"Doing well" in the Teaching for Robust Understanding approach revealed by the lens of the semiotic potential of tasks with the GGBot

Anna Baccaglini-Frank, Maria Alessandra Mariotti

"Doing well" in the Teaching for Robust Understanding approach revealed by the lens of the semiotic potential of tasks with the GGBot

Anna Baccaglini-Frank[1] and Maria Alessandra Mariotti[2]

[1]University of Pisa, Department of Mathematics, Pisa, Italy; anna.baccaglinifrank@unipi.it

[2]University of Siena, Italy; mariotti21@unisi.it

In this paper we report on experiences we have provided students in a grade-3 classroom with the aim of introducing concepts in plane geometry through interaction with the GGBot, a drawing robot that can be programmed with SNAP! Blocks. We do this to explore the effectiveness of the use of a new theoretical approach that combines the Teaching for Robust Understanding Framework with the Theory of Semiotic Mediation. We claim that the articulation between the two frames is not only feasible but also insightful, providing an a priori analysis of two tasks, and a detailed analysis of a short excerpt from the corresponding Didactic Cycle.

Keywords: Coding, GGBot, geometry, personal meanings, situated signs, STEM education, Theory of Semiotic Mediation (TSM).

A STEM scenario: the position of mathematics when coding with the GGBot.

More attention is being placed on educational experiences in the context of STEM, as shown, for example, by the increasing importance and number of international STEM education journals (Li et al., 2020). However, STEM disciplines are not yet treated in a coherent way in educational settings; moreover, the role of mathematics in this panorama has been problematized by Li and Schoenfeld (2019) who advance an interesting and constructive proposal about the positioning of mathematics within STEM contexts within the Teaching for Robust Understanding (TRU) Framework (Shoenfeld, 2013). Presenting the framework, thew authors exemplify how it can be applied to mathematics, in order to help students "develop into powerful thinkers" (2019, p. 8).

In this paper we are interested in exploring the appropriateness of certain mathematical experiences we have offered primary school students using the GGBot, a drawing robot, in terms of STEM education and of the overarching goal of fostering their development into powerful thinkers. Specifically, we conceptualized the GGBot and students' experiences with it with the aim of fostering sense-making in a STEM perspective. Coherently with the TRU Framework, we followed Schoenfeld's suggestion "to operationalize what was meant by “doing well” in each of the five dimensions, and to see how “doing well” related to student outcomes." (Schoenfeld, p. 492, 2018).

However, given the breadth and generality of such a framework, in order to design and implement experiences with the GGBot, we felt the need of a more specific, though coherent, framework that describes the didactical potential of the GGBot, allowing a reformulation of chosen dimensions of the TRU Framework upon which we will focus our analyses. The specific framework we choose for this description is the Theory of Semiotic Mediation. Now we introduce our conceptual framework.

Teaching for Robust Understanding (TRU) and the Theory of Semiotic Mediation (TSM).

As explained by Li and Schoenfeld (2019), the Teaching for Robust Understanding (TRU) Framework is comprised of the following five dimensions, that can be introduced through guiding questions: 1) *content*: is it conceptualized as something rich and connected that can be experienced and codified in meaningful ways? 2) *cognitive demand*: what opportunities do students have to do that kind of sense-making and codification? 3) *equitable access to content*: who has such opportunities: is there equitable access to the core ideas? 4) *agency, ownership, and identity*: do students encounter the discipline in ways that enable them to see themselves as sense makers, building both agency and positive disciplinary identities? 5) *formative assessment*: does instruction routinely use formative assessment, allowing student thinking to become public so that instruction can be adjusted accordingly?

In presenting the framework as an alternative to viewing mathematics as "given" or "fixed" and arguing how such a framework also applies to STEM education, Li and Schoenfeld propose a conception of mathematics as "empirical". That is, mathematics can be (and should be) seen as products created through experience (as opposed to pre-existing). This perspective focusses on students' experience, in which a shift needs to take place from "instruction conceived as "what should the teacher do" to instruction conceived as "what mathematical experiences should students have in order for them to develop into powerful thinkers?"" (Li & Schoenfeld, 2019, p. 8). For mathematical experiences to accomplish this, the authors argue that they need to provide not only opportunities for making sense of the mathematics at stake, but for *sense-making* processes (McCallum, 2018), highlighting "the importance for students to experience mathematics through creating, designing, developing, and connecting mathematical ideas" (Li & Schoenfeld, 2019, p. 6).

While the TRU seems quite appealing for framing mathematical experiences that strive to find a place and identity in STEM scenarios, we find it too broad and general to actually help gain insight into how specific mathematical experiences are (or not) well-designed and, when carried out, whether they help students become powerful mathematical thinkers. Moreover, the context we are interested in studying – the learning of geometry through coding of a drawing robot – involves an artefact. Therefore, an appropriate framework that seems to compensate for the generality of the TRU Framework and allow for more detailed analyses in this setting is the Theory of Semiotic Mediation (TSM). Indeed, the TSM has its roots in Vygotskian socio-constructivism, according to which students are guided by their teacher to construct mathematical knowledge by solving appropriately designed tasks with the use of appropriately designed and chosen artefacts (Bartolini Bussi & Mariotti, 2008). Students' activity with artefacts allows the unfolding of their *semiotic potential* leading to students to produce personal signs that are closely related to the task and to the artefact, and that gradually, through mathematical discussions orchestrated by the teacher, are transformed into shared signs. The shared signs are generalizations of the situated personal signs, and they are more closely related to the mathematical signs belonging to the concepts being taught.

Focusing especially on three dimensions of the TRU Framework, we need to draw connections between such a broad framework and the TSM. The notion of semiotic potential expresses the relationship between the personal meanings emerging from the experience of acting with the artefact and the mathematical meanings recognizable by the expert in such actions; its strict dependency on the task to be accomplished by the students makes it the key tool for designing appropriate tasks. In doing so, it operationalizes a link between dimension 1 (Mathematical content) and dimension 2 (Cognitive demand) of the TRU Framework.

The structure of the iteration of the Didactic Cycle (work on a task with an artefact, individual production of signs, collective production of signs through mathematical discussion) organizes the implementation of teaching sequence in the classroom, and it is in line with dimension 4 of the TRU: the potential of agency ownership, and identity. For instance, typically the first cycle is characterized by a "discovery task" that foresees guided exploration of the artefact, in which the teacher's intervention is expected in order to prepare students to the following tasks, where their autonomous work is encouraged, with the aim of making personal meanings emerge and the semiotic potential unfold. Students' personal meanings can emerge in their conversation with peers during the solution of the task, providing a base for the following collective discussion (e.g., Mariotti, 2009) during which the semiotic production can (and should) foster the evolution of the expected mathematical meanings.

Objective and Methodology.

The broad objective of our work with robotic toys (e.g., Bartolini & Baccaglini-Frank, 2015; Baccaglini-Frank et al., 2020) is to study their educational potential with respect to specific geometrical concepts and with respect to significant reasoning processes in a STEM educational perspective. As for this paper, we explore the effectiveness of the use of a new theoretical approach that combines the TRU Framework and the TSM both for the design and the analysis of classroom activities. In order to show that the articulation between the two frames is not only feasible but also insightful, we provide an a priori analysis of two tasks to be carried out in a grade-3 classroom, and a detailed analysis of a short excerpt from the corresponding Didactic Cycle.

We will use a fine-grained analysis of a short excerpt, paying particular attention to the unfolding of the semiotic potential and to the teacher's actions, to show how the activity placed the students in a meaningful (to them) situation, in which they felt the need to explore and express mathematical ideas.

Presentation of the GGBot.

The GGBot (short for "GREATGeometryBot") builds on the convergence of physical and digital affordances, combining the well-known strengths and opportunities offered by Papert's original robotic drawing-turtle (more recently developed into robotic toys like the "Bee-bot") and LOGO programming with those of the block-based programming language SNAP!. The GGBot can hold a marker between its wheels (Figure 1a) that draws out its path as it moves on a sheet of paper on the floor, as well as a marker at the front, on its "nose", to highlight its movement when it changes direction (Figure 1b). Such traces provide situated signs that can be elaborated into geometrical notions – such as segment, vertex, angle, rotation, polygon – while still carrying the situatedness given by the real movement of the physical artefact. Commands are given to the GGbot through an SNAP! interface that was customarily designed, and they can be gradually added based on the teacher's needs. Other than the possibility of holding two markers, the way in which commands are given to the GGBot is quite different from other robotic toys like the Bee-bot, because the blocks represent commands (in the machine's language) that can be given to the GGbot by putting them together into sequences or *codes* (figure 2c) that are transmitted to the GGbot via a wifi module. Although these blocks are virtual objects that "live" on a screen (touch-screen of an interactive white board, tablet, or computer screen), they are concrete enough to be accessible to and shared by the whole class, and by each student–consistently with respect to TRU dim. 4, on agency, ownership, and identity–when engaged in tasks such as the following.

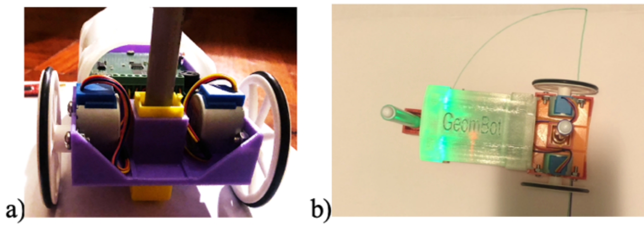


Figure 1*:* a) back view of the GGBot; b) top view of the GGBot

Though the GGBot also has a completely digital version (https://sprintingkiwi.github.io/virtual-geombot-snap), that is more similar to the LOGO turtle (though still in the block-based SNAP! environment), the physical artefact offers (especially younger) students a very different experience.

The semiotic potential of the figure-to-code and code-to-figure tasks.

A *figure-to-code task* consists in giving students the name of a figure and asking them to work in pairs and use the blocks to produce a code, so that when sent, the GGbot draws the required figure. The task we will be considering is: "Make a code so that when it is sent, the GGBot draws a square".

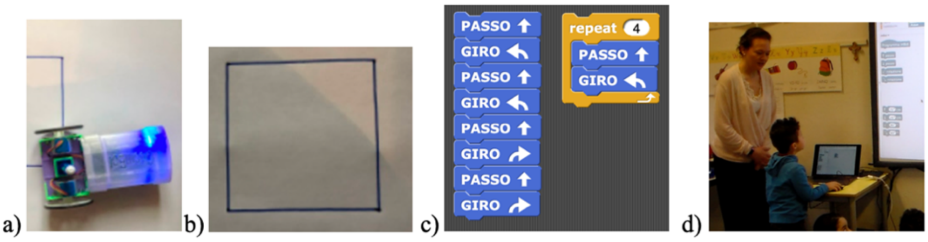


Figure 2: a) GGBot's moves; b) GGBot's trace; c) examples of codes; d) collective discussion of codes

In order to accomplish the task, it is necessary for the students to engage in a process that relates the GGBot's affordances with their conceptualization of "square shape". The shape needs to become an envisioned contour (e.g., figure 2b), a path along its border that corresponds to the GGBot's trace mark as it moves along such a path. Then, through the lens of the available commands (blocks), such a contour/path must be seen as a sequence of steps, leading to the realization of a code (such as those in figure 2c) to be communicated to GGBot. A spontaneous approach (e.g., Clements & Battista, 2001) consists in imagining to walk along the border of the figure and, based on such a simulation, planning the code to send the robot. While it is straightforward to identify the four *segments* constituting the sides of the square, and relate them to the step ↑ commands, it might be more challenging to decide how to connect these four steps. Indeed, it requires mastering the complex meaning of the Turn ↱ command (whether it is to the right or to the left) in order to relate it to the interpretation of each connecting point as "turn on the spot". Such points correspond to the thick dots left by the GGBot as it executes the turn ↱ commands, making the robot change direction before it takes another step forward to continue moving along the path. Moreover, a consistent interpretation, leading to complete the drawing, needs putting these points in relation with one another, as *the vertices of the square*, and as centers of the *rotations of the external angles of that polygon.* So, an essential feature of the semiotic potential of this artefact is its building on the relationship between the global movement and its breaking up into steps and turning points and the geometrical meaning of a polygon at a global and an analytical level. From a cognitive point of view (and this relates to TRU dim. 2, on cognitive demand), the task consists in breaking down a path that is imagined to be generated through physical continuous motion, into geometrical elements (TRU dim. 1) of a different nature: they are static and discrete.

Producing the code involves the use of signs corresponding to the required commands. Such signs refer both to the GGBot's movement and to the trace mark left, and their efficiency depends on the relationship between the drawing produced and the breaking up of the movement into its constituent elements and the corresponding commands. Comparing different codes produced in response to a figure-to-code task opens the way to the dual type of task: *code-to-figure*, that consists in focusing on a given code and asking the students to predict the trace mark that will be left on the paper by the GGbot when it executes such a code. This type of task asks to invert the relationship as the one described above between blocks in a code, the movements of the GGBot and the trace mark left. A potential challenge for students (TRU dim. 2) lies in finding a cognitive harmony between specific movements induced by a single command in the code and the continuous movement leading to the global trace mark imagined.

Design of the experiment and data collection.

We focus on a grade-3 class that worked with the GGBot over 3 sessions of one period (45 min) each. The students had been introduced to the bee-bot in earlier grades, but they had not previously worked with the GGBot. They had learned about plane geometry figures (though not specifically the notion of angle), but not in the context of coding with drawing robots. First, the students discovered and explored the GGBot, through the questions: What is it? What is it for? Why does is do it? How does it do it? The sessions were conducted by the first author in collaboration with the classroom teacher. After the initial discovery of the artefact (first cycle), the first author was prepared to propose both figure-to-code and code-to-figure tasks in the next cycles, based on students' responses. No rigid sequence of tasks (after the first exploration) was decided a priori. This design choice was made to increase the potential of agency, ownership, and identity (dim. 4), because the flow of the experience could better match students' actions, curiosity and involvement by building on the situated signs actually produced. Consistently with this purpose, the tasks proposed were intended to foster the unfolding of the semiotic potential: from the situated signs carrying the students' personal meanings the teacher would foster the development of mathematical signs. In the following section, we present an episode to illustrate a key moment of the unfolding of the semiotic potential, when the teacher changed the task from a figure-to-code to a code-to-figure task. In the analysis presented in the next section, we will consider a code-to-figure task.

"Doing well" in the TRU approach revealed by the lens of the semiotic potential.

After considering some of the codes produced in response to the figure-to-code task "Draw a square", the teacher selected the following: "step, left turn, step, left turn, step, right turn, step, right turn"; the teacher shared it on the main screen (figure 2a) and asked: "What will the GGBot draw if we give it this code?", posing a *code-to-figure task*. She then called on various students, asking them to share their predictions with the class. Below we present an excerpt from a student's response (among the six other ones provided by other students contributing to the discussion) concerning the interpretation of this code. We chose this excerpt because of how it shows the sprouting of an apparent cognitive conflict that triggers the unfolding of the semiotic potential of the GGBot. We use the example to show how the evolution of the semiotic mediation process is consistent with the TRU approach.

Silvia's prediction: the unfolding of the semiotic potential through a cognitive conflict.

Table 1 shows Silvia's prediction, including both words and gestures.

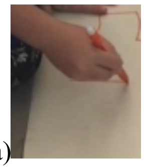
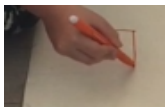
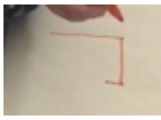
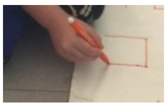
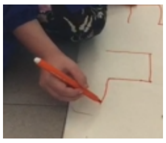
L.	S.'s words	S.'s gestures
1 2	So, he took a step forward (a), no? a rotation (b)	(a)  (b) 
3	then another, a ...another step forward, so (without touching the paper with the tip of the marker) the rotation...(c)	 [oscillating over the second segment] (c)
4	yes well, the, the step forward (c)	oscillating the marker over the two segments drawn
5	a rotation [(c) to (d)]	draws over the third short segment
6 7 8	and then after the rotation again a step forward (d) then he put the rotation the opposite way, and so like this (e) and like this (f)	   (d) (e) (f)

Table 1: Silvia's prediction expressed through words, gestures, and drawing

The task appears to be demanding for Silvia: she seems to be struggling as she thinks aloud (TRU dim. 2). Even though she had already envisaged a figure, she still seems hesitant about the "rotation", to which she returns again and again (lines 2, 3, 5). Silvia keeps on moving the marker on the paper as she says "rotation" (it. "rotazione") the first time (line 2), which suggests that she has not yet conceptualized a turn/rotation as a change in direction without continuing the movement in the new direction. Indeed, in real life, one does not rotate without continuing to walk in the new direction! However, there seems to be a seed of conflict (line 5), that might bloom into the mathematical notion of angle as rotation (dim. 1), by engaging in productive struggle (TRU dim. 2). The conflict remains unresolved, as Silvia then continues to draw a longer segment as she mentions again "rotation" and "step forward" (line 6). We interpret this as a global perception that seems to dominate over Silvia's analytic prediction: she starts off looking at each block on the screen and looking for a correspondence with the trace mark, but then her global perception takes over, making her lose control of the analytic relationship between blocks and the parts of the predicted trace mark.

We interpret Silvia's conflict as being (at least in part) due to her relying on personal meanings that are based on a physical everyday experience of "turning" during a walk, that is never accomplished as a rotation without moving along a straight trajectory "before" and "after" the turn". In this sense, the seed of the mathematical concept of *angle* is "empirical" in nature, and full of aspects that gradually will be "weeded out" as this (and other) mathematical discussion(s) move away from situated signs and towards mathematical signs, pointing to more abstract mathematical meanings. On the one hand, designing tasks that foster sense making through the production of personal meanings that live in a sort of "empirical" mathematics, seems to be very much in line with what Li and Schoenfeld (2019) argue for. On the other hand, being aware of the hybrid nature of the personal meanings produced by students like Silvia can help the teacher better exploit the semiotic potential of the tasks accomplished with the GGBot.

In terms of signs produced, we notice that Silvia produces distinct signs (word and gesture) only for the step ↑ block, which is a straight segment of a fixed length. While her drawing shows, in a global way, the "change in direction" in the form of perpendicularity between consecutive segments. However, though her struggle in trying to elaborate a sign for interpreting the Turn ↱ block remains unsolved, she successfully contributes to the conversation about mathematical ideas (dim. 4).

Discussion and conclusions.

The dual set of tasks proposed by the teacher led the students (not only Silvia, but all her classmates, too) to producing a very rich set of situated signs corresponding to personal meanings, that helped the students gain insight into the relationship between the single blocks, the robot's movement and its trace marks. Beyond the opportunity of reconceptualizing the square to be drawn as a contour, many students were led to conceiving such a contour as being made up of segments and points/turns (some spoke of "turning points") where the idea of *rotation* is associated to a specific point around which the robot stops and turns/rotates without moving forward.

The excerpt analyzed certainly shows that tasks such as those described have a high potential for mediating geometrical concepts. However, experiences such as those presented with the GGBot (and more in general with an artefact), may not be purely mathematical. Indeed, the cognitive complexity of tasks with a physical artefact such as the GGBot calls into play personal meanings that are indeed seeds of formal mathematical meanings, but initially they are hybrid, with an important "empirical" component. Such a component can be insightfully woven into the didactical cycle by the teacher, who can elaborate on it helping the students construct more formal mathematical meanings related to certain mathematical notions. Moreover, the way in which such an empirical component contributed to Silvia's cognitive conflict, fuels the notion of semiotic potential at a theoretical level. Indeed, according to its definition the notion of semiotic potential alone describes the possible coherence of the meanings emerging from the use of an artefact with the expected mathematical meanings. However, not always does such a coherence occur, and conflicts may arise; studying connections between the semiotic potential and cognitive conflicts that can emerge as students carry out the tasks, opens new theoretical and practical avenues.

Overall, this contribution shows that the articulation between the TRU Framework and the TSM, combined as we have suggested, is not only feasible but also insightful; and it operationalizes "doing well" in (3 of) the 5 dimensions in the context of coding with the GGBot.

Acknowledgment

The GGBot is an invention of Eric Frank and the first author, with patent n. 102019000018254; it is produced by GREATRobotics Srl. The SNAP! commands are developed by Alessandro Norfo under the collaboration with ASPHI onlus within the PerContare Project, co-funded by the Fondazione per la Scuola della Compagnia di San Paolo di Torino.

References

Baccaglini-Frank, A., Santi, G., Del Zozzo, A., & Frank, E. (2020). Teachers' perspectives on the intertwining of tangible and digital modes of activity with a drawing robot for geometry. *Education Sciences, 10*(12), 387. https://doi.org/10.3390/educsci10120387

Bartolini Bussi, M. G., & Mariotti, M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective. In L. English, M. Bartolini Bussi, G. Jones, R. Lesh, & D. Tirosh (Eds.), *Handbook of international research in mathematics education, second revised edition* (pp. 746–783).

Bartolini Bussi, M.G. & Baccaglini-Frank, A. (2015). Geometry in early years: sowing the seeds towards a mathematical definition of squares and rectangles. *ZDM Mathematics Education, 47*(3), 391–405. https://doi.org/10.1007/s11858-014-0636-5

Clements, D.H. & Battista, M.T. (2001). Logo and Geometry. *Journal for Research in Mathematics Education, Monograph 10*. https://doi.org/10.2307/749924

Li, Y., Wang, K., Xiao, Y., & Froyd, J.E., (2020). Research and trends in STEM education: a systematic review of journal publications. *International Journal of STEM Education, 7*(11), 1–16. https://doi.org/10.1186/s40594-020-00207-6

Li, Y., & Schoenfeld, A. H. (2019). Problematizing teaching and learning mathematics as "given" in STEM education. *International Journal of STEM Education, 6*(44), 1–13. https://doi.org/10.1186/s40594-019-0197-9

Mariotti, M.A. (2009). Artifacts and signs after a Vygotskian perspective: the role of the teacher. *ZDM Mathematics Education, 41*(4), 427–440. https://doi.org/10.1007/s11858-009-0199-z

McCallum, W. (2018). *Sense-making and making sense*. https://blogs.ams.org/matheducation/2018/12/05/sense-making-and-making-sense/ Retrieved on August 10, 2021.

Schoenfeld, A. H. (2013). Classroom observations in theory and practice. *ZDM Mathematics Education, 45*(4), 607–621. https://doi.org/10.1007/s11858-012-0483-1

Schoenfeld, A. H. (2018). Video analyses for research and professional development: the teaching for robust understanding (TRU) framework. *ZDM Mathematics Education, 50*(3), 491–506. https://doi.org/10.1007/s11858-017-0908-y