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# Measurement of the CKM angle  $\varphi_1$  in  $B^0 \to \bar{D}^{(*)0} h^0, \bar{D}^0 \to K^0_S \pi^+ \pi^-$  decays with time-dependent binned Dalitz plot analysis

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We report a measurement of the CP-violation parameter  $\varphi_1$  obtained in a time-dependent analysis of  $B^0 \to \bar{D}^{(*)0}h^0$  decays followed by  $\bar{D}^0 \to K_S^0\pi^+\pi^-$  decay. A model-independent measurement is performed using the binned Dalitz plot technique. The measured value is  $\varphi_1 = 11.7^\circ \pm 7.8^\circ \text{(stat)} \pm 2.1^\circ \text{(syst)}$ . Treating sin  $2\varphi_1$  and  $\cos 2\varphi_1$  as independent parameters, we obtain  $\sin 2\varphi_1 = 0.43 \pm 0.27$ (stat)  $\pm$ 0.08(syst) and  $\cos 2\varphi_1 = 1.06 \pm 0.33$ (stat)<sup>+0.21</sup>(syst). The results are obtained with a full data sample of 772  $\times$  10<sup>6</sup>BB pairs collected near the  $\Upsilon(4S)$  resonance with the Belle detector at the KEKB asymmetricenergy  $e^+e^-$  collider.

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# I. INTRODUCTION

The study of CP symmetry provides valuable insight into the structure and dynamics of matter from the subatomic to the cosmic scale. CP violation is a necessary ingredient for baryogenesis and explaining the state of matter in the observable Universe [\[1\]](#page-13-0). The Standard Model (SM) of particle physics accounts for CP violation using the mechanism proposed by Kobayashi and Maskawa (KM) [\[2\]](#page-13-1). A unitary matrix of quark flavor mixing, referred to as the Cabibbo-Kobayashi-Maskawa (CKM) [\[2,3\]](#page-13-1) matrix, encodes this mechanism. The CKM matrix makes charged weak currents noninvariant under CP transformation. The SM does not predict the values of the elements of the CKM matrix, but theoretical predictions estimate that the amount of CP violation introduced by the SM is too feeble to explain the baryon asymmetry of the Universe [\[4\]](#page-13-2). Thus, it is important to test the KM mechanism and search for new sources of CP violation.

Unitarity of the CKM matrix implies several relations among its elements that can be represented as triangles in the complex plane. In particular, the relation formed by the elements of the first and the third columns, referred to as the unitarity triangle (UT) [\[5\]](#page-13-3), is the most accessible for experimental tests.

The CP-violation parameter  $\varphi_1 = \arg(-V_{cd}V_{cb}^*/V_{td}V_{tb}^*)$ ,<br>pere V<sub>td</sub> is an element of the CKM matrix is one of the where  $V_{ij}$  is an element of the CKM matrix, is one of the angles of the UT.<sup>1</sup> The value of  $\sin 2\varphi_1$  has been measured precisely in  $b \rightarrow c\bar{c}s$  transitions by Belle, BABAR and LHCb [\[6\].](#page-13-4) Two discrete ambiguities remain with the known value of sin  $2\varphi_1$ :  $\varphi_1 \to \varphi_1 + \pi$  and  $\varphi_1 \to \pi/2 - \varphi_1$ . Currently, no theoretical approach is available to resolve the former ambiguity, but the latter can be resolved by measuring  $\cos 2\varphi_1$ . Existing measurements of  $\cos 2\varphi_1$  in  $b \to c\bar{u}d$  [\[7,8\]](#page-13-5) and  $b \rightarrow c\bar{c}s$  [\[9,10\]](#page-13-6) transitions are much less precise and, in most cases, model dependent.

Here, we present a model-independent measurement of the angle  $\varphi_1$  in  $b \to c \bar{u} d$  transitions [Fig. [1\(a\)](#page-2-0)] governing  $B^0 \to \bar{D}^{(*)0} h^0$  decays with subsequent  $\bar{D}^0 \to K_S^0 \pi^+ \pi^$ decay,<sup>2</sup> where  $h^0$  is a light unflavored meson. This measurement is based on a data sample twice as large as that used in the previous  $\varphi_1$  measurement using  $B^0 \rightarrow \overline{D}^{(*)0}h^0$  decays at Belle [\[8\]](#page-13-7). The technique of a binned Dalitz plot analysis is applied to the  $\varphi_1$  measurement for the first time.

### A. Formalism

This section describes the technique used to measure the angle  $\varphi_1$  at an asymmetric-energy  $e^+e^-$  collider operating at center-of-mass (c.m.) energy near the  $\Upsilon(4S)$  resonance

<span id="page-2-0"></span>

FIG. 1. (a)  $\bar{b} \rightarrow \bar{c} u \bar{d}$  transition leading to  $B^0 \rightarrow \bar{D}^0 h^0$  decay, and (b)  $\bar{b} \rightarrow \bar{u}c\bar{d}$  transition leading to  $B^0 \rightarrow D^0h^0$  decay.

[\[11,12\]](#page-13-8). When a pair of neutral  $B$  mesons is produced, they oscillate coherently until one decays. Therefore, at the moment of a flavor-specific decay of one of the B mesons [in the  $\Upsilon(4S)$  rest frame], the flavor of the other B meson is fixed. The former  $B$  meson is referred to as the tagging  $B$ meson and the latter as the *signal B* meson. The tagging and signal B mesons decay at proper times  $t_{\text{tag}}$  and  $t_{\text{sig}}$ , respectively.

The longitudinal distance  $\Delta z$  along the beam axis between the decay vertices of the signal and tagging  $B$ mesons in the lab frame is measured. Since the B mesons are produced almost at rest in the c.m. frame, their momentum can be neglected and the approximation  $\Delta t \approx$  $\Delta z/(c\beta\gamma)$  can be used, where  $\Delta t = t_{\text{sig}} - t_{\text{tag}}$ , and  $\beta$  and  $\gamma$ are the Lorentz factors of the  $\Upsilon(4S)$  parent.

<span id="page-2-1"></span>If the amplitudes  $\mathcal{A}(B^0 \to f) \equiv \mathcal{A}_f$  and  $\mathcal{A}(\bar{B}^0 \to f) \equiv$  $\bar{\mathcal{A}}_f$  are nonzero for some final state f, then the distribution of the decay time difference, attributed to the interference of the processes  $B^0 \rightarrow f$  and  $B^0 \rightarrow \bar{B}^0 \rightarrow f$ , is [\[11\]](#page-13-8)

$$
\mathcal{P}(\Delta t) = h_1 e^{-\frac{|\Delta t|}{\tau_B}} \left[ 1 + \frac{1 - |\lambda_f|^2}{1 + |\lambda_f|^2} \cos(\Delta m_B \Delta t) - \frac{2\text{Im}\lambda_f}{1 + |\lambda_f|^2} \sin(\Delta m_B \Delta t) \right],
$$
  

$$
\lambda_f = \frac{q}{p} \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f},
$$
 (1)

where  $p$  and  $q$  are the coefficients relating the mass and flavor B-meson eigenstates to each other,  $\tau_B$  is the neutral B-meson lifetime (assumed to be the same for both mass eigenstates),  $\Delta m_B$  is the mass difference between the mass eigenstates, and  $h_1$  is a normalizing constant. In the following, we assume the absence of CP violation in mixing and a null CP-violating weak phase in the B-meson decay amplitudes:

$$
\frac{q}{p} = e^{-i2\varphi_1}, \qquad \arg\left(\frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f}\right) = \Delta \delta_f, \tag{2}
$$

so that

$$
\text{Im}\lambda_f = \left| \frac{\bar{\mathcal{A}}_f}{\mathcal{A}_f} \right| \sin \left( \Delta \delta_f - 2\varphi_1 \right); \tag{3}
$$

<sup>&</sup>lt;sup>1</sup>Another naming convention,  $\beta$  ( $\equiv \varphi_1$ ), is also used in the literature.

Throughout this paper, the inclusion of the charge-conjugate decay mode is implied unless otherwise stated.

here,  $\Delta \delta_f$  is the difference in strong phases, which does not change sign under a CP transformation. Consideration of the CP-conjugated process, in which the CP-violating phase  $\varphi_1$  is replaced by  $-\varphi_1$ , allows one to distinguish between the weak  $(2\varphi_1)$  and strong  $(\Delta \delta_f)$  phases.

<span id="page-3-0"></span>For  $B^0 \to \bar{D}^{(*)0} h^0$  decays, the amplitudes  $\mathcal{A}_f$  and  $\bar{\mathcal{A}}_f$  can be expressed as

$$
\mathcal{A}_f = \alpha_B \bar{\mathcal{A}}_D, \qquad \bar{\mathcal{A}}_f = \alpha_B \xi_{h^0} (-1)^L \mathcal{A}_D, \qquad (4)
$$

where  $\xi_{h^0}$  is the CP eigenvalue of the  $h^0$  meson, L is the relative angular momentum in the  $D^{(*)0}h^0$  system,  $\mathcal{A}_D$  $(\bar{\mathcal{A}}_D)$  is the  $D^0$  ( $\bar{D}^0$ ) decay amplitude into the final state  $f_D$ , and  $\alpha_B$  is a complex coefficient. The charm mixing and possible CP violation in the D-meson decays are neglected in Eq.  $(4)$ . With the existing *B*-factories statistics, the  $B^0 \rightarrow D^0 h^0$  decay amplitude [Fig. [1\(b\)](#page-2-0)] can be neglected with respect to the  $B^0 \rightarrow \bar{D}^0 h^0$  decay amplitude [Fig. [1\(a\)\]](#page-2-0) because it is suppressed by  $|V_{ub}V_{cd}/V_{cb}V_{ud}| \approx 0.02$ .

If the state  $f<sub>D</sub>$  is a CP eigenstate, then the entire state f is CP eigenstate (except for the  $\bar{D}^{*0}h^0$  state with a vector  $h^0$ meson) as well, and the phase  $\Delta \delta_f$  equals 0 or  $\pi$ . This exposes a sensitivity to  $\sin 2\varphi_1$  but not  $\cos 2\varphi_1$  and provides the best way to measure  $\sin 2\varphi_1$  in  $b \rightarrow c\bar{u}d$ transitions [\[13\]](#page-13-9).

The three-body state  $f_D = K_S^0 \pi^+ \pi^-$  is not a CP eigen-<br>the so the phase  $\Delta \delta_0$  is not limited to the values 0 and  $\pi$ state, so the phase  $\Delta \delta_f$  is not limited to the values 0 and  $\pi$ . As a consequence, this state provides sensitivity to both  $\sin 2\varphi_1$  and  $\cos 2\varphi_1$ . The amplitude of  $D^0 \to K_S^0 \pi^+ \pi^$ decay can be expressed as a function  $A_D(m_+^2, m_-^2)$  of two Dalitz-distribution variables [\[14\],](#page-13-10) where  $m_{\pm} =$  $m(K_S^0 \pi^{\pm})$  are the invariant masses. The amplitude  $\bar{\mathcal{A}}_D$  of  $\bar{\mathcal{D}}^0 \rightarrow K^0 \pi^+ \pi^-$  decay can be obtained by transposing the  $\bar{D}^0 \to K_S^0 \pi^+ \pi^-$  decay can be obtained by transposing the Dalitz variables:  $\bar{\mathcal{A}}_D(m_+^2, m_-^2) \equiv \mathcal{A}_D(m_-^2, m_+^2)$ . Therefore, the phase difference  $\Delta \delta$ , is a function of the Dalitz the phase difference  $\Delta \delta_f$  is a function of the Dalitz variables:

$$
\Delta \delta_f(m_+^2, m_-^2) = \arg \left( \xi_{h^0} (-1)^L \right) - \Delta \delta_D(m_+^2, m_-^2),
$$
  

$$
\Delta \delta_D(m_+^2, m_-^2) = \arg \left( \frac{\mathcal{A}_D(m_+^2, m_+^2)}{\mathcal{A}_D(m_+^2, m_-^2)} \right).
$$
 (5)

For the  $f_D = K_S^0 \pi^+ \pi^-$  final state, the strong phase  $\Delta \delta_D$ <br>nnot be measured at each noint in the phase space: cannot be measured at each point in the phase space: additional information is necessary. An approach based on an isobar model of the D-meson decay amplitude was proposed in Ref. [\[15\]](#page-13-11) and used in the measurement of the CKM angle  $\varphi_1$  performed by *BABAR* [\[7\]](#page-13-5) and Belle [\[8\]](#page-13-7). Alternatively, we use here a method that is independent of the decay model, as described below.

#### B. Time-dependent binned Dalitz plot analysis

Our measurement is based on the binned Dalitz distribution approach. This idea was proposed in Ref. [\[16\]](#page-13-12) to measure the CKM angle  $\varphi_3$  and further developed for several applications in Refs. [\[17](#page-13-13)–19]. We extend this approach to measure the angle  $\varphi_1$  in the time-dependent analysis of  $B^0 \to \bar{D}^{(*)0}h^0$ ,  $\bar{D}^0 \to K_S^0 \pi^+\pi^-$  decays. The Dalitz plot is divided into 16 bins  $(2\mathcal{N}$  in the general case) symmetrically with respect to  $m_+^2 \leftrightarrow m_-^2$  exchange. The bin index *i* lies between  $-8$  and 8, excluding 0;  $m_+^2 \leftrightarrow m_-^2$  exchange<br>corresponds to the sign inversion  $i \rightarrow -i$ corresponds to the sign inversion  $i \rightarrow -i$ .

<span id="page-3-2"></span>Several parameters related to a Dalitz plot bin on the Dalitz plane  $D$  are introduced. These are the probability for the  $\bar{D}^0$  meson to decay into the phase space region  $\mathcal{D}_i$  of the Dalitz plot bin  $i$ ,

$$
K_i = \int_{\mathcal{D}_i} |\mathcal{A}_D(m^2, m^2_+)|^2 dm^2_+ dm^2_- \tag{6}
$$

(normalized by  $\sum_{i=-8}^{8} K_i = 1$ ), and the weighted averages<br>of the sine and cosine of the phase difference between  $\bar{D}^0$ of the sine and cosine of the phase difference between  $D^0$ and  $D^0$  decay amplitudes  $\Delta \delta_D(m_+^2, m_-^2)$  over the *i*th Dalitz plot bin: plot bin:

$$
C_i = \frac{\int_{\mathcal{D}_i} |\mathcal{A}_D| |\bar{\mathcal{A}}_D| \cos \Delta \delta_D dm_+^2 dm_-^2}{\sqrt{K_i K_{-i}}},
$$

$$
S_i = \frac{\int_{\mathcal{D}_i} |\mathcal{A}_D| |\bar{\mathcal{A}}_D| \sin \Delta \delta_D dm_+^2 dm_-^2}{\sqrt{K_i K_{-i}}}.
$$
(7)

<span id="page-3-1"></span>The binning method yields the relations  $C_i = C_{-i}$  and  $S_i = -S_{-i}$ . Equation [\(1\)](#page-2-1) can be expressed in the form appropriate for a time-dependent binned analysis:

$$
\mathcal{P}_{i}(\Delta t, \varphi_{1}) = h_{2} e^{-\frac{|\Delta t|}{\tau_{B}}} \bigg[ 1 + q_{B} \frac{K_{i} - K_{-i}}{K_{i} + K_{-i}} \cos (\Delta m_{B} \Delta t) + 2q_{B} \xi_{h^{0}} (-1)^{L} \frac{\sqrt{K_{i} K_{-i}}}{K_{i} + K_{-i}} \sin (\Delta m_{B} \Delta t) \times (S_{i} \cos 2\varphi_{1} + C_{i} \sin 2\varphi_{1}) \bigg], \tag{8}
$$

where  $q_B = -1$  (+1) corresponds to a signal  $B^0$  ( $\bar{B}^0$ ) meson and  $h_2$  is a normalizing constant.

The knowledge of the signal-event distribution over the Dalitz plot bins for both B-meson flavors is necessary for the fit that extracts the CP-violation parameters. The expected fraction  $n_{i,q_B}$  of signal events for the *i*th Dalitz plot bin and signal B flavor  $q_B$  is

$$
n_{i,q_B} = \frac{K_i + K_{-i}}{2} + \frac{q_B}{1 + (\tau_B \Delta m_B)^2} \cdot \frac{K_i - K_{-i}}{2}.
$$
 (9)

This formula is obtained by integrating Eq. [\(8\)](#page-3-1) over  $\Delta t$ .

In principle, each pair  $(i, -i)$  of bins provides enough information to measure the CP-violation parameters if the values of parameters  $K_{+i}$ ,  $C_i$ , and  $S_i$  are known and do not equal zero.

<span id="page-4-0"></span>

FIG. 2. (a) Dalitz plot distribution and (b) equal-phase binning obtained with the amplitude model of  $\bar{D}^0 \to K_S^0 \pi^+ \pi^-$  decay from Ref. [\[21\].](#page-14-1)

For a given binning of the Dalitz plot, the parameters  $K_i$ can be measured with a set of flavor-tagged neutral D mesons such as  $D^{*+} \to D^0 \pi^+$  or  $B^+ \to \overline{D}{}^0 \pi^+$  decays, by measuring signal yield in each Dalitz plot bin. The measurement of the phase parameters  $C_i$  and  $S_i$  is more complicated and can be done with coherent decays of  $D^0\bar{D}^0$ pairs [\[20\]](#page-14-0).

Measurement of the CP-violation parameters is possible for an arbitrary binning of the Dalitz plot, but usage of the realistic decay amplitude model allows one to optimize the binning to approach the maximal statistical sensitivity. In particular, the equal-phase binning method [\[17\]](#page-13-13) suggests the following rule for  $i > 0$  and  $m_+^2 < m_-^2$ :

$$
\frac{\pi(i-3/2)}{4} < \Delta \delta_D(m_+^2, m_-^2) < \frac{\pi(i-1/2)}{4}.\tag{10}
$$

This binning and the  $D^0 \to K_S^0 \pi^+ \pi^-$  decay amplitude model reported in Ref. [\[21\]](#page-14-1) (see Fig. [2](#page-4-0)) are employed in the analysis presented here. The analysis uses the values of K<sub>i</sub> extracted from the  $B^+ \rightarrow \bar{D}^0 \pi^+$  sample, as described in Sec. [IV,](#page-6-0) and the values of  $C_i$  and  $S_i$  parameters measured by CLEO-c [\[20\]](#page-14-0), as listed in Table [I](#page-4-1).

<span id="page-4-1"></span>TABLE I. The values of the parameters  $C_i$  and  $S_i$  measured by CLEO-c [\[20\]](#page-14-0) for equal-phase Dalitz-plot binning according to the  $\bar{D}^0 \to K_S^0 \pi^+ \pi^-$  decay model obtained in Ref. [\[21\]](#page-14-1).

$\operatorname{Bin}(i)$	$C_i$	$S_i$
	$0.710 \pm 0.034 \pm 0.038$	$-0.013 \pm 0.097 \pm 0.031$
2	$0.481 \pm 0.080 \pm 0.070$	$-0.147 \pm 0.177 \pm 0.107$
3	$0.008 \pm 0.080 \pm 0.087$	$0.938 \pm 0.120 \pm 0.047$
$\overline{4}$	$-0.757 + 0.099 + 0.065$	$0.386 \pm 0.208 \pm 0.067$
5	$-0.884 + 0.056 + 0.054$	$-0.162 \pm 0.130 \pm 0.041$
6	$-0.462 \pm 0.100 \pm 0.082$	$-0.616 \pm 0.188 \pm 0.052$
7	$0.106 \pm 0.105 \pm 0.100$	$-1.063 \pm 0.174 \pm 0.066$
8	$0.365 \pm 0.071 \pm 0.078$	$-0.179 \pm 0.166 \pm 0.048$

Model-inspired binning of the Dalitz plot does not lead to a bias in the measured parameters, because of the excellent invariant mass resolution of the detector. Therefore, an alternative binning derived from a model that parametrized the data poorly would only reduce the statistical sensitivity of the measurement.

# II. BELLE DETECTOR

This measurement is based on a data sample that contains  $772 \times 10^6 B\overline{B}$  pairs, collected with the Belle detector at the KEKB asymmetric-energy  $e^+e^-$  (3.5 on 8 GeV) collider [\[22\]](#page-14-2) operated near the  $\Upsilon(4S)$  resonance.

The Belle detector is a large-solid-angle magnetic spectrometer that consists of a silicon vertex detector (SVD) featuring the double-sided silicon strip devices, a 50-layer central drift chamber (CDC), an array of aerogel threshold Cherenkov counters, a barrel-like arrangement of time-of-flight scintillation counters, and an electromagnetic calorimeter composed of CsI(Tl) crystals located inside a superconducting solenoid coil that provides a 1.5 T magnetic field. An iron flux return located outside of the coil is instrumented to detect  $K_L^0$  mesons and to identify muons. The detector is described in detail elsewhere [\[23\]](#page-14-3). Two inner detector configurations were used. A 2.0 cm radius beampipe and a three-layer silicon vertex detector were used for the first sample of  $152 \times 10^6 B\overline{B}$  pairs, while a 1.5 cm radius beampipe, a four-layer silicon vertex detector and a small-cell inner drift chamber were used to record the remaining  $620 \times 10^6 B\bar{B}$  pairs [\[24\]](#page-14-4).

# III. EVENT SELECTION

Six  $B^0 \to \bar{D}^{(*)0}h^0$  decay modes,  $\bar{D}^0\pi^0$ ,  $\bar{D}^0\eta$ ,  $\bar{D}^0\eta'$ ,  $\bar{D}^0\omega$ ,  $\bar{D}^{*0}\pi^0$ , and  $\bar{D}^{*0}\eta$ , with subsequent decays  $\bar{D}^0 \to K_S^0 \pi^+ \pi^-$ ,  $\eta \to \gamma \gamma$  or  $\pi^+ \pi^- \pi^0$ ,  $\omega \to \pi^+ \pi^- \pi^0$ ,  $\eta' \to [\gamma \gamma]_n \pi^+ \pi^-$ , and  $\bar{D}^{*0} \to \bar{D}^0 \pi^0$ , are used in this analysis. Only  $\eta \to \gamma \gamma$  is

considered for the  $\bar{D}^0 \eta'$  and  $\bar{D}^{*0} \eta$  modes. Charged B-meson decay  $B^+ \to \bar{D}^0 \pi^+$  followed by  $\bar{D}^0 \to K_S^0 \pi^+ \pi^-$  is used to measure the parameters  $K_i$ .

The charged pion candidates are selected from the reconstructed tracks and are required to have both z and  $r\varphi$  hits in at least one layer and at least one additional layer with a  $z$  hit. The impact parameters of the tracks with respect to the beam interaction point in the longitudinal and transverse projections are required to satisfy  $|dz| < 5$  cm and  $dr < 2$  cm, respectively. The transverse momentum  $p_t$ is required to be greater than 50 MeV/c (100 MeV/c) for pions produced in  $D^0 \to K_S^0 \pi^+ \pi^ (h^0 \to \pi^+ \pi^- \pi^0)$  decay. These requirements are not applied for the pions that are daughters of  $K_S^0$  candidates.

The  $K_S^0 \to \pi^+ \pi^-$  candidates are reconstructed from two oppositely charged tracks using two artificial neural networks (NN). The first NN is trained to suppress the combinatorial background and fake tracks: it uses the track impact parameters with respect to the beam interaction point, the azimuthal angle between the  $K_S^0$  momentum and the decay-vertex vectors, the distance between the tracks, the  $K_S^0$  flight length in the x-y plane, the  $K_S^0$  momentum, the distance between the beam interaction point and the tracks, the angle between the  $K_S^0$  and pion flight directions, the presence of the SVD hits and the number of CDC hits on the tracks. The second NN is trained to suppress the background from  $\Lambda \to p\pi^-$  decays: it uses the reconstructed mass with the lambda hypothesis, the absolute values of the track momenta, the track-momenta polar angles and the particle identification parameter distinguishing pions from protons. Further details of the procedure are described in Ref. [\[25\].](#page-14-5) The invariant mass of the selected candidates is required to be between 488.5 and 506.5 MeV/ $c^2$ . This mass interval, as well as any other mass interval used in the analysis (unless explicitly stated otherwise), corresponds to  $\pm 3$  standard deviations from the nominal value.

The  $\pi^0$  candidates are formed from photon pairs with an invariant mass between 115.7 and 153.7 MeV/ $c^2$ . The photon energy is required to be greater than 40 MeV. The energy of the  $\pi^0$  candidate from  $h^0 \to \pi^+ \pi^- \pi^0$  ( $h^0 = \eta$ ) and  $\omega$ ) decay must be greater than 200 MeV.

The  $\eta \rightarrow \gamma \gamma$  candidates are formed from photon pairs with an invariant mass between 530.0 and 573.7 MeV/ $c^2$ . The photon energy is required to be greater than 80 MeV.

The  $h^0 \to \pi^+ \pi^- \pi^0$  candidates, where  $h^0 = \eta$  or  $\omega$ , are formed from a  $\pi^0$  candidate and two oppositely charged tracks with invariant mass between 537.6 and 557.4 MeV/ $c^2$  for  $\eta$  and between 760.4 and 803.9 MeV/ $c^2$  for  $\omega$ . For the  $\omega$  candidates, the absolute value of the cosine of the helicity angle  $\theta_{hel}$  (the angle between the  $B^0$  flight direction and the normal to the  $\omega$  decay plane in the  $\omega$  rest frame) is required to be greater than 0.2.

The  $\eta' \to \eta \pi^+ \pi^-$  candidates are formed from a  $\eta \to \gamma \gamma$ candidate and two oppositely charged tracks, both treated as pions. The invariant mass difference  $\Delta m_{\eta} \equiv m(\eta') - m(\eta)$ is required to lie between 401.7 and 417.7 MeV/ $c^2$ .

The  $D^0 \to K_S^0 \pi^+ \pi^-$  candidates are formed from a  $K_S^0$ candidate and two oppositely charged tracks, both treated as pions, with an invariant mass between 1.8516 and 1.8783 GeV/ $c^2$ .

The  $D^{*0} \to D^0 \pi^0$  candidates are formed from a  $D^0$ candidate and a neutral pion candidate. The invariant mass difference  $\Delta m_D \equiv m(\overline{D}^{*0}) - m(D^0)$  must lie between 140.2 and 144.2 MeV/ $c^2$ .

The selection of  $B^0$  and  $B^{\pm}$  candidates is based on the variables  $\Delta E = E_B^{\text{c.m.}} - E_{\text{beam}}^{\text{c.m.}}$ , the energy difference between the signal  $B$  candidate and beam in the c.m. frame, and  $M_{bc} = \sqrt{(E_{\text{beam}}^{\text{c.m.}}/c^2)^2 - (p_B^{\text{c.m.}}/c)^2}$ , the beam-<br>energy constrained mass of the signal *B* candidate. The candidates satisfying  $|\Delta E| < 0.3$  GeV and  $M_{\text{bc}} >$ 5.2 GeV/ $c^2$  are retained for further analysis.

The vertex-constrained kinematic fit is applied to the signal and tagging B candidates and to the  $D^0$  candidates. We require  $\chi^2$ /n.d.f. < 500 for the vertex-constrained fit of the  $D^0$  meson candidates, where n.d.f. denotes the number of degrees of freedom.

When  $h^0$  is a  $\pi^0$  or  $\eta \to \gamma \gamma$  candidate, the  $B^0 \to \bar{D}^{(*)0} h^0$ decay has no charged particle originating from the primary  $B$  decay vertex. In this case, the  $B$  decay vertex is determined by projecting the  $D^0$ -candidate trajectory onto the beam-interaction profile. The estimated longitudinal resolution  $\sigma_z$  of a such vertex, obtained from the fit, is required to be less than 0.5 mm. This requirement is also imposed on the tagging B-decay vertices obtained by projecting a single track onto the beam interaction profile.

The vertex-constrained kinematic fit for other signal B-decay modes requires that the D-candidate trajectory and the two tracks from the  $h^0$  decay originate from a common vertex and applies the Gaussian constraints on the position of this vertex based on the geometry of the beam interaction profile. The requirements  $\sigma_z < 0.2$  mm and  $\chi^2 / n.d.f. < 50$ for the vertex quality are imposed, where  $\chi^2$ /n.d.f. is calculated without taking into account the beam interaction profile constraint. These requirements are alsoimposed onthetagging B-decay vertices reconstructed with more than one track.

The vertex position for the tagging  $B$  candidate is determined from the kinematic fit of well-reconstructed tracks that are not assigned to the signal B-candidate decay chain [\[26\]](#page-14-6).

The momentum of the  $\pi^0$ ,  $K^0_S$ , and  $\eta \to \gamma \gamma$  candidates, with the invariant mass constrained to its nominal value [\[27\]](#page-14-7), is used to improve the  $\Delta E$  resolution. The momenta of the  $D^0$  daughters obtained by a mass-constraint fit to the  $D^0$ candidate are used to calculate the Dalitz variables.

The continuum background arising from  $e^+e^- \rightarrow q\bar{q}$ (where  $q = u, d, s, c$ ) events is suppressed with the procedure described in Refs. [\[28,29\]](#page-14-8) and with the BDT [\[30,31\]](#page-14-9) algorithm implemented within the TMVA [\[32\]](#page-14-10) package.

The  $b$  flavor of the tagging  $B$  meson is identified from inclusive properties of particles that are not associated with the signal  $B$  candidate [\[33\]](#page-14-11). The tagging information is represented by two parameters: the  $b$ -flavor charge  $q$ and the purity  $r$ . The parameter  $r$  is an event-by-event, MC-determined flavor-tagging dilution factor that ranges from  $r = 0$  for no flavor discrimination to  $r = 1$  for unambiguous flavor assignment. The data are sorted into seven intervals of r. For events with  $r > 0.1$ , the wrong-tag fractions for six r intervals,  $w_l$  ( $l = 1, 2, ..., 6$ ), and their differences between  $B^0$  and  $\bar{B}^0$  decays,  $\Delta w_i$ , are determined from semileptonic and hadronic  $b \rightarrow c$  decays [\[34\].](#page-14-12) If  $r \leq 0.1$ , the wrong-tag fraction is set to 0.5, and the tagging information is not used. The total effective tagging efficiency,  $\varepsilon_{\text{eff}} = \sum (f_l \times (1 - 2w_l)^2)$ , is 0.3, where  $f_l$  is the fraction of events in the category *l*. The parameter  $Q_B$  =  $q_B(1 - 2w)/(1 - q_B\Delta w)$  is used instead of the parameter  $q_B$ , defined in Eq. [\(8\),](#page-3-1) to account for the wrong tag.

The signal yields of  $B^0 \to \bar{D}^{(*)0} h^0$  modes are obtained from an extended unbinned maximum likelihood fit of the  $\Delta E$ - $M_{\text{bc}}$  two-dimensional distribution in the region  $\Delta E \in (-0.15 \text{ GeV}, 0.30 \text{ GeV}) \cap M_{bc} \in (5.20 \text{ GeV}/c^2,$ 5.29 GeV/c<sup>2</sup>). The signal yield of  $B^+ \rightarrow \bar{D}^0 \pi^+$  events is obtained from an extended unbinned maximum likelihood fit of the  $\Delta E$  distribution in the region (-0.10 GeV, 0.15 GeV) for  $M_{bc} \in (5.272 \text{ GeV}/c^2, 5.287 \text{ GeV}/c^2)$ .

The sideband region is defined as the union of two rectangular regions in the  $\Delta E - M_{bc}$  plane:  $M_{bc}$  ∈ (5.23 GeV/ $c^2$ , 5.26 GeV/ $c^2$ ) ∩  $\Delta E$  ∈ (-0.15 GeV, 0.30 GeV), and  $M_{bc} \in (5.26 \text{ GeV}/c^2, 5.29 \text{ GeV}/c^2) \cap$  $\Delta E \in (0.12 \text{ GeV}, 0.30 \text{ GeV}).$ 

The selection criteria and the analysis procedure are tested using the Monte Carlo (MC) simulation and fixed before performing the fit of the CP-violation parameters. The MC events are generated with EvtGen [\[35\].](#page-14-13) Finalstate radiation from charged particles is simulated during the event generation using PHOTOS [\[36\]](#page-14-14). The generated events are processed through the detailed detector simulation based on GEANT3 [\[37\]](#page-14-15).

# IV.  $B^+ \rightarrow \bar{D}^0 \pi^+$  SAMPLE

<span id="page-6-0"></span>The  $B^+ \rightarrow \bar{D}^0 \pi^+$  control sample is experimentally clean and has kinematic properties and detection efficiency similar to the  $B^0 \to \bar{D}^{(*)0} h^0$  decay. We use this process to select a sample of D mesons in the flavor eigenstate and to measure the parameters  $K_i$  defined in Eq. [\(6\).](#page-3-2)

#### A. Signal yield

Three components are included in the fit of the  $\Delta E$ distribution: signal,  $B^+ \rightarrow \bar{D}^0 K^+$  background and combinatorial background.

The signal distribution is parametrized by the sum of a Gaussian and two Crystal Ball functions [\[38\]](#page-14-16) with a common peak position. The mean and the Gaussian width are free-fit parameters, while the other parameters are fixed to the values obtained from simulation. Background from the  $B^+ \rightarrow \bar{D}^0 K^+$  decays is parametrized by a Gaussian function with all parameters fixed from simulation. Combinatorial background is parametrized by a secondorder Chebyshev polynomial. The parameters of the combinatorial background shape are obtained from the fit. The  $\Delta E$  distribution for  $B^+ \to \bar{D}^0 \pi^+$  candidates and the results of the fit are shown in Fig. [3](#page-6-1). Yields of the signal and background components are listed in Table [II.](#page-7-0)

The parameters  $K_i$  are measured using the events in the  $\Delta E$  interval between -30 and 40 MeV. This interval is optimized to suppress the background from  $B^+ \to \bar{D}^0 K^+$ events without significant signal-efficiency loss.

# B. Measurement of parameters  $K_i$

The charged pion from the  $B^+ \rightarrow \bar{D}^0 \pi^+$  decay tags the flavor of the D meson. Therefore, the fraction of the signal events corresponding to the *i*th Dalitz plot bin equals  $K_i$ .

The Dalitz distribution for  $D \to K_S^0 \pi^+ \pi^-$  in the signal  $\Delta E$  range, where the D meson is produced in  $B^+ \to \bar{D}^0 \pi^+$ 

<span id="page-6-1"></span>

FIG. 3.  $\Delta E$  distribution for  $B^+ \rightarrow \bar{D}^0 \pi^+$  candidates. Black circles with error bars show data, the solid blue line is the complete fit function, the dashed blue line is the signal component, the dashed black line is the background from  $B^+ \rightarrow \bar{D}^0 K^+$ decays, and the dashed brown line is the combinatorial background. Vertical red lines show the signal area. A histogram with the pulls of the data with respect to the fit curve is shown at the bottom (with horizontal blue dashed lines at pull values of  $\pm 3$ ).

<span id="page-7-0"></span>TABLE II. Fit results of the  $\Delta E$  distribution for  $B^+ \to \bar{D}^0 \pi^+$ candidates. The numbers of events and the fraction of signal events are shown for the signal  $\Delta E$  region.

Value	
$(1.375 \pm 0.014) \times 10^4$	
$18.7 \pm 9.8$	
$1295 + 79$	
$91.3 \pm 0.9$	

decays, is shown in Fig. [4\(a\).](#page-7-1) The fraction of signal,  $f_{\text{sig}} = (91.3 \pm 0.9)\%$ , is obtained from a fit of the  $\Delta E$ distribution. The Dalitz plot for events from the  $\Delta E - M_{\text{bc}}$ sideband is shown in Fig. [4\(b\)](#page-7-1). The binned background distribution is obtained from these data.

The values of the parameters  $K_i$  are listed in Table [III](#page-7-2). The uncertainties shown include the statistical uncertainty of the signal sample and the uncertainty due to background evaluation, added in quadrature. The systematic uncertainties associated with the background Dalitz plot distribution are neglected because the background fraction is very small.

# V.  $B^0 \rightarrow \bar{D}^{(*)0} h^0$  SAMPLE

### A. Background components

Three background components are considered for the  $B^0 \rightarrow \bar{D}^{(*)0} h^0$  candidates:

- (i) Combinatorial background from nonresonant light quark production (continuum background).
- (ii) Combinatorial background from  $BB$  events.

(iii) Background from partially reconstructed  $B$  decays. Background from partially reconstructed decays is dominated by  $B \to \bar{D}^0 \rho$  and  $B \to \bar{D}^* \pi^0$  for the  $B^0 \to \bar{D}^0 \pi^0$ mode, and by  $B \to \bar{D}^* \rho$  for the  $B^0 \to \bar{D}^{*0} \pi^0$  mode. These

<span id="page-7-2"></span>TABLE III. The values of the parameters  $K_i$  measured with the  $B^+ \rightarrow \bar{D}^0 \pi^+$  data sample. The values are not corrected for the detection efficiency. The precision is limited by statistics, while the systematic uncertainty is found to be negligible.

$\operatorname{Bin}(i)$	$K_i$ (%)	$K_{-i}$ (%)
	$17.42 \pm 0.32$	$7.81 \pm 0.25$
2	$7.51 \pm 0.22$	$1.29 \pm 0.10$
$\mathcal{E}$	$10.24 \pm 0.26$	$2.58 \pm 0.14$
$\overline{4}$	$2.85 \pm 0.14$	$1.16 \pm 0.10$
5	$9.45 \pm 0.25$	$4.25 \pm 0.17$
6	$7.31 \pm 0.22$	$1.73 \pm 0.11$
	$10.48 \pm 0.26$	$1.18 \pm 0.10$
8	$12.46 \pm 0.28$	$2.38 \pm 0.14$

processes, reconstructed with one missing pion, lead to a concentration below  $-0.1$  GeV in the  $\Delta E$  distribution. The background in all other channels is dominated by the combinatorial contribution with featureless  $\Delta E$ distribution.

The background contribution from charmless  $B^0$  decays is suppressed by requiring the presence of a  $D^0$  candidate, and thus is found to be negligible in this measurement.

#### B. Signal yield

A two-dimensional unbinned maximum-likelihood fit of the  $\Delta E$ - $M_{bc}$  distribution is performed for each signal mode. The probability density function (PDF) contains four components, corresponding to the signal and three backgrounds introduced above.

The signal  $\Delta E$  distributions are parametrized by the sum of a Gaussian and two Crystal Ball functions with a common peak position. The signal  $M<sub>bc</sub>$  distributions are parametrized by a function introduced in Ref. [\[39\]](#page-14-17) and referred to as the Novosibirsk distribution. The peak

<span id="page-7-1"></span>

FIG. 4. Dalitz plot distributions for  $D \to K_S^0 \pi^+ \pi^-$  candidates with D from  $B^+ \to \bar{D}^0 \pi^+$  decay in the (a) signal and (b) sideband areas.

position in the  $\Delta E$ - $M_{bc}$  plane is obtained from the fit, while the other parameters are fixed at the values obtained from simulation.

The  $\Delta E$  distributions for events from the continuum background are parametrized by a second-order Chebyshev polynomial. The  $\Delta E$  distributions of the combinatorial background from the  $B\bar{B}$  events are parametrized by an exponential function. The  $M_{bc}$  distributions of the combinatorial backgrounds are parametrized by an ARGUS function [\[40\].](#page-14-18) The parameters of the  $\Delta E$  PDF are obtained from the fit, while those of the  $M_{bc}$  PDF are fixed at the values obtained from simulation.

The  $\Delta E$  distributions of the background from partially reconstructed B decays are parametrized by the following function:

$$
p_{\text{pr}}(\Delta E) \propto 1 + \zeta_1(\Delta E - \Delta E_0)
$$
  
+  $s \ln \left( 1 + b \exp \left[ \frac{(\zeta_r - \zeta_1)(\Delta E - \Delta E_0)}{s} \right] \right).$  (11)

This function describes two asymptotically straight lines smoothly merged near the point given by the  $\Delta E_0$  parameter whose slopes are given by  $\zeta_{\{r, l\}}$ . The parameter s determines the curvature at the junction. If the B-candidate decay chain contains a  $\pi^0$  or  $\eta$  reconstructed in the  $\gamma\gamma$  final state, the  $M_{bc}$  distribution of the background from partially reconstructed B decays is parametrized by the Novosibirsk function; otherwise, it is parametrized by the sum of ARGUS and Gaussian functions. All parameters are fixed at the values obtained from simulation except for the values of the  $\Delta E_0$  parameter for the  $B^0 \to \bar{D}^0 \pi^0$  and  $B^0 \to \bar{D}^{*0} \pi^0$ modes that are obtained from the fit.

Several correlations between the  $\Delta E$  and  $M_{bc}$  distributions are taken into account. A left-side tail of the signal  $\Delta E$  distribution is due to a  $\pi^0$  or  $\eta$  candidate where only one photon was identified correctly. This partially wrong combination leads to correlated shift both in  $\Delta E$  and  $M_{\text{bc}}$ . A similar correlation appears in the distributions of the background from partially reconstructed  $B$  decays. The width of the signal  $\Delta E$  distribution for the B candidates with the  $\eta$  or  $\omega$  reconstructed in the  $\pi^+\pi^-\pi^0$  final state is determined by the charged final-state particle momentum resolution if both final-state photons are correctly assigned. For such candidates, the  $\Delta E$  and  $M_{bc}$  distributions are correlated. That correlation is accommodated by introducing a  $\Delta E$  dependence of the signal  $M_{bc}$  PDF parameters. This parametrization is equivalent to a two-dimensional Gaussian function. No significant correlation is found for the combinatorial background. The values of parameters required to employ the correlations are obtained from simulation.

The fit projections for the  $B^0 \rightarrow \bar{D}^0 \pi^0$  and  $B^0 \rightarrow \bar{D}^0 \omega$ modes are shown in Fig. [5.](#page-9-0) The fit projections for the other signal modes are shown in Fig. [6.](#page-10-0) The fractions of background from partially reconstructed  $B$  decays are small for all modes except  $B^0 \to \bar{D}^0 \pi^0$  and  $B^0 \to \bar{D}^{*0} \pi^0$  (compare the  $\Delta E$  distributions below -0.1 GeV for  $B^0 \rightarrow \bar{D}^0 \pi^0$  and  $B^0 \rightarrow \bar{D}^0 \omega$  in Fig. [5,](#page-9-0) for example) and cannot be determined from the fit. These fractions are fixed relative to the fractions of combinatorial background from  $B\bar{B}$  events using the values obtained from MC simulation.

The ellipses in the  $\Delta E$ - $M_{bc}$  plane inscribed in the rectangular areas marked by the vertical red lines in Figs. [5](#page-9-0) and [6](#page-10-0) are defined for each signal mode and are referred to as signal regions. The events in these signal regions are used in the fit of the CP-violation parameters. The signal yields  $N_{sig}$  and fractions  $f_{sig}$  of signal events for each signal region obtained from the  $\Delta E - M_{bc}$  fit are listed in Table [IV.](#page-10-1) The Dalitz plots for events with a wrong-tag probability of under 23% are shown in Fig. [7.](#page-11-0)

# VI. DETERMINATION OF THE CP-VIOLATION PARAMETERS

<span id="page-8-0"></span>The CP-violation parameters are measured using the unbinned maximum-likelihood fit of the  $\Delta t$  distribution. The likelihood function is defined as

$$
\mathcal{L} = \prod_{j=1}^{N} \left[ (1 - f_{\text{bkg},j}) p_{\text{sig}}(\Delta t_j) + f_{\text{bkg},j} p_{\text{bkg}}(\Delta t_j) \right],\tag{12}
$$

where the product is evaluated over all  $N$  events in the sample,  $f_{bkg,j}$  is the event-dependent background fraction obtained from the  $\Delta E$ - $M_{\text{bc}}$  fit,  $p_{\text{sig}}$  is the signal PDF, and  $p_{\text{bkg}}$  is the background PDF.

The background  $\Delta t$  distributions are parametrized by convolving the function

$$
f_{\delta}\delta(\Delta t) + (1 - f_{\delta})2\tau_{\text{bkg}}e^{-|\Delta t|/\tau_{\text{bkg}}}
$$
 (13)

with a double-Gaussian function; here,  $\delta$  is the Dirac delta function and  $\tau_{bkg}$  is the effective lifetime for background events. The widths of the double-Gaussian function are event dependent and proportional to the estimated vertex resolution obtained from the vertex-constrained kinematic fits. The parameters  $f_{\delta}$  and  $\tau_{bkg}$  are obtained from simulation, while the parameters of the double-Gaussian function are obtained from the fit of the  $\Delta t$  distribution in the  $\Delta E$ - $M_{\text{bc}}$  sideband. The  $\Delta t$  distributions for background from  $B\bar{B}$  events and from continuum events are parametrized separately.

The signal  $\Delta t$  distribution is parametrized by convolving Eq. [\(8\)](#page-3-1) with a resolution function. The resolution function is described in Ref. [\[41\]](#page-14-19). It is tuned for each event using information obtained from the vertex-constrained kinematic fits.

<span id="page-9-0"></span>

FIG. 5.  $\Delta E$  fit projections for the signal  $M_{bc}$  regions (a,c) and  $M_{bc}$  fit projections for the signal  $\Delta E$  regions (b,d) for the  $B^0 \rightarrow \overline{D}^0 \pi^0$ (a,b) and  $B^0 \rightarrow \bar{D}^0 \omega$  (c,d) candidates. Black circles with errors show data, continuous blue lines show projections of complete fit functions, dashed blue lines show signal components, dashed black lines show continuum background components, dashed brown lines show background from partially reconstructed B decays and dot-dashed lines show combinatorial background from  $B\bar{B}$  events. Histograms with the pulls of the data with respect to the fit curves are shown at the bottom of each plot (with horizontal blue dashed lines at pull values of  $\pm 3$ ).

<span id="page-10-0"></span>

FIG. 6.  $\Delta E$  fit projections for the signal  $M_{bc}$  regions (a–e) and  $M_{bc}$  fit projections for the signal  $\Delta E$  regions (f–j) for the  $B^0 \to \bar{D}^0 \eta$ ,  $\eta \to \gamma \gamma$  (a,f),  $B^0 \to \bar{D}^0 \eta$ ,  $\eta \to \pi^+ \pi^- \pi^0$  (b,g),  $B^0 \to \bar{D}^0 \eta'$  (c,h),  $B^0 \to \bar{D}^{*0} \pi^0$  (d,i), and  $B^0 \to \bar{D}^{*0} \eta$  (e,j) candidates. Black circles with errors show data, continuous blue lines show projections of complete fit functions, dashed blue lines show signal components, dashed black lines show continuum background components, dashed brown lines show background from partially reconstructed B decays and dot-dashed lines show combinatorial background from  $BB$  events.

Table [V](#page-11-1) shows results of the fit of the CP-violation parameters, where  $\sin 2\varphi_1$  and  $\cos 2\varphi_1$  are treated as independent variables. The correlation coefficient of  $\sin 2\varphi_1$  and  $\cos 2\varphi_1$  is about  $-3\%$ .

The combined fit of all signal modes, with the parameters  $\sin 2\varphi_1$  and  $\cos 2\varphi_1$  considered as functions of the angle  $\varphi_1$ , results in

$$
\varphi_1 = 11.7^{\circ} \pm 7.8^{\circ} \text{(stat)}.
$$
 (14)

For illustration, the raw CP asymmetries for the Dalitz plot bins most sensitive to  $\sin 2\varphi_1$  are shown in Fig. [8.](#page-11-2) The

<span id="page-10-1"></span>TABLE IV. Results of the  $\Delta E \text{-} M_{bc}$  fit for  $B^0 \to \bar{D}^{(*)0} h^0$  data. The numbers of events  $N_{sig}$  and the fractions  $f_{sig}$  of signal events obtained from the fit for the signal  $\Delta E$ - $M_{bc}$  regions are shown.

Mode	$N_{\text{sig}}$	$f_{\text{sig}}(\%)$
$B^0\to \bar{D}^0\pi^0$	$464 \pm 26$	$72.1 \pm 4.1$
$B^0 \to \bar{D}^0 \eta_{\gamma\gamma}$	$99 \pm 14$	$50.5 \pm 7.0$
$B^0 \to \bar{D}^0 \eta_{\pi^+\pi^-\pi^0}$	$51.3 \pm 8.8$	$66 \pm 11$
$B^0\to \bar{D}^0\omega$	$182 \pm 18$	$58.4 \pm 5.7$
$B^0 \rightarrow \bar{D}^0 \eta'$	$28.2 \pm 6.4$	$70 \pm 16$
$B^0 \rightarrow \bar{D}^{*0} \pi^0$	$103 \pm 17$	$44.1 \pm 7.4$
$B^0 \rightarrow \bar{D}^{*0} \eta$	$36.1 \pm 7.6$	$64 \pm 13$
Total	$962 \pm 41$	$61 \pm 2.6$

 $\Delta t$  distributions for the Dalitz plot bins most sensitive to  $\cos 2\varphi_1$  are shown in Fig. [9](#page-11-3).

#### VII. SYSTEMATIC UNCERTAINTIES

Table [VI](#page-12-0) provides the estimates of the systematic uncertainties in the measured values of the CP-violation parameters.

The uncertainty due to the experimental resolution for the Dalitz variables is evaluated using the large sample of simulated signal events. The fit results are compared for the CP-violation fit performed using the reconstructed and the generated Dalitz-variables values. The uncertainty due to the detection-efficiency variation over the Dalitz plot is also evaluated using the simulated signal events. The fit results are compared for the CP-violation fit performed with and without the efficiency correction.

The systematic uncertainty related to the signal  $\Delta t$ parametrization is estimated by varying each resolution parameter by  $\pm \sigma_+$  ( $\pm 2\sigma_+$  for parameters obtained from MC simulation) and repeating the fit.

<span id="page-10-2"></span>Other contributions to the systematic uncertainty (items 4–10 in Table [VI](#page-12-0)) are evaluated simultaneously from the fit performed with nuisance parameters and the likelihood function expressed as follows:

$$
-2\log \mathcal{L}_n = -2\log \mathcal{L} + \sum_{j,k} (p_j - p_j^0) \mathcal{K}_{jk} (p_k - p_k^0), \quad (15)
$$

<span id="page-11-0"></span>

FIG. 7. Dalitz distributions for D mesons produced in tagged  $B^0 \to \bar{D}^{(*)0} h^0$  decays with wrong-tag probability of less than 23%. The signal B meson is tagged as (a)  $B^0$  and (b)  $\bar{B}^0$ .

where  $\mathcal L$  is defined in Eq. [\(12\)](#page-8-0),  $p_i$  and  $p_i^0$  are the current and central values of the jth nuisance parameter, respectively,  $K$  is the inverse covariance matrix for the nuisance parameters, and the sum is evaluated over all nuisance parameters. The following nuisance parameters are introduced to evaluate the systematic uncertainty:

- (i) The parameters  $C_i$  and  $S_i$  that give the dominant contribution (with the covariance matrix taken from the supplementary materials for Ref. [\[20\]\)](#page-14-0).
- (ii) The parameters  $K_i$  with the uncertainties shown in Table [III.](#page-7-2)
- (iii) The yield of signal events in each Dalitz plot bin for each signal mode, with the value and uncertainty obtained from the  $\Delta E$ - $M_{bc}$  fit.
- (iv) The background  $\Delta t$  PDF parameters, with the values and uncertainties obtained from the fit of the  $\Delta t$ distribution in the  $\Delta E$ - $M_{\text{bc}}$  sideband.
- (v) The parameters  $\tau_B$  and  $\Delta m_B$  with values and uncertainties taken from Ref. [\[42\].](#page-14-20)
- (iv) The average bias in the wrong-tag probability, with the uncertainty obtained using the results from Ref. [\[34\].](#page-14-12)

The flavor-tagging procedure and the uncertainties in the  $\Delta m_B$  and  $\tau_B$  values give negligible contributions to the systematic uncertainty.

<span id="page-11-1"></span>TABLE V. Fit of the CP-violation parameters. Only statistical uncertainty is shown.

Signal mode	$\sin 2\varphi_1$	$\cos 2\varphi_1$
$B^0 \rightarrow \bar{D}^0 \pi^0$	$0.61 \pm 0.37$	$0.88^{+0.46}_{-0.52}$
$B^0 \rightarrow \bar{D}^0 \omega$	$-0.12 \pm 0.58$	$1.28_{-0.69}^{+0.62}$
Other modes	$0.44 \pm 0.51$	$0.89^{+0.49}_{-0.55}$
All modes	$0.41 \pm 0.27$	$0.97 \pm 0.33$

<span id="page-11-4"></span>Frequentist confidence intervals for the CP-violation parameters are evaluated using the profile likelihood method with likelihood ratios [\[43\]](#page-14-21)

$$
\lambda(\xi) = \frac{\mathcal{L}(\xi, \hat{\hat{p}})}{\mathcal{L}(\hat{\xi}, \hat{p})},\tag{16}
$$

<span id="page-11-2"></span>

FIG. 8. Raw CP-asymmetry distributions for the  $B^0 \to \bar{D}^{(*)0} h^0$ candidates in the (a)  $\pm 1$ st and (b)  $\pm 5$ th  $D \rightarrow K_5^0 \pi^+ \pi^-$  decay<br>Dalitz plot bins. Red lines are the result of the CP-violation fit Dalitz plot bins. Red lines are the result of the CP-violation fit performed with the full data sample. The asymmetry for the  $\bar{B}^0 \rightarrow \bar{D}^{*0} h^0$  candidates is taken with inverted sign.

<span id="page-11-3"></span>

FIG. 9.  $\Delta t$  distributions for the  $B^0 \rightarrow \bar{D}^0 h^0$  candidates with wrong-tag probability of less than 23% in (a) the  $-3rd$  D  $\rightarrow$  $K_S^0 \pi^+ \pi^-$  Dalitz plot bin tagged as  $B^0$  and (b) the 7th  $D \to$  $K_S^{\tilde{0}} \pi^+ \pi^-$  Dalitz plot bin tagged as  $\bar{B}^0$ . Continuous blue lines are the result of the CP-violation fit performed with the full data sample. Dashed red lines are obtained with  $\varphi_1 = 68.1^{\circ}$ .

<span id="page-12-1"></span>

FIG. 10. Negative double logarithm of profiled likelihood ratio Eq. [\(16\)](#page-11-4) for (a) sin  $2\varphi_1$ , (b) cos  $2\varphi_1$  and (c)  $\varphi_1$  obtained with the Minos algorithm [\[44\]](#page-14-22). Black squares mark  $n\sigma$  standard confidence intervals corresponding to statistical uncertainty, while blue circles mark  $n\sigma$ standard confidence intervals corresponding to the overall uncertainty. Continuous blue and dashed black lines show (a, b) fourth- and (c) fifth-order polynomial fits.

where  $\xi$  is  $\sin 2\varphi_1$  or  $\cos 2\varphi_1$  or  $\varphi_1$ ,  $\hat{\xi}$  is the optimal value,  $\hat{p}$ represents the optimal values of all other parameters corresponding to  $\hat{\xi}$ , and  $\hat{\hat{p}}$  represents the optimal values of all other parameters corresponding to the  $\xi$  value. Negative double logarithms of the likelihood ratios are shown in Fig. [10](#page-12-1).

The dominant uncertainties shown in Table [VI](#page-12-0) could be reduced in high-statistics measurements at Belle II. Indeed, the uncertainties associated with the parameters  $K_i$ , the  $\Delta t$  parametrization and the  $\Delta E - M_{\text{bc}}$  fit are determined by the size of the data sample. The parameters  $C_i$  and  $S_i$  can be measured more precisely with a large data set of coherently produced  $D^0\bar{D}^0$  pairs collected by the BES-III experiment.

<span id="page-12-0"></span>TABLE VI. The sources and estimates of the systematic uncertainties for the CP-violation parameters measured in the  $B^0 \rightarrow \bar{D}^{(*)0}h^0$  decays. The uncertainty  $\sigma_{\text{nuis}}$  due to sources 4–10 is evaluated from the single fit varying all the nuisance parameters and using the likelihood function Eq. [\(15\)](#page-10-2). The total systematic uncertainty  $\sigma_{syst}$  is calculated as  $\sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \sigma_{nus}^2}$ . The values related to sources 4.10 are shown for illustration. values related to sources 4–10 are shown for illustration.

Source	$\delta_{\sin 2\varphi_1}$ (%)	$\delta_{\cos 2\varphi_1}$ (%)	$\delta_{\varphi_1}$ (deg)
1. Dalitz variables resol.	0.3	0.7	0.1
2. Detection efficiency	0.6	0.8	0.2
3. $\Delta t$ resolution	3.8	6.7	1.2
4. Flavor tagging	0.1	0.1	< 0.1
5. $\Delta m_R$	0.1	0.1	< 0.1
6. $\tau_B$	0.1	0.1	< 0.1
7. $M_{\text{bc}}$ - $\Delta E$ fit	3.4	1.9	0.8
8. Bkg. $\Delta t$ param.	3.6	3.1	0.7
$9. K_i$	3.2	2.0	0.7
10. $C_i$ and $S_i$	7.6	$+20$ $-13$	1.1
$\sigma_{\text{nuis}}$	7.6	$+20$ $-13$	1.6
Total $\sigma_{\text{syst}}$	8.5	$+21$ $-1.5$	2.1
Stat. error for comparison	27	33	7.8

## VIII. CONCLUSIONS

A novel model-independent approach for measuring the CKM angle  $\varphi_1$  has been developed and applied to the full data set of the Belle experiment. The following results are obtained:

$$
\sin 2\varphi_1 = 0.43 \pm 0.27 \text{(stat)} \pm 0.08 \text{(syst)},
$$
  
\n
$$
\cos 2\varphi_1 = 1.06 \pm 0.33 \text{(stat)}^{+0.21}_{-0.15} \text{(syst)},
$$
  
\n
$$
\varphi_1 = 11.7^\circ \pm 7.8^\circ \text{(stat)} \pm 2.1^\circ \text{(syst)}.
$$
 (17)

The value  $\sin 2\varphi_1 = 0.691 \pm 0.017$  measured in  $b \rightarrow$  $c\bar{c}s$  transitions determines the absolute value of cos  $2\varphi_1$ , leading to two possible solutions in the  $0^{\circ} \leq \varphi_1 < 180^{\circ}$ range. Our measurement is inconsistent with the negative solution, corresponding to the value  $\varphi_1 = 68.1^{\circ}$  at the level of 5.1 standard deviations, but in agreement with the positive solution, corresponding to the value  $\varphi_1 = 21.9^\circ$ at 1.3 standard deviations. Thus, this measurement clearly resolves the ambiguity in  $\varphi_1$  inherent in the measurement of  $\sin 2\varphi_1$  using the  $b \to c\bar{c}s$  transition.

This measurement supersedes the previous measurement of the sin  $2\varphi_1$  and cos  $2\varphi_1$  in  $B^0 \to \bar{D}^{(*)0}h^0$  decays at Belle [\[8\]](#page-13-7). Nevertheless, it should be emphasized that a different analysis technique is used here. Furthermore, experimental information from  $B^+ \to \bar{D}^0 \pi^+$  decays and from Ref. [\[20\]](#page-14-0) is used in this analysis but not in Ref. [\[8\]](#page-13-7).

The binned Dalitz plot approach could be used for precise  $\varphi_1$  measurements in  $B^0 \to \bar{D}^{(*)0} h^0$  followed by  $\bar{D}^0 \to K_S^0 \pi^+ \pi^-$  decays with the high-statistics data from the Belle II experiment. The dominant systematic uncertainties could be reduced with this larger data sample. Also, abundant coherently produced  $D^0\bar{D}^0$  pairs collected by the BES-III experiment can be used to improve our knowledge of the phase parameters  $C_i$  and  $S_i$ . The number of Dalitz plot bins can be increased in future measurements to improve the statistical sensitivity to the CP-violation parameters.

Some NP models predict the magnitude of CP violation to differ from the SM expectations [\[45\]](#page-14-23). The difference may vary for different quark transitions. Thus, it would be interesting to compare the sin  $2\varphi_1$  value precisely measured in the  $b \to c\bar{u}d$  transitions governing the  $B^0 \to \bar{D}^{(*)0}h^0$ decays with the  $\sin 2\varphi_1$  value precisely measured in the  $b \rightarrow c\bar{c}s$  transitions.

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- <span id="page-13-0"></span>[1] A. Sakharov, Zh. Eksp. Teor. Fiz. 49, 345 (1965) [Sov. J. Exp. Theor. Phys. 22, 241 (1966)].
- <span id="page-13-1"></span>[2] M. Kobayashi and T. Maskawa, [Prog. Theor. Phys.](http://dx.doi.org/10.1143/PTP.49.652) 49, 652 [\(1973\).](http://dx.doi.org/10.1143/PTP.49.652)
- [3] N. Cabibbo, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.10.531) **10**, 531 (1963).
- <span id="page-13-2"></span>[4] A. G. Cohen, D. B. Kaplan, and A. E. Nelson, [Annu. Rev.](http://dx.doi.org/10.1146/annurev.ns.43.120193.000331) [Nucl. Part. Sci.](http://dx.doi.org/10.1146/annurev.ns.43.120193.000331) 43, 27 (1993).
- <span id="page-13-3"></span>[5] A. J. Buras and R. Fleischer, [Adv. Ser. Dir. High Energy](http://dx.doi.org/10.1142/ASDHEP) Phys. 15[, 65 \(1998\)](http://dx.doi.org/10.1142/ASDHEP).
- <span id="page-13-4"></span>[6] The first sin  $2\varphi_1$  measurements at *B* factories are published in B. Aubert et al. (BABAR Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.87.091801) 87[, 091801 \(2001\)](http://dx.doi.org/10.1103/PhysRevLett.87.091801); K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 87[, 091802 \(2001\)](http://dx.doi.org/10.1103/PhysRevLett.87.091802); The most precise results at the moment are B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 79[, 072009 \(2009\)](http://dx.doi.org/10.1103/PhysRevD.79.072009); I. Adachi et al. (Belle Collaboration), Phys. Rev. Lett. 108[, 171802 \(2012\)](http://dx.doi.org/10.1103/PhysRevLett.108.171802); R. Aaij et al. (LHCb Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.115.031601) 115, [031601 \(2015\).](http://dx.doi.org/10.1103/PhysRevLett.115.031601)
- <span id="page-13-5"></span>[7] B. Aubert et al. (BABAR Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.99.231802) 99, [231802 \(2007\).](http://dx.doi.org/10.1103/PhysRevLett.99.231802)
- <span id="page-13-7"></span>[8] P. Krokovny et al. (Belle Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.97.081801) 97[, 081801 \(2006\).](http://dx.doi.org/10.1103/PhysRevLett.97.081801)
- <span id="page-13-6"></span>[9] R. Itoh et al. (Belle Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.95.091601) 95, [091601 \(2005\)](http://dx.doi.org/10.1103/PhysRevLett.95.091601); B. Aubert et al. (BABAR Collaboration), Phys. Rev. D 71[, 032005 \(2005\)](http://dx.doi.org/10.1103/PhysRevD.71.032005).
- [10] B. Aubert et al. (BABAR Collaboration), [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.74.091101) 74, [091101\(R\) \(2006\)](http://dx.doi.org/10.1103/PhysRevD.74.091101); J. Dalseno et al. (Belle Collaboration), Phys. Rev. D 76[, 072004 \(2007\)](http://dx.doi.org/10.1103/PhysRevD.76.072004).
- <span id="page-13-8"></span>[11] A. B. Carter and A. I. Sanda, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.45.952) 45, 952 [\(1980\);](http://dx.doi.org/10.1103/PhysRevLett.45.952) Phys. Rev. D 23[, 1567 \(1981\)](http://dx.doi.org/10.1103/PhysRevD.23.1567); I. I. Bigi and A. I. Sanda, [Nucl. Phys.](http://dx.doi.org/10.1016/0550-3213(81)90519-8) 193, 85 (1981).
- [12] A. J. Bevan, B. Golob, Th. Mannel, S. Prell, and B. D. Yabsley, [Eur. Phys. J. C](http://dx.doi.org/10.1140/epjc/s10052-014-3026-9) 74, 3026 (2014).
- <span id="page-13-9"></span>[13] A. Abdesselam et al. (BABAR and Belle collaborations) Phys. Rev. Lett. 115[, 121604 \(2015\).](http://dx.doi.org/10.1103/PhysRevLett.115.121604)
- <span id="page-13-10"></span>[14] R. H. Dalitz, Phys. Rev. 94[, 1046 \(1954\).](http://dx.doi.org/10.1103/PhysRev.94.1046)
- <span id="page-13-11"></span>[15] A. Bondar, T. Gershon, and P. Krokovny, [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2005.07.053) 624, [1 \(2005\).](http://dx.doi.org/10.1016/j.physletb.2005.07.053)
- <span id="page-13-12"></span>[16] A. Giri, Y. Grossman, A. Soffer, and J. Zupan, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.68.054018) 68[, 054018 \(2003\).](http://dx.doi.org/10.1103/PhysRevD.68.054018)
- <span id="page-13-13"></span>[17] A. Bondar and A. Poluektov, [Eur. Phys. J. C](http://dx.doi.org/10.1140/epjc/s2006-02590-x) 47, 347 (2006); 55[, 51 \(2008\).](http://dx.doi.org/10.1140/epjc/s10052-008-0600-z)
- [18] A. Bondar, A. Poluektov, and V. Vorobiev, [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.82.034033) 82, [034033 \(2010\).](http://dx.doi.org/10.1103/PhysRevD.82.034033)
- [19] S. Harnew and J. Rademacker, [Phys. Lett. B](http://dx.doi.org/10.1016/j.physletb.2013.11.065) 728, 296 [\(2014\);](http://dx.doi.org/10.1016/j.physletb.2013.11.065) [J. High Energy Phys. 03 \(2015\) 169.](http://dx.doi.org/10.1007/JHEP03(2015)169)
- <span id="page-14-0"></span>[20] J. Libby et al. (CLEO Collaboration), [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.82.112006) 82, [112006 \(2010\).](http://dx.doi.org/10.1103/PhysRevD.82.112006)
- <span id="page-14-1"></span>[21] A. Poluektov et al. (Belle Collaboration), [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.81.112002) 81, [112002 \(2010\).](http://dx.doi.org/10.1103/PhysRevD.81.112002)
- <span id="page-14-2"></span>[22] S. Kurokawa and E. Kikutani, [Nucl. Instrum. Methods](http://dx.doi.org/10.1016/S0168-9002(02)01771-0) [Phys. Res., Sect. A](http://dx.doi.org/10.1016/S0168-9002(02)01771-0) 499, 1 (2003), and other papers included in this volume.
- <span id="page-14-3"></span>[23] A. Abashian et al. (Belle Collaboration), [Nucl. Instrum.](http://dx.doi.org/10.1016/S0168-9002(01)02013-7) [Methods Phys. Res., Sect. A](http://dx.doi.org/10.1016/S0168-9002(01)02013-7) 479, 117 (2002).
- <span id="page-14-4"></span>[24] Z. Natkaniec et al. (Belle SVD2 Group), [Nucl. Instrum.](http://dx.doi.org/10.1016/j.nima.2005.11.228) [Methods Phys. Res., Sect. A](http://dx.doi.org/10.1016/j.nima.2005.11.228) 560, 1 (2006).
- <span id="page-14-5"></span>[25] H. Nakano, Ph.D thesis, Tohoku University (2014), Chap. 4 (unpublished).
- <span id="page-14-6"></span>[26] K. F. Chen et al. (Belle Collaboration), [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.72.012004) 72, [012004 \(2005\).](http://dx.doi.org/10.1103/PhysRevD.72.012004)
- <span id="page-14-7"></span>[27] K. A. Olive et al. (Particle Data Group), [Chin. Phys. C](http://dx.doi.org/10.1088/1674-1137/38/9/090001) 38, [090001 \(2014\)](http://dx.doi.org/10.1088/1674-1137/38/9/090001) and 2015 update.
- <span id="page-14-8"></span>[28] The Fox-Wolfram moments were introduced in G. C. Fox and S. Wolfram, [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.41.1581) 41, 1581 (1978); The Fisher discriminant used by Belle, based on modified Fox-Wolfram moments (SFW), is described in K. Abe et al. (Belle Collaboration), Phys. Rev. Lett. 87[, 101801 \(2001\)](http://dx.doi.org/10.1103/PhysRevLett.87.101801); K. Abe et al. (Belle Collaboration), [Phys. Lett. B](http://dx.doi.org/10.1016/S0370-2693(01)00626-8) 511, 151 [\(2001\)](http://dx.doi.org/10.1016/S0370-2693(01)00626-8).
- [29] S. H. Lee et al. (Belle Collaboration), [Phys. Rev. Lett.](http://dx.doi.org/10.1103/PhysRevLett.91.261801) 91, [261801 \(2003\).](http://dx.doi.org/10.1103/PhysRevLett.91.261801)
- <span id="page-14-9"></span>[30] L. Breiman, J. H. Friedman, R. A. Olshen, and C. J. Stone, Classification and Regression Trees (Wadsworth International Group, Belmont, CA, 1984).
- [31] R. E. Schapire and Y. Freund, [J. Comput. Syst. Sci.](http://dx.doi.org/10.1006/jcss.1997.1504) 55, 119 [\(1997\).](http://dx.doi.org/10.1006/jcss.1997.1504)
- <span id="page-14-10"></span>[32] A. Hoecker et al., Proc. Sci., ACAT040 (2007) 040.
- <span id="page-14-11"></span>[33] H. Kakuno et al., [Nucl. Instrum. Methods Phys. Res., Sect.](http://dx.doi.org/10.1016/j.nima.2004.06.159) A 533[, 516 \(2004\).](http://dx.doi.org/10.1016/j.nima.2004.06.159)
- <span id="page-14-12"></span>[34] K. Abe et al. (Belle Collaboration), [Phys. Rev. D](http://dx.doi.org/10.1103/PhysRevD.71.072003) 71, [072003 \(2005\)](http://dx.doi.org/10.1103/PhysRevD.71.072003); K. F. Chen et al. (Belle Collaboration), Phys. Rev. D 72[, 012004 \(2005\);](http://dx.doi.org/10.1103/PhysRevD.72.012004) H. Sahoo et al. (Belle Collaboration), Phys. Rev. D 77[, 091103 \(2008\)](http://dx.doi.org/10.1103/PhysRevD.77.091103).
- <span id="page-14-13"></span>[35] D. J. Lange, [Nucl. Instrum. Methods Phys. Res., Sect. A](http://dx.doi.org/10.1016/S0168-9002(01)00089-4) 462[, 152 \(2001\)](http://dx.doi.org/10.1016/S0168-9002(01)00089-4).
- <span id="page-14-14"></span>[36] T. Sjöstrand, [Comput. Phys. Commun.](http://dx.doi.org/10.1016/0010-4655(94)90132-5) 82, 74 (1994).
- <span id="page-14-15"></span>[37] R. Brun et al., GEANT 3.21, CERN Report No. DD/EE/84-1, 1984.
- <span id="page-14-16"></span>[38] T. Skwarnicki, Ph.D thesis, DESY F31-86-02 (1986), Appendix E.
- <span id="page-14-17"></span>[39] H. Ikeda et al. (Belle Collaboration), [Nucl. Instrum.](http://dx.doi.org/10.1016/S0168-9002(99)00992-4) [Methods Phys. Res., Sect. A](http://dx.doi.org/10.1016/S0168-9002(99)00992-4) 441, 401 (2000).
- <span id="page-14-18"></span>[40] H. Albrecht et al. (ARGUS Collaboration), [Phys. Lett. B](http://dx.doi.org/10.1016/0370-2693(90)91293-K) 241[, 278 \(1990\)](http://dx.doi.org/10.1016/0370-2693(90)91293-K).
- <span id="page-14-19"></span>[41] M. Röhrken, Time-Dependent CP Violation Measurements (Springer, Berlin, 2014), pp. 71–106.
- <span id="page-14-20"></span>[42] K. A. Olive et al. (Particle Data Group), [Chin. Phys. C](http://dx.doi.org/10.1088/1674-1137/38/9/090001) 38, [090001 \(2014\).](http://dx.doi.org/10.1088/1674-1137/38/9/090001)
- <span id="page-14-21"></span>[43] G. Bohm and G. Zech, Introduction to Statistics and Data Analysis for Physicists (Verlag Deutsches Elektronen-Synchrotron, Hamburg, 2010).
- <span id="page-14-22"></span>[44] B. A. Murtagh and M. A. Saunders, [Math. Program.](http://dx.doi.org/10.1007/BF01588950) 14, 41 [\(1978\);](http://dx.doi.org/10.1007/BF01588950) 16[, 84 \(1982\)](http://dx.doi.org/).
- <span id="page-14-23"></span>[45] Y. Grossman and M.P. Worah, [Phys. Lett. B](http://dx.doi.org/10.1016/S0370-2693(97)00068-3) 395, 241 [\(1997\).](http://dx.doi.org/10.1016/S0370-2693(97)00068-3)