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Age and convective core overshooting calibrations in CPD-54 810 binary system

Statistical investigation on the solution robustness

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ABSTRACT

Aims. Relying on recent very precise observations for the CPD-54 810 binary system, we investigate on the robustness of the estimated age and convective core overshooting for a system with both stars in the main sequence (MS). Our main aim is to assess the variability in the results, accounting for different statistical and systematic sources of uncertainty.
Methods. We adopt the SCEPtER pipeline, a well established maximum likelihood technique, based on fine grids of stellar models computed for different initial chemical composition and convective core overshooting efficiency.
Results. We performed different fits of the system, under different assumptions. The base fit suggests a common age of 3.02±0.15 Gyr, in agreement with recent literature. This estimated convective core overshooting parameter is $\beta = 0.09 \pm 0.01$, with a corresponding convective core mass $M_c = 0.059^{+0.017}_{-0.021}$ $M_{\odot}$. The robustness of these estimates were tested assuming a narrow constraint on the helium-to-metal enrichment ratio, in agreement with recently published results on the Hyades cluster. Under this constraint the chemical solution of the system changes, but the age and the overshooting parameter are almost unchanged ($3.08^{+0.17}_{-0.14}$ Gyr and 0.09 ± 0.01). In a further test we halved the uncertainty in the effective temperature of both stars and again the estimated parameter shows only small variations (3.02 ± 0.12 Gyr and 0.09 ± 0.01).
Conclusions. This low variability suggest that the age of the system with both stars in MS can be reliably estimate at 5% level, but also points out that the power of the investigation is probably low, because it is possible to find a satisfactory fit in several different configurations by only varying the initial chemical composition within its uncertainty. Despite the great increase in the observational constraints precision, the results support the conclusions of previous theoretical works on the stellar parameter calibration with double MS star binary systems.

Key words. Binaries: eclipsing – Stars: fundamental parameters – methods: statistical – stars: evolution – stars: interiors

1. Introduction

While stellar model prediction became very accurate in the last decades, they are still nonetheless affected by some weakness, one of the major being the lack of a rigorous treatment of convective transport (see Viallet et al. 2015, for a comprehensive introduction). This limitation hampers a firm prediction of the dimension of the convective core. Stellar modellers usually compute its extension beside the classical Schwarzschild convective region allowing for an overshooting zone, whose extension is a function of a free parameter to be empirically calibrated. However, it has been shown in the literature (Claret & Torres 2017; Valle et al. 2015a, 2016, 2017) that only very precise observations of binary systems could, in principle, be used for model calibration.

The outstanding improvement in the radii and masses precision for stars in binary system – thanks to satellite missions such as Kepler and TESS (Borucki et al. 2010; Ricker et al. 2015) – coupled with those in the methods of estimating the stellar effective temperatures (Miller et al. 2020), allow nowadays to achieve a precision in the measured parameters below 1% for well studied targets. This relevant improvement might modify the conclusions of studies even from the recent pasts which adopted much larger uncertainties in their computations (e.g. Valle et al. 2016).

An ideal target to investigate on the robustness and variability of fundamental parameter estimates was recently identified in the CPD-54 810 binary system, extensively analysed by Miller et al. (2022), who adopted a powerful technique to estimate the stellar effective temperatures, relying on estimates of masses and radii at the 0.1% level. The system is composed by two stars in the main-sequence (MS) evolutionary stage and is therefore a perfect target to investigate on the possible changes in the (Valle et al. 2016) conclusions – who advised against adopting targets in MS for calibration purposes – thanks to the improvement of an order of magnitude in the observational uncertainties.

A preliminary attempt to constrain the age of the system, profiting from the achieved precision, was performed by Miller et al. (2022)- However the focus of that paper was not on the stellar evolution calibration. Here we specifically address this topic by investigating different scenarios. In fact, the main interest in performing a fit of such a system, besides the obvious interest in the age estimation, is to establish the robustness of the fitted stellar parameters.

Therefore we investigate in this paper several systematic effects that can bias the results, owing to different but legitimate decisions of the researchers in the fitting process. It should how-

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ever be noted that we perform our analysis fixing the input physics of the adopted theoretical stellar models. A non negligible variability is expected when results from different pipelines are compared on common targets (see e.g. Reese et al. 2016; Stancliffe et al. 2016; Silva Aguirre et al. 2017; Valle et al. 2017). Therefore our results can be considered as representative of the random uncertainty achievable with a single pipeline approach. In the concluding section we discuss further on what is expected from a multi pipeline analysis.

The structure of the paper is as follows. In Sect. 2, we discuss the method and the grids used in the estimation process. The fit of the system is presented in Sect. 3, with an analysis of the impact of different assumptions in the adopted grids and observational uncertainties. Some concluding remarks and a comparison with the literature can be found in Sect. 4.

2. Methods and observational constraints

2.1. Fitting technique

The analysis is conducted adopting the SCEPtER pipeline[1]. This technique is well tested and adopted in the literature for single stars and binary systems (e.g. Valle et al. 2014, 2015c,b,a, 2017). The procedure provides estimates of the parameters of interest (age, initial helium abundance, initial metallicity, core overshooting parameter and extension of the convective core) adopting a maximum likelihood over a grid approach.

The method is explained in detail in Valle et al. (2015a), we provide here only a brief summary for reader's convenience. Basically, for every j-th point in the estimation grid of pre-computed stellar models a likelihood estimate is obtained for both stars

$$\mathcal{L}^{1,2}{}_j = \left(\prod_{i=1}^{n} \frac{1}{\sqrt{2\pi}\sigma_i} \right) \times \exp\left(-\frac{\chi^2}{2} \right), \quad (1)$$

where

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{o_i - g_i^j}{\sigma_i} \right)^2 . \quad (2)$$

o_i are the n observational constraints, g_i^j the j-th grid point corresponding values, and σ_i the observational uncertainties.

Then, the joint likelihood of the system is computed as the product of the single star likelihood functions. It is possible to obtain the estimates for both the two individual components and for the whole system. In the former case, the fits for the two stars are obtained independently, while in the latter case the two objects must have a common age (with a tolerance of 1 Myr), identical initial helium abundance and initial metallicity.

The error on the estimated parameters is obtained by means of Monte Carlo simulations. We generate $N = 10\,000$ artificial binary systems, sampling from a multivariate Gaussian distribution centred on the observational data, taking into account the correlation structure among the two star observational data. As in Valle et al. (2017), we assume a correlation of 0.95 between the primary and secondary effective temperatures, and 0.95 between the metallicities of the two stars. Regarding mass and radius correlations, the high precision of the estimates makes these parameters of no importance, but we set it at 0.8 for the masses and -0.9 for the radii, typical values for this class of stars (Valle et al. 2015a, 2017). We explicitly verified that different adoptions of masses and radii correlation lead to negligible modifications in the results.

Table 1. Observational constraints for the CPD binary system from Miller et al. (2022), with increased uncertainty in the effective temperatures.

	primary	secondary
M ($M_\odot$)	1.3094 ± 0.0051	1.0896 ± 0.0034
R ($R_\odot$)	1.9288 ± 0.0030	1.1815 ± 0.0037
$T_{\rm eff}$ (K)	6462 ± 100	6331 ± 100
[Fe/H]	0.0 ± 0.2	0.0 ± 0.2

2.2. Observational data

As observational constraints, we use the masses, radii, metallicities [Fe/H] and effective temperatures of both stars. The adopted values and their uncertainties, reported in Table 1, are taken from Miller et al. (2022).

The uncertainty in the effective temperature reported in Miller et al. (2022) is 43 K, but as it is discussed extensively in that paper there are possible systematic effects that might modify the calibration scale. Therefore as a reference scenario we assume a prudential estimate of 100 K as error in $T_{\rm eff}$ for both stars. We discuss in Sect. 3.3 the modification of the parameter estimates assuming a more precise effective temperature constraint.

2.3. Stellar models grid

The grids of models were computed for the mass in the ranges [1.08, 1.10] $M_\odot$ and [1.30, 1.32] $M_\odot$ with a step of 0.002 $M_\odot$, from the pre MS up to the start of the red-giant branch (RGB). The initial metallicity [Fe/H] was varied from −0.4 dex to 0.3 dex, with a step of 0.05 dex. The solar heavy-element mixture by Asplund et al. (2009) was adopted[2]. Several initial helium abundances were considered at fixed metallicity by adopting the commonly used linear relation $Y = Y_p + \frac{\Delta Y}{\Delta Z} Z$ with the primordial abundance $Y_p = 0.2485$ from WMAP (Peimbert et al. 2007a,b). The helium-to-metal enrichment ratio $\Delta Y/\Delta Z$ was varied from 1.45 to 2.55 with a step of 0.15, centred around 2.0 (Tognelli et al. 2021).

Models were computed with FRANEC code, in the same configuration as was adopted to compute the Pisa Stellar Evolution Data Base[3] for low-mass stars (Dell'Omodarme et al. 2012). The models were computed by assuming the solar-scaled mixing-length parameter $\alpha_{\rm ml} = 1.74$. The extension of the extra-mixing region beyond the Schwarzschild border was considered only for the primary star and was parametrized in terms of the pressure scale height $H_{\rm p}$: $l_{\rm ov} = \beta H_{\rm p}$, with β in the range [0.00; 0.28] with a step of 0.01. The code adopts an instantaneous mixing in the overshooting treatment. Atmospheric models by Brott & Hauschildt (2005), computed using the PHOENIX code (Hauschildt et al. 1999, 2003), available in the range 3000 K ≤ $T_{\rm eff}$ ≤ 10000 K, 0.0 ≤ $\log g$ (cm s^{-2}) ≤ 5.0, and −4.0 ≤ [M/H] ≤ 0.5 where adopted. In the range 10000 K ≤ $T_{\rm eff}$ ≤ 50000 K, 0.0 ≤ $\log g$ (cm s^{-2}) ≤ 5.0, and −2.5 ≤ [M/H] ≤ 0.5, where models from Brott & Hauschildt (2005) are unavailable,

[1] Publicly available on CRAN: http://CRAN.R-project.org/package=SCEPtER, http://CRAN.R-project.org/package=SCEPtERbinary

[2] A reduced test was conducted adopting the Grevesse & Sauval (1998) heavy-element mixture, with negligible differences in the results.

[3] http://astro.df.unipi.it/stellar-models/

Table 2. Results of the CPD-54 810 simultaneous binary system fitting.

	q_{16}	q_{50}	q_{84}
Y	0.268	0.280	0.289
Z	0.0103	0.0128	0.0158
β	0.080	0.090	0.105
M_c ($M_\odot$)	0.038	0.059	0.076
age (Gyr)	2.87	3.02	3.17
	Fit parameters		
$T_{\rm eff,1}$ (K)		6449	
$T_{\rm eff,2}$ (K)		6251	
M_1 ($M_\odot$)		1.310	
M_2 ($M_\odot$)		1.090	
R_1 ($R_\odot$)		1.928	
R_2 ($R_\odot$)		1.183	
[Fe/H]$_1$		−0.13	
[Fe/H]$_2$		−0.12	
χ^2		1.6	

models by Castelli & Kurucz (2003) are used. Further details on the stellar models are fully described in Valle et al. (2015c,a) and references therein.

3. Results

3.1. Single stars and overall system fit

The results of the estimation procedure applied to the whole system and to the individual stars are reported in Table 2 and 3, respectively.

The simultaneous fit of both stars of the system suggests a common age of 3.02 ± 0.15 Gyr, with initial [Fe/H] = 0.0 and initial helium abundance $Y = 0.28$. Therefore a helium-to-metal enrichment ratio $\Delta Y/\Delta Z = 2.5^{+0.1}_{-0.65}$ is preferred. The position of the data with respect to the best fit evolutionary tracks is displayed in the Fig. 1 (left panel) in the radius versus effective temperature plane. The figure shows a good agreement between theoretical models and observational data (the corresponding best fit models are not shown to improve the figure readability, and corresponds to the points where the error bars cross the evolutionary tracks). Qualitatively the fit agrees with the one proposed by Miller et al. (2022), with a primary star just above the hook in the overall contraction phase. To perform a formal assessment of the goodness of the fit we evaluated a χ^2 considering the differences between the fit values and the observational constraints, weighted by the observational uncertainties. A $\chi^2 = 1.6$ is obtained for 8 variables and 4 parameters, suggesting a good fit.

The right panel in Fig. 1 shows the probability density for the age estimates for the whole system and for the two individual stars, independently considered (see Sect. 2 for a detailed explanation of the differences in the fit constraints). The evolutionary stage of the two stars allows an unambiguous fit with a single peak in the age estimation. It is apparent that the age of the system is mainly dictated by the primary star fit, a result in agreement with those in Valle et al. (2015a, 2016). The result is clearly due to the fact that the primary star is in a faster evolutionary phase than the secondary, thus allowing a better constraint on its age than the latter.

The system fit points toward a non negligible overshooting parameter $\beta = 0.09 \pm 0.01$. However, the meaning of this parameter is relative to the actual overshooting scheme implemented in the stellar evolutionary code. Many different choices exist in the literature, from a diffusive approach to a capped estimate in the extension of overshooting region with respect to the core extension (e.g. Weiss & Schlattl 2008; Paxton et al. 2013, 2018; Hidalgo et al. 2018; Nguyen et al. 2022). The different implementations are still quite arbitrary (see e.g. the discussion in Salaris & Cassisi 2017), therefore a better alternative is to focus on the extension of the convective core mass M_c. From the system fit a value of M_c of about 0.06 $M_\odot$ is obtained. Left panel in Fig. 2 shows the bi-dimensional density of probability in the M_c vs β plane; as expected there is a positive correlation between the two parameters, but – apart two negligible islands of solution at the grid edges – the density shows a clear single and narrow peak. The fact that the evolutionary stage of the system is unambiguously identified in the fit is also apparent in the right panel of the figure which shows the dependence of the estimated age on the reconstructed convective core overshooting parameter. The joint density of probability is elongated in age for a nearly constant β parameter, suggesting that the uncertainty on this latter is not the direct cause of the uncertainty in the final age estimate.

A limiting factor in the system fit is the large uncertainty in the observed metallicity [Fe/H], which allows for a wide range of solutions at different initial metallicities. Therefore we tested the sensitivity to this parameter by repeating the fit halving the [Fe/H] uncertainty to 0.1 dex. The fitted system age increases to 3.12 Gyr, and this is an expected results because the preferred [Fe/H] solution in Tab. 2 is slightly below 1σ from the observational constraint. Shifting to an higher metallicity causes an increase in the age. Other changes occur in the β parameter median value, to $\beta = 0.095$, and a corresponding median core mass $M_c = 0.055$ $M_\odot$. Overall, the uncertainty in the recovered parameters do not change, therefore a more precise metallicity does not directly translate in more precise estimates for this particular system.

The individual fits of the two stars shows some interesting characteristics which is worth discussing. Basically the parameters inferred from them are in good agreement, and in fact the joint fit discussed above is satisfactory. However, as it is shown in Tab. 3, the initial metallicity obtained from the primary star ($Z = 0.0143$) is quite higher than that from the secondary ($Z = 0.0128$), with a consequent higher estimated age for the primary star (3.08 vs 2.95 Gyr). The most interesting fact is the estimated extension of the convective core for the primary star, which is $M_c = 0.019$ $M_\odot$ clearly lower than that obtained from the joint stars estimate. Moreover, for the two dimensional density of probability in the M_c vs β plane shown in Fig. 3, it is apparent that the median value of the estimated core is influenced by the very long tail in the distribution towards high β and M_c values, while the distribution is peaked around $M_c = 0.015$ $M_\odot$. The possibility to fit the primary star in quite different configurations confirms the results in Valle et al. (2015a): the degeneracy between the initial chemical composition and the core overshooting efficiency significantly reduces the power of the analysis when the two stars are both in the MS.

3.2. A tighter constraint in the helium-to-metal enrichment ratio

The two stars in the CPD-54 810 system are too cold for a spectroscopic measurement of their helium content. Therefore both the current and the initial helium abundance must be estimated in the fit process. This introduces a non negligible uncertainty source, due to the concurrent interplay of metallicity, initial helium abundance and core overshooting efficiency in setting the pace of the stellar evolution.

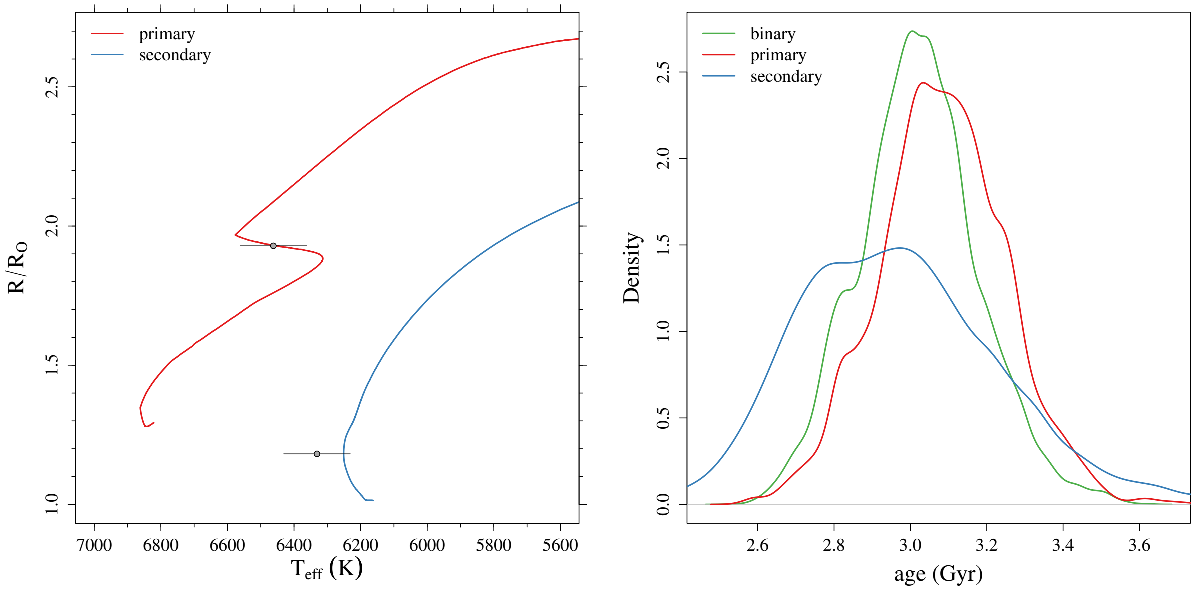

Fig. 1. *Left*: comparison between the observational values of effective temperature and radius of the two stars (grey circle) and the evolutionary tracks for the best solutions found in the analysis. The error bars correspond to 1 σ errors. *Right*: density of probability for the estimated age of the system, and for the two individual stars.

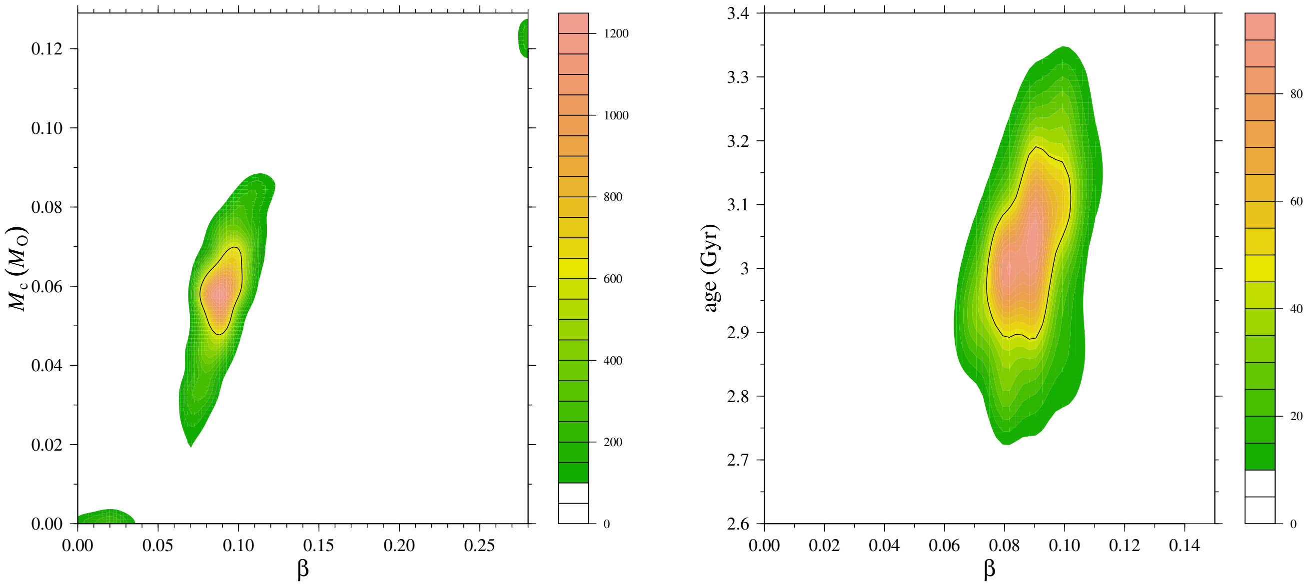

Fig. 2. *Left*: joint two-dimensional density of probability for the estimated overshooting parameter β and the convective core mass of the binary system. The solid black line corresponds to points for which the density is half of the maximum value. *Right*: same as in the left panel but in the age vs β plane.

To fit the system we adopted a linear relation between initial helium abundance and initial Z metallicity, a common choice among stellar modellers: $Y = Y_p + \frac{\Delta Y}{\Delta Z} Z$. Adopting in the fit a grid with multiple $\Delta Y/\Delta Z$ values, the estimating pipeline proposed, as discussed in the previous section, a value of the helium-to-metal enrichment ratio close to 2.5. This value is higher than that obtained by a recent investigation performed on the Hyades cluster (Tognelli et al. 2021). That paper significantly improved the measurement precision of this parameter, getting a value of $\Delta Y/\Delta Z = 2.03 \pm 0.15$. It is therefore interesting to analyse the changes in the results when adopting a fitting grid with fixed $\Delta Y/\Delta Z = 2.0$.

The results, presented in Tab. 4, show remarkable agreement with those obtained in the previous section with a loose constraint in the helium-to-metal enrichment ratio. The pipeline is able to find a good fit also restricting the model hyperspace ($\chi^2 = 2.1$), at age 3.08 Gyr (range [2.94, 3.25] Gyr), very close to the value obtained with the full grid. The same agreement holds for the convective core extension and the overshooting parameter, 0.062 $M_{\odot}$ and 0.09 respectively. The position of the observational data with respect to the evolutionary tracks with $\Delta Y/\Delta Z = 2.0$ are shown in Fig. 4, where the position of best fit models are not marked for better readability but corresponds to the points where the theoretical evolutionary tracks cross the error bars of observational data.

Table 3. Result of the CPD-54 810 individual stars fitting.

	primary			secondary		
	q_{16}	q_{50}	q_{84}	q_{16}	q_{50}	q_{84}
Y	0.270	0.281	0.291	0.270	0.276	0.282
Z	0.0104	0.0143	0.0183	0.0103	0.0128	0.0131
β	0.041	0.062	0.103			
M_c ($M_\odot$)	0.008	0.019	0.070			
age (Gyr)	2.92	3.08	3.25	2.71	2.95	3.23

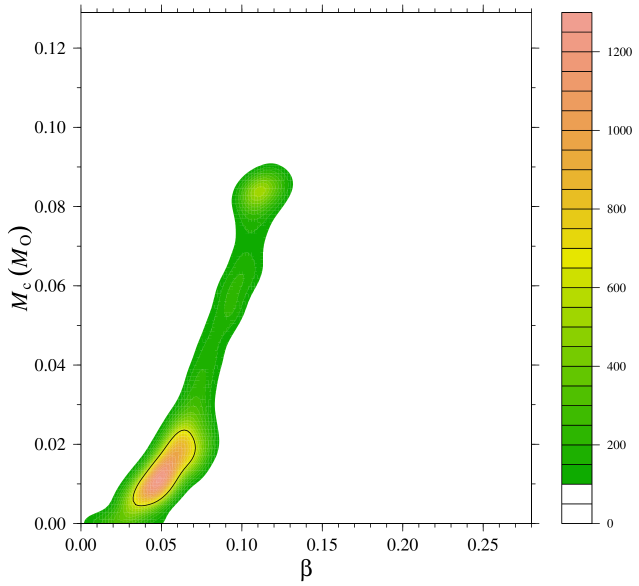

Fig. 3. Same as in the left panel in Fig. 2, but for the single fit of the primary star.

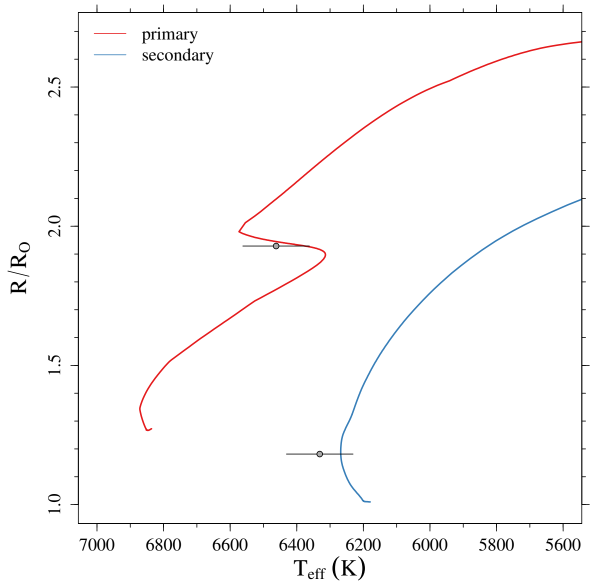

Fig. 4. As in the left panel of Fig. 1, but for the results obtained adopting a fitting grid with fixed $\Delta Y/\Delta Z$ = 2.0.

Table 4. Result of the CPD-54 810 binary system fitting imposing the $\Delta Y/\Delta Z$ = 2.0 constraint.

	q_{16}	q_{50}	q_{84}
Y	0.269	0.272	0.274
Z	0.0104	0.0116	0.0129
β	0.080	0.090	0.105
M_c ($M_\odot$)	0.046	0.062	0.090
age (Gyr)	2.94	3.08	3.25
Fit parameters			
$T_{\rm eff,1}$ (K)		6483	
$T_{\rm eff,2}$ (K)		6240	
M_1 ($M_\odot$)		1.308	
M_2 ($M_\odot$)		1.090	
R_1 ($R_\odot$)		1.929	
R_2 ($R_\odot$)		1.181	
[Fe/H]$_1$		−0.15	
[Fe/H]$_2$		−0.14	
χ^2		2.1	

The possibility to have a satisfactory fit in several similar configurations is partially due to the large uncertainty in the system metallicity [Fe/H]. However it also confirms that when both stars are in the MS, even if their masses and radii are measured with an awesome precision, they still allow for a large solution hyperspace. A system with a star in a more evolved and faster evolutionary phase would allow for a much more narrow solution space, possibly highlighting some discrepancy between theoretical models and observations. On the contrary, while the discriminating power achievable from a system with both stars in MS is low, nonetheless the estimated fundamental parameters (i.e. the age and the core overshooting parameter in the present paper) appear to be very robust against different assumption and constraints in the fitting process.

3.3. Impact of the effective temperature uncertainty

The uncertainty in the effective temperature reported by Miller et al. (2022) is about one half of that adopted in the previous section. They obtained such an high precision combining the measurement of the stellar radii with Gaia parallaxes, and with an accurate knowledge of the bolometric flux (Miller et al. 2020). As briefly mentioned in Sect. 2 our choice is mainly dictated by a cautious approach to this observational constraint. In fact the absolute calibration of the effective temperature is still affected by large uncertainties. It is not uncommon to find different works in the literature claiming an accuracy of some tenth of K; however a comparison of results by different authors on the same stars often shows differences larger than 100 K (see e.g. Ramírez & Meléndez 2005; Schmidt et al. 2016).

Indeed this is the case for the system under investigation. The results by Ratajczak et al. (2021), who presented an analysis of this system based on a reduced data set, significantly disagree with those by Miller et al. (2022). In fact the former stellar effective temperatures are about 500 K cooler than the latter. We are not interested here in discussing the possible origins of this

Table 5. Result of the CPD-54 810 binary system fitting with halved uncertainty in $T_{\rm eff}$.

	q_{16}	q_{50}	q_{84}
Y	0.269	0.270	0.272
Z	0.0104	0.0108	0.0116
β	0.090	0.092	0.100
M_c ($M_\odot$)	0.057	0.064	0.073
age (Gyr)	2.92	3.02	3.14
	$\Delta Y/\Delta Z$ = 2.0		
Y	0.269	0.270	0.272
Z	0.0104	0.0108	0.0116
β	0.090	0.090	0.100
M_c ($M_\odot$)	0.059	0.065	0.073
age (Gyr)	2.92	3.01	3.13

discrepancy, and we refer the interested reader to Miller et al. (2022), who discuss this topic in detail. However the large discrepancy suggested us the cautious approach adopted in the fit.

However it is nonetheless interesting to analyse how the uncertainties in the $T_{\rm eff}$ constraints propagate into the final result. Therefore we repeated the analysis of the system (both with the full and the reduced $\Delta Y/\Delta Z = 2.0$ grids) but adopting an uncertainty of 50 K in $T_{\rm eff}$ that is, one half of that adopted in the previous fits. This value is close to that from Miller et al. (2022) – i.e. 43 K – when adding the systematic uncertainty quoted in that paper (13 K).

The results of the whole system fit with full and reduced grids are reported in Tab. 5. Both fits are satisfactory with a $\chi^2 = 2.8$. The most interesting fact is that they remarkably agree each other. When the algorithm is forced to refine the effective temperatures of the proposed solution at high accuracy the differences between the two grid solutions disappear. Moreover the initial chemical abundances of the fits are more similar to those in Tab. 4 (for the $\Delta Y/\Delta Z = 2.0$ scenario) than to those in Tab. 2. The estimated age is 3.02 Gyr, the same results obtained with a full grid (see Tab. 2). The extension of the convective core ($M_c \approx 0.064\ M_\odot$) and the overshooting parameters ($\beta \approx 0.090$) agree well with the results presented above.

In summary, an improvement in the accuracy of the effective temperatures impacts more on the determination of the initial chemical abundances than on the age of the system.

4. Discussion and conclusions

Profiting of the recent availability of very high accuracy observational data for the binary system CPD-54 810 (Miller et al. 2022) we attempted a fit of this system to investigate on the robustness of the age estimate under different assumptions. To do this, we used the SCEPtER pipeline (Valle et al. 2014, 2015c,a), a maximum likelihood procedure, on a dense grid of stellar models computed ad-hoc.

Relying on the observational constraint by Miller et al. (2022), but adopting a conservative uncertainty of 100 K in the effective temperatures, we obtained a satisfactory simultaneous fit of the system at 3.02±0.15 Gyr. The awesome precision in the radii and masses measurements allowed for an outstanding 5% precision in the age estimate, an unusual results for MS stars, an evolutionary phase where the power of the investigation is usually quite low (see e.g. Valle et al. 2015a, 2016).

Taking advantage of the very precise observational data we also tried to constrain the efficiency of the convective core overshooting in the primary star. We obtained an overshooting parameter $\beta = 0.09 \pm 0.01$, with a corresponding convective core mass $M_c = 0.059^{+0.017}_{-0.021}\ M_\odot$.

The binary age estimate proposed in this paper is 7% higher than the 2.83 Gyr reported by Miller et al. (2022). This difference is at 1σ level or below; since Miller et al. (2022) do not report the error in the age estimate it is impossible to precisely quantify the significance of the discrepancy. It is also impossible to unambiguously identify the origin of this difference because the stellar models adopted in that paper differ from those adopted here in many aspects. In particular, the treatment of the convective core overshooting is different because we do not allow for it for masses below 1.1 $M_\odot$ (i.e. it does not affect the secondary star), while it is adopted even in this range for the models used in Miller et al. (2022). Moreover, the scheme of overshooting implementation itself is different. While we adopt a step overshooting, a diffusive approach is used in Miller et al. (2022). While it does not directly lead to differences in the estimated age when the other model input are the same (see Valle et al. 2017, for a comaprison of overshooting scheme in a controlled environment), it adds to others differences between the evolutionary tracks.

Another recent estimation of the system age was proposed by Ratajczak et al. (2021), who estimated it at $3.4 - 4.0^{+0.15}_{-0.25}$ Gyr adopting PARSEC and YY models, respectively (Yi et al. 2001; Bressan et al. 2012). While the former estimate marginally agrees with our result at 1σ level, the latter is significantly higher. However Ratajczak et al. (2021) estimates adopt stellar effective temperatures lower by about 500 K than those adopted in our investigation. This difference forces the fit towards higher metallicity and lower initial helium abundance, and therefore higher ages, with respect to our models.

As a difference with previous results in the literature we explicitly tested here the possible dependence of the estimated age on the efficiency of the convective core overshooting. This results may help shedding some light in the ongoing discussion about the dependence of the overshooting efficiency with the stellar mass (Stancliffe et al. 2015; Claret & Torres 2016; Constantino & Baraffe 2018; Anders & Pedersen 2023). While some works suggest such a dependence exists, others do not agree with this claim. Calibrations from systems with a high precision in the radii and effective temperature measurements are of paramount importance to get further insight into this topic.

The main interest of the present works is possibly the investigation on the robustness of the age and overshooting parameter estimates. Firstly, we tested their variability when imposing a very strong prior on the helium-to metal enrichment ratio, in practice adopting an estimation grid restricted to only stellar models computed assuming $\Delta Y/\Delta Z = 2$, in agreement with recent investigations (Tognelli et al. 2021). A second test was performed adopting the full grid of models but reducing the uncertainty in the effective temperatures to 50 K, as in Miller et al. (2022). The results from these different analyses were in a very good agreement each others. The proposed age is particularly robust and only showed variations of few tenths of Myr in the different tests performed.

While this very low variability is reassuring, it also points out that the power of the investigation is probably low. With both stars in MS it was possible to find a satisfactory fit in several different configurations by only changing the initial chemical composition of the proposed solution of the system within the current uncertainty. On the other hands, this means that in a system with both stars in MS, the estimated fundamental parameters (i.e. the age and the core overshooting parameter) appear to be very robust against different assumption and constraints in the fitting process, contrarily to systems with a star in a more evolved phase.

The robustness of the age estimate and the low uncertainty in its value ($\approx$ 5%) should be however cautiously interpreted. The results presented in this paper are obtained with a fixed grid of stellar models, all computed with identical assumptions in the input physics. A larger variability is expected when comparing results from various pipelines (Reese et al. 2016; Stancliffe et al. 2016; Silva Aguirre et al. 2017; Valle et al. 2017) due to the different but legitimate choices of stellar modellers. Ultimately, in the light of the obtained results and of those in the literature, a conservative but realistic estimate of the precision achievable for a binary system age with both stars in MS, even when their fundamental parameters are measured at very high precision, is probably about 7 - 10%.

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