https://doi.org/10.1142/9789811287183_0028

28

Attitudes in Mathematics Education

Pietro Di Martino[1]

ABSTRACT Attitudes towards mathematics has a long history in mathematics education research. Over the time, research on attitudes and, more in general, on affective aspects developed a wide range of methodologies and perspectives in mathematics education, playing a growing role in the field. In this chapter, I will describe the development of the research about attitude in mathematics education, discussing the main issues emerged in this field. In particular, I will discuss the definition problem, that is the emergence of the need for a clear definition of the construct, and the ground for the development of *our* (TMA) three-dimensional model of attitude (Di Martino and Zan, 2010). In the last part of the chapter, some fields of application of the TMA model will also be discussed.

Keywords: Attitude towards mathematics; Affect in mathematics education; Qualitative research.

1. Introduction

The awareness that the learning process of mathematics is strongly affected by affective factors was born and developed in the field of Mathematics Education during the Eighties.

Mason, Burton and Stacey (1982) published the book "Thinking Mathematically". The main aim of the book was to unfold the processes which lie at the heart of mathematics, and, within this scope, authors underlined as their "*experience with students of all ages has convinced us that mathematical thinking can be improved by (…) linking feelings with action*" (ibidem, p. ix).

The role of affect in the specific development of mathematical thinking was stated so clearly for the first time and the emotion management was described as a fundamental part for the thinking development: "*Probably the single most important lesson to be learned is that being stuck is an honourable state and an essential part of improving thinking*" (ibidem, p. ix).

This is a significant breakthrough in the field of mathematics education: affect was no longer considered "auxiliaries" in the learning process of mathematics, rather a crucial part for the development of mathematical thinking and, therefore, an important key for the interpretation of the widespread students' difficulties in mathematics.

In the same period, several scholars arrived at the same conclusion because of their research about problem solving. At the end of a long research into human problem-

[1] Department of Mathematics, Università di Pisa, Pisa, Italy. E-mail: pietro.di.martino@unipi.it

solving process, Schoenfeld (1983, p. 330) stated: "*The point here is simply that "purely cognitive" behavior — the kind of intellectual performance characterized by discussion of resources, heuristics, and control alone — is rare. The performance of most intellectual tasks takes place within the context established by one's perspective regarding the nature of those tasks. Belief systems shape cognition, even when one is not consciously aware of holding those beliefs*".

This stance marks the end of a naïve era, the overcoming of the assumption that the mathematical thinking and its quality is determined only by cognitive elements.

This awareness finds its definitive consecration in 1989, when the book "Affect and Mathematical problem solving" was published (Adams and McLeod, 1989). The book was developed in a particular period: on the one hand, most research on problem solving were developed in the wake of Polya's seminal work, giving no attention to affective issues, on the other hand, many results of the research about problem-solving showed the limits of a purely cognitive approach.

Based on these results, McLeod (1989, p. 23) clearly underlined how: "*Limiting one's research perspective to the purely cognitive seems acceptable for those interested mainly in the performance of machines; however, researchers who are interested in human performance need to go beyond the purely cognitive if their theories and investigations are to be important for problem solving in classrooms* […] *Affective issues play a central role in mathematics learning and instruction* [...] *If research on learning and instruction wants to maximize its impact on students and teachers, affective issues need to occupy a more central position in the minds of researchers*".

In the first chapter of the book, Mandler (1989, p. 3) was clear about the need to seriously develop research investigating the role of affect in problem-solving and, more in general, in the teaching and learning of the mathematics: "*The* problem-solving *and* teaching-and-learning *literature is full of remarks that have a single message: <Someday soon — maybe tomorrow — we must get around to doing something about affect and emotion>. I am delighted to see that* tomorrow *has come".*

Few years later, McLeod (1992) depicted the state of the art of the research on affect in mathematics education, trying to systemize the field.

McLeod recognized three main constructs emerging in the field of affect: emotions, beliefs and attitudes. These constructs differ in the stability of the affective responses and in the level of intensity of the affects that they represent (Fig. 1).

Fig. 1. The degree of stability and intensity of beliefs, attitudes and emotions

In McLeod's view, emotions, beliefs and attitudes also differ for their "cognitive component". However, the basic assumption was that all of them have a strong relationship with cognition and research in the field of mathematics education should have been investigate the dynamics of this relationship.

To do that, the first needed step was to ground affect and its main constructs in a strongest theoretical foundation.

Some year later, Goldin (2004) underlined how this need did not appear to be fully satisfied, since a precise and shared language for describing the affective domain was still missing.

However, if some progress had been made in describing emotions (Zan et al., 2006) and beliefs (Törner, 2002), this was not the case for the concept of attitudes: "*probably the most problematic concept in McLeod's framework*" (Hannula et al., 2011, p. 38).

In this context and in an interpretivist perspective, Rosetta Zan and I began to get interested in the definition of the construct "attitude towards mathematics", imagining the potential of the construct for a better and more complete interpretation of the students' difficulties *in* mathematics (Fig. 2).

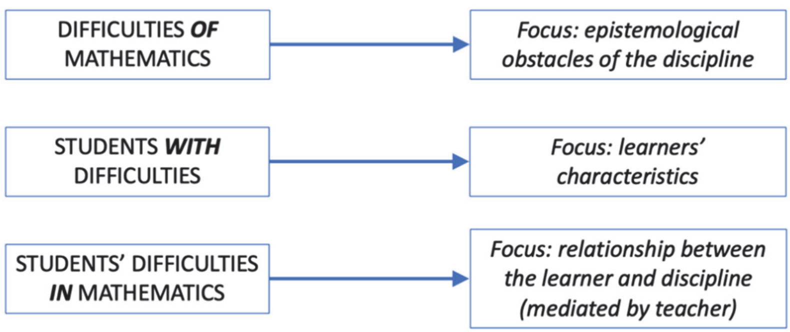

Fig. 2. The different kinds of difficulties and their focus

2. Toward a (Working) Definition of Attitude towards Mathematics

According to Boero and Szendrei (1998, p. 199): "If we claim that research in mathematics education must be similar to research in any "normal" science, "cumulation" and "universality" of research results are needed, and the existence of the progress must be evaluated comparing new results with previous ones".

In line with this characterization of research in mathematics education, in 2001 we developed a critical overview of the existing literature about attitude (Di Martino and Zan, 2001), trying to reconstruct if and how the several studies about attitude in mathematics education answered to the following question: what is attitude towards mathematics? The answer to this question is also crucial for characterizing what are a positive and a negative attitude towards mathematics.

It emerged that a large portion of studies about attitude did not provide a clear definition of the construct itself. Attitude tended rather to be defined implicitly and a posteriori through the instruments used to measure it. However, in the studies where an explicit definition of attitude was given, we recognized three main typologies of definition (Fig. 3).

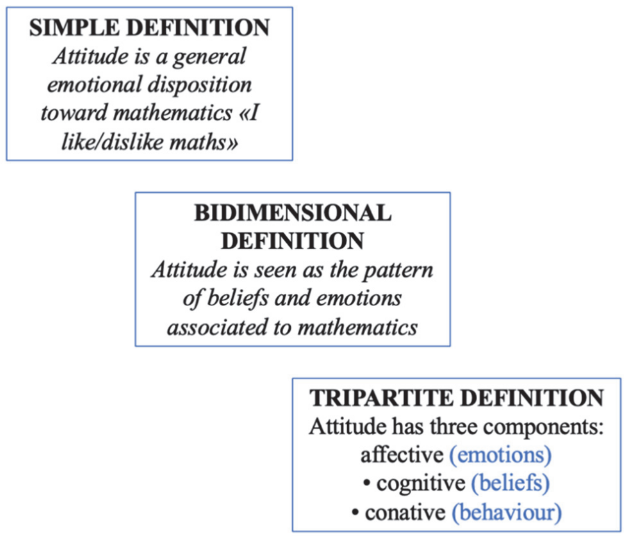

Fig. 3. The different characterizations of attitude in 2001

Considering this variety of approach, the debate about which is the correct definition was particularly intense. On the other hand, according to Kulm's position (1980, p. 358) "*It is probably not possible to offer a definition of attitude toward mathematics that would be suitable for all situations, and even if one were agreed on, it would probably be too general to be useful*", Daskalogianni and Simpson (2000) suggested to consider the definition of attitude as a working definition. Attitude is considered as "*a construct of the observer's desire to formulate a story to account for observation*" rather than "*a quality of an individual*" (Ruffel et al., 1998, p. 1).

In this perspective, the development of the (working) definition of attitude must be related and functional to the research problem and, therefore, the real question we needed to reply was the following: which is the most adequate definition of attitude for our research interests?

Being particularly interested in the third kind of difficulties in Fig. 2 — students' difficulties in mathematics — we needed to develop a definition of "attitude towards mathematics" strongly related to students' relationship with mathematics, as well as, to teachers' practice. We had the ambition to characterize in an operative way what "positive" and "negative" attitude toward mathematics are, developing a theoretical tool for the interpretation of students' difficulties in mathematics, capable to suggest didactical strategies to overcome these difficulties.

According to this aim, a first investigation about teachers' use of the diagnosis 'negative attitude' in their school practice was developed by Polo and Zan (2005) within an Italian research project.

It emerged that the diagnosis "*This student has a negative attitude toward mathematics*" was frequently (85.6% of the sample) used by teachers regardless of their school level. On the other hand, several different meanings of the diagnosis "negative attitude" emerged (Fig. 4).

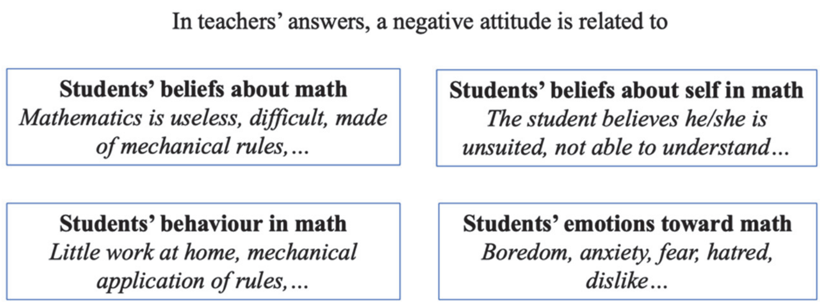

Fig. 4. The different meanings of "negative attitude" in teachers' view

Another result of the research conducted by Polo and Zan was particularly interesting: it emerged how the diagnosis "that student has a negative attitude towards mathematics" usually represents a sort of claim of surrender of the teacher in the face of students' difficulties in mathematics rather than an interpretation for steering didactical intervention. The "negative attitude" diagnosis was a black box in a nutshell.

In our view, clarify the meaning of (positive/negative) attitude from a theoretical viewpoint was the key to open the black box, turning the "negative attitude" diagnosis into a useful instrument for teachers and researchers.

To do that, we developed our main research about attitude (Di Martino and Zan, 2010, 2011), studying students' relationship with mathematics at school through the collection of several autobiographical essays: *Me and mathematics: my relationship with maths up to now* (Fig. 5).

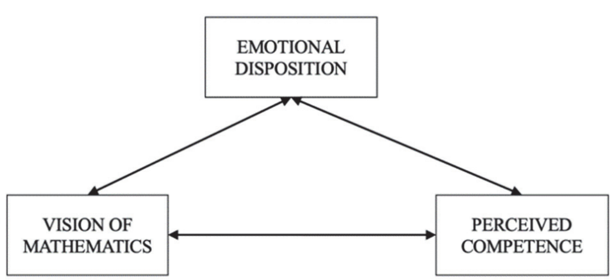

Fig. 5. The description of the data collected

Lieblich et al. (1998) describe different approaches for the analysis of narrative materials. They identify two main independent dimensions: holistic vs categorical (this dimension refers to the chosen unit of analysis: the complete narrative or a specific part of the story), and content vs form. Combining these dimension results in four modes of analyzing narrative data, each of which responds to a specific research interest.

Through a categorical-content approach, we found three recurrent expressions in students' narrative: "I like/dislike mathematics", "I am able/ unable to do mathematics", "mathematics is". These three expressions identified three core themes: the emotional disposition towards mathematics, the vision of mathematics, the perceived competence in mathematics. At the end of the analysis, we obtained only 32 essays (less than 2 % of the entire sample) that did not refer to at least one of these three themes.

Coherently with our initial assumptions, we developed our Three-dimensional Model for Attitude (TMA) based on the themes students used to describe their relationship with mathematics (Fig. 6).

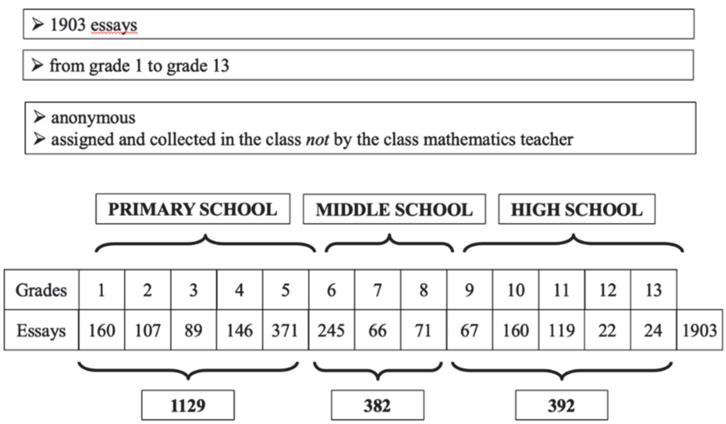

Fig. 6. The three-dimensional model for attitude (Di Martino and Zan, 2010)

The multidimensionality of the model suggests the development of different profiles of attitude towards mathematics. The multidimensionality therefore underlines the inadequacy of the positive/negative dichotomy for attitude referred only to the emotional dimension, suggesting to consider an attitude as *negative*, when at least one the dimensions is *negative*. In this way we can outline different *profiles* of negative attitude, depending on the dimension that appears to be *negative*.

On the other hand, the developed definition of attitude could be a valuable tool for didactical diagnosis and intervention if and only if its complexity is limited and reasonable. In this perspective, we decided to reduce the complexity of each of the three dimensions in the model to the following dichotomies:

- Emotional disposition: positive/negative
- Vision of mathematics: relational/instrumental (Skemp, 1976)
- Perceived competence: high/low

In this way, TMA model identifies eight different profiles of attitude towards mathematics (Fig. 7): a unique profile of positive attitude towards mathematics and seven different profiles of negative attitude, depending on the dimension that appears to be negative.

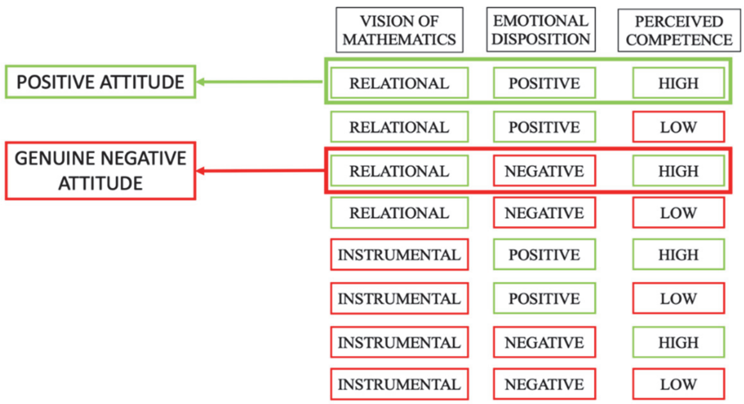

Fig. 7. The profiles of attitude

The so called *genuine* negative attitude towards mathematics — the profile characterized by a relational view of math, a high perceived competence but a negative emotional disposition — is particularly interesting, It is based on an epistemologically correct vision of mathematics and it is not related to a story of difficulties with mathematics, therefore, in our view, the negative emotional disposition is genuine, a sort of personal taste that we should accept despite our passion for mathematics (this means that I believe a didactical intervention is not needed).

From a quantitative point of view, it is important to underline that this *genuine* profile was not represented in our data, and it is not represented now either collecting other essays (up to now we collected almost 2000 essays).

3. The Possible Uses of the TMA Framework

The developed TMA framework has the potential to be used for multiple research interests, appropriately modifing the involved variables (Fig. 8).

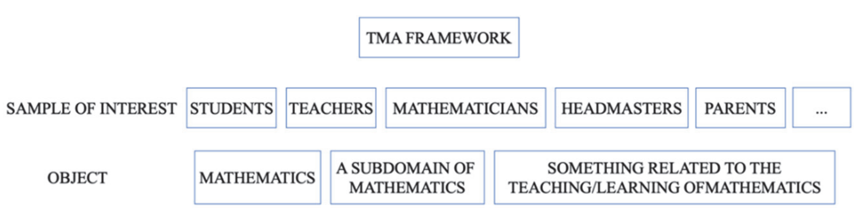

Fig. 8. The possible research variables

The first variable is the sample of interest, that is the group of people we are interested in. Several possibilities exist: students, teachers, mathematicians, students' parents, headmasters, politicians, etcetera.

The second variable is the *object*. It can be mathematics as well as a *subdomain* of mathematics (geometry, algebra, problem solving, etcetera) or something related to mathematics, for example the attitude towards the teaching of mathematics. However, the TMA framework can be also used for objects different from mathematics: for example, Bocchialini and Ronchini used TMA framework for assessing business students' attitudes toward finance (Bocchialini and Ronchini, 2019).

The third variable is represented by the dichotomy between static vs developmental research on attitude. In the first case, the focus is on the current attitude of a certain sample, in the latter case, the focus is on the evolution over a period of attitude (typically this period is characterized by a didactical intervention or by a discontinuity in the educational path, for example the transition between two different school levels).

I want to conclude this chapter with a synthesis of the research I developed (with different colleagues) using the TMA theoretical framework, in addition to the study of students' attitude towards mathematics in the Italian context.

First, the TMA theoretical framework has been used for studying the attitude towards having to teach mathematics of pre-service primary teachers. These studies confirmed how negative attitudes towards mathematics and towards the idea having to teach mathematics are very common among the future teachers in primary school. On the other hand, we discovered and described an interesting phenomenon that we called the *desire for math-redemption*: it happens when there is a positive attitude towards the *challenge* of having to teach mathematics, motivated by the desire of a personal reconstruction of the (negative) attitude towards mathematics developed during the school experience as student (Coppola et al. 2013).

The TMA theoretical framework has been also used for studying the in-service teachers' attitude towards the national and international standardized assessments (Di Martino and Signorini, 2019). The results of the study — that involved of all school levels — confirmed a generalized What emerges is a complex picture that includes positions of principle against the standardized assessments and their uses, but also more specific criticism towards the design of the test. Understanding the teachers' attitude towards this kind of assessment appears to be crucial also to exploit the informational and developmental potential of the standardized assessment (Di Martino and Baccaglini-Frank, 2017).

In the last period, I have applied the TMA framework to the evolution of attitude in two different cases. The first one concerns the evolution of pupils' attitude towards problems in the period from kindergarten to the end of primary school (Di Martino, 2019). The results of the study showed a worrisome evolution of the pupils' attitude towards problems, in terms of all the three components of TMA. Kindergarten pupils reported a very promising view of problems, not fixed to a stereotypical model: this view is held until the end of grade 1 in primary school, then a deterioration of this idea begins, and this deterioration appears to be linked to precise didactic choices.

Currently, I am using the TMA framework for analyzing the mathematical crisis in secondary-tertiary transition (Di Martino and Gregorio, 2019). The secondary-tertiary transition in mathematics is described by Clark and Lovric (2008; 2009) as a rite of passage. This rite of passage consists of three stages: the separation stage, the liminal stage, the incorporation stage. The liminal stage is characterized by several individual's crisis and it is very interesting to analysis the evolution of attitude from the separation stage to the liminal stage.

To conclude, I strongly believe that research on attitude still can and must say much about several significant phenomenon for mathematics education, and the TMA framework has the potential to give a contribute.

References

E. Bocchialini and B. Ronchini (2019). A Pilot Study Assessing Attitudes toward Finance among Italian Business Students. *International Journal of Business and Management*, 14(10), 1–44.

P. Boero and J. Szendrei (1998). Research and results in mathematics education: Some contradictory aspects. In J. Kilpatrick & A. Sierpinska (eds.), *Mathematics Education as a Research Domain: The Search for Identity* (Vol. 1, pp. 197–212). Dordrecht: Kluwer.

M. Clark and M. Lovric (2008). Suggestion for a theoretical model for secondary-tertiary transition in mathematics. *Mathematics Education Research Journal*, 20(2), 25–37.

M. Clark and M. Lovric (2009) Understanding secondary–tertiary transition in mathematics. *International Journal of Mathematical Education in Science and Technology*, 40(6), 755–776. https://doi.org/10.1080/00207390902912878

C. Coppola, P. Di Martino, T. Pacelli, and C. Sabena (2013). Inside teachers' affect: teaching as an occasion for math-redemption. In M. Hannula, P. Portaankorva-Koivisto, A. Laine, and L. Naveri (eds.), *Current State of Research on Mathematical Beliefs* (pp. 203–216). University of Helsinki.

K. Daskalogianni and A. Simpson (2000). Towards a definition of attitude: The relationship between the affective and the cognitive in pre-university students. In T. Nakahara and M. Koyama (eds.), *Proceedings of the 24th Conference of the IGPME* (Vol. 2, pp. 217–224). PME: Hiroshima, Japan.

P. Di Martino and R. Zan (2001). Attitude toward mathematics: some theoretical issues, in M. van den Heuvel-Panhuizen (ed.), *Proceedings of the 25th Conference of the IGPME*, (Vol. 3, pp. 209–216). PME: Utrecht, The Netherlands.

P. Di Martino and R. Zan (2010). "Me and maths": Towards a definition of attitude grounded on students' narratives. *Journal of Mathematics Teacher Education,* 13(1), 27–48.

P. Di Martino and R. Zan (2011). Attitude towards mathematics: A bridge between beliefs and emotions. *ZDM — The International Journal on Mathematics Education,* 43(4), 471–482.

P. Di Martino and A. Baccaglini-Frank (2017). Beyond performance results: Analyzing the informational and developmental potentials of standardized mathematics test. *For the Learning of Mathematics*, 37(3), 39–44.

P. Di Martino (2019). Pupils' view of problems: The evolution from kindergarten to the end of primary school. *Educational Studies in Mathematics,* 100, 291–307. https://doi.org/ 10.1007/s10649-018-9850-3

P. Di Martino and G. Signorini (2019). Teachers and standardized assessments in mathematics: An affective perspective. In M. Graven, H. Venkat, A. Essien, P. Vale (eds.), *Proceedings of the 43rd Conference of the IGPME*, Vol. 2, pp. 185–192. PME: Pretoria, South Africa.

P. Di Martino and F. Gregorio (2019). The Mathematical Crisis in Secondary-Tertiary Transition. *International Journal of Science and Mathematics Education,* 17, 825–843. https://doi.org/10.1007/s10763-018-9894-y

G. Goldin (2004). Characteristic of affect as a system of representation. In M. Johnsen Hoines and A. Fuglestad (eds.), *Proceedings of the 28th Congress of the IGPME*, (Vol. 1, pp. 109–114). PME: Bergen, Norway.

M. Hannula (2011). The structure and dynamics of affect in mathematical thinking and learning. In M. Pytlak, T. Rowland, and E. Swoboda (eds.), *Proceedings of the 7th Conference of ERME* (pp. 34–60). Rzeszow, Poland.

G. Kulm (1980). Research on mathematics attitude. In R. J. Shumway (ed.), *Research in Mathematics Education* (pp. 356–387). Reston: NCTM.

J. Mason, L. Burton and K. Stacey (1982). *Thinking Mathematically*. London: Addison-Wesley.

D. McLeod and V. Adams (eds.). (1989). *Affect and Mathematical Problem Solving: A New Perspective*. New York: Springer.

D. McLeod (1989). The role of affect in mathematical problem solving. In D. McLeod and V. Adams (eds.), *Affect and Mathematical Problem Solving: A New Perspective* (pp. 20–36). New York: Springer.

D. McLeod (1992). Research on affect in mathematics education: A reconceptualization. In D. Grouws (ed.), *Handbook of Research on Mathematics Learning and Teaching* (pp. 575−596). New York: MacMillan.

G. Mandler (1989). Affect and learning: Causes and consequences of emotional interactions. In D. McLeod and V. Adams (eds.), *Affect and Mathematical Problem Solving: A New Perspective* (pp. 3–19). New York: Springer.

M. Polo and R. Zan (2005). Teachers' use of the construct "attitude". Preliminary research findings. In M. Bosch (ed.), *Proceedings of the 4th Conference of ERME* (pp. 265–274). Sant Feliu de Guíxols, Spain.

M. Ruffel, J. Mason and B. Allen (1998). Studying attitude to mathematics. *Educational Studies in Mathematics,* 35, 1–18.

A. Schoenfeld (1983). Beyond the purely cognitive: Belief systems, social cognitions, and metacognition as driving forces in intellectual performance. *Cognitive Science*, 7, 329–363.

R. Skemp (1976). Relational understanding and instrumental understanding. *Mathematics Teaching,* 77, 20–26.

G. Törner (2002). Mathematical beliefs–a search for a common ground. In G. C. Leder, E. Pehkonen and G. Törner (eds.), *Beliefs: A Hidden Variable in Mathematics Education?* (pp. 73–94). Dordrecht: Kluwer Academic Publishers.

R. Zan, L. Brown, J. Evans and M. Hannula (2006). Affect in mathematics education: An introduction. *Educational Studies in Mathematics*, 63(2), 113–121.