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# Government size and speculative attacks on public debt

(Revised version of: "Composition effect of the primary surplus and speculative attacks on public debt", resubmitted on August 25, 2020)

**October 14, 2020**

**Abstract**

In this paper I set up a *primary surplus* target zone model—differently from some previous papers that used instead an *interest rate* target zone model— both to interpret the public debt euro area crisis and, more importantly, to draw some more general conclusions as to the role of the primary surplus in preventing speculative attacks against public debt. This *primary surplus* target zone model also allows considering the effects resulting from either a small or a big government. If a small government determines a higher GDP growth than a big one, then a higher steady state public debt-to-GDP level can be obtained in the first case. However, this paper shows that in the case of a credible fiscal upper target it also turns out that such a difference does not affect the size of the expectation effect determining the 'honeymoon'. Moreover, when the primary surplus reaches its upper boundary—something that will occur inevitably, irrespective of the size of the government determining the primary surplus— monetary policy and the presence of a lender of last resort remain essential to stabilize the public debt and avoid speculative attacks.

**Keywords**: Euro area crisis, primary surplus, government size, target zones, public debt, speculative attacks

**JEL Classification:** E65, F34, F36

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In this paper I set up a *primary surplus* target zone model—differently from some previous papers that used instead an *interest rate* target zone model— both to interpret the public debt euro area crisis and, more importantly, to draw some more general conclusions as to the role of the primary surplus in preventing speculative attacks against public debt. This *primary surplus* target zone model also allows considering the effects resulting from either a small or a large government. If a small government determines a higher GDP growth than a large one, then a higher steady state public debt-to-GDP level can be obtained in the first case. However, this paper shows that in the case of a credible fiscal upper target it also turns out that such a difference does not affect the size of the expectation effect determining the 'honeymoon'. Moreover, when the primary surplus reaches its upper boundary—something that will occur inevitably, irrespective of the size of the government determining the primary surplus— monetary policy and the presence of a lender of last resort remain essential to stabilize the public debt and avoid speculative attacks.



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## 1. Introduction[1]

The euro area crisis was characterized by a convex non-linear relationship between interest rate and public debt, with the former increasing more than the linear relationship with the latter would have suggested. Such a non-linearity has been explained using a model that applied the exchange rates target zone methodology[2] to interest rates while considering the case of a limited fiscal space (Della Posta, 2018). This means that the primary surplus that has to be run in order to stabilize the public debt may be subject to an upper feasibility constraint, thereby setting an upper limit also on the interest rate guaranteeing public debt stability. It is the lack of credibility of such an upper boundary that explains the speculative attack on public debt. An alternative explanation, still within an *interest rate* target zone model, focused on the availability of the monetary instrument (Della Posta, 2019). Such a second approach shows that a speculative attack can be avoided if the monetary authorities can provide the necessary stabilizing additional resources thanks to the role of lenders of last resort they play. The availability of the monetary instrument also helps justifying the fact that some countries were not attacked, while some others were attacked despite an equally large public debt. De Grauwe (2012) and De Grauwe and Ji (2013) argued that this phenomenon could be explained by the fact that stand-alone countries reassured their debtors, since their domestic central bank acting as a lender of last resort guaranteed the repayment of their public debt, contrary to what characterized countries belonging to EMU.

This paper follows an alternative and even more intuitive approach to interpret the euro area crisis and to draw some more general conclusions on speculative attacks against public debt. This is done still using a target zone model, but by focusing directly on the upper threshold represented by the government's *primary surplus*, rather than by the *interest rate* on public debt.

Referring directly to a *primary surplus* target allows analyzing the different effects produced by large or small governments. As it will be shown, even in the case in which small governments determine a higher GDP growth compared to large governments, the role of a monetary authority playing the role of lender of last resort remains essential in order to assure

[1] Thanks to an anonymous referee for valuable comments that have helped me to better clarify and improve my article.

[2] The seminal contribution in the target zone literature is due to Krugman (1991). Krugman and Miller (eds) (1992) contains a first set of important contributions to the literature on exchange rate target zones.

public debt stability and resist speculative attacks, unless the difference between market interest rate and GDP growth is not positive.

The paper is structured as follows. In Section 2 I discuss public debt stability. In particular, I introduce the role played by the primary surplus on public debt (Section 2.1) and define its upper feasibility threshold (Section 2.2). Section 3 presents the model to be solved when considering respectively the 'honeymoon' case, in which the stability primary surplus threshold is expected to be met and public debt will be stabilized (3.1) and the 'divorce' case, in which the feasible stability primary surplus is expected instead to be overtaken, so that public debt becomes unsustainable (3.2). Section 4 analyzes the different stabilizing effects of a primary surplus obtained in the case of a small size and a large size government and the implications for the role of lender of last resort to be played by monetary policy. Some final remarks close the paper in Section 5.

## 2. The public debt stability condition

### 2.1 The stability condition and the role of the government's primary surplus

The term $b_t$ indicates the public debt-to-GDP ratio. Its dynamics, namely its continuous time variation, $db_t$, is determined by the following equation:

$$db_t = -(s_t + m_t)dt + (i_t - g_t)b_t dt + \sigma dz \qquad (1)$$

The term $s_t$ is the primary surplus-to-GDP ratio (namely $t_t - e_t$, where $t_t$ is the government revenues-to-GDP ratio and $e_t$ is government's fiscal expenditure-to-GDP ratio with the exclusion of the service on public debt), and the term $m_t$ is the monetization rate of public debt, still as a ratio of GDP. The term $(i_t - g_t)b_t$ is the contribution to $db_t$ resulting from the nominal interest rate, $i_t$, net of the nominal GDP rate of growth, $g_t$, which is charged on the public debt-to-GDP ratio. The term $dt$ indicates the instantaneous time variation. The last term on the right hand side represents a stochastic component of public debt-to-GDP dynamics, which is supposed to follow the Brownian motion process $\sigma dz$. Parameter $\sigma$ represents the instantaneous standard deviation of the Brownian motion and the term *dz* is the Brownian motion variation, which is so characterized:

$$dz = \chi\sqrt{dt}, \qquad (2)$$

where $\chi$ is a random variable which is independently, identically and normally distributed, with 0 mean and variance equal to 1.

If, in Equation (1) above:

$$s_t = -m_t + (i_t - g_t)b_t. \quad (3)$$

public debt would only evolve stochastically, so that Equation (1) becomes:

$$db_t = \sigma dz. \quad (1')$$

The nominal interest rate on public debt, $i_t$, results from an interest rate arbitrage. It is determined, then, by a riskless reference interest rate which is set by the central bank, $r_t$, and by a risk premium, $RP_t \geq 0$, implying that Equation (3) can be written as:

$$s_t = -m_t + (r_t - g_t)b_t + RP_t b_t \quad (4)$$

The risk premium on interest rates, $RP_t b_t$, has two components. The first one depends on the size of the public debt-to-GDP ratio, so it can be defined as fundamentals-driven. [3] Alcidi and Gros (2018), IMF (2011), European Commission (2014) suggest that the risk premium on public debt increases when the public debt-to-GDP ratio exceeds a given threshold which is assumed to be risk free. In particular, the IMF (having in mind mostly emerging countries) assumes that the interest rate on public debt increases by .04% for any percentage point of the public debt-to-GDP ratio exceeding 60%, while the European Commission, referring to developed European countries, considers a .03% increase in the risk premium for any percentage point of the public debt-to-GDP ratio exceeding 60%.

The second component, instead, has an expectations-driven self-fulfilling nature, which is a customary feature of many financial variables (not only interest rates, but also exchange rates and inflation rates, for example).[4] Such self-fulfilling expectations are normally based on the expected worsening of the state of economic fundamentals, requiring in our case the coverage of some additional primary surplus, if public debt is to

[3] Thanks to an anonymous referee for pointing this aspect to my attention.

[4] An example relative to exchange rates is given by Krugman (1979), where the current value of the exchange rate also depends on its expected change. The case of the inflation rate is well represented by Barro and Gordon (1983) and the use that in that article is made of the Phillips curve, where the current inflation rate also depends on the expected inflation rate for the future.

be kept stable. Expectations are defined as self-fulfilling since they may self-validate, thereby causing a crisis that would not occur otherwise.[5]

Considering, for the sake of simplicity, the assumption that an exogenously given $\delta$ will be charged as a risk premium on any positive level (rather than only above the 60% level mentioned above) of public debt-to-GDP ratio, it can be argued, then, that:

$$RP_t b_t = \delta b_t + \beta E \frac{d(s_t)}{dt}. \quad (5)$$

Parameter $\beta$ expresses the sensitivity of the risk premium with respect to self-fulfilling expectations. It tells us, then, how much the future expected worsening of economic fundamentals – requiring some future expected additional stability primary surplus - reflects in the current interest rate and therefore in the current stability primary surplus.

As a result, we have that:

$$s_t = -m_t + (r_t + \delta - g_t) b_t + \beta E \frac{d(s_t)}{dt}. \quad (6)$$

Equation (6) says that the primary surplus granting a deterministic stability of public debt decreases with the monetization of public debt operated by the central bank, $m_t$ and increases with the financing needs of both the current net service on public debt,

[5] See De Grauwe and Ji (2013) and Della Posta (2016) for an interpretation of the euro area crisis in terms of self-fulfilling speculative attacks. In the first paper, the conclusion that speculative attacks against public debt in the euro area had a self-fulfilling nature is obtained empirically, rather than through a theoretical model. Della Posta (2016) applies to public debt the approach relative to fixed exchange rates synthesized by Morris and Shin (1998). While the initial contributions of the literature on self-fulfilling speculative attacks on fixed exchange rates seemed to conclude that market expectations always have the power to destabilize an otherwise stable situation (Obstfeld, 1986, 1996), subsequent theoretical developments made clear that self-fulfilling expectations can determine a crisis only in the presence of an already fragile environment: speculators would be incurring a loss in attacking (i.e. selling) a stable currency or a strong asset, so even if their attack could be successful in defeating the resistance of the central bank or of the government, they would refrain from it (Morris and Shin, 1998, Della Posta, 2016). Morris and Shin (1998) even showed, in a model characterized by the lack of common knowledge of the state of economic fundamentals, that the region of multiple equilibria, whose actual realization would be subject to the state of market expectations, vanishes and that a unique value of the economic fundamental emerges, separating the stable from the unstable region. The first applications to public debt of the literature on self-fulfilling speculative attack on fixed exchange rates are due to Calvo (1988) and Cole and Kehoe (2000). Differently from them, in the present model I am considering market expectations as driven by the presence of an upper target on the primary surplus, whose credibility is tested by considering also the availability of the additional stabilizing instrument represented by monetary policy.

$(r_t + \delta - g_t)b_t$, and the amount of the risk premium resulting from the expected public debt instability and the additional primary surplus required to stabilize it.

For future use, the value of $s_t$ in the absence of expectations can be defined as $\bar{s}_t$, that can also be interpreted as the steady state solution of (6), occurring with zero shocks to $b_t$ and no expected change in $s_t$ [6] and whose value is given by the following linear equation:

$$\bar{s}_t = -m_t + (r_t + \delta - g_t)b_t. \quad (6')$$

From Equation (6') it follows that the steady state value of the public debt-to-GDP ratio, $\bar{b}_t$, corresponding to the value of the primary surplus, $\bar{s}_t$ for given values of the other variables and parameters, is:

$$\bar{b}_t = \frac{\bar{s}_t + m_t}{(r_t + \delta - g_t)} \quad (7)$$

## 2.2 The upper threshold on the primary surplus

As argued, among others, by Heller (2005), Tamborini (2015), Ghosh et al. (2013), the fiscal space, i.e. "the ability of a government to serve its own debt" (Kose et al., 2017, p. 2) is not unlimited. One can think that it is determined as a result of an optimization process by comparing the social cost of the primary surplus (positively correlated to $s_t$ since a higher primary surplus implies higher taxation and/or lower public spending, implying in both cases some obvious social costs), with the risk of default (negatively correlated to $s_t$, since the higher the primary surplus the lower the probability of a default of the public debt), so as to be determined at the point where the two marginal values are equal (Tamborini, 2015). In addition, a fiscal contraction that is too high, implemented in order to guarantee the stability of the public debt, could also be counterproductive from an economic point of view, since the cut in productive spending, as for example in the health sector, would prove to be a false economy because it would entail higher future costs. If this were the case, then, there would be no fiscal space and such a cut in spending would not be feasible.[7]

[6] Thanks to an anonymous referee for suggesting this interpretation.

[7] Viegas and Ribeiro (2016) analyze the welfare consequences of some fiscal consolidation episodes in the European Union. Kose et al. (2017) provide a database of the fiscal space available in 202 countries.

The resulting government's maximum feasible long-run primary surplus - which in this model is assumed to be known with certainty by market participants, differently from what Tamborini (2015) does in his heterogeneous agents model - is represented by $s^*$.[8]

The government's commitment to stabilize public debt at the level $b^*$ then, will only be credible if:

$$s_t \leq s^*. \qquad (8)$$

If that is the case, it is known that there is sufficient room to guarantee public debt stabilization and, in turn, this information will have a stabilizing effect on the dynamics of the primary surplus, as it will be shown below.

Since we are considering the case of a restrictive fiscal policy which is necessary to guarantee the stability of an existing public debt, we only concentrate on the upper bound for the primary surplus and we can ignore its lower bound.[9]

## 3. The primary surplus target zones model

According to Equation (1'), if the government stabilizes the fundamentals-driven component of public debt by running a feasible primary surplus $s_t \leq s^*$, given $g_t$, $m_t$, $r_t$, $\delta$ and given the effect of a future expected variation of the primary surplus itself, $\beta E\frac{d(s_t)}{dt}$, the only variability of the otherwise stable public debt is due to the stochastic component.

Public debt, then, may still increase above its steady state value because of a process of stochastic shocks hitting it and in turn this might drive the stability primary surplus above the level that the government can credibly stand.[10]

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[8] As a result, considering Equation (7) above, $b^* = \frac{s^*+m_t}{(r_t+\delta-g_t)}$ is the steady state level of public debt-to-GDP ratio, given the upper feasible limit for the primary surplus, $s^*$, and given $m_t$, $r_t$ and $g_t$.

[9] Needless to say, a lower bound on the primary surplus (or, which is the same, an upper limit to the value that a fiscal deficit can take), may also well exist, but since it is irrelevant for the issues addressed in the current paper, discussing the conditions to be satisfied to guarantee public debt stability, it can be neglected, as I am going to do in the rest of the paper.

[10] As we will be discussing, one possibility for the stability primary surplus to remain below $s^*$ is that the central bank stands ready to buy public debt.

To summarize, then, our (credible) primary surplus target zone model is composed by the following equations:

$$s_t = -m_t + (r_t + \delta - g_t)b_t + \beta E\frac{d(s_t)}{dt} \tag{6}$$

$$s_t \leq s^* \tag{8}$$

$$db_t = \sigma dz \tag{1'}$$

To analyze the case of speculative attacks on public debt characterizing the euro area crisis, Della Posta (2018 and 2019) uses an *interest rate* target zone model rather than the *primary surplus* target zone model which is proposed in this paper. As we will see, using this different modelling strategy allows analyzing the different implications of a primary surplus obtained in a context of large government size (in which the primary surplus results from the positive difference between relatively high taxes and relatively high public spending), as opposed to a situation of a government of low size (in which the same level of primary surplus is due to the positive difference between relatively low taxes and relatively low public spending).

### 3.1 Feasibility of government intervention: the 'honeymoon'.

To solve Equation (6) given Equation (8) and (1'), we can assume $s_t$ to be a function of the debt-to-GDP ratio, $b_t$.

$$s_t = q(b_t) \tag{9}$$

This equation can be used to calculate the expected variation of the primary surplus of Equation (6). To do so, let us expand our (stochastic) equation in a Taylor-type series, by calculating Ito's differential:

$$ds_t = q'(b_t)E(db_t) + \frac{1}{2}q''(b_t)E(db_t)^2 \tag{10}$$

From the definition of $db_t$ in (1'), it turns out that $(db_t)^2 = \sigma^2\chi^2 dt$. By considering expected values and by dividing by the infinitesimal temporal variation, we obtain Ito's Lemma:

$$\frac{E(ds_t)}{d_t} = \frac{1}{2}q''(b_t)\sigma^2, \tag{11}$$

given that $\frac{E[db_t]}{dt} = 0$ and $\frac{E[db_t]^2}{dt} = \sigma^2$. Replacing (11) into (6) we have, then:

$$s_t = q(b_t) = -m_t + (r_t + \delta - g_t)b_t + \beta \frac{1}{2} q''(b_t)\sigma^2 \quad (12)$$

This is now a differential equation of the second order, whose generic solution is of the kind:

$$s_t = q(b_t) = -m_t + (r_t + \delta - g_t)b_t + A_1 e^{\lambda_1 b_t} + A_2 e^{\lambda_2 b_t}$$

Since we are ignoring the lower bound, however, we can remove the last term of the generic solution, $A_2 e^{\lambda_2 b_t}$, and consider (13) below:

$$s_t = q(b_t) = -m_t + (r_t + \delta - g_t)b_t + A_1 e^{\lambda_1 b_t} \quad (13)$$

Let's take the second order derivative of (13) above in order to have a value for:

$$q''(b_t) = \lambda_1^2 A_1 e^{\lambda_1 b_t} \quad (14)$$

So that by replacing it in (12), it gives:

$$s_t = q(b_t) \ = -m_t + (r_t + \delta - g_t)b_t + \beta \frac{\sigma^2}{2} \lambda_1^2 A_1 e^{\lambda_1 b_t}. \quad (15)$$

Comparing (15) with (13), we obtain:

$$A_1 e^{\lambda_1 b_t} \left( \beta \frac{\sigma^2}{2} \lambda_1^2 - 1 \right) = 0. \quad (16)$$

We can calculate now $\lambda_1$ by solving the equation: $\left( \lambda_1^2 \beta \frac{\sigma^2}{2} - 1 \right) = 0$, whose solution is:

$$\lambda_1 = \sqrt{\frac{2}{\beta \sigma^2}} \quad (17)$$

Let us determine, now, the constant $A_1$ in Equation (13). In order to close the model in the case that we are considering, in which we assume that there is room for the primary surplus to stabilize public debt (for given values of the other variables), we can use the so called 'smooth pasting' condition, that was also used to close the first generation of target zone models initiated by Krugman (1991). This is realized by the tangency condition described below (see Figure 1 for a graphical representation).

Intuitively, the expectation that the primary surplus $s_t$ will not be allowed to exceed the level $s^*$ is based on the expectation that the government will be capable to run a feasible primary surplus large enough to stabilize public debt. If that is the case, then, the variable $s_t$ remains within the band, no matter whether $b_t$ will be increasing or decreasing: this can only be the case if a tangency condition occurs, as Figure 1 shows clearly.

So, let us impose the 'smooth pasting' condition to (13), applying it at time *T*, when $s_T$ touches $s^*$:

$$\frac{ds_T}{db_T} = (r_T + \delta - g_T) + \lambda_1 A_1 e^{\lambda_1 b_T} = 0 \tag{18}$$

From which it follows, for $(r_T + \delta - g_T) > 0$, that:

$$A_1 = \frac{-(r_T+\delta-g_T)e^{-\lambda_1 b_T}}{\lambda_1} < 0. \tag{19}$$

Using (19) into (13) we have, then, the value taken by $s_T^H$ at time *T*, in which 'smooth pasting' occurs and a 'honeymoon' (as a similar result was dubbed in the case of exchange rate target zones) obtains so as to match exactly $s^*$:

$$s_T^H = s^* = -m_T + (r_T + \delta - g_T)b_T^H - \frac{(r_T+\delta-g_T)}{\lambda_1}. \tag{20}$$

In the equation above, where superscript $H$ provides a label for the 'honeymoon' case, $b_T^H$ indicates the maximum value of a stable public debt-to-GDP ratio at time *T* when such a situation obtains (see Figure 1).

Recalling (6'), from which we know that it must also be that $-m_T + (r_T + \delta - g_T)b_T^H = \bar{s}_T^H$, we can conclude that:

$$s_T^H = s^* = \bar{s}_T^H - \frac{(r_T+\delta-g_T)}{\lambda_1} < \bar{s}_T^H. \tag{20'}$$

Rewriting Equation (20), we have that:

$$b_T^H = \frac{s^*+m_T}{(r_T+\delta-g_T)} + \frac{1}{\lambda_1} \tag{20''}$$

and considering Equation (7) in the 'honeymoon' case, from which it follows that $b^* = \frac{s^*+m_T}{(r_T+\delta-g_T)}$, it also turns out that:

$$b_T^H = b^* + \frac{1}{\lambda_1} = b^* + \sqrt{\frac{\beta\sigma^2}{2}} > b^* \tag{21}$$

We have, then, that the difference between $b_T^H$ and $b^*$, which is given by $\frac{1}{\lambda_1} = \sqrt{\frac{\beta\sigma^2}{2}}$, produces a 'honeymoon' effect and tells us by how much the public debt-to-GDP ratio can exceed the one resulting from the maximum feasible primary surplus, thanks to the stabilizing role of expectations, while keeping $s_T \leq s^*$ (see Figure 1).

It should be noticed that such a ‘honeymoon’ effect is only positively related to $\beta$ and $\sigma$, something we will go back to when considering the different cases of a small and a large government.

What the ‘smooth pasting’ condition suggests, then, is that the more $b_t$ increases, the more the government is expected to intervene increasing the primary surplus, so as to guarantee public debt stability (until it reaches its upper feasibility limit). Such an expected credible stabilizing intervention reassures the markets and reduces the risk premium (and therefore the interest rate) below the level it would take in its absence. In turn, this reduces the stability primary surplus, $s_t$, and produces a concave non-linearity between $b_t$ and $s_t$.

Figure 1 provides a graphical representation of these results.

All of this would only apply, however, if there is a sufficient fiscal space, namely if there is an expected room for the primary surplus to stabilize public debt (for given values of the other variables and parameters).

When this is not the case, we need to consider a different closing condition, that I am going to deal with in the next paragraph.

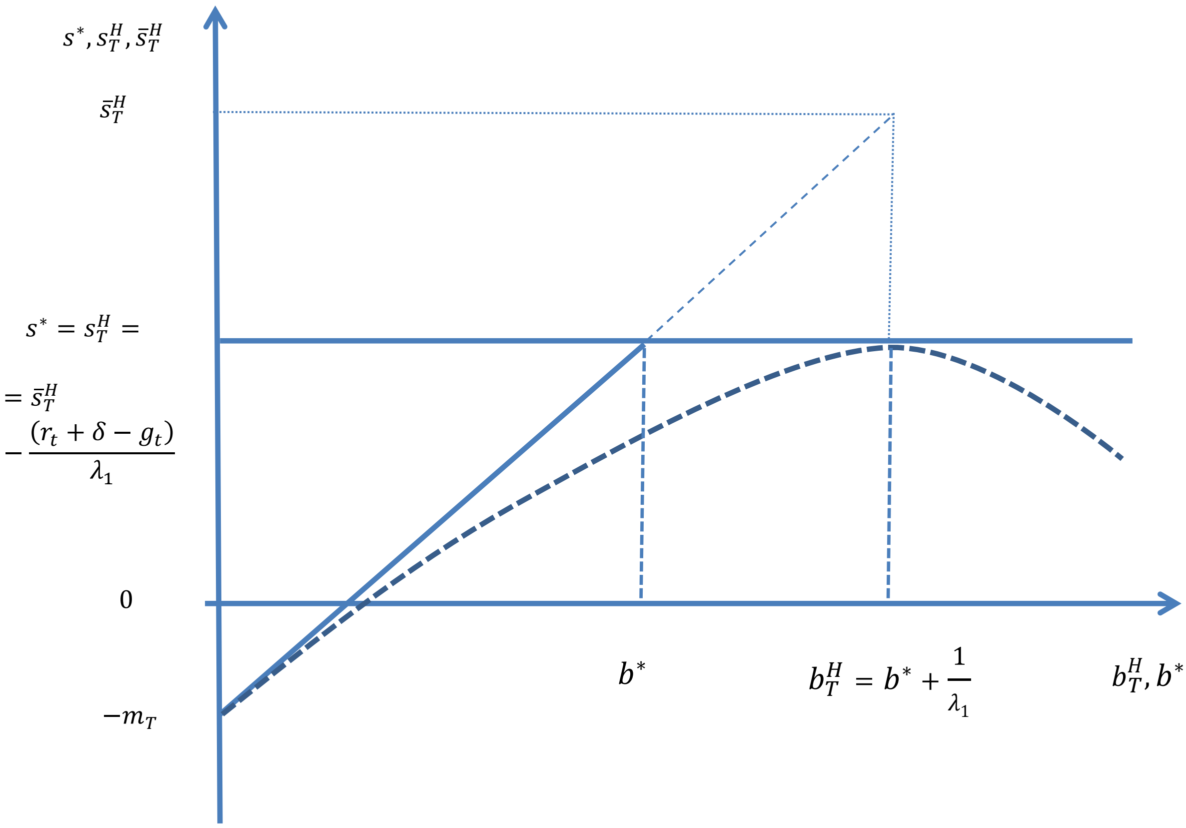


**Figure 1: The 'smooth pasting' condition.**

### 3.2 Non-feasibility of government and central bank intervention: the 'divorce' effect

The literature on exchange rate target zones also helps us understanding what the outcome would be in the situation in which the government is not expected to be able to intervene with a sufficiently large stabilizing primary surplus (given the other values of the public debt stability constraint). It can be argued, for example, that when, in the euro area crisis countries, public debt was growing (for a given market interest rate and a given GDP rate of growth), the primary surplus that was required for its deterministic stabilization, $s_t$, was approaching its upper feasibility limit, $s^*$. The more $b_t$ was being hit by a stochastic shock (requiring a higher primary surplus to be stabilized), then, the lower the confidence in public debt stabilization and the more likely the speculative attack was (in the absence of a central bank playing the role of lender of last resort). In other words, the more public debt increases, the higher the risk premium charged by the markets, reflecting in a higher interest rate and therefore in a higher $s_t$: this is why, this time, a convex (rather than concave) non-linearity between stability primary surplus and public debt realizes.

When there is no certainty that the stability primary surplus will be feasible, namely in the situation in which the fiscal surplus that would be necessary to assure public debt stability, $s_t$, will be higher than the largest one that the country can stand, $s^*$, the debt stability condition will not be satisfied. This expectation produces destabilizing rather than stabilizing effects, namely a 'divorce', rather than a 'honeymoon'.

Following what Bertola and Caballero (1992) did in the case of exchange rate target zones, the public debt-to-GDP ratio can be assumed as fluctuating between 0 and the maximum level of public debt ($b^*$) which is obtained in correspondence with the maximum feasible primary surplus that assures public debt stability ($s_t = s^*$), while the center of the fluctuation band of the economic fundamental can be taken as equal to $b^*/2$.

An arbitrage argument provides the closing equation. When the primary surplus reaches its upper threshold, $s^*$, its value has to be equal to the expected one resulting from the weighted probabilities of the two different events that may take place. There is a probability $p$ that there will not be enough resources to avoid the stability primary surplus, $s_t$, to exceed the feasible stability threshold, $s^*$ . This implies that public debt may be allowed to exceed its stability target, and the primary surplus that would be necessary to guarantee stability will jump accordingly, with no chances to be satisfied.

So, one possibility is that when the value of public debt-to-GDP ratio reaches $b^*$, it will move up by the size $\gamma > 0$ – thereby not being stable anymore – and it will jump to the center of a new fluctuation band, namely to $b^* + \gamma/2$.

With complementary probability (1-*p*) the public debt-to-GDP ratio will not be allowed to increase, remaining at or below $b^*$. The stabilizing intervention – whatever the origin or form it can take - might be such as to move its floating band down by $\varepsilon$. The public debt-to-GDP ratio will then go back to the center of a new floating band, $b^* - \frac{\varepsilon}{2}$, which is included between $b^* - \varepsilon$ and $b^*$.

It turns out, then, that the arbitrage equation is as follows:

$$p\, s_t(b^* + \tfrac{\gamma}{2}, b^* + \tfrac{\gamma}{2}) + (1-p) s_t(b^* - \tfrac{\varepsilon}{2}, b^* - \tfrac{\varepsilon}{2}) = s_t(b^*, \tfrac{b^*}{2}). \qquad (22)$$

In the expression $s_t(x, y)$, $x$ refers to the current value taken by the fundamental, and $y$ refers to the value taken by the fundamental at the center, respectively of the new and the old band.

Still following Bertola and Caballero (1992) and considering only one band, as it was also done in order to solve for the 'smooth pasting' case, centered on point $c$, we have:

$$s_t(b^*, c) = -m_t + (r_t + \delta - g_t)b^* + Ae^{\lambda(b^* - c)}. \quad (23)$$

Considering Equation (22) and Equation (23) we have, then, that:

$$s_t(b^*, \frac{b^*}{2}) = p[-m_t + (r_t + \delta - g_t)\left(b^* + \frac{\gamma}{2}\right) + A] + (1-p)[-m_t + (r_t + \delta - g_t)\left(b^* - \frac{\varepsilon}{2}\right) + A] = -m_t + (r_t + \delta - g_t)b^* + Ae^{\lambda\frac{b^*}{2}}, \quad (24)$$

from which it follows that:

$$A = \frac{(r_t + \delta - g_t)[p\left(\frac{\gamma+\varepsilon}{2}\right) - \frac{\varepsilon}{2}]}{e^{\lambda\frac{b^*}{2}} - 1}. \quad (25)$$

This also means that $A \geq 0$ iff $[p\left(\frac{\gamma+\varepsilon}{2}\right) - \frac{\varepsilon}{2}] \geq 0$, that is iff:

$$p \ \geq \frac{\varepsilon}{\gamma+\varepsilon}, \quad (26)$$

which corresponds to the case made by Bertola and Caballero (1992) for $\gamma = \varepsilon = b^*$ in the different context of an exchange rate target zone. [11]

The economic intuition for Equation (26) is straightforward. The more public debt is expected to increase above its upper target (i.e. the larger $\gamma$), given the non-feasibility of the fiscal consolidation that would be necessary to guarantee public debt stability, the lower the $p$ required for it to increase above its stability level as a result of a speculative attack.

Figure 2 represents in a simplified way what has been shown analytically above. A positive (negative) value of $A$ implies a convex (concave) dynamics for the stability primary surplus in its move towards the feasible primary surplus as a response to the stochastic move of public debt.

Using Equation (23) it is also possible to calculate the size of the 'divorce', since we obtain that:

$$s_T^V = s^* = -m_T + (r_T + \delta - g_T)b_T^V + Ae^{\lambda\frac{b_T^V}{2}}. \quad (27)$$

[11] Della Posta (2019) reaches a similar conclusion. He does it, however, in the different context of an *interest rate* target zones model.

Still considering Equation (6'), then, with superscript $V$ referring to the 'divorce' effect, since we know that in the case of 'divorce'— namely with $p \geq \frac{\varepsilon}{\gamma+\varepsilon}$ — it turns out that $A \geq 0$, we have that:

$$s_T^V = s^* = \bar{s}_T^V + Ae^{\lambda\frac{b_T^V}{2}} > \bar{s}_T^V. \quad (28)$$

It also turns out that:

$$s_T^V = b^* - Ae^{\lambda\frac{b_T^V}{2}} < b^*. \quad (29)$$

Figure 2 represents in graphical terms what has been shown analytically above.

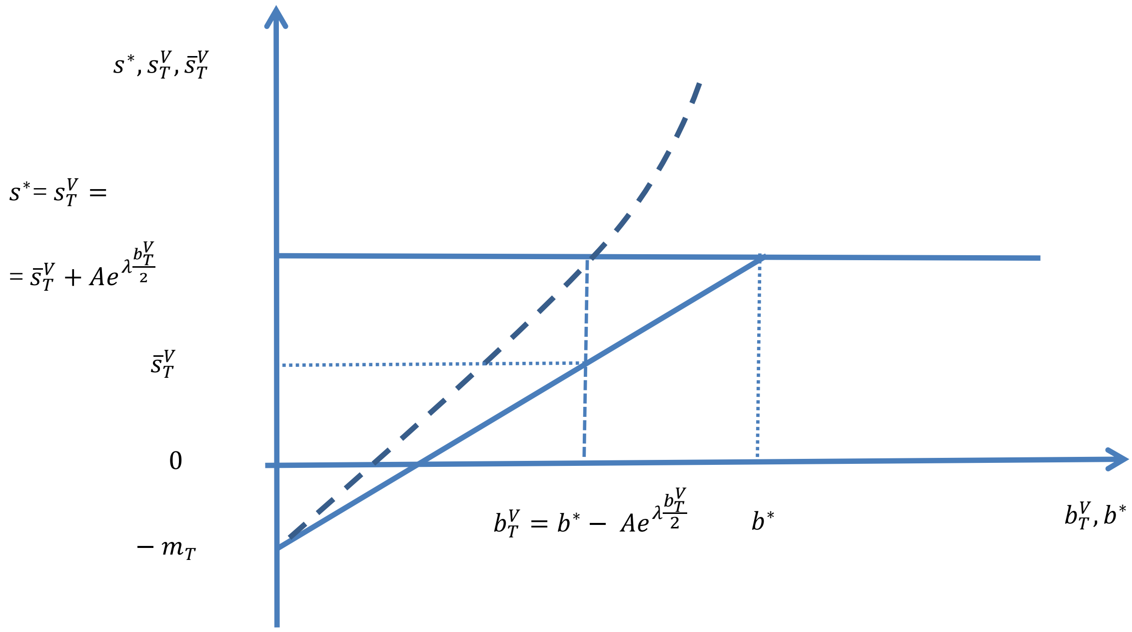


**Figure 2: The 'divorce' effect (*V*) in the case of a non-feasible fiscal consolidation (in which $A > 0$).**

The case of 'divorce' and its implications need to be discussed further. The expectation that the upper feasible boundary on the primary surplus will not be sufficient to stabilize public debt determines a convex non-linearity: the primary surplus which is necessary to assure public debt stability increases more than its linear relationship with the public debt-to-GDP ratio would suggest. This is due to the fact that the risk premium on the service of public debt increases with the stability primary surplus, since it is recognized that it will meet its upper boundary soon and, in the absence of other instruments, public debt stability could only be assured if it were allowed to exceed it. When that is the case, then, announcing primary surpluses that will clearly be insufficient to stabilize public debt would have a counterproductive effect and the risk premium and interest rates will increase rather than decrease. This is also the conclusion that Tamborini (2015) (referring also to Ghosh *et al.* 2010, and to their idea of a limited fiscal space), reached in a different modelling environment characterized by the presence of heterogeneous agents.

Such conclusions are the opposite of those, relative to the expansionary effect of fiscal consolidations reached instead by Giavazzi and Pagano (1990, 1996), who are the first ones to introduce the idea of anti-Keynesian fiscal effects due to the role played by expectations, Alesina *et al.* (2015) and Corsetti *et al.* (2013). In particular, Alesina *et al.* (2015) argue that in spite of the possibly negative effects produced by the fiscal multiplier on GDP, a fiscal consolidation obtaining thorough spending cuts is relatively more expansionary than one based on higher taxes, since in the first case future taxes are expected to decrease and consumers and investors will look more confidently into the future. Corsetti *et al.* (2013) observe that the private sector's spread is correlated with the sovereign spread for both 'weak' and 'strong' countries of the euro area. According to their 'rationality presumption, then, fiscal consolidation, by reducing *with certainty* the sovereign spread, favors the private sector demand for consumption and above all for investments.

The 'rationality presumption' invoked by Corsetti *et al.* (2013), however, assumes implicitly that the primary surplus run by the government is far away from its upper boundary. When that is not the case, however, the opposite result obtains.

Moreover, the presumed anti-Keynesian effects described above were certainly disproved in the case of the fiscal consolidation undertaken during the euro area crisis, where, as also acknowledged by Blanchard and Leigh (2013), the contractionary effect of the traditional Keynesian fiscal multiplier prevailed over the alleged expansionary effect driven by favorable expectations.

An additional point that could be investigated is the effect of the government size on public debt stability, given the presumed larger positive impact on GDP growth of a small government compared to a large one.

This is what I am going to discuss in the next section.

## 4. Government size and primary surplus

The model presented so far encompasses the conclusions reached by Della Posta (2018 and 2019) within the context of an *interest rate* target zones model and reinterprets them in terms of a *primary surplus* target zone model. Such a different approach, however, also allows comparing the effects obtained when a given primary surplus—which is necessarily bounded above, given the existence of an upper limit to the imposition of taxes and of a lower limit to the reduction of expenditure— is run by a government characterized by a large public sector (high taxes and high public spending) or by a government with a small public sector (low taxes and low public spending).

A prominent view in the literature argues that large governments on average generate lower GDP growth than small governments (see, for example, Bergh and Karlsson, 2010). In spite of the many objections that may be raised against this conclusion, as accounted by the comprehensive study by Bergh and Henrekson (2011) [12], let us accept it and let us see what its implications would be in this model.

I can represent the assumption above with the inequality $g_l < g_s$, where $g_l$ ($g_s$) is the GDP growth of a large (small) government. It turns out, then, that:

$$\bar{b}_T^l = \frac{\bar{s}_T + m_T}{(r_T + \delta - g_l)} < \bar{b}_T^s = \frac{\bar{s}_T + m_T}{(r_T + \delta - g_s)}. \quad (30)$$

Equation (30) says that the steady state level of public debt would be larger if the primary surplus is produced by a small government, as opposed to the case of a large government.[13]

What precedes, considering Equation (6) also implies that the slope of the steady state primary surplus with respect to public debt flattens when the primary surplus is obtained by small governments compared to large governments.

As Figure (3) shows clearly, it must also be the case that:

$$s^* = -m_T + (r_T + \delta - g_l)b^{*l} = -m_T + (r_T + \delta - g_s)b^{*s}. \quad (31)$$

[12] One objection, for example is given by the observation that open economies are usually characterized by large governments, that are necessary to compensate the losers of globalization and to make openness socially acceptable. More open economies, however, are usually characterized by a higher GDP growth than closed ones.

[13] Notice, however, that Equation (30) also implies that if $g_s$ is large enough to match $r_T - \delta$, the level of the sustainable public debt becomes theoretically unbounded.

From Equation (21) we also know, however, that the size of the 'honeymoon' is not affected by the stability primary surplus $s^*$, since it only depends on $\beta$ and $\sigma$. This means that a larger or smaller government size, although affecting the public debt sustainability level (see Equation 30), has no effect on the 'honeymoon'.

Moreover, the largest sustainable level of public debt that can be reached thanks to the 'honeymoon' has to be guaranteed by the remaining stabilizing tools implied by the public debt stability constraint given by Equation (31) (see also Della Posta, 2018 based on Krugman and Rotemberg (1992) who endogenize the 'honeymoon' and explain it in terms of the availability of a stabilizing buffer stock).

This means that once the feasible primary surplus $s^*$ has been reached (let us recall that $s^* = s_T^l = s_T^s$), the largest possible steady state value of the public debt-to-GDP ratio in the case of a small size government, $b_T^s$ , for example, can be credibly reached only if there is the availability of some extra room for monetary expansion, $m_T^s > m_T$ to finance the public debt, namely if $b_T^s = \frac{s_T^s + m_T^s}{(r_T + \delta - g_s)}$.

The amount of monetary 'buffer stock' making possible the realization of a 'honeymoon' is easily calculated:

$$b_T^s - b^{*s} = \frac{s^* + m_T^s}{r_T + \delta - g_s} - \frac{s^* + m_T}{r_T + \delta - g_s} = \frac{m_T^s - m_T}{r_T + \delta - g_s} = \frac{1}{\lambda}. \quad (34)$$

Similarly, in the case in which the primary surplus is obtained, instead, in the case of a large government, we have that:

$$b_T^l - b^{*l} = \frac{s^* + m_T^l}{r_T + \delta - g_l} - \frac{s^* + m_T}{r_T + \delta - g_l} = \frac{m_T^l - m_T}{r_T + \delta - g_l} = \frac{1}{\lambda}. \quad (35)$$

By comparing the two different cases, the first one characterized by a small size government and the other one characterized by the opposite situation, as represented in Eqs. (34) and (35) above, we have that:

$$\frac{m_T^s - m_T}{r_T + \delta - g_s} = \frac{1}{\lambda} = \frac{m_T^l - m_T}{r_T + \delta - g_l}. \quad (36)$$

Since we know that $g_s > g_l$, it follows that:

$$m_T^s < m_T^l. \quad (37)$$

This is a quite important and still very intuitive result: if a primary surplus obtained by a small government implies a larger GDP growth compared to the case of a large government, the need for monetary stabilization gets reduced in the former case.

A further observation, however, needs to be made: if no more fiscal space is available, as at some point it will inevitably be the case, a monetary stabilization buffer stock (thereby exceeding $m_T$) still needs to be in place in order to guarantee the 'honeymoon' of size $\frac{1}{\lambda}$ (unless GDP growth is as large as $r_T + \delta$).

Even if the stabilizing primary surplus is obtained by a small government rather than by a large one, then, when the upper boundary level of the sustainable primary surplus has been inevitably reached and unless $r_T + \delta \leq g_S$, a central bank playing the role of a lender of last resort still needs to be in place, in order to respond to stochastic shocks hitting the public debt and to stabilize the latter.

While the reduction of the risk premium depends on the credibility of the upper boundary of the stability primary surplus, then, it is not affected by the size of the government running it. So, once such an upper limit has been reached and there is the expectation that it would be overtaken, not only the risk premium does not fall, but it even reacts perversely, thereby increasing, irrespective of the size of the government running the primary surplus:  even  a higher GDP growth, which presumably would result from a small size government, does not produce any positive expectational effect when the upper limit of the primary surplus granting public debt stability loses its credibility.

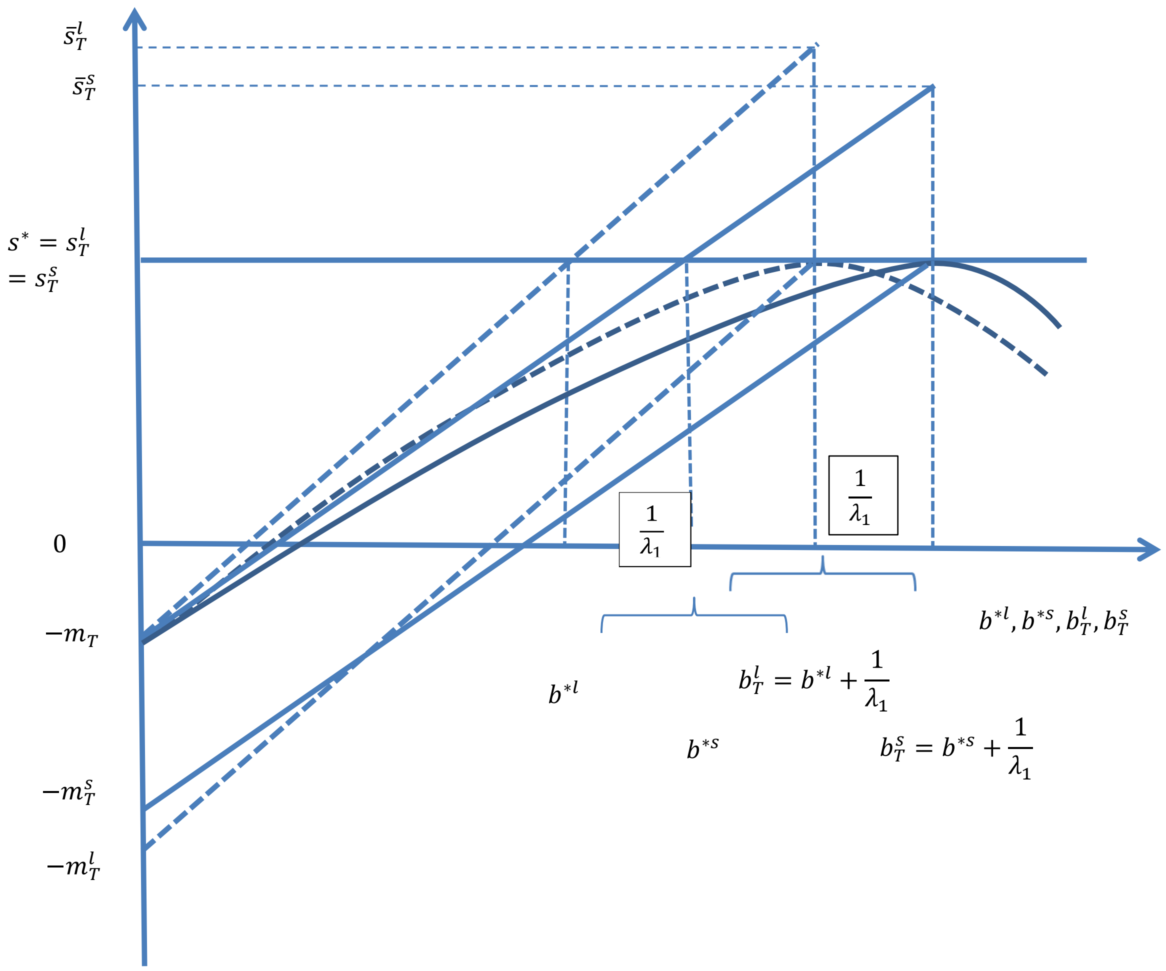


**Figure 3: The different effects of a primary surplus obtained by a large or a small government.**

## 5. Concluding remarks

In this article I have shown that the exchange rate target zone models developed in the 1990s could be applied to speculative attacks on public debt that characterized the euro area crisis, not only using the *interest rate* target zone approach proposed by Della Posta (2018 and 2019), but also adopting a new *primary surplus* target zone model. This allows both to show the flexibility of the target zone approach and, more importantly, to focus on the specific role played by different market responses to the implementation of a stabilizing primary surplus by a small or large government. In the former case, for example, if we accept the conclusion that small governments produce a higher GDP growth than large ones, as argued by Bergh and Karlsson (2010), a higher value of a stable government debt-to-GDP ratio is obtained. The paper also shows that the size of the monetary 'reserve stock' needed to ensure the stability of public debt is reduced when the primary surplus is achieved by a small government with respect to the one needed by a large government (still assuming the validity of the conclusion that the former is characterized by a higher GDP growth than the latter).

A noteworthy observation, however, is that the size of the 'honeymoon' is not affected by the different size of the government running the primary surplus and that the stabilizing role of a lender of last resort is still necessary to ensure the stability of public debt.

The only exception to the need for monetary stabilization would be in the event that the expansionary effect on GDP is at least as large as the market interest rate to be paid on public debt.

The results obtained confirm, then, both the non-linearity of the behavior of the primary stability surplus for some euro area countries, resulting from a non-linearity of interest rates due to the expected lack of stability of public debt, and the crucial role for stability played by a central bank acting as lender of last resort.

Thanks to the stabilizing role of monetary policy, for example, a 'honeymoon' characterized the United Kingdom and the United States, while a 'divorce' prevailed with the euro area countries in crisis, given the constraints on the ECB monetary policy before Draghi's celebrated 'whatever it takes'.

This article also sheds light on the implicit debate between a position that the euro area crisis was caused by the lack of credibility of fiscal consolidation (Tamborini, 2015) and one claiming instead that it was due to the lack of credibility of monetary stabilization (De Grauwe, 2012, De Grauwe and Ji, 2013). While the primary surplus always finds an upper limit, regardless of the size of the government running it, so as to conclude that it cannot be a reliable tool to stabilize public debt, monetary policy is potentially unlimited and is therefore the only dependable and necessary protection against speculative attacks, as the euro area crisis clearly demonstrated.

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