R&D subsidies in a duopoly market with outsourcing to the rival firm

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Abstract This paper investigates the effects of a public uniform R&D subsidy policy in a downstream duopoly market where a non-integrated firm, which faces a lower marginal cost, outsources inputs from its vertically integrated rival. It is shown that, in this market structure, such a policy has relevant effects largely differentiated between downstream competitors as it can significantly modify the relative market shares and profitability of competing firms. Unlike the standard Cournot setting augmented with R&D, results show that the subsidy policy can have different (counterintuitive) effects on R&D investments, output and profits of the vertically integrated producer and the vertically separated firm, which hold in both cases of exogenous and endogenous (optimal) subsidy. Our findings offer some testable implications and suggest that a subsidy policy in a market with outsourcing to a rival should also consider its different effects on competitors.

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1. Introduction

The study of vertical industries has increasingly become popular over the last decade. Among the various structures characterising vertical relationships, the one in which the production of key inputs is outsourced to external suppliers is widespread in the modern economy.

A relevant case of outsourcing in many important industries is the one in which outsourcing involves retail competitors: for instance, the presence in a duopoly market of a firm outsourcing input supplies from its downstream integrated rival. This kind of outsourcing characterizes industries such as telecommunications, energy and transportation, where the crucial input (represented by the access to network infrastructures) is provided by a vertically integrated incumbent to retail competitors (e.g., Bourreau et al., 2011).

Popular examples in Italy are the landline industry (with Telecom as input supplier to retail competitors such as Wind and Vodafone) and the rail transport industry (with Trenitalia as input supplier[1] – indirectly through RFI, being Trenitalia and RFI subsidiaries of the Ferrovie dello Stato Italiane holding – to the retail competitor, NTV). This type of outsourcing is particularly observed in highly innovative industries in which operators significantly invest in R&D activities to enhance their organisational arrangements and introduce new processes to lower their costs.

Theoretical models dealing with a duopoly market with outsourcing to the rival firm have been developed, among others, by Kamien et al. (1989), Spiegel (1993), Baake et al. (1999), Shy and Stenbacka (2003), Van Long (2005), Arya et al. (2008), and Fanti and Scrimitore (2019). The first five works focus on the effects of nonlinear cost structures, whereas the last two ones consider the strategic decision between price and quantity. However, all of them abstract from the issue of R&D investments as well as R&D subsidies.

Regarding the latter point, we note that, in the real world, policies supporting R&D investments are high in the political agenda of industrialised countries, especially in recent years, in the light of the Fourth Industrial Revolution, also known as Industry 4.0, in which industrial demands and technological advances act together as a driving force to reduce operating costs and improving production processes (Deloitte, 2018). Therefore, governments have started providing R&D subsidies which have often been crucial in promoting many technological advances (e.g., biotechnology). For example, in April 2015, the French Government has identified 34 industrial plans and

[1] However, for the sake of realism, we note that the price NTV (and formally also by Trenitalia) pays to RFI for the access to the railway infrastructures is regulated by the Transport Authority and, in the year 2014, passed from 13 euro/Km to 8.7 euro/km for each train. However, anectodical evidence shows that also RFI may affect the input price through indirect practices as the following: at the Rimini railway station RFI delayed the renovation of platforms to prevent the arrivals of NTV trains and thus NTV offered 500,000 euros to make the renovation. Therefore, the assumption of a monopolistic choice of the input price by the vertically integrated firm amounts, loosely speaking, to implicitly assume, in a regulated industry, a full (resp. zero) “political” power of such a firm (resp. outsourcing firm).

launched the programme “Industrie du Futur” (Industry of the Future). The goal of this plan is to give support to the French companies in developing digital technologies, renovating their business models and upgrading production processes. A backbone of this initiative is to support companies with research funding, subsidies and loan to introduce cutting-edge technologies (Yang and Gu, 2021). Facing the increasing importance of sectors with highly innovative activities, the issue of designing an effective technological policy for such sectors through old and new tools is increasingly relevant (e.g., Hart 1998).

Therefore, this paper is motivated by the aim of filling this gap by extending the above-mentioned models with R&D investments. As the received literature has not yet investigated the potential role of public policies towards R&D, the present work aims at developing a positive analysis of the effects of R&D subsidies by considering a duopoly market with outsourcing to a rival.

The current evidence of R&D subsidy, essentially based on standard Cournot models, is generally targeted to increase R&D investments, outputs, profits and welfare. In this regard, so far, some papers have analysed the effectiveness of public policies aimed to stimulate business-performed R&D in many different contexts, such as Hinloopen (1997, 2000a, 2000b), Haaland and Kind (2006), Liao (2007), Gil-Moltó et al. (2011) and Michalsen (2012).

Specifically, the three works of Hinloopen essentially regard the analysis of cooperative and non-cooperative R&D with spillovers in a Cournot setting à la d’Aspremont and Jacquemin (1988, 1990), with the first and the second ones that are more targeted to R&D subsidisation. In those articles, Hinloopen compared the effects of subsidising private R&D with R&D cooperation, showing that the preferred R&S stimulating policy should be that of considering the opportunity of optimally subsidising an agreement in which firms disclose the outcome of their R&D research.

Subsequently, Haaland and Kind (2006) show that coordinating R&D policies within an economic union can help overcome some problems about firms’ competitiveness in the cases of horizontal and vertical differentiation, whereas Liao (2007) opens the route for studying R&D subsidisation in international markets by investigating non-cooperative and jointly optimal R&D policies in the presence of R&D spillovers. Later, Gil-Moltó et al. (2011) examine the effects of R&D subsidies in private and mixed (private/public) duopolies, showing that the optimal subsidy increases with the extent of R&D disclosure, but it is lower in the private duopoly. Then, it has the standard effects of increasing total R&D and production. Finally, Michalsen (2012) concentrates on a vertically related market and considers the role of an R&D active upstream supplier in a four-stage R&D model by considering direct public funding to firms’ R&D activity. This policy has a positive impact on R&D investments.

More recently, Amir et al. (2019) and Buccella et al. (2021) extended the literature on R&D subsidies in a duopoly à la d’Aspremont and Jacquemin (1988, 1990) showing, among other things, that 1) in the absence of spillovers, market outcomes (i.e., the full non-cooperative scenario and the joint lab scenario) and the second-best planner’s solution coincide, but with spillovers, the three scenarios allow getting R&D investments that fall short of the socially optimal level (Amir et al., 2019), and

2) in the absence of R&D subsidies the unique (Pareto inefficient) Nash equilibrium of the non-cooperative disclosure decision game is the non-disclosure scenario as it is the sole dominant strategy of the game, but R&D subsidisation can dramatically change this result as a Pareto efficient Nash equilibrium in which both firms disclose can be achieved in the market. However, none of them considers the market structure with outsourcing to retail competitors.

Although R&D subsidisation has the general aim of increasing R&D effort, outputs, profits and welfare, in an economy with outsourcing where vertically integrated firms compete with non-integrated firms, R&D subsidies may not achieve all these goals.

To show these outcomes, we develop a simple model, drawing, essentially, from Arya et al. (2008) and Fanti and Scrimitore (2019), to illustrate formally simply and intuitively the effectiveness of publicly provided R&D subsidies. This work studies the effects of a public uniform R&D subsidy policy, building a downstream duopoly market in which a non-integrated firm, which faces a lower downstream marginal cost, outsources inputs from a vertically integrated rival firm. To the best of our knowledge, the present paper is the first attempt to investigate such a policy in a wide typology of vertical industry, and thus different from the above-mentioned contributions in terms of specific objectives, analysis and results. It thoughtfully shows – in the cases of exogenous and endogenous (optimal) R&D subsidisation – that the subsidy policy affects differently R&D investments, output and profits of the vertically integrated producer and those of the vertically separated firm. The former generally gain by the R&D subsidisation. The latter instead can be worse off as output and profits can be reduced by the introduction of the subsidy.

From a broader perspective, our paper belongs to the tradition of public economics dealing with markets and taxation. The paper may also be viewed as a contribution to the wide literature on the role of public intervention in the Industrial Organization literature.

The remainder of the paper is organised as follows. In Section 2, we present a description of the model and its solution. In Section 3, we analyse its implications as to the R&D subsidy policy. Concluding remarks are contained in Section 4.

2. The model

The market demand in the case of a homogeneous product is given by

$$p(Q) = 1 - Q \tag{1}$$

where, as usual, p denotes the product price, and $Q = q_1 + q_2$ is the industry output, given by the sum of the quantities produced by firm 1 and 2, respectively. Following the baseline model of duopoly competition by Arya et al. (2008), firm 1 is a vertically integrated producer (VI) which is the sole producer of an input that is essential for retail production, and whose marginal cost of production is assumed equal to zero, for simplicity. This means that firm 2 buys the input by firm 1 (i.e., a vertically separated firm, VS). The latter charges its retail rival firm the unit price z for the input. As usual in the literature on vertical relationships it is assumed that one unit of input is embodied in each unit of output (i.e., perfect vertical complementarity).

Moreover, both firms conduct process R&D that reduces their marginal production cost by an amount x_i, $0 < x_i < c_i$, $i = 1, 2$. We assume[2] that such an activity is perfectly protected against imitation and its cost is given by $x_i = x_i^2$, which entails diminishing returns to the level of R&D expenditure. Further, firms receive a subsidy to R&D investments, sx_i, where $s \in [0,1]$ is the subsidy per-unit of investment.

When both firms produce retail output $Q = q_1 + q_2$, the retail price is p, the input price is z, and costs to produce retail output are c_1 and c_2, respectively, with $1 > c_1, c_2$, profits are:

$$\pi_1 = zq_2 + (1 - q_1 - q_2)q_1 - (c_1 - x_1)q_1 - x_1^2 + sx_1 \quad (2)$$

$$\pi_2 = (1 - q_1 - q_2)q_2 - (c_2 - x_2)q_2 - x_2^2 + sx_2 - zq_2. \quad (3)$$

As the right side of the equality in Eq. (2) shows, firm 1's profit is given by: 1) a first term representing firm 1's profit from selling the input to firm 2; 2) a second term capturing firm 1's revenue from its retail sales, 3) a third term which is given by its downstream production cost (c_1) net of the reduction due to the R&D investment, x_1, 4) the quadratic cost of R&D, and 5) the R&D subsidy. Firm 2's profit in Eq. (3) derives from its retail sales, its input cost (z), the incremental downstream production cost (c_2) net of the reduction due to the R&D investment, x_2, the quadratic cost of R&D and the R&D subsidy. We assume $c_1 > c_2$, that is the VI firm 1 is less efficient than the rival VS firm 2. This may be coherent with the idea that the VI firm is the "established" firm, while the VS firm - buying inputs from the "established" firm - is the "younger" (a start-up) firm, an assumption made in the recent theory of regulation of entry (Fumagalli et al., 2018; Fumagalli and Motta, 2020).[3]

We develop a three-stage game whose timing is as follows. At stage 1, firms choose R&D levels. At stage 2, firm 1 sets the input price it will charge to firm 2. Finally, at stage 3, under Cournot competition, firms 1 and 2 choose their retail output levels simultaneously and independently. This timing implies that the setting of price input is a choice of a shorter period than that of R&D investment.[4] Solving the game by

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[2] These assumptions strictly follow d'Aspremont and Jacquemin (1988, 1990), which early introduced cost-reducing R&D investments in the basic Cournot duopoly model.

[3] By contrast, Arya et al. (2008) assume $c_1 \leq c_2$; however, this assumption, apart from the problem of its realism, strongly works for a foreclosure of firm 2. In fact, in the case of homogeneous product, as postulated in this paper, we would have only a monopolistic market because firm 2 would always have to close.

[4] However, although maybe less realistic, in principle also reversed stages (i.e., the length of input price contracts is higher (for instance two years) than the length of the validity of a choice of a R&D subsidy rate (for instance, on yearly basis according to which the Budget Law) may be assumed. This analysis is left for future research.

backward induction, standard calculations lead to the following equilibrium outcomes:

$$x_1 = 0.49 - 0.95c_1 + 0.46c_2 + 0.74s \quad (4)$$
$$x_2 = -0.09 + 0.37c_1 - 0.28c_2 + 0.45s \quad (5)$$
$$q_1 = 0.86 - 1.44c_1 + 0.58c_2 + 0.43s \quad (6)$$
$$q_2 = -0.23 + 0.93c_1 - 0.7c_2 - 0.12s \quad (7)$$
$$z = 0.51 - 0.05c_1 - 0.46c_2 + 0.26s \quad (8)$$
$$\pi_1 = 0.35s^2 + (0.38 - 0.53c_1 + 0.15c_2)s + 0.45c_2^{\ 2} + (0.3 - 1.19c_1)c_2 + 0.38 - 1.06c_1 + 1.13c_1^{\ 2} \quad (9)$$
$$\pi_2 = 0.26s^2 + (0.04 - 0.18c_1 + 0.14c_2)s + 0.41c_2^{\ 2} + (0.27 - 1.09c_1)c_2 + 0.04 - 0.36c_1 + 0.73c_1^{\ 2} \quad (10).$$

3. The analysis of a R&D subsidy policy

In the relevant range of the parameters of the model, there is a set of conditions that need to be satisfied. First, the non-negativity on output, specifically, the non-negativity condition on q_2, such that the non-foreclosure condition ensuring that the duopoly structure is always preserved. Second, the non-negativity of retail costs after their reduction due to the R&D investments, that is, $(c_i - x_i) \geq 0, i = 1,2$. Third, the non-negativity of the cost of the investment, $x_i \geq 0, i = 1,2$. Formally, those conditions are enunciated in the following Lemmas.

Lemma 1. a) $x_1 \geq 0, \Leftrightarrow c_1 \leq c_{1,0} = 0.51 + 0.48c_2 + 0.78s$; b) $q_1 \geq 0 \Leftrightarrow c_1 \leq c_{1,1} = 0.59 + 0.4c_2 + 0.3s$; c) $(c_1 - x_1) \geq 0 \Leftrightarrow c_1 \geq c_{1,2} = 0.25 + 0.24c_2 + 0.38s$.
Proof: see the Appendix.

Lemma 2. a) $x_2 \geq 0 \Leftrightarrow c_2 \leq c_{2,0} = -0.3 + 1.33c_1 + 1.62s$; b) $q_2 \geq 0 \Leftrightarrow c_2 \leq c_{2,1} = -0.33 + 1.33c_1 - 0.17s$; c) $(c_2 - x_2) \geq 0 \Leftrightarrow c_2 \geq c_{2,2} = -0.07 + 0.29c_1 + 0.35s$.
Proof: see the Appendix.

Lemma 3. a) $c_{1,1} \leq c_{1,0}$ if $c_1 \geq 1 - 2s$; b) $c_{1,2} < c_{1,1}$ always; c) $c_{2,1} < c_{2,0}$ always; d) $c_{2,2} \leq c_{2,1}$ if $c_1 \geq \frac{1}{4} + \frac{1}{2}s$; e) $(c_1 - x_1) \geq 0 \wedge (c_2 - x_2) \geq 0$ if $c_1 \geq \frac{1}{4} + \frac{1}{2}s$; f) $q_1 \geq 0 \wedge q_2 \geq 0$ if $c_1 \leq 1$.
Proof: see the Appendix.

Lemma 1 reveals that the marginal production costs of firm 1, c_1, is, for a given c_2, neither too high nor too low, that is, it is included in the interval $c_{1,2} \leq c_1 \leq c_{1,1}$. Lemma 2 shows a similar constraint for firm 2: the marginal production cost, c_2, for a given c_1, must be included in the interval $c_{2,2} \leq c_2 \leq c_{2,1}$.

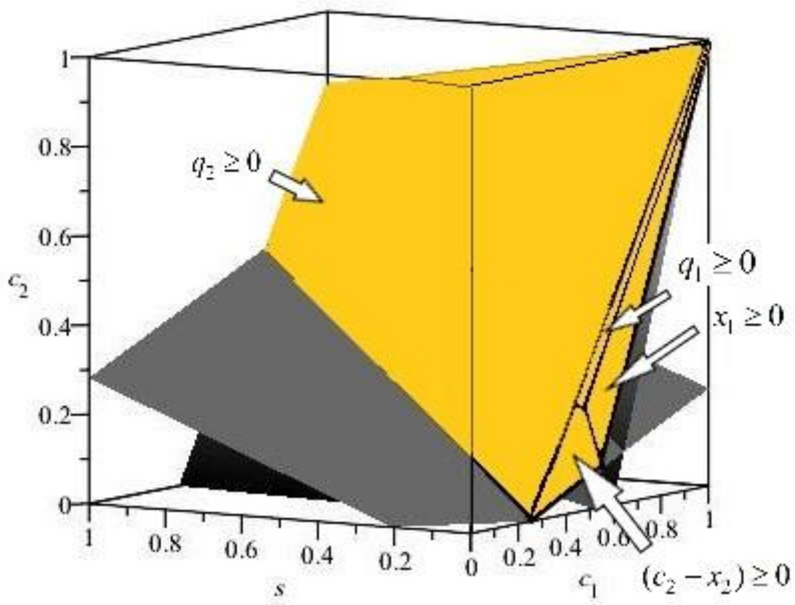

Fig. 1. Feasible area (yellow) of the model in the $(c_1, c_2, s)-$space.

Lemma 3 reveals that: 1) the condition $q_1 \geq 0$ is stricter than the condition $x_1 \geq 0$ if the cost of the VI firm are adequately high, otherwise the condition $x_1 \geq 0$ is the more stringent one. This fact introduces a first restriction to the area of feasibility of our model; 2) the condition $q_2 \geq 0$ is always stricter than the condition $x_2 \geq 0$ for the VS firm; 3) the simultaneous non-negativity conditions of retail costs (after their reduction due to the R&D investments) and of output respectively define the lower and upper borders of the feasible space of the model. Figure 1 gives a simplified graphical representation of this feasible space.

Lemma 4. Under Cournot retail competition, firm 1 forecloses firm 2 (i.e., $q_2 = 0$) if and only if $c_2 = c_{2,1}$.
Proof: see the Appendix.

In the absence of the subsidy, foreclosure of firm 2 is always prevented, and it is more likely when the subsidy is introduced. This is because the subsidy reduces the production of firm 2 which implies a lower wholesale profit for firm 1; therefore, the VI firm becomes less willing to support the VS firm's operation. If firm 2 is foreclosed, then the industry becomes a vertically integrated monopoly. Hence, we follow the line of reasoning of Aryia et al. (2008, 4): because our primary purpose is to study the competitive interactions between a vertically integrated producer and its retail rival, we will abstract from the foreclosure setting. Therefore, in the rest of the work, we assume that the non-foreclosure condition, $c_2 < c_{2,1}$ holds.

Note that we can see, from the expressions for $c_{2,1}$ and $c_{2,2}$, that the R&D subsidy works for reducing such an interval, and thus also the region of existence of a feasible duopolistic market, favouring the appearance of a monopoly of the VI firm.
Preliminarily, we recall the standard results of the basic duopoly model (i.e., without outsourcing) extended with R&D (d'Aspremont and Jacquemin, 1988, 1990) in the following remark.

Remark: The R&D subsidy increases R&D investments, output and profits of both firms.

When outsourcing to the rival is introduced, the role played by the R&D subsidy on the R&D investments, output and profits becomes richer than in the d'Aspremont and Jacquemin (1988, 1990)'s context, strongly modifying the common wisdom, as shown below.

Lemma 5. The input price is increased by the subsidy rate and reduced by both costs increases.
Proof: $\frac{\partial z}{\partial s} > 0, \frac{\partial z}{\partial c_1} < 0, \frac{\partial z}{\partial c_2} < 0$.

The economic intuition behind Lemma 5 is as follows. The maximization problem of the VI firm with respect to the input price leads to $\frac{\partial \pi_1}{\partial z} \geq 0 \Rightarrow z = z(x_1(s), x_2(s), c_1, c_2)$. The analysis of total derivative of the input price with respect to the subsidy reveals that $\frac{dz}{ds} = \overset{+}{\frac{\partial z}{\partial x_1}} \overset{+}{\frac{dx_1}{ds}} + \overset{+}{\frac{\partial z}{\partial x_2}} \overset{+}{\frac{dx_2}{ds}} > 0$. As expected, $\frac{dx_i}{ds} > 0$ informs us that an increase in the subsidy incentivizes firms to engage in R&D game which leads to a reduction in the marginal costs of both downstream firms. Therefore, downstream duopoly rents enlarge and the VI firm can extract a larger share of the VS firm's rent by increasing the input price. With a similar reasoning, an increase in the exogenous costs shrink those rents, generating the opposite result.

Result 1. The introduction of a R&D subsidy always reduces firm 2's output and increases firm 1's output.
Proof: $\frac{\partial q_2}{\partial s} < 0; \frac{\partial q_1}{\partial s} > 0.$

The rationale for Result 1 is the following. From the above analysis we know that $\frac{dx_i}{ds} > 0$; therefore, both downstream firms reduce their marginal costs. However, $\left|\frac{dx_1}{ds}\right| > \left|\frac{dx_2}{ds}\right|$: that is, an increasing subsidy always increases the R&D investment of firm 1 more than that of firm 2. With the provision of a subsidy, duopoly rents enlarge and, besides the input price, the VI firm can further extract rents of the VS firm expanding output in the downstream market. Therefore, the VI firm is more incentivized than the VS firm to reduce the marginal costs of its affiliated downstream firm, which leads to an output expansion. The reaction function of the affiliated downstream VI firm shifts outward; in order not to allow a drastic fall of the retail price, the VS firm reduces its production.

Result 2. In the feasible space, profits of the vertically separated firm 2 are reduced by the introduction of a R&D subsidy, and this reductive effect persists until such a subsidy reaches a sufficiently high level, beyond which firm 2's profits becomes increasing with an increasing subsidy (i.e., a U-shaped relationship between firm 2's profits and R&D subsidy). Moreover, in any case the positive effects on firm 2's profits is always lower than that on firm 1's profit.
Proof: $\frac{\partial \pi_2}{\partial s} \gtrless 0 \Leftrightarrow s \gtrless s^{\circ} = -0.09 + 0.35c_1 - 0.26c_2$, and $\frac{\partial \pi_1}{\partial s} > \left| \frac{\partial \pi_2}{\partial s} \right|$.

The reason for Result 2 is as follows. From Lemma 2 and Result 2, one can observe that an increase in the subsidy has two opposite effects on the VS firm. On the one hand, a subsidy increase allows the VI firm to charge a higher price for the outsourced input of the VS firm, reducing output and profitability. On the other hand, the provision of a subsidy increases the R&D investment of the VS firm, which leads to a decrease of marginal cost, and therefore improving its profits. When the amount of the subsidy is low, the former effect dominates, while the opposite holds after the threshold s°.

Result 3. a) In the absence of the R&D subsidy, the R&D investments of the VS firm 2 are larger than those of the VI firm 1 if the marginal cost differential is adequately high. b) An increase in the subsidy enlarges the R&D investments differential in favour of the VI firm.
Proof: a) $(x_1 - x_2)|_{s=0} = -1.32c_1 + 0.74c_2 + 0.58$; because $c_1, c_2 > 1$, then it holds that $x_1 < x_2$ if $c_1 > 0.56c_2 + 0.44$ and $x_1 \geq x_2$ if $c_2 < c_1 \leq 0.56c_2 + 0.44$.
b) $(x_1 - x_2) \equiv -1.32c_1 + 0.29s + 0.74c_2 + 0.58 \gtrless 0 \Leftrightarrow s \gtrless s^{\circ\circ} = -2 + 4.56c_1 - 2.56c_2$; $\frac{\partial (x_1 - x_2)}{\partial s} > 0.$

The intuition behind Result 3 is the following. Lemma 5 has revealed that $\frac{dx_i}{ds} > 0$: a subsidy induces both firms to invest more in R&D. When the subsidy provided is of a small amount, the more efficient VS firm further reduces its marginal cost and gains market shares in the downstream sector; however, the VI firm prefers to use the input price to extract duopoly rents from the VS firm. Nonetheless, Result 1 has shown that $\left| \frac{dx_1}{ds} \right| > \left| \frac{dx_2}{ds} \right|$; therefore, when the size of the subsidy increases, the marginal cost reduction of the VI's affiliated downstream due to the R&D becomes substantial. As discussed above, though keeping extracting rents from the VS firm via the input price, the VI firm finds more profitable to regain market shares in the downstream market.

Result 4. a) In the absence of the R&D subsidy, the quantity of the VS firm 2 is larger than those of the VI firm 1 if the marginal cost differential is sufficiently high. b) An increase in the subsidy enlarges the output differential in favour of the VI firm.

Proof: a) $(q_1 - q_2) = -2.37c_1 + 1.27c_2 + 1.09$; because $c_1, c_2 > 1$, then it holds that $q_1 < q_2 \Leftrightarrow c_1 > 0.54c_2 + 0.46$ and $q_1 \geq q_2$ if $c_2 < c_1 \leq 0.54c_2 + 0.46$.

b) $(q_1 - q_2) \equiv -2.37c_1 + 0.55s + 1.3c_2 + 1.09 \underset{<}{\overset{>}{=}} 0 \Leftrightarrow s \underset{<}{\overset{>}{=}} s^{\circ\circ\circ} = -1.98 + 4.3c_1 - 2.36c_2$; $\frac{\partial(q_1 - q_2)}{\partial s} > 0$.

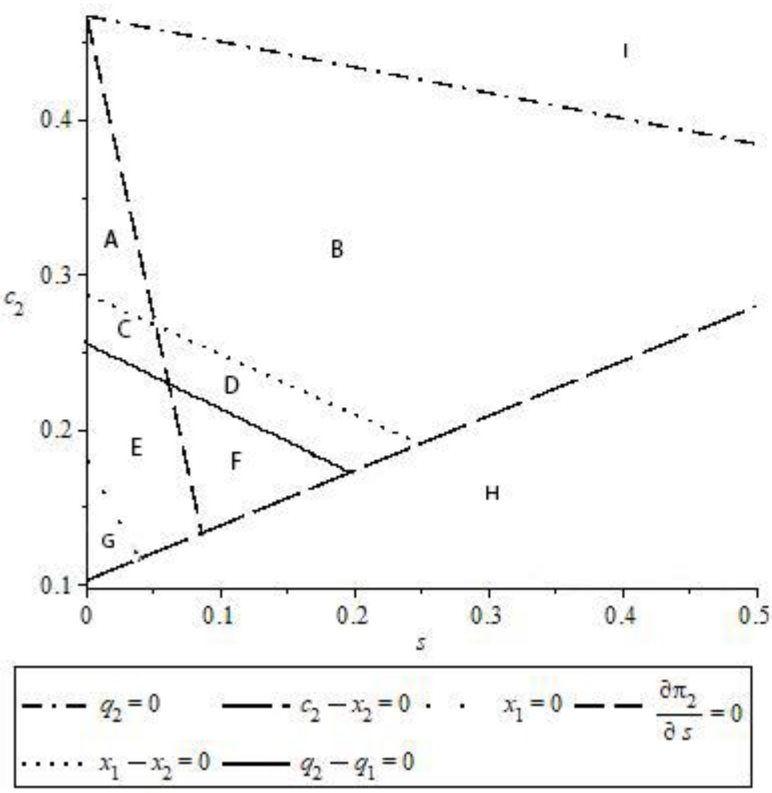

Fig. 2. Feasible area in the $(s, c_2)-$space, for $c_1 = 0.6$. Legend: the following inequalities hold in these regions: in region A, $\frac{\partial \pi_2}{\partial s} < 0, q_1 > q_2, x_1 > x_2$; in region B, $\frac{\partial \pi_2}{\partial s} > 0, q_1 > q_2, x_1 > x_2$; in region C, $\frac{\partial \pi_2}{\partial s} < 0, q_1 > q_2, x_1 < x_2$; in region D, $\frac{\partial \pi_2}{\partial s} > 0, q_1 > q_2, x_1 < x_2$; in region E, $\frac{\partial \pi_2}{\partial s} < 0, q_1 < q_2, x_1 < x_2$; in region F, $\frac{\partial \pi_2}{\partial s} > 0, q_1 < q_2, x_1 < x_2$; in region G, unfeasible region, $x_1 < 0$; in region H, unfeasible region, $c_2 - x_2 < 0$; in region I, unfeasible region, $q_2 < 0$.

The rationale for Result 4 directly follows from the line of reasoning of Lemma 5 and Results 1-3. Then, it is clear how the R&D subsidy policy affects the relative market shares of firms and their relative profitability: it always transfers market shares from firm 2 to firm 1 and, at least for sufficiently low levels of subsidy, firm 1's profit increases to the detriment of firm 2’s profit.[5] However, the total industry quantity is always increasing with an increasing subsidy (i.e. $\frac{\partial Q}{\partial s} > 0$ because $\frac{\partial q_1}{\partial s} > \left|\frac{\partial q_2}{\partial s}\right|$); as expected, thus the consumer surplus is always enhanced by a R&D subsidy policy: from the consumers’ perspective, this policy always improves their well-being, and represents a rationale for the government’s intervention.

Figure 2, drawn for a given value of $c_1 = 0.6$, quantitatively assesses the content of the Results. It is easy to see that the introduction of the subsidy and its increase up to a value of $s = 0.3$ can trigger all the effects enunciated in Results above.

The previous findings can be summarized in the next Proposition.

Proposition 1. In the feasible space of the analysis, the provision of an R&D subsidy leads to: a) an increase in R&D investments, total output, and total profits; and b) an increase in overall social welfare if the subsidy is of an adequately small amount.

Proof: a) $\frac{dx_i}{ds} > 0$; $\frac{\partial Q}{\partial s} > 0$; $\frac{\partial \pi_1}{\partial s} > 0, \frac{\partial \pi_2}{\partial s} \gtrless 0$; from Result 2 it follows that $\frac{\partial \Pi}{\partial s} > 0$, where $\Pi = \pi_1 + \pi_2$; b) defining social welfare as $SW = \pi_1 + \pi_2 + CS - S$, where $CS = \frac{Q^2}{2}$ is consumer surplus and $S = s(x_1 + x_2)$ the total subsidy government’s expenditure, simple differentiation shows that $\frac{\partial SW}{\partial s} \gtrless 0$ if $s \lessgtr s^{\circ\circ\circ\circ} = .21 - .27c_1 + .06c_2$.

The intuition behind point b) of the Proposition is straightforward. The provision of a subsidy has two opposite effects on overall welfare: on the one hand, a positive impact on total production, and therefore on profits and consumer surplus; on the other hand, this represents a cost for the government. If the amount of the subsidy is adequately small, the former effect dominates the latter, otherwise, the opposite holds.

To sum up, the basic intuition behind these findings is as follows. The presence of a subsidy allows the VI firm to set a higher input price, and, through this channel, it reduces firm 2’s quantity. On the other hand, the subsidy also allows for a larger R&D investment of firm 2 with the consequent cost-reduction effect. These counterbalancing forces on quantities and profits originated by the subsidy are

[5] By using the Italian rail transport sector for illustrative purposes, we may observe that a R&D subsidy policy would always expand the market share of Trenitalia to the detriment of NTV, of course, broadly speaking and under the extreme assumptions of full political power of FS on the decision on the regulated input price and a relatively higher cost-efficiency of NTV.

ultimately responsible for 1) the non-monotonicity of firm 2's profits about the subsidy; and 2) more in general, a positive effect of the subsidy on firm 2's profits less than that on firm'1 profits.
So far, we have considered the subsidy as exogenously given. However, point c) in Proposition 1 reveals that there is room for social welfare maximising government (the benevolent social planner) to choose optimally a positive (second-best) R&D subsidy. The Appendix examines this case and shows that the model's qualitative results are confirmed in such a framework in which the subsidy is financed, in the usual simple way, by a lump-sum tax on consumers.

4. Conclusions

This paper studies the effects of a public R&D subsidy policy in a duopoly market of a firm outsourcing input supplies (VS) from its downstream integrated rival (VI), which is a widespread vertical market structure. The main finding of the paper is that, although there exists a uniform subsidy rate to competing firms, a publicly financed R&D subsidy policy has relevant effects in the presence of outsourcing to the rival. These effects are different between "competitors". Then, such a policy influences the market structure, in the sense that it may strongly modify the relative market shares and profitability of the competing firms. In particular, the ultimate effect is to determine R&D investments relatively larger in the VI firm and to shift market shares in favour of the VI firms, with the possible consequence even of a transfer of profits from the VS to the VI firm.[6] Therefore, these findings offer some testable implications, and suggest that a subsidy policy in a market with outsourcing to a rival should take also into account of its differential effects on the "competitors".
This paper opens several issues for further research, e.g., to consider a cooperative R&D choice by rival firms or the creation of a research joint venture between them.

Appendix

1. Proof of Lemmas 1-4.

Proof of Lemma 1

a) From (4), $x_1 \geq 0$ if $0.49 - 0.95c_1 + 0.46c_2 + 0.74s \geq 0$. Solving for c_1, one gets $c_1 \leq c_{1,0} = 0.51 + 0.48c_2 + 0.78s$;
b) From (6), $q_1 \geq 0$ if $0.86 - 1.44c_1 + 0.58c_2 + 0.43s \geq 0$. Solving for c_1, one gets $c_1 \leq c_{1,1} = 0.59 + 0.4c_2 + 0.3s$;

[6] For instance, always referring to the metaphor of the Italian rail industry, the introduction of a subsidy for promoting the innovation in the retail services uniform for both competitors would have the effects of 1) promoting cost-reducing innovation more in Trenitalia than in Ntv; 2) increasing Trenitalia's profits and reduce Ntv's profits; and 3) shifting market shares towards Trenitalia to the detriment of Ntv.

c) Substituting (4) into the non-negativity condition of retail costs after their reduction due to the R&D investments, $(c_1 - x_1) \geq 0$, one obtains $1.95c_1 - 0.46c_2 - 0.74s - 0.49 \geq 0$. The latter expression, solved for c_1, leads to $c_1 \geq c_{1,2} = 0.25 + 0.24c_2 + 0.38s$.

Proof of Lemma 2

a) From (5), $x_2 \geq 0$ if $-0.09 + 0.37c_1 - 0.27c_2 + 0.45s \geq 0$. Solving for c_2, one gets $c_2 \leq c_{2,0} = -0.33 + 1.33c_1 + 1.62s$;

b) From (7), $q_2 \geq 0$ if $-0.23 + 0.93c_1 - 0.7c_2 - 0.12s \geq 0$. Solving for c_2, one obtains $c_2 \leq c_{2,1} = -0.33 + 1.33c_1 - 0.17s$;

c) Substituting (5) into the non-negativity condition of retail costs after R&D investments, $(c_2 - x_2) \geq 0$, it is obtained $1.28c_2 - 0.37c_1 - 0.45s + 0.09 \geq 0$, which solved for c_2 yields $c_2 \geq c_{2,2} = -0.07 + 0.29c_1 + 0.35s$.

Proof of Lemma 3

a) From point (a) of Lemma 1, one can rewrite the expression of $c_{1,0}$ as $c_2 = -\frac{21}{20} - \frac{8}{5}s + \frac{41}{20}c_1$. Likewise, from point (b) of Lemma 1, one can rearrange the equation of $c_{1,1}$ as $c_2 = -\frac{37}{25} - \frac{37}{50}s + -\frac{62}{25}c_1$. Combining the two expressions and solving the inequality for c_1, the result follows;

b) the equation of $c_{1,1}$ can be rewritten as $c_1 = \frac{37}{62} + \frac{25}{62}c_2 + \frac{37}{124}s$. Similarly, from point (c) of Lemma 1, the constraint $c_{1,2}$ can be expressed as $c_1 = \frac{1}{4} + \frac{8}{21}s + \frac{5}{21}c_2$. Equating the two expressions and solving, it is obtained that for $s \in [0,1]$ the inequality $c_{1,2} < c_{1,1}$ always holds: in fact, $c_{1,2} = c_{1,1}$ for $c_2 \leq \frac{1}{2}s - \frac{21}{10}$, which is always negative for $s \in [0,1]$;

c) From point (a) of Lemma 2, the expression of $c_{2,0}$ can be written as $c_2 = -\frac{1}{3} + \frac{13}{8}s + \frac{4}{3}c_1$. In a similar vein, from point (b) of Lemma 2, the equation of $c_{2,1}$ can be rearranged as $c_2 = -\frac{1}{3} - \frac{1}{6}s + -\frac{4}{3}c_1$. By using the two expressions and solving for the relevant variable, the inequality $c_{2,1} < c_{2,0}$ always holds for $s \in [0,1]$;

d) from the previous point, the constraint $c_{2,1}$ can be rewritten as $c_2 = -\frac{1}{3} - \frac{1}{6}s + -\frac{4}{3}c_1$. Similarly, from point (c) of Lemma 2, the expression of the constraint $c_{2,2}$ can be reformulated as $c_2 = -\frac{4}{55} + \frac{39}{110}s + \frac{16}{55}c_1$. Combining the two expressions and solving for c_1 the inequality holds;

e) Rewriting $c_{1,2}$ as $c_2(c_1)$ one gets $c_2 = -\frac{21}{20} - \frac{8}{5}s + \frac{21}{5}c_1$. Solving for c_1 the system composed by the latter expression and that of $c_{2,2}$ in point d) above leads directly to the inequality;

f) Making use of the expression of $c_{1,1}$ as $c_2(c_1)$ in point (a) above, i.e. $c_2 = -\frac{37}{25} - \frac{37}{50}s + -\frac{62}{25}c_1$, and that of $c_{2,1}$ in point (c) above, i.e. $c_2 = -\frac{1}{3} - \frac{1}{6}s + -\frac{4}{3}c_1$, and solving for c_1 the system composed by those expressions one obtains $c_1 \leq 1 + \frac{1}{2}s$ which is always larger than 1 for $s \in [0,1]$.

Proof of Lemma 4

From Lemma 2, point (b), one obtains that $q_2 = 0$ when $c_2 = c_{2,1}$. Because Lemma 3, point (c), reveals that for the VS firm the constraint $q_2 \geq 0$ is the strictest operative condition, the result directly follows.

2. Endogenous optimal choice of the subsidy by the Government

We consider here the case in which the government optimally chooses the amount of the subsidy instrument to maximize social welfare.

The government's optimization problem is to choose the uniform subsidy level that maximizes the following expression:

$$SW = \pi_1 + \pi_2 + CS - S \qquad \text{(A.1)}$$

where $CS = \frac{(q_1 + q_2)^2}{2}$ is the expression of the consumer surplus, and $S = s(x_1 + x_2)$ the sum of the R&D subsides provided by the government to firms in the market. This subsidy towards firm 1 and firm 2 is financed at a balanced budget with a uniform non-distorting lump-sum tax on the side of consumers (i.e., the tax does not cause violations of the conditions for social efficiency). The available post-tax exogenous nominal income of the representative consumer is assumed to be high enough to avoid corner solutions.

The second order condition of (A.1) is satisfied, i.e., $\frac{\partial^2 SW}{\partial s^2} < 0$; therefore, the government's maximization problem leads to the following optimal subsidy

$$s^* = 0.21 - 0.27c_1 + 0.06c_2; \quad s^* \geq 0 \Leftrightarrow c_2 \geq 4.68c_1 - 3.68 \qquad \text{(A.2)}$$

Substitution of s^* into the output and cost reduction expressions leads to:

$$q_1^* = 0.95 - 1.56c_1 + 0.6c_2; \quad q_1^* \geq 0 \Leftrightarrow c_2 \geq 2.57c_1 - 1.57 \tag{A.3}$$
$$q_2^* = -0.25 + 0.96c_1 - 0.7c_2; \quad q_2^* \geq 0 \Leftrightarrow c_2 \leq 1.36c_1 - 0.36 \tag{A.4}$$
$$x_1^* = 0.64 - 1.15c_1 + 0.5c_2; \quad x_1^* \geq 0 \Leftrightarrow c_2 \geq 2.27c_1 - 1.27 \tag{A.5}$$
$$x_2^* = 0.003 + 0.24c_1 - 0.25c_2; \quad x_2^* \geq 0 \Leftrightarrow c_2 \leq 0.98c_1 + 0.13 \tag{A.6}$$

to which one needs to add the conditions

$$(c_1 - x_1) \geq 0 \Leftrightarrow c_2 \geq 4.24c_1 - 1.27 \tag{A.7}$$
$$(c_2 - x_2) \geq 0 \Leftrightarrow c_2 \geq 0.2c_1 + 0.002 \tag{A.8}$$

A preliminary inspection reveals that: 1) with regard to the VI firm, condition (A.5) is the more stringent; 2) with regard to the VS firm, condition (A.4) is the more stringent; 3) given (A.4) and (A.5), condition (A.8) is more stringent than (A.7); 4) given (A.4), (A.5) and (A.8), condition (A.2) is always satisfied; 5) the condition $c_1 > c_2$ is always met.
Having restricted the feasibility area to the portion of the plane defined by conditions (A.4), (A.5) and (A.8), it is possible to re-examine all results in the main text.

Result A.1 In the feasible set, given the optimal R&D subsidy, a decrease of firm 2's cost leads to an output expansion.
Proof: $\frac{\partial q_2^*}{\partial c_2} < 0$.

This is the standard result that a decrease in the own costs leads to output expansion.

Result A.2 In the feasible set, given the optimal R&D subsidy, a decrease of firm 2's cost leads to a decrease of its profit.
Proof: $\frac{\partial \pi_2^*}{\partial c_2} \geq 0$ if restriction (A.4) holds.

The reason for Result A.2 is as follows. Result A.1 shows that a cost reduction leads the VS firm to expand output; however, the VI firm extract the VS firm's duopoly rents via the input price.

Result A.3 The R&D investments of firm 2 can be larger/lower than those of firm 1 depending on the cost differential .
Proof: $(x_1 - x_2)\big|_{s=s^*} = -1.4c_1 + 0.76c_2 + 0.64 \Rightarrow (x_1 - x_2) \underset{<}{\geq} 0 \Leftrightarrow c_2 \geq 1.84c_1 - 0.84$.

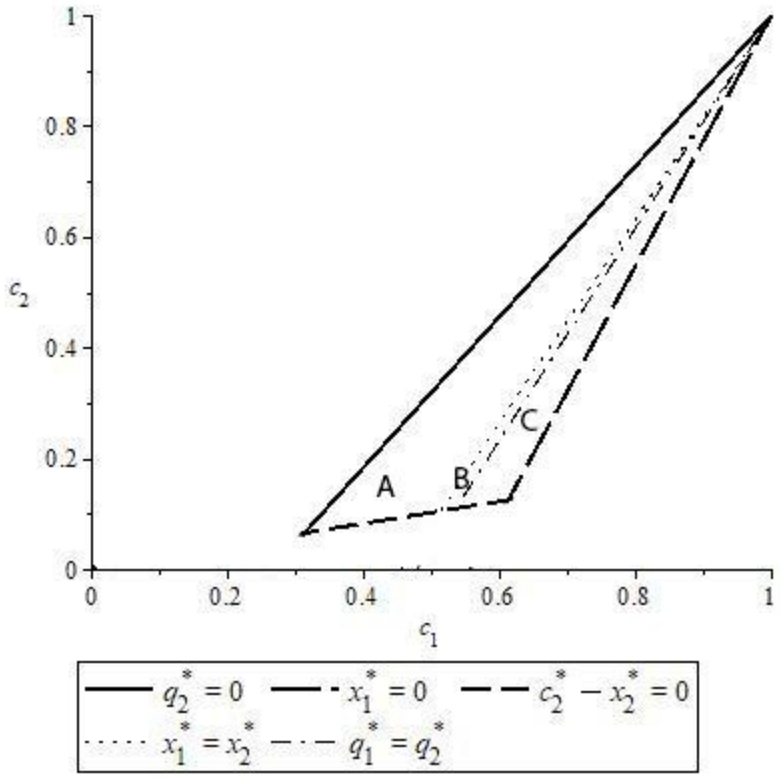

Fig. A.1. Feasible set of the model with endogenous R&D subsidy. Legend: the following inequalities hold in these regions: in region A, $q_1 > q_2, x_1 > x_2$; in region B, $q_1 > q_2, x_1 < x_2$; in region C, $q_1 < q_2, x_1 < x_2$.

Result A.4 In the feasible set, given the optimal R&D subsidy, the quantity of firm 2 can be larger smaller than that of firm 1 depending on the cost differential.

Proof: $(q_1 - q_2)\big|_{s=s^*} = -2.52c_1 + 1.31c_2 + 1.2 \Rightarrow (q_1 - q_2) \underset{<}{\overset{>}{-}} 0 \Leftrightarrow c_2 \geq 1.92c_1 - 0.92$

The above results are summarized in the next proposition.

Proposition A.1. In the feasible space, the provision of an optimal R&D subsidy leads to an increase in total R&D investments, output, and profits and consequently of the overall social welfare.

Figure A.1 qualitatively mirrors Figure 2 in the main text, confirming the robustness of the model to the endogenous choice of the subsidy rate.

References

Amir, R., Liu, H., Machowska, D., Resende, J., 2019. Spillovers, subsidies, and second-best socially optimal R&D. *Journal of Public Economic Theory*, 21, 1200–1220.

Arya, A., Mittendorf, B., Sappington, D.E.M., 2008. Outsourcing, vertical integration, and price vs. quantity competition. *International Journal of Industrial Organization*, 26, 1–16.

Baake, P., Oechssler, J., Schenk, C., 1999. Explaining cross-supplies. *Journal of Economics*, 70, 37–60.

Bourreau, M., Hombert, J., Pouyet, J., Schutz, N., 2011. Upstream competition between vertically integrated firms. *Journal of Industrial Economics*, 59, 677–713.

Buccella, D., Fanti, L., Gori, L., 2021. A contribution to the theory of R&D investments. GLO Discussion Paper No. 940.

d'Aspremont C, Jacquemin A., 1988. Cooperative and noncooperative R&D in duopoly with spillovers. *American Economic Review*, 78, 1133–1137.

d'Aspremont, C., Jacquemin, A., 1990. Cooperative and noncooperative R&D in duopoly with spillovers: Erratum. *American Economic Review* 80, 641–642.

Deloitte, 2018. Toward the next horizon of Industry 4.0. Available online at https://www2.deloitte.com/content/dam/insights/us/articles/IL327_Toward-the-next-horizon/IL327_Toward-the-next-horizon.pdf

Fanti, L., Scrimitore M., 2019. How to compete? Cournot versus Bertrand in a vertical structure with an integrated input supplier. *Southern Economic Journal*, 85, 796–820

Fumagalli, C., Motta, M., Calcagno, C., 2018. *Exclusionary practices: The economics of monopolisation and abuse of dominance*. Cambridge University Press.

Fumagalli, C., Motta, M., 2020. Dynamic vertical foreclosure. *Journal of Law and Economics*, 63, 763–812.

Gil-Moltó, M.J., Poyago-Theotoky, J., Zikos, V., 2011. R&D subsidies, spillovers, and privatization in mixed markets. Southern Economic Journal, 78, 233–255.

Haaland, J., Kind, H. J., 2006. Cooperative and non-cooperative R&D policy in an Economic Union. *Review of World Economics*, 142, 721–45.

Hart, M. D., 1998. US Technology Policy: New Tools for New Times. http://www.nira.go.jp/publ/review/98summer/hart.html, Summer.

Hinloopen, J., 1997. Subsidizing cooperative and noncooperative R&D in duopoly with spillovers. *Journal of Economics*, 66, 151–175.

Hinloopen, J., 2000a. More on subsidizing cooperative and noncooperative R&D in duopoly with spillovers. *Journal of Economics*, 3, 295–308.

Hinloopen, J., 2000b. Strategic R&D co-operatives. *Research in Economics*, 54, 153–85.

Kamien, M., Li, L., Samet, D., 1989. Bertrand competition with subcontracting. *Rand Journal of Economics,* 20, 553–567.

Liao, P. C., 2007. International R&D rivalry with spillovers and policy cooperation in R&D subsidies and taxes. *International Economic Journal*, 21, 399–417.

Michalsen A., 2012. R&D policy in a vertically related industry. *Economics of Innovation and New Technology*, 21, 737-751.

Shy, O., Stenbacka, R., 2003. Strategic outsourcing. *Journal of Economic Behavior and Organization*, 50, 203–224.

Spiegel, Y., 1993. Horizontal subcontracting. *Rand Journal of Economics*, 24, 570–590.

Van Long, N., 2005. Outsourcing and technology spillovers. *International Review of Economics & Finance*, 14, 297–304.

Yang, F., Gu, S., 2021. Industry 4.0, a revolution that requires technology and national strategies. *Complex & Intelligent Systems*, 7, 1311–1325.