly) invest in R&D they would have obtained a higher profit by producing a smaller amount of product, reducing consumers' surplus and avoiding the incidence of R&D costs. Thus, by making decisions that guarantee each firm (individually) the best outcome, both players are worse off than under cooperation (in the absence of binding contracts), i.e., the pursuit of individual success leads to collective failure. May the players, knowing that they can achieve this disappointing outcome, reach an agreement to which they both should not invest in R&D? In this scenario the players' choices are compatible when firms choose to invest in R&D so that the (inefficient) Nash equilibrium outcome is a conflict that creates a dilemma. This holds when each firm cannot get the advantages of product differentiation (higher market power) and the spreading of knowledge (positive externality). The scenario can change following, ceteris paribus, an exogenous increase in β that reduces the incentive to be the only firm to invest in R&D (by reducing its profits) and consequently increases the profits of the non-investing rival (that can benefit from a greater positive externality). However, the increased spill-over allows both firms to make a higher profit than if they did not invest. This is because of the benefits that they can jointly obtain when they spread their technology to the rival, which helps to solve the dilemma.

Other aspects being equal, product differentiation, by increasing firm's profits, helps to relax the tightness of the relevant constraints in the (β, g) space and contributes to pushing down the stability conditions and the R&D cost conditions, thereby allowing firms to increase the efficiency of the R&D activity (lower values of g) and reduce the need to sharing R&D related information (lower values of β). This situation eventually opens the route for a solution to the prisoner's dilemma identified by Bacchiega et a. (2010) *irrespective of the R&D spill-over*. This dramatic change allows for a scenario in which product differentiation increases profits of every firm irrespective of whether each player is playing I or NI. This is because of the increased market power that product differentiation allows for. The percentage increase in profits when both firms invest in R&D is substantial, i.e., the R&D investment strengthens the increase in the market power of each firm due to product differentiation and contributes to solving the dilemma. Firms therefore jointly have an incentive to invest in R&D even when the degree of spillovers is low or zero.

Figures 1A and 1B provide qualitative support to the analysis carried out so far by helping to clarify (for $w = 0.8$) the role of product differentiation in the AJ setting, showing specifically the role of the stability conditions and the R&D cost conditions in determining the Nash equilibrium outcomes of the game in the case of homogeneous products ($d = 1$) and heterogeneous products ($d < 1$). Amongst other information, the figures clearly show that product differentiation pushes all constraints downward, thus increasing the potential of technological improvements ($g \downarrow$) for any given value of the extent of the spillovers.

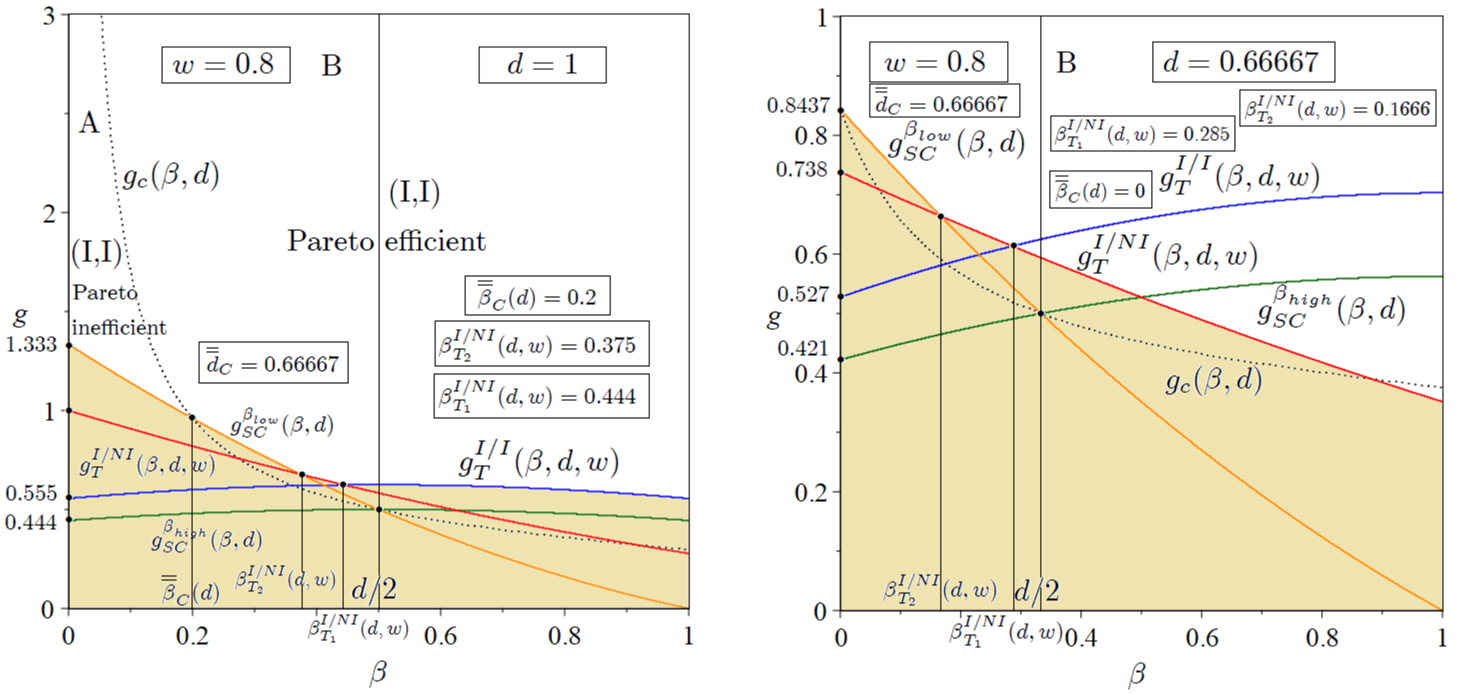
When the degree of product differentiation is low ($d \uparrow$), the results of Bacchiega et al. (2010) hold, so that, for a given value of the efficiency of R&D activity, g, the Nash equilibrium outcome is Pareto inefficient when β is sufficiently low and Pareto efficient when β becomes larger. However, unlike the case of homogeneous products studied by Bacchiega et al. (2010), there exists a finite value of g such that the game turns to be a deadlock *also in the absence of spill-over effects* ($\beta = 0$). Let us begin with a parametric configuration such that the R&D game is a prisoner's dilemma with a low spill-over effect. In that case, I is a dominant strategy of each rational and selfish player (allowing us to obtain the best outcome regardless of the rival's choice). However, the Nash equilibrium (I,I) is sub-optimal, as firms have a joint incentive to coordinate towards NI, though each of them has a unilateral incentive to play I and invest in R&D. Given the payoff matrix, no one is interested in playing NI if the rival plays NI. This is because everyone prefers to unilaterally invest in process innovation, increase output, and obtain a higher profit. Also, no one is interested in playing NI, even when the rival plays I. This is because each player prefers to forgo being NI rather than be the only one to play NI, which leads to the worst possible outcome, as a portion of its profits is eroded by the rival that is investing in R&D. Thus, regardless of the rival's activity, no one will play NI, and everyone will forgo obtaining a higher profit by becoming an investing firm. However, if both players decide to cooperate in becoming a non-investing firm, they will be better off. Thus, by making decisions that guarantee each player the best outcome unilaterally, both players are worse off than they would have been if they had both chosen to play NI. The pursuit of individual success can thus lead to a collective failure. However, we cannot then expect that both players, aware that they may face disappointment by this result, will reach an agreement to jointly play NI. Players' choices are consistent if and only if no one should regret them after learning of the rivals' strategy. In an R&D game with this parameter configuration, players make consistent decisions when they choose to play I. After both firms have chosen to play I, no one will regret it, as anyone who had decided to play NI unilaterally would have been worse off. In contrast, players would have made conflicting decisions if they had both chosen to cooperate and play NI. In this case, each would have regretted their choice as playing I unilaterally would have been better off resulting in a higher payoff. On the one hand, we must expect players to be able to achieve an agreement prescribing consistent choices (R&D investments), because everyone is aware that no one after the agreement will be interested in the violation if the rival complies with it. On the other hand, we should not expect players to be able to achieve an agreement prescribing choices that are not mutually consistent (no R&D investment). This is because everyone is aware that no one will be interested in complying with that agreement if the rival complies with it.

In this setting, an increase in the degree of R&D spillovers allows (i) an increase in profits of the non-investing firm in the asymmetric sub-game, and (ii) a reduction in profits of the investing firm in the asymmetric sub-game. Indeed, the former player gains from the free-riding activity at the expense of the investing rival (the latter). However, both increase their profits under the strategic profile I. Therefore, the game becomes a deadlock, and there is no conflict between self-interest and mutual benefit to undertake cost-reducing R&D.

When the degree of product differentiation increases ($d \downarrow$), the market share (in the markets for the relevant products) of each firm tends to increase, and firms' profits become larger than when the degree of product differentiation was lower. Under this parameter configuration, there is no need to disclose that both firms invest in R&D, and the R&D game becomes a deadlock irrespective of the parameter scale.

It is now important to turn to the study of social welfare following the results of Proposition 3, i.e., $W^{*I/I} > W^{*NI/NI}$ for any $0 \leq \beta \leq 1$. Indeed, though in the absence of spillovers and homogeneous products R&D investments improve social welfare, firms are worse off (the game is a prisoner's dilemma). Differently, product differentiation represents a win-win result, as social welfare under I/I is larger than under NI/NI, but firms are better off (the game is an anti-prisoner's dilemma).

In addition, the government should intervene to favour product differentiation. This is because $W^{*I/I}$ is a monotonic decreasing function of d, so that increasing the degree of product differentiation ($d \downarrow$) is welfare improving. This is because both profits of the investing firms and consumers' surplus increase when d decreases, though the latter is the result of two opposing effects (in the case of linear demand): 1) it reduces due to the reduction in the number of available goods, and 2) it increases due to the increase in the variety of products available in the markets.



(A) (B)

Figure 1. The R&D investment decision game: Nash equilibrium outcomes when $\frac{2}{3} \leq w < 1$ ($w = 0.8$) and $d = 1 > \bar{\bar{d}}_C = \frac{2}{3}$ (Panel A) and $d = \bar{\bar{d}}_C = \frac{2}{3}$ (Panel B). The sand-coloured region represents the parametric area of unfeasibility in the (β, g) space. In Panel A, the R&D game is a prisoner's dilemma (area A) and an anti-prisoner's dilemma (area B). In Panel B, the prisoner's dilemma is solved, and the R&D game is an anti-prisoner's dilemma regardless of the parameter scale (area B).

4. R&D subsidies as a welfare maximising tool in an environment with product differentiation

This section aims at complementing the analysis made so far with some policy prescriptions. The main result of the R&D game with product differentiation is related to the solution of the prisoner's dilemma emerging in Bacchiega et al. (2010). In this regard, product differentiation allows R&D investing firms to increase their market power that, in turn, causes an increase in profits letting the game become a deadlock instead of a prisoner's dilemma, irrespective of the main parameters of the model. This opens the route to policies that favour horizontal product differentiation as social welfare under I/I is greater than social welfare under NI/NI. Indeed, these policies may require designing an ad hoc public incentive towards advertising investments – which should therefore be conducted by the firm in addition to the R&D effort by endogenizing d, but may be difficult to implement as product differentiation is a marketing strategy related to customers' preferences aimed at reducing the relative degree of competition amongst firms. Amongst other things, in fact, this kind of policy requires identifying several product characteristics and information that may be very costly for the public authority.

Alternatively, a policy instrument that can be used as a welfare-maximising tool in this environment is represented by the standard R&D subsidy aimed at incentivising the private R&D effort along the line of Hinloopen (1997), Hinloopen (2000) and Amir et al. (2019). Interestingly, this instrument does not change the nature of the R&D investment decision game (which continues to be a deadlock with investing firms irrespective of the parameter scale in either case of exogenous and optimal subsidy) and the role of product differentiation in the model. Therefore, in what follows, we briefly present the structure of the R&D incentive scheme in the subgame I/I and compare the welfare outcomes.

The public policy to incentivise the firm's R&D effort directly follows the model of Amir et al. (2019) with homogeneous products and aims at showing that product differentiation plays a relevant at the optimum. Specifically, 1) unlike Amir et al. (2019), when products are differentiated the optimal subsidy is positive also in the absence of R&D spillovers ($\beta = 0$), 2) when products are homogeneous (Amir et al., 2019) the optimal subsidy is positive if and only if $\beta > 0$.

From a modelling perspective, we assume that the R&D subsidies towards firm i ($\Sigma_i > 0$) and firm j ($\Sigma_j > 0$) are financed at a balanced budget with a uniform non-distorting lump-sum tax ($T > 0$) on the side of consumers (i.e., the tax does not cause violations of the conditions for social efficiency). The available post-tax exogenous nominal income of the representative consumer ($M - T > 0$) is high enough to avoid corner solutions. Therefore, the government budget constraint reads as follows:

$$T = \Sigma_i + \Sigma_j, \tag{30}$$

where $\Sigma_i = \sigma \frac{g}{2} x_i^2$, $\Sigma_j = \sigma \frac{g}{2} x_j^2$ and $0 \leq \sigma \leq 1$ is the subsidy rate. Definitively, the government first announces the policy, and then let firms be engaged in the R&D investment decision game discussed so far.

Firm i's profits in the I/I sub-game become:

$$\Pi_i^{I/I} = \left(1 - q_i - dq_j\right)q_i - \left(w - x_i - \beta x_j\right)q_i - \frac{g}{2}x_i^2(1 - \sigma). \tag{31}$$

Once one gets the main equilibrium outcome of the subgame, social welfare at the equilibrium can be computed as $W^{*I/I}(\sigma) = CS^{*I/I}(\sigma) + PS^{*I/I}(\sigma) - T^{*I/I}(\sigma)$. The analysis of $W^{*I/I}(\sigma)$ leads to the following proposition.

Proposition 5 [Policy]. Introducing R&D subsidies when products are horizontally differentiated is welfare improving, and there exists a welfare-maximising optimal policy if and only if:

$$\sigma = \sigma^{OPT}(d) := \frac{(2+d)[(1+\beta)(1-d)+2\beta]}{(3+d)(2-d)(1+\beta)} < 1, \tag{32}$$

where (1) $\sigma^{OPT}(d) > 0$ for any $0 \leq \beta \leq 1$ if $d < 1$, (2) $\sigma^{OPT}(1) := \frac{3\beta}{2(1+\beta)} > 0$ for any $0 < \beta \leq 1$ if $d = 1$ and $\sigma^{OPT}(1) = 0$ if $\beta = 0$ and $d = 1$, and (3) $W^{*I/I}(\sigma)$ is a monotonic decreasing function if $\beta = 0$ and $d = 1$.

Proof. Differentiating $W^{*I/I}$ for σ one gets:

$$\left.\frac{\partial W^{*I/I}(\sigma)}{\partial \sigma}\right|_{\sigma=0} = \frac{4g^2(1-w)^2(2-d)(2+d)^3(2-d\beta)[(1+\beta)(1-d)+2\beta]}{[g(2-d)(2+d)^2-2(1+\beta)(2-d\beta)]^3} > 0, \tag{33}$$

for any $g > g_{SC}^{\beta_{high}}(\beta, d)$, and

$$\frac{\partial W^{*I/I}(\sigma)}{\partial \sigma} = \frac{4g^2(1-w)^2(2-d)(2+d)^2(2-d\beta)\{(2+d)[(1+\beta)(1-d)+2\beta]-\sigma(3+d)(2-d)(1+\beta)\}}{[g(2-d)(2+d)^2(1-\sigma)-2(1+\beta)(2-d\beta)]^3}. \tag{34}$$

Then, $\frac{\partial W^{*I/I}(\sigma)}{\partial \sigma} = 0$ if and only if $\sigma = \sigma^{OPT}(d)$. Therefore, $\frac{\partial W^{*I/I}(\sigma)}{\partial \sigma} > 0$ if $\sigma < \sigma^{OPT}(d)$ and $\frac{\partial W^{*I/I}(\sigma)}{\partial \sigma} < 0$ if $\sigma > \sigma^{OPT}(d)$ for any $g > \frac{g_{SC}^{\beta_{high}}(\beta,d)}{1-\sigma}$. Point 1 immediately follows from (32). Point 2 follows from (32) by setting $\beta = 0$ and $d = 1$. Point 3 follows from (34) by setting $\beta = 0$ and $d = 1$, that is $\frac{\partial W^{*I/I}(\sigma)}{\partial \sigma} = \frac{-288g^2(1-w)^2\sigma}{[9g(1-\sigma)-4]^3} < 0$. **Q.E.D.**

Proposition 5 resembles and complements Amir et al. (2019) under product differentiation. Policy applications are relevant especially because, *unlike Amir et al. (2019)*, the optimal subsidy is positive also in the absence of R&D spillovers. Public policies aiming at subsiding R&D effort generate a clear trade-off between consumers' surplus and producers' surplus. On the consumer side, the policy provides benefit by allowing an increase in output production. On the producer side, the effect of the policy is *a priori* uncertain as it may increase firms' profits until the percentage increase in output is greater than the percentage reduction in the market price that consumers are willing to pay (the market demand is downward sloping). This effect of course depends on the firm's market power that, in turn, is affected by product differentiation. However, profits might reduce if the percentage reduction in the market price is large enough to more than compensate for the positive effects on profits due to the increase in output production. If marginal benefits (as measured by the increase in both consumers' surplus and producers' surplus followed by the augmented output production) exceed marginal costs (as measured by both the societal cost of financing the policy and the reduction in the producers' surplus caused by the reduction in the market price), then it is convenient to increase the R&D subsidy to increase welfare. In contrast, when economic costs outweigh economic benefits, it is convenient to reduce the R&D subsidy to increase welfare. Social welfare is maximised only when marginal benefits equal the corresponding marginal costs.

At this point, it might be interesting to graphically compare the social welfare obtained in the subgame I/I (with and without the R&D subsidy), which represents the welfare level corresponding to the Nash equilibrium, with the social welfare obtained in the subgame in which both firms do not invest in R&D (NI/NI). This comparison clarifies the role played by product differentiation relative to the spill-over rate. The existence of product differentiation opens the route to the introduction of R&D subsidies as a welfare-maximising tool also in the *absence of R&D spillovers* (which is the robust outcome to a correct game-theoretic approach based on the AJ setting; see Buccella et al. (2021), as the increase in profits (along with the consumers' surplus) more than compensates the societal costs to finance the policy. The main results are depicted in Figure 2. Specifically, Panel A and Panel B refer to product homogeneity ($d = 1$) and are plotted for $\beta = 0$ and $\beta = 0.2$, respectively. Differently, Panel C and Panel D refer to product substitutability ($d = 0.5$) and are plotted for $\beta = 0$ and $\beta = 0.2$, respectively. The figures clearly show that under product substitutability there exists a welfare-maximising subsidy rate also in the case of perfect patenting, in turn, opening the route for the use of a fiscal tool to incentivise the R&D effort of selfish firms when there is no spreading of knowledge.

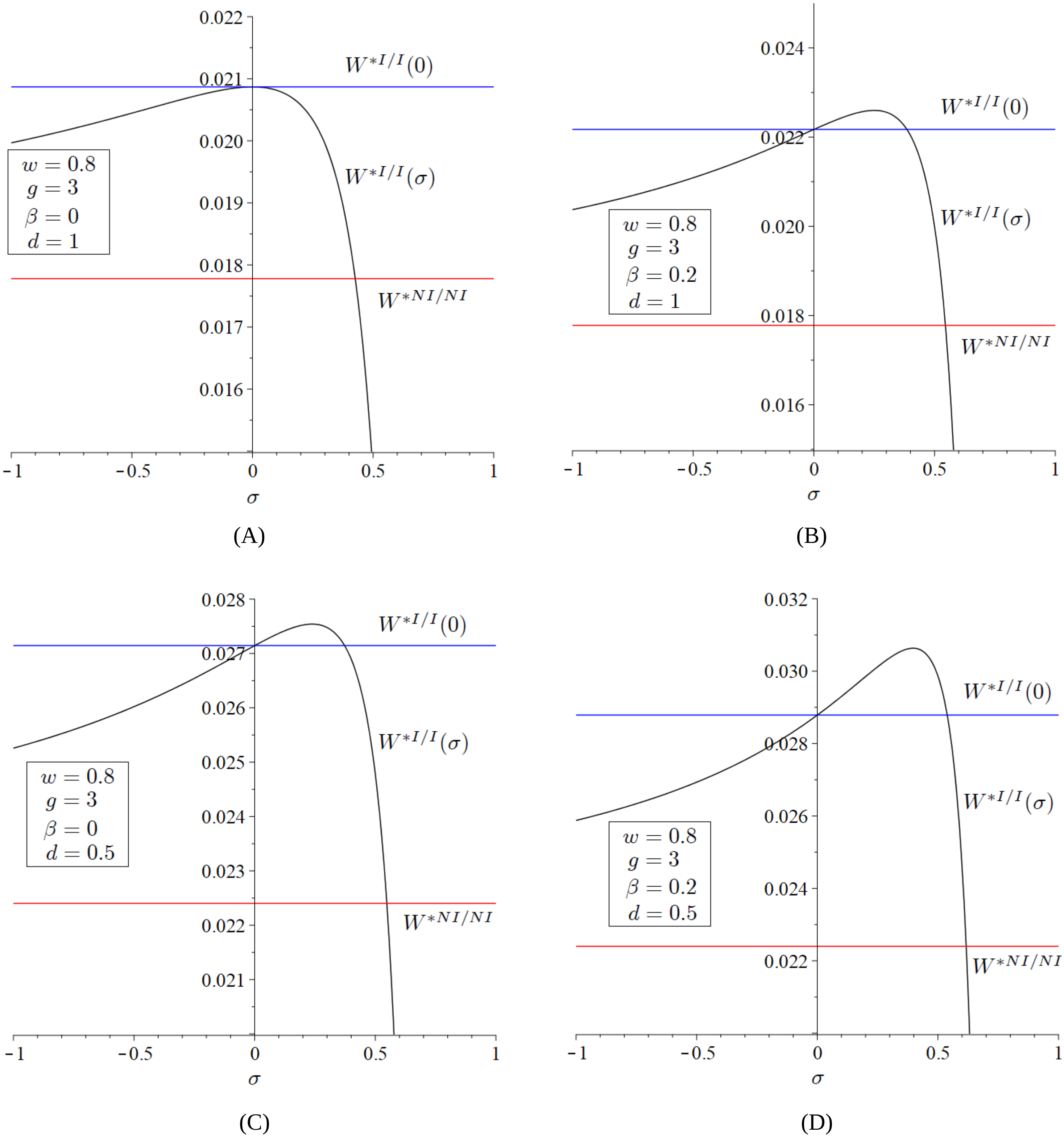


Figure 2. Social welfare and R&D subsidy when products are homogeneous ($d = 1$), Panel A and Panel B, and heterogeneous ($d = 0.5$), Panel C and Panel D. Panels A and C are plotted for $\beta = 0$. Panel B and Panel D are plotted for $\beta = 0.2$. Negative values of σ are shown for clarity of presentation only.

5. Conclusions

This article departs from d'Aspremont and Jacquemin (1988, 1990) and deepens the analysis of the non-cooperative version of the cost-reducing R&D model with spillovers using a game-theoretic approach. The work aims at augmenting Bacchiega et al. (2010), who concentrate on homogeneous products and identify an interplay between the extent of technological spillovers and the efficiency of the R&D activity in determining whether the game is a prisoner's dilemma or an anti-prisoner's dilemma among firms investing in R&D. However, in their work, 1) the prisoner's dilemma vanishes if and only if the extent of technological spillovers is sufficiently high, which would require firms to disclose (or equivalently to be unable to keep undisclosed) the information on the results of their R&D investment, and 2) this scenario cannot emerge as a Nash equilibrium outcome in which non-cooperative investing firms choose whether to disclose the R&D-related results to the rival. Generalising on the assumption of product differentiation à la Singh and Vives (1984) allows solving the dilemma also in the *absence of R&D spillovers* (which is the robust outcome to a correct game-theoretic approach based on the AJ setting) in both cases of quantity and price competition.

Unlike the previous literature, this article (i) provides a thoughtful and detailed analysis of the role of the constraints needed to define the feasibility of the non-cooperative version of the R&D investment decision game à la AJ with homogeneous and heterogeneous products, (ii) identifies the parametric regions in which the game is a prisoner's dilemma or an anti-prisoner's dilemma, and (iii) shows the conditions for making the prisoner's dilemma disappear *in the absence of R&D spillovers*.

The work additionally shows the effects of public subsidies to the R&D effort. This fiscal tool can be used in a simpler way than an incentive policy for product differentiation, which is much more difficult to implement considering that it involves consumers' preferences so that regulator would need to have fine-tuned information to calibrate an incentive subsidy for firms' advertisement investments. Unlike the existing related literature (e.g., Amir et al., 2019), product differentiation allows the R&D policy to be effective also in the absence of R&D spill-over.

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Compliance with ethical standards

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