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Model independent bounds in direct dark matter detection

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Summary. — We review a streamlined method and a complete set of numerical tools to set bounds from direct searches experiments on virtually any arbitrary model of dark matter elastically scattering on target nuclei.

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1. – Introduction

The nature of Dark Matter (DM) is one of the most exciting open questions at the interface between cosmology and particle physics. Since several decades, we have a compelling evidence of unseen mass at different scales. Nevertheless, a non gravitational manifestation of DM is yet to be identified. Direct, indirect and collider searches may shed light on the nature of DM. Hence, a model independent study of its phenomenology is crucial. For a review see, *e.g.*, [1].

Direct detection for DM aim at detecting the feeble kick to an atomic nucleus in underground detectors. DM direct searches experiments are achieving unprecedented sensitivity to DM detection. Indeed, in addition to the long standing DAMA results [2], nowadays there are other three experiments (CoGeNT [3,4], CRESST-II [5] and CDMS-II-Si [6]) with data that may have the right properties to be potentially ascribed to a DM interaction. DAMA [2] and CoGeNT [4] observe an annual modulation in their counting rates, while CRESST-II [5] and CDMS-II-Si [6] report an excess of events above their estimated backgrounds. However, we are far from a definitive and clear discovery because other experiments do not observe any significant excess above their expected backgrounds. The most stringent bounds for the spin-independent interactions are set by LUX [7], XENON100 [8], CDMS-II-Ge [9] and CDMSlite [10], while SIMPLE [11], PICASSO [12] and COUPP [13] setting relevant limits for the DM-p spin-dependent interactions.

Nevertheless, when interpreting the different experimental data, one has always to bear well in mind at least two main caveats. The first is that the fine experimental details must be treated with great care. The second caveat is instead associated with the choice of a very simple DM model for interpreting the experimental data (*e.g.* the DM-nucleus spin independent interactions). Upon relaxing one or more of these assumptions, the complicated experimental puzzle can perhaps be solved.

The scope of this work is to review a streamlined method and a complete set of numerical tools to derive the limits from direct detection experiments on virtually any arbitrary model of DM elastically scattering on target nuclei. In particular, in sect. **2**, we present an innovative and more general approach to study signals in this field based on non-relativistic operators. Then, we describe how to write down the main observables as a linear function of a manipulation of the form factor (provided in the appendix of ref. [14]), which take into account the non-relativistic physics of the DM-nucleus interaction. Finally, we show in sect. **3** that it is possible to set limits in this field in a completely model independent way thanks to the formalism described in sect. **2**.

2. – Basics and formalism of non-relativistic operators

When computing scattering cross sections at direct searches experiments, the main quantity that one has to compute is the differential rate of nuclear recoil measured in cpd/kg/keV. For a target nuclide $\mathcal{N}$ at rest, recoiling with energy $E_{\rm R}$ with a DM particle with initial velocity v and mass $m_{\rm DM}$, it reads

$$\frac{{\rm d}R_{\mathcal{N}}}{{\rm d}E_{\rm R}} = \frac{\xi_{\mathcal{N}}}{m_{\mathcal{N}}}\frac{\rho_\odot}{m_{\rm DM}}\int_{v_{\rm min}(E_{\rm R})}^{v_{\rm esc}} {\rm d}^3v\, v\, f_{\rm E}(\vec{v})\frac{{\rm d}\sigma_{\mathcal{N}}}{{\rm d}E_{\rm R}}(v, E_{\rm R}), \tag{1}$$

where $\rho_\odot$ is the local DM energy density, $m_{\mathcal{N}}$ is the mass of the target nuclide and $\xi_{\mathcal{N}}$ is its mass fraction in the detector. Here the differential cross section ${\rm d}\sigma_{\mathcal{N}}/{\rm d}E_{\rm R}$ is weighted with the DM velocity distribution in the Earth's frame $f_{\rm E}(\vec{v})$ which is modulated in time due to the Earth's motion around the Sun during the years [15]. In the velocity integral above, $v_{\rm min}(E_{\rm R})$ is the minimal velocity providing a nuclear recoil $E_{\rm R}$ of the nuclide and $v_{\rm esc}$ is the Milky Way's escape velocity. For elastic collision, $v_{\rm min}(E_{\rm R}) = \sqrt{m_{\mathcal{N}}E_{\rm R}/(2\mu_{\mathcal{N}}^2)}$, where $\mu_{\mathcal{N}} = m_{\rm DM}m_{\mathcal{N}}/(m_{\rm DM}+m_{\mathcal{N}})$ is the DM-nucleus reduced mass.

In general, the differential cross section in the non-relativistic regime, writes

$$\frac{{\rm d}\sigma_{\mathcal{N}}}{{\rm d}E_{\rm R}}(v, E_{\rm R}) = \frac{1}{32\pi}\frac{1}{m_{\rm DM}^2 m_{\mathcal{N}}}\frac{1}{v^2}\overline{|\mathcal{M}_{\mathcal{N}}|^2}, \tag{2}$$

where $\overline{|\mathcal{M}_{\mathcal{N}}|^2}$ is the square of the DM-nucleus matrix element that contains all the information related to the nature of the interactions and the nuclear physics.

We know that the local DM velocity is much smaller than the speed of light, therefore the right formalism that let better describes the physics of the scattering is the one of non-relativistic operators. In fact, since for elastic collisions, the relevant degrees of freedom are the exchanged momentum $\vec{q}$, the DM-nucleon relative velocity $\vec{v}$, the nucleon spin $\vec{s}_N$ ($N = p, n$ can be proton or neutron) and eventually the DM one $\vec{s}_\chi$ if different from zero, the scattering amplitude at the level of the nucleons will be a rotationally invariant function of these variables. In this regards, a basis of 16 Galilean invariant operators ($\mathcal{O}_i^{\rm NR}$) can be constructed and therefore the DM-nucleon matrix element $\mathcal{M}_N$ can be expressed as a linear combination of them. In particular $\mathcal{M}_N = \sum_{i=1}^{16} \mathfrak{c}_i^N(\lambda, m_{\rm DM})\, \mathcal{O}_i^{\rm NR}$, where the coefficients $\mathfrak{c}_i^N(\lambda, m_{\rm DM})$ are functions of the free parameters of the underlying relativistic theory (collectively denoted by λ), and the mass of the DM particle $m_{\rm DM}$.

A complete list and the numbering of these Galilean invariant combinations can be found for example in refs. [14, 17], where the reader can find further details.

Since now the nucleus is made of neutrons and protons, one has to correct the DM-nucleon matrix element with the nuclear responses that take into account the finite size of the target. According to eq. (55) of [14] we can write the spin-averaged amplitude squared for scattering off a target nucleus as

$$\overline{|\mathcal{M}_\mathcal{N}|^2} = \frac{m_\mathcal{N}^2}{m_N^2} \sum_{i,j=1}^{16} \sum_{N,N'=p,n} \mathfrak{c}_i^N \mathfrak{c}_j^{N'} F_{i,j}^{(N,N')}(v, E_\mathrm{R}, \mathcal{N}). \tag{3}$$

The functions $F_{i,j}^{(N,N')}(v, E_\mathrm{R}, \mathcal{N})$ are the nuclear responses and they encode all the information coming from the non-relativistic nuclear physics. A complete set of them for each pair of operators (i, j), pair of nucleons (N, N'), and for several target nuclei $\mathcal{N}$, has been for the first time provided in the appendices of ref. [14]. This is extremely useful because in this way all the possible non-relativistic DM-nucleus interactions can be studied.

Plugging back eq. (3) in eq. (2), the differential rate of nuclear recoil can be cast in a very general way. Following refs. [17, 18], it writes

$$\frac{\mathrm{d}R_\mathcal{N}}{\mathrm{d}E_\mathrm{R}} = X\,\xi_\mathcal{N} \sum_{i,j=1}^{16} \sum_{N,N'=p,n} \mathfrak{c}_i^N(\lambda, m_\mathrm{DM})\, \mathfrak{c}_j^{N'}(\lambda, m_\mathrm{DM})\, \mathcal{F}_{i,j}^{(N,N')}(E_\mathrm{R}, \mathcal{N}), \tag{4}$$

where the constant $X \equiv \rho_\odot/(32\pi) \cdot 1/(m_\mathrm{DM}^3 m_N^2)$ and

$$\mathcal{F}_{i,j}^{(N,N')}(E_\mathrm{R}, \mathcal{N}) \equiv \int_{v_\mathrm{min}(E_\mathrm{R})}^{v_\mathrm{esc}} \mathrm{d}^3 v\, \frac{1}{v}\, f_\mathrm{G}(\vec{v} + \vec{v}_\mathrm{E}(t))\, F_{i,j}^{(N,N')}(v, E_\mathrm{R}, \mathcal{N}). \tag{5}$$

To properly reproduce now the measured recoil rate and in turn the expected number of events in a given experiment, we need to take into account the characteristics of the detector. In so doing, we have to convolve eq. (4) with the resolution of the detector $\mathcal{K}_\mathcal{N}(E_\mathrm{R}, E')$ and the efficiency function $\epsilon(E')$. Collected the elements commented upon above, we can finally write the expected number of events in the k-th energy bin of the detector as

$$N_k^\mathrm{th} = X \sum_{i,j=1}^{16} \sum_{N,N'=p,n} \mathfrak{c}_i^N(\lambda, m_\mathrm{DM})\, \mathfrak{c}_j^{N'}(\lambda, m_\mathrm{DM})\, \tilde{\mathcal{F}}_{i,j}^{(N,N')}(m_\mathrm{DM}, k). \tag{6}$$

where the functions

$$\tilde{\mathcal{F}}_{i,j}^{(N,N')}(m_\chi, k) = w_k \sum_T \xi_T \int_{\Delta E_k} \mathrm{d}E'\, \epsilon(E') \int_0^\infty \mathrm{d}E_\mathrm{R}\, \mathcal{K}_\mathcal{N}(E_\mathrm{R}, E')\, \mathcal{F}_{i,j}^{(N,N')}(E_\mathrm{R}, \mathcal{N}), \tag{7}$$

are a sort of *integrated form factors* that encodes all the information related to astrophysics (in the velocity distribution), nuclear physics (in the nuclear responses) and the detector dependency of the rate. Here w_k is the exposure (expressed in kg per days) and ΔE_k is the width of the k-th energy bin. There is just one of these factors for each energy bin k of a given experiment, and for each pair of operators (i, j) and pair of nucleons (N, N'). Therefore, once one has computed all of these finite number of *integrated* *form factors* the expected number of events can be obtained for any kind of interactions whose particle physics is completely encapsulated in the coefficient $\mathfrak{c}_i^N(\lambda, m_{\rm DM})$. In this way, the model dependent results presented by the experimental collaboration in terms of the "standard" spin-independent and spin-dependent cross section can also be applied for other class of models characterized by a different DM-nucleus interaction. Several authors (see, *e.g.*, refs. [16-20]) have used this new formalism in order to explore how different DM-nucleus interactions, described in the non-relativistic limit by different operators and in turn *integrated form factors*, can alter the allowed regions of the positive results experiments and the constraints coming from null results. In particular, ref. [21] found that a DM particle interacting with ordinary matter via the exchange of a light pseudo-scalar (this model in the non-relativistic limit is described by the operator $\mathcal{O}_6^{\rm NR} = (\vec{s}_\chi \cdot \vec{q})(\vec{s}_N \cdot \vec{q})$), can accommodate the DAMA data while being compatible with all null direct DM searches.

3. – Model independent bounds in direct DM searches

In this section we show how to use the formalism commented in the previous section in order to derive a bound on the physics parameters λ; *i.e.* on the underlying relativistic theory. First we derive a bound for a benchmark model. Then we discuss how to translate this bound in order to set a bound on the free parameter of another DM-nucleus interaction described in the non-relativistic limit by different operators and in turn different *integrated form factors*.

The starting point is the definition of our benchmark model that will constitute the basic brick of our statistical analysis. Any model can be choose as benchmark and we use here the simplest one; namely a DM model in which the DM particles interact with only protons with a constant cross section proportional to the square number of them. In that case, the DM-nucleon matrix element is $\mathcal{M}_{p\rm B} = \lambda_{\rm B}\mathcal{O}_1^{\rm NR}$; *i.e.* $\mathfrak{c}_1^p = \lambda_{\rm B}$, $\mathcal{O}_1^{\rm NR} = \mathbb{1}$, while all the other $\mathfrak{c}_i^N = 0$. By means of eq. (6), the number of events can be cast as $N_k^{\rm th,B} = X\lambda_{\rm B}^2\,\tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\chi, k)$, where $\lambda_{\rm B}$ is the free parameter of the benchmark model we want to constraint. To this aim we adopt the customary Likelihood ratio statistical test,

$$\mathrm{TS}(\lambda_{\rm B}, m_{\rm DM}) = -2\ln\left(\mathcal{L}(\vec{N}^{\rm obs}|\lambda_{\rm B})/\mathcal{L}_{\rm bkg}\right), \tag{8}$$

where $\mathcal{L}(\vec{N}^{\rm obs}|\lambda_{\rm B})$ is the likelihood of obtaining the set of experimentally observed data $\vec{N}^{\rm obs}$, while $\mathcal{L}_{\rm bkg} \equiv \mathcal{L}(\vec{N}^{\rm obs}|0)$ is the background likelihood. For any $m_{\rm DM}$, we can then extract the maximal value of the parameter $\lambda_{\rm B}$ allowed by the experimental data $\vec{N}^{\rm obs}$. For example, since the function TS has an approximated χ^2 distribution with a number of degrees of freedom equal to the number of free parameters of the model, a 90% CL lower bound on the parameter $\lambda_{\rm B}$ can be obtained by solving, for any $m_{\rm DM}$, the equation $\mathrm{TS}(\lambda_{\rm B}, m_{\rm DM}) = 2.71$. On the website: `http://www.marcocirelli.net/NRopsDD.html`, we provide the functions $\mathrm{TS}(\lambda_{\rm B}, m_{\rm DM})$ computed in a broad range of DM masses, for the six experiments (LUX, XENON100, CDMS-II-Ge, SuperCDMS, PICASSO and COUPP) considered in our analysis. In fig. 1 of ref. [17], we show the bound on the parameter $\lambda_{\rm B}$ at 90% CL as a function of the DM mass.

Having now at our disposal the functions TS, we show how to scale this bound in order to get a limit on the free parameters λ of another DM-nucleus interaction. To this aim, since the constraint must be drawn at the same CL, once the limit on $\lambda_{\rm B}(m_{\rm DM})$ is known, a bound on the free parameters λ is trivially obtained by equating $\mathrm{TS}(\lambda, m_{\rm DM}) =$

$\mathrm{TS}(\lambda_{\mathrm{B}}(m_{\mathrm{DM}}), m_{\mathrm{DM}})$. This trivial relation is actually very powerful, because for null results experiments leads to the following quadric form:

$$\sum_{i,j=1}^{16} \sum_{N,N'=p,n} \mathfrak{c}_i^N(\lambda, m_\chi)\, \mathfrak{c}_j^{N'}(\lambda, m_\chi)\, \mathcal{Y}_{i,j}^{(N,N')}(m_\chi) = \lambda_{\mathrm{B}}(m_\chi)^2, \tag{9}$$

where the model dependent part of the problem, encapsulated in the coefficient $\mathfrak{c}_i^N(\lambda, m_\chi)$, is completely separated from the model independent one in the rescaling functions

$$\mathcal{Y}_{i,j}^{(N,N')}(m_\chi) = \frac{\sum_k \tilde{\mathcal{F}}_{i,j}^{(N,N')}(m_\chi, k)}{\sum_k \tilde{\mathcal{F}}_{1,1}^{(p,p)}(m_\chi, k)}. \tag{10}$$

These functions are provided on the website: `http://www.marcocirelli.net/NRopsDD.html` and are shown in figs. 2 to 6 of ref. [17].

Summarizing on the website we provide as Mathematica interpolated functions:

- ◇ The function $\mathrm{TS}(\lambda_{\mathrm{B}}(m_{\mathrm{DM}}), m_{\mathrm{DM}})$ that allows the users to derive the bound on the benchmark parameter at the desired CL.
- ◇ The functions $\mathcal{Y}_{i,j}^{(N,N')}(m_\chi)$ for each pair of operators (i, j), pair of nucleons (N, N') and for the six experiments considered in our analysis that allow the users to scale the bound $\lambda_{\mathrm{B}}(m_{\mathrm{DM}})$ to a bound on the free parameters λ of another interaction.
- ◇ A mathematica sample which illustrate the usage of the files.

With these ingredients, eq. (9) allows to virtually set a bound on the free parameters λ of any DM-nucleus interactions (meaning any possible choice of the coefficient $\mathfrak{c}_i^N(\lambda, m_{\mathrm{DM}})$). We show few explicit examples (*e.g.* “standard” spin-independent and spin-dependent interactions described in the non-relativistic limit by the operators $\mathcal{O}_1^{\mathrm{NR}} = \mathbb{1}$ and $\mathcal{O}_4^{\mathrm{NR}} = \vec{s}_\chi \cdot \vec{s}_N$, respectively) in sect. 6 of ref. [17] where the reader can find further details.

4. – Conclusions

In this work we have reviewed an innovative method and a self-contained set of numerical tools to derive the bounds from some current direct DM searches experiments on virtually any arbitrary model of DM elastically scatter on target nuclei. The method is based on the formalism of non-relativistic operators and it incorporates into the nuclear responses all the necessary astrophysical and detector ingredients. Our main outputs are provided as Mathematica interpolated functions and allow the users to scale a bound given on a certain benchmark interaction, in order to set limits on the free parameters of another DM-nucleus interaction. Finally, it is worth stressing that since the formalism of non-relativistic operators describes all the possible DM-nucleus interactions, the method and the numerical tools developed in ref. [17] are fully model independent and therefore we encourage a synergy between the experimentalists and nuclear physicists in order to provide the function TS for a given benchmark model and a complete set of rescaling functions like we have done on the website: `http://www.marcocirelli.net/NRopsDD.html`.

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