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EDITORS

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Research Reports (A - H)



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TURNING MOMENTS: THE CROSSROADS OF THE PROSPECTIVE SECONDARY TEACHERS' ATTITUDE TOWARDS MATH

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Relationship with mathematics strongly affects teachers' practices. Specialist mathematics teachers' positive relationship is often taken for granted, although recent studies suggest that this may not be the case. In this paper we present a narrative research aimed at investigating which events do prospective secondary teachers recognise as crucial in the development of their relationship with mathematics. The analysis reveals that experiences of success/failure in mathematics, teacher's opinion, teacher's charisma, and experiences of helping someone with maths are frequent factors influencing these events, impacting on prospective teachers' emotions, perceived competences, and view of mathematics.

INTRODUCTION AND THEORETICAL FRAMEWORK

According to Nias (1996, p. 293) "affectivity is of fundamental importance in teaching and to teachers". Several studies discussed how teachers' relationship with mathematics (in terms of beliefs, identity, emotions, and attitudes towards mathematics) can strongly affect their decisions and their teaching style (De Simone, 2014). How Zembylas (2005) underlines:

Teacher knowledge is located in 'the lived lives of teachers, in the values, beliefs, and deep convictions enacted in practice [...].' These values, beliefs and *emotions* come into play as teachers make decisions, act and reflect on the different purposes, methods and meanings of teaching (p. 467).

Therefore, it appears to be particularly significant to analyse prospective teachers' relationship with mathematics and its development during the school experience. This knowledge is crucial to understanding if and how teacher education programs should be designed to positively affect this relationship. However, research on teacher affectivity in mathematics education has been mainly focused on prospective and in-service primary teachers (Martinez Sierra et al., 2021). These studies have shown that many of them have developed a very bad relationship with mathematics during their school experience (Hannula et al., 2007). For this reason, it appears significant to fill this gap by considering an affective perspective also in the professional development of prospective secondary school mathematics teachers (PT). Unlike future primary teachers, many secondary teachers have a degree in Mathematics: they are therefore specialist mathematics teachers and their passion for mathematics is, in a certain sense, taken for granted. On the other hand, as shown by recent studies, several

brilliant students live “crisis moments” during their university experience in a mathematics degree, also developing strong negative feelings towards mathematics (Di Martino & Gregorio, 2019).

Referring to the Three-dimensional Model for Attitude (TMA, see Fig. 1) introduced by Di Martino and Zan (2010) and within a larger study, we conducted a narrative study among PTs to identify crucial events in the development of their relationship with mathematics.

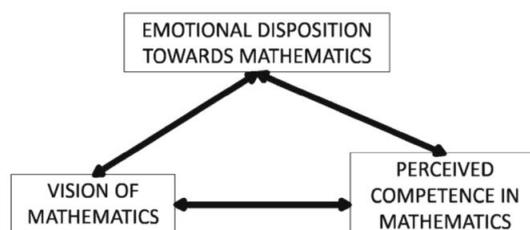


Figure 1: The Three-dimensional Model of Attitude (Di Martino & Zan, 2010).

We were inspired by Bruner’s conceptualisation of the idea of *turning points* (1991) in autobiographical accounts. According to Bruner, the marking of a turning point is a narrator’s device to signal a rupture inside a habitual and expectable routine. Turning points consist in an inner transformation of the narrator, a change in intentional states, linked to a particular external event or experience:

By “turning points” I mean those episodes in which, as if to underline the power of the agent’s intentional states, the narrator attributes a crucial change or stance in the protagonist’s story to a belief, a conviction, a thought (Bruner, 1991, p. 73).

Turning point narratives are characterised by mental verbs indicating internal transformations of which the narrator expresses awareness. We will call *turning moment* an episode, a school period, or a particular experience which PT considers to be determinant in her personal relationship with mathematics and its teaching, since it involved a particular internal change.

Our study was guided by the following research question: *which events do PTs recognise as crucial in the development of their relationship with mathematics?*

METHODOLOGY

The sample and the data collection

The sample of our study consists of 62 students attending the course of Mathematics Education for the master’s degree in Mathematics in an Italian university.

In line with well-established methods of narrative data collection (Kaasila, 2007), students were asked to answer the following open prompt in the first session of the course: *Tell us about an episode from your scholastic past career (at any school level, from primary to university) that you consider particularly significant for the development of your relationship with mathematics. Based on what you remember,*

include in your story the details of the situation you experienced, the emotions you felt and finally explain why you consider the episode significant.

PTs could use as much time as they wanted within the two-hour session and their productions were collected in an anonymous way. According to Connelly and Clandinin (1990):

Humans are storytelling organisms who, individually and socially, lead storied lives. The study of narrative, therefore, is the study of the ways humans experience the world [...] teachers and learners are storytellers and characters in their own and other's stories (p. 2).

In this frame, how (form) and why (reason) the narrator describes her experiences matter more than a present objectivity of the narrated facts.

Approach to the data

To analyse the collected narratives, we referred to two main independent dimensions, *holistic vs categorical* and *content vs form*:

The first dimension refers to the unit of analysis, whether an utterance or section abstracted from a complete text or the narrative as a whole. [...] The second dimension, that is, the distinction between the content and form of a story, refers to the traditional dichotomy made in literary reading of texts. (Lieblich et al., 1998, p. 12).

A purely categorical or holistic approach is not practically possible, whereas the combination of the different dimensions allows for a deeper and differentiated understanding of the narratives. In particular, we developed a holistic approach to the data to identify narratives containing events considered crucial by the narrator for her relationship with mathematics and its teaching. Regarding the content/form dimension, the content was the main focus of our analysis to recognize turning moments in the crucial events that participants reported; however, the recurrence of some expressions was considered. After this analysis, we identified recurrent themes characterising the turning moments through a categorical approach, discussing which dimensions of the TMA were involved. This process of analysis was conducted and finally discussed using an investigator triangulation method.

RESULTS

Five PTs did not respond to the given prompt. Four PTs described a stable relationship with mathematics during the school experience. These narratives do not include any event recognized as crucial for the development of the relationship with math, although, in one case, fluctuating emotions are reported (PT11: “over the years it has been a relationship of *odi et amo*”). Thus, the final corpus was composed of 53 narratives of events. Within them, we recognized two types of events:

- Single *episodes* identified as crucial for the relationship with mathematics.
- *Periods* perceived as crucial for the relationship with mathematics. Periods can be short-lived, such as the preparation for the high school diploma, of

medium duration, such as the encounter with a particular teacher, or of longer duration, such as the experience during a whole school level.

The corpus contains a total of 68 narrated events: 39 episodes and 29 periods (some PTs reported more than one event). For both types of events, we determined the school levels in which they occurred (Table 1). It is conceivable that, given the prompt, all the 53 selected narratives include episodes or periods that are crucial for the narrator. However, in this paper, we discuss more in depth those narratives in which the narrator explicitly identifies an episode or a period as a *turning moment* for her relationship with mathematics. 39 turning moments were selected according to the above criteria.

Account type	Primary	Middle	High	University
Episodes	8	3	22	6
Periods	1	4	19	5
Turning moments	5	4	19	11

Table 1: School levels of the accounts, according to the different typologies.

From the point of view of *form*, to turning moments group belong narratives of events in which *mental verbs* appear (Bruner, 1991), indicating the narrator's access to new consciousness, such as, for example, the *emergence of an intention for the future* (PT53: "So once I overcame the initial difficulties due to the new approach to the discipline, I understood that mathematics would be present in my future"), the *achievement of a certain view of mathematics* (PT45: "This made me understand something important: you can always find a solution"), or a *new awareness in the perceived competences*, in mathematics or in its teaching (PT8: "this made me think that maybe I have the gift of a good teacher). In addition, narratives in which there are expressions that indicate a *beginning*, in the context of the relationship with mathematics, also fall into this category: as an example, the adoption of a different method of study (PT49: "Since then I have changed, I started to apply myself more to mathematics") or a new emotional disposition (PT18: "it was from that moment that I started to have fun").

Analysing the *content* of the turning moments' narratives, four main themes emerge as factors influencing the turning moments: experiences of success/failure in mathematics, teacher's opinion about PT's mathematical competence, teacher's charisma, and experiences of helping someone with maths. The related categories of narratives are not disjointed from each other: PTs often refer to more than one factor. We present each of them in detail, also identifying which dimensions of the TMA model are involved.

Experiences of success/failure in mathematics influence about half of the turning moments narrated. In most cases the *success* experience is linked to a school test or a mathematical competition, resulting in a *positive change in the student's perceived competences*. As we could expect, from these accounts *positive emotions* emerge,

sometimes very strong (PT56: “that feeling was unforgettable and indescribable”), although often preceded by strongly described moods of *anxiety* (PT41: “panic while waiting [for an evaluation]”; PT56: “heart in the throat while waiting”). Instead, in some cases, the success experiences lead to *contrasting emotional states* (PT18: “this is where my troubled love-hate relationship with mathematics began”).

Among the accounts of *failures* only one is from primary school, all the others regard the transition from one school level to another, in particular the transition to high school or university. They are almost always characterised by an *abrupt negative change in the perceived competence*, caused by a bad result in a school test or in a university exam. Nevertheless, although often involving strong negative emotions, such as *sadness*, *anger*, *sorrow*, none results in surrender, but all have a *story of redemption* as consequence (PT19: “I felt somehow encouraged to study mathematics in order to succeed, not to feel inferior to others”). In these stories, PTs claim to have changed their method of study, to have worked hard in a different way, to have felt spurred on.

Teacher’s role emerges as decisive in almost half of the turning moment narratives and the main factors influencing them are teacher’s opinion about PT’s competence in mathematics and/or teacher’s charisma.

Teacher’s opinion about PT’s competence in mathematics is a very recurrent theme in the narrated turning moments. In many cases, PTs refer to *teachers’ trust*, which determines the turning moment in different ways. Great trust is narrated as being associated with positive emotions, such as *pride*, or with the *desire not to disappoint* the teacher’s expectations (PT26: “Even on the graduation day, my teacher shook my hand and said ‘Be sure to enrol in maths’. Some trusted me, I won’t let them down”). On the other hand, cases in which the PT feels that she has disappointed the teacher’s trust are associated with very negative emotions, such as great *bitterness*, or with a *desire to recover* that trust. Also, the turning moments are narrated as being caused by the teacher’s *attribution of innate capacities* to the PT as student, determining student’s perceived high competence, or on the contrary by the teacher’s perception of their *lack*, always leading to the PT’s *will to prove the opposite*, through commitment and determination. Sometimes, PTs consider the *admiration* of the teacher for an outstanding performance as decisive for a turning moment. This situation is described as involving positive emotions, such as a *sense of reassurance*, great *gratification* or even a long-term change in the emotional disposition (PT23: “From that moment on, my teacher too changed her opinion of me and I maybe began to love mathematics a little more”).

Teachers are undoubtedly a main actor in PT’s narratives. This very particular sample of students – they are enrolled in a math degree – usually judged their school teachers as “good”, “excellent”, “fascinating”, “passionate”, “enthusiastic”, “open to dialogue with students”. The teachers are narrated as able to arouse students’ *interest* and *passion* for mathematics, or a feeling of *reassurance*, or to spur a *less algorithmic view*

of mathematics. In some cases, a significant *teacher's charisma* is recognized: the feelings induced by the teacher and her acts are described as very intense. This fascination is very often recognized as the main factor for a turning moment (PT29: “his lessons were a joy; they gave me that something new: you know when you fall in love for the first time?!”).

The opposite scenario –a negative experience with a math teacher as the main reason for the distaste for mathematics as described in previous studies involving prospective primary teachers (Coppola et al., 2015)– is surely less frequent in PTs' narratives analysed in this work. However, in some narratives it emerges. In these cases, PTs reported a *loss of enthusiasm* or a *worsening of perceived competence* (then overcome). An interesting case is that of a PT who, having become aware of her teacher's lack of inclusiveness, talks about her motivation to seek redemption in her future teaching activity (PT24: “I realised that this teacher had brought forward four or five ‘elect’ [...] leaving all the others behind. I realised then that I was going to study mathematics, I was going to teach, and I was going to worry about all my students”).

Some PTs report an episode or a period in which they experienced *helping someone with maths* as a turning moment that led them to choose teaching mathematics as their *future profession*. In such accounts, PTs report good perceived teaching skills and positive emotions such as *gratification* and *joy* (PT32: “My desire to teach what little I knew, to notice that after I explained, they were able to finish the exercises, aroused great joy in me and slowly my dream grew”).

Focusing again on the *form* of the turning moments narratives, it is worth observing how in many cases the narratives are ‘teacher-centred’ and that the student appears to have a more passive role. In many cases, teachers' exact *words* are quoted (PT40: “At that point, the professor made a weird face and said, ‘I doubt you'll be able to do it!’”; PT55: “She looked at me smugly, commented ‘my future colleague’”) and teachers' emotions as *disappointment* or *satisfaction* are reported. Moreover, many words are spent on describing the teacher as passionate, enthusiastic, “austere at the right point” (PT13) and taken as a model. Sometimes the passive form of verbs is used, almost as if some student's internal changes have been heavily influenced by the teacher's action. Still on the form, in several cases PTs use strong expressions such as “the straightway” or “my way” when reporting the intention to put more effort into mathematics or to choose a degree course in mathematics.

DISCUSSION AND CONCLUSIONS

In this paper we presented a narrative study aimed at investigating which events prospective secondary teachers recognise as crucial in the development of their relationship with mathematics, as a part of a wider study about this topic. The analysis shows that more than half of these events are turning moments (Bruner, 1991), most of them occurring during high school.

We recognized four categories in the narrated turning moments: success/failure experiences, teacher's opinion about PT's mathematical competence, teacher's charisma, and PTs' helping experiences of someone with maths. In undertaking this study, we expected some different results from previous studies on crucial events with prospective primary teachers (Coppola et al., 2015) as our new sample consisted of people who had already earned a bachelor's degree in Mathematics and would be future specialist mathematics teachers. Contrary to what we might expect, conflicting emotions often emerge in PT's accounts. In some cases, "troubled" or "odi et amo" relationships are narrated and very strong negative emotions, such as anxiety and panic, are often reported alongside positive emotions. As for the future primary teachers, very frequent in the factors influencing the turning moments are the experiences of failure associated, also for PTs, with very strong negative emotions and a sudden lowering in perceived competence. For future primary teachers they very often resulted in life choices aimed at avoiding mathematics. In contrast, for our new sample, these experiences, although causing feelings of crisis, mistrust, and uncertainty, were taken as a *challenge*, either to themselves or to the teacher, which was then won. Many *stories of redemption* therefore emerge. The theme of redemption had emerged with future primary teachers too, but only in the form of a desire for the future and to be realised not so much in the study of mathematics but in teaching it. It had been called a "desire for math-redemption" (Di Martino et al., 2013).

Success/failure experiences leading to turning moments are in most cases linked to moments of official assessment (in particular to grading) and only rarely to different moments of mathematical activity. This could be indicative of how much importance is given to assessment in the educational system of our sample of PTs -and how this is often restricted to the attribution of a numerical grade to the student's performance. The teacher appears to play a primary role in determining the turning moments. As recalled above, the teacher's opinion about PT's competence in mathematics and the teachers' charisma are recurrent factors in the turning moments accounts and many narratives are 'teacher-centred' in the form. Moreover, classmates appear only in a few cases in the accounts (and never as peers, but only as learners or almost antagonists of the protagonist). This seems to us indicative of 'traditional school' experiences, in which the control of class activity is almost entirely in the hands of the teacher.

To conclude, we believe that the collected narratives describe a picture of the PT's attitude towards mathematics that is more complex than it might sound. While prospective primary teachers' attitudes towards mathematics are widely negative and strongly marked by negative experiences with school mathematics (Di Martino et al., 2013), our data showed that PT's experiences are not simply the other side of the coin. The variety of the collected narratives and the different phenomena that emerged from our data suggest the need for further studies to describe possible recurrent paths in prospective secondary teachers' attitudes with mathematics. It seems evident that these paths could be strongly affected by socio-cultural issues: therefore, the development of comparative studies between different countries is strongly encouraged.

This description is not an end in itself: as teacher educators, we strongly believe that knowing and understanding prospective teachers' past is crucial for developing effective training programs.

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