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Anisotropy of the $Q\overline{Q}$ potential in a magnetic field

C. Bonati(1), M. D'Elia(1), M. Mariti(1), M. Mesiti(1), F. Negro(1) and F. Sanfilippo(2)

(1) *Dipartimento di Fisica dell'Università di Pisa and INFN, Sezione di Pisa, Largo Pontecorvo 3, I-56127 Pisa, Italy*
(2) *School of Physics and Astronomy, University of Southampton, SO17 1BJ Southampton, UK*

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Summary. — We study how the static quark-antiquark potential for $N_f = 2+1$ QCD at the physical point gets modified by the presence of a constant and uniform magnetic field. We observe an anisotropy to appear in the potential: it gets steeper in the directions transverse to the magnetic field than in the longitudinal one. By comparing to the case with zero magnetic field, we show that the string tension increases (decreases) in the perpendicular (parallel) direction, while the absolute value of the Coulomb coupling and the Sommer parameter show the opposite behavior.

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1. – Introduction and motivations

The properties of strong interactions in the presence of an intense magnetic background field have attracted much interest in the recent past [1]. Such properties are expected to be relevant for the physics of compact astrophysical objects, of non-central heavy ion collisions (HIC) [2] and of the early Universe. In such contexts the magnitude of the field spans from 10^{10} Tesla up to 10^{16} Tesla ($|e|B \sim 1\,\text{GeV}^2$). Gluon fields, though not directly coupled to electromagnetic fields, may be subjected to significant modifications, through effective QED-QCD interactions mediated by quark loop effects. In this paper, we study in a nonperturbative way how the confining inter-quark potential, that emerges as a property of the gauge fields only, depends on a magnetic field $|e|B$ [3].

2. – Numerical results

We adopted the lattice QCD approach [4] in order to obtain fully non-perturbative results. We made use of a state-of-art discretization of $N_f = 2+1$ QCD by considering the stout smearing improved fermionic action and the tree-level improved Symanzik gauge action (see [3] for the discretization details). A constant and uniform magnetic

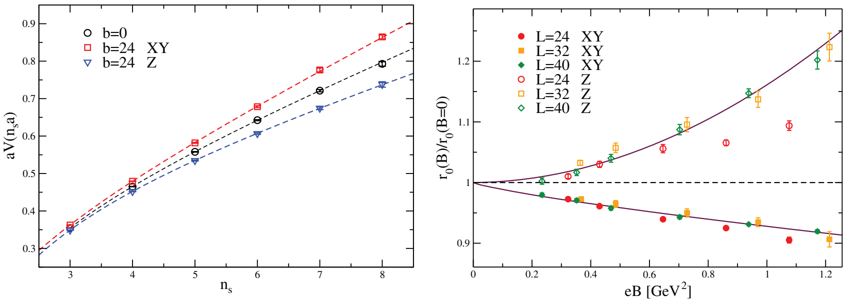

Fig. 1. – Left: $Q\overline{Q}$ potential both for $|e|B = 0$ and for $eB = 0.7\,\mathrm{GeV}^2$ on the finest 40^4 lattice. Right: The Sommer parameter ratio defined in eq. (3) as a function of $|e|B$.

field enters the QCD Lagrangian by modifying the quark covariant derivatives. On the lattice, that amounts to adding proper $U(1)$ phases to the usual $SU(3)$ links entering the Dirac operator. We included the electromagnetic Abelian field as in [5]. We considered $T = 0$, three lattice spacings $a = 0.2173\,\mathrm{fm}$, $0.1535\,\mathrm{fm}$, $0.1249\,\mathrm{fm}$ and a symmetric lattice of fixed physical volume $V = (a\ L)^4 \simeq (5\,\mathrm{fm})^4$, $L = 24,\ 32,\ 40$. We varied the magnetic field, oriented along the Z-axis, going up to $|e|B \sim 1\,\mathrm{GeV}^2$.

We evaluated the expectation values of rectangular Wilson loops $W(a\vec{n}, an_t)$ and determined from them the static potential $aV(a\vec{n})$ (see [4] for a description of the method):

$$aV(a\vec{n}) = \lim_{n_t\to\infty} \log\left(\frac{\langle W(a\vec{n}, an_t)\rangle}{\langle W(a\vec{n}, a(n_t+1))\rangle}\right), \tag{1}$$

where $\vec{n}$ and n_t are the spatial and temporal sides of the loop in lattice units. In fig. 1, left, we show the $Q\overline{Q}$ potential as a function of the quark separation both at zero and nonzero magnetic field on the 40^4 lattice. In the case of a nonzero field we observe a clear anisotropic behavior, with a remarkable separation of the values of the potential measured along the Z or XY directions. A comparison with the $|e|B = 0$ case shows that the potential increases in the transverse directions and decreases in the longitudinal one. This is observed for all the explored setups, starting from $|e|B \simeq 0.2\,\mathrm{GeV}^2$. We fitted the potential, for each value of $|e|B$, according to the Cornell parameterization:

$$aV(an\hat{d}) = \hat{c}_d + \hat{\sigma}_d n + \frac{\alpha_d}{n}, \tag{2}$$

where $\hat{d}$ is a versor along the Z or XY directions, $\hat{\sigma}_d$ is the string tension, α_d the Coulomb coupling. We can keep track of the dependence of the fitted parameters on $|e|B$, by normalizing them to their values at zero field and at the same lattice spacing:

$$R^{\mathcal{O}_d} = \mathcal{O}_d(|e|B)/\mathcal{O}(|e|B = 0). \tag{3}$$

These ratios are shown in figs. 1, right, 2, left and 2, right, respectively for the Sommer parameter (see ref. [4]) $\mathcal{O}_d = \hat{r}_{0d}$, the string tension $\hat{\sigma}_d$ and the Coulomb term α_d. Due to the anisotropy of $aV(a\vec{n})$, we get significant splittings of the ratios, which are of the order of 10–20%. In particular, the string tension increases (decreases), as a function of eB, in the trasverse (longitudinal) direction, while $\hat{r}_0$ and α show an opposite behavior. A similar scenario has been recently observed by means of an holographic approach [6].

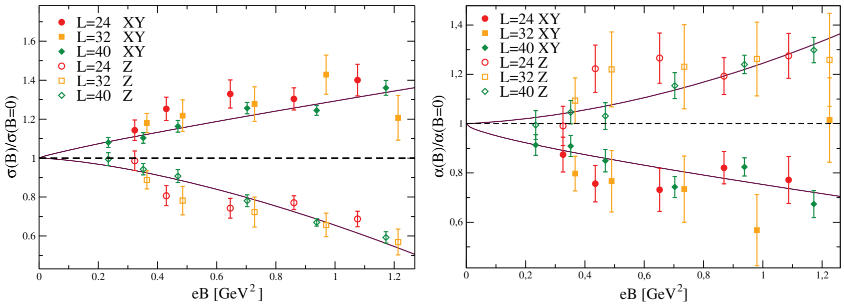

Fig. 2. – The string tension (left) and the Coulomb term (right) ratios defined in eq. (3) as a function of $|e|B$.

Our results show a mild dependence on the lattice spacing; however our present accuracy does not permit a proper continuum extrapolation. We fitted the $L = 40$ data with the following ansatz for the $|e|B-$dependence of the ratios: $R^{\mathcal{O}_d} = 1 + A^{\mathcal{O}_d}\,(|e|B)^{C^{\mathcal{O}_d}}$. The best fit results are reported in [3] and shown as solid lines in the figures.

3. – Discussion

Our findings might be of phenomenological relevance for non-central events in HIC experiments. For example, it was recently observed that the spectrum of mesons gets modified by the presence of a strong magnetic background [7]. Moreover, even slight changes in the energy levels induce sizeable corrections to cross sections, production and decay rates [8]. Our result is relevant in this context: we can expect the spectrum to be furtherly modified by the anisotropy of the $Q\overline{Q}$ potential. Anyhow, in order to have valid predictions, it is necessary to extend the investigation to finite temperature.

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