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Some arguments against the possibility of an infinite past

Luca Bellotti

Abstract: In this brief note we discuss some arguments against the purely conceptual possibility of an infinite past, arguing that they are ungrounded and showing how some points of the contemporary debate can be found in some mid-thirteenth-century controversies on the topic.

Keywords: infinite past, eternity of the world, philosophy of time

We will consider some classical arguments proposed by various scholars, including in particular G.J. Whitrow (1966, 1980), P. Huby (1971), W.L. Craig (2000), D.A. Conway (1974), which should demonstrate the purely conceptual impossibility of an infinite past. We believe that these arguments are ungrounded, either because they invalidly draw conclusions from true assumptions, or because they are based on false assumptions. Starting from the critical discussion of these arguments carried out by Q. Smith (1987) and R. Sorabji (2006), now classical in the vast literature on the topic, we will see how the arguments (some of which have, for better or worse, an undeniably Zenonian flavour) are articulated and why they are untenable. We will also show how some fundamental aspects of the contemporary discussion can be found, in all their precision, in the mid-thirteenth-century controversy between Bonaventure and Thomas Aquinas on the problem of the eternity of the world, as well as in the treatise *De aeternitate mundi* by Boethius of Dacia (*circa* 1270).

The arguments we will examine should show, perhaps unsurprisingly, that there is no reason, particularly in the light of the mathematics of infinity of the last 150 years (purely set-theoretic notions will prove to be sufficient below - without denying the possible relevance of mereological, topological, metrical, or measure-theoretic ones for the problem), to deny the purely conceptual possibility of an infinite past. Distinct, and certainly relevant, is the problem of the physical possibility of an infinite past; however, it seems that this problem is on a decidedly different level from the one we want to discuss here, a level in which

considerations based purely on the analysis of concepts are insufficient. What we can conclude in the present context is that the question of the infinity of the past does not seem apt to be decided in the negative with purely *a priori* arguments.

Our main motivation is mixed, both theoretical and comparative (though not strictly historical). Although some more recent literature in the philosophy of time on the question of a possibly infinite past shows interests which are admittedly different from ours, the persistence of arguments like those discussed below in recent debates, with proposals which do not withstand mathematical scrutiny and should, in our opinion, be simply ruled out, shows perhaps the necessity of recalling again some fundamental points (not yet taken as uncontroversial) before any further refinement.

We will consider and re-evaluate the six classical fundamental arguments against the conceptual possibility of an infinite past which were collected and systematized by Smith (1987); these arguments are substantially similar to those examined by Sorabji (2006, Part III, Ch. 4, especially pp. 219-224), but the two discussions are independent. We will see how the two authors critically analyze these arguments, we will compare their solutions, which in more than one case diverge (or in any case start from different points of view), and we will see to what extent they are tenable. Of course, we will refer to further, more recent contributions to the debate when necessary.

1. (Whitrow 1966). If the series of past events is infinite, it must constitute an actual infinity, since the events really happened; but an actual infinity of past events is impossible: there would be events of the past separated from the present by an infinity of intermediate events.

Following Smith (1987), we immediately notice that the argument equivocates about 'actual'. Initially, 'actual' is opposed to 'potential' in the sense of the relationship between the concepts of act and power; subsequently 'actual' refers to the infinity of a series of events such that some of them are separated from the present by an infinite number of intermediate events. But it is well possible that there is an infinite series of events that actually happened, such that each is separated from the present by a finite number of intermediate events. A model of such a series is simply the set of negative integers in their usual order.

2. (Whitrow 1966, Huby 1971, Craig 2000). Recall that \aleph_0 is the smallest infinite cardinal number, i.e. the cardinality of the set of natural numbers, as well as of any countable set. The argument is as follows: (1) \aleph_0 events happened before the present; (2) Events divided from the present by \aleph_0 events occurred; (3) From an event divided from the present by \aleph_0 events, this could not have been reached. It is believed that (1) implies (2), which in turn implies (3), whence the absurdity that the present cannot be reached.

Smith (*op. cit.*) notes the incorrectness of the inference from (1) to (2), which is evident once again considering the model of negative integers in their natural order: they constitute a set of cardinality \aleph_0 in which any element is separated from zero ('the present') by a finite number of elements. Note that the order of the elements is important: e.g., if we order the negative integers in such a way that all the even ones precede all the odd ones, we will certainly have elements that are separated from others by \aleph_0 elements; but this is not the relevant order for the argument. The fact that we deal with the same set is irrelevant: the order properties of a set are in general completely independent from those of an extensionally identical set ordered in a different way, since different orders can be defined on the same set; if two sets are placed in one-to-one correspondence, the order properties of a certain element of the first set are in general independent of the order properties of the corresponding element of the second set.

Sorabji's formulation (*op. cit.*) of the argument we are discussing (further examined and criticized, more recently, by Puryear 2014; see also Morrison 2022) is substantially analogous to Smith's formulation: if an infinity of days had passed before the present day, the latter would never have been able to occur. Sorabji replies that this would be true if there were a first day followed by an infinity of days before reaching today; but those who maintain the possibility of an eternal world *a parte ante* just do not admit the existence of a first day.

It is interesting to note that an argument against the existence of the world *ab aeterno* that falls within this setting can be found in Bonaventure (*Commentarius in quatuor libros Sententiarum Petri Lombardi*, II, dist. 1, pars 1, art. 1, q. 2, pp. 12-17) and is contested by Thomas Aquinas (*Summa contra gentiles*, II, 38, arg. 4; see also *Summa theologiae*, I, 46, 2 and *Scriptum super libros Sententiarum* II, d. 1, q. 1, a. 5; the tract *De aeternitate mundi contra murmurantes* of 1270 is of course also important on the subject). Bonaventure's argument is substantially based on the classic Aristotelian principle (see e.g. Aristotle, *De caelo*, I, 4, 272a3) *impossibile est infinita pertransiri*, which in our case would imply, if the past were infinite, the unreachability of the present. The use of the Aristotelian idea that an actual infinity cannot be traversed, in order to demonstrate the impossibility of an infinite past, actually dates back to Johannes Philoponus, in particular to his *De aeternitate mundi contra Proclum* of 529 AD (see Sorabji, *loc. cit.*, for references). Now, Aquinas accepts Bonaventure's two assumptions: (1) the eternity of the world implies that the present day has been preceded by an infinite number of days; (2) infinity cannot be crossed. However, the conclusion is not what Bonaventure would like: every given day in the past is in fact separated from the present by a finite number of intermediate days. Aquinas's argument is this: if the world is eternal, the past days can be taken either simultaneously, or in succession; if they are taken simultaneously, there is no question

of ‘crossing’, since a starting point is missing; if instead they are taken in succession, we can designate one of the past days as the starting point, but in this case the days that must be crossed are finite in number. What therefore divides Bonaventure and Aquinas is a point which, as we have seen above, remains fundamental even in contemporary discussions on the possibility of an infinite past: Bonaventure believes that an infinite series of past days must contain days that are separated from the present by an infinite number of intermediate days; Thomas believes, on the contrary, that an infinity of past days can be real and yet each past day remains separated from the present by a finite number of days, however large it may be.

We must remark, at this point, that the post-Cantorian mathematics of infinity, in its less controversial aspects from a foundational point of view, aspects which are currently widely accepted, cannot avoid to agree with Aquinas in the present controversy. If the pure conceivability of an infinite past is at stake, then there seems to be no reasonable doubt that contemporary set theory offers models such that an infinite past is simply a priori not impossible.

Sorabji (*op. cit.*) attributes the argument just discussed to Bonaventure in the following form: if we think ‘backwards’, starting from the present, we will never find a year at an infinite distance from the present; then the past years are finite in number. It is a question, Sorabji observes, of *ignoratio elenchi*: in fact, no one claims that there are years in the past that are infinitely distant from the present; we have a set of years that are all finitely distant from the present, and nevertheless this set is infinite. This is exactly the argument used by Aquinas against Bonaventure. Sorabji, however, thinks that Aquinas has only partially grasped the truth, in his objection to Bonaventure: Aquinas would have correctly seen that the distance between the present and any past year is in any case finite, but he would have incorrectly deduced that in a universe without a beginning no infinity of years would be gone through. We do not understand what is wrong with this deduction, once we accept the premise that any crossing requires two extremes to be fixed, one initial and one final, a premise in fact assumed by Aquinas, that seems entirely reasonable.

Still in this order of ideas, Sorabji presents the following argument (reported in Sorabji 2006, p. 221, attributing it to P. Huby). An infinity of future years from the present will always remain potential and will never be completed. Why shouldn’t we say the same of an infinity of past years? The answer starts from the lack of analogy between past and future, consisting in the fact that the past does not start now, although certainly our thoughts on the past do. But then, when does the past begin? The answer, consistently with what we have been saying so far, should be clear: the past, under the hypothesis we are considering, does not begin. Not that past and future are inherently asymmetrical;

it is their being ‘crossed’ that presents a crucial difference: while the crossing of future years starting from the present is still something that has two extremes, two boundaries, the ‘crossing’ of the past we are now dealing with has only one boundary, which is the one *a parte post*, and no boundary *a parte ante*. Thus we speak of ‘crossing’ in a metaphorical sense, since, as we have seen, a real crossing requires two boundaries. Arguably, it is already the very notion of ‘crossing’ that can be considered no more than a metaphor; but at least in this context its features are sufficiently clear, intuitively, to allow the refutation of the argument proposed.

Having replied along these lines, Sorabji (*loc. cit.*) makes a remark that seems rather strange: a set of future years starting from the present would become infinite in actuality only if it reached a year infinitely distant from the present; but the same cannot be said of the past. This seems to contradict Aquinas’s argument that the actual infinity of a set does not require that there are elements of the set ‘at infinity’, an argument which, as we have seen, is difficult to refute, given that we have the succession of natural numbers as a simple example. Sorabji’s claim remains all the more strange, since the preceding argument seems correct and does not depend on it in any way, since it is based on the difference, which certainly exists, between going through the past in its totality and going through the future in the sense of pushing forward into the future indefinitely.

3. (Conway 1974, Craig 2000). The set of past events is never complete, but new events are always being added to it; therefore there cannot exist in the past an actual or complete infinity of events. A model could be a library of \aleph_0 volumes in which each volume is marked with a natural number: it would be impossible to add a volume to this library. Two assumptions seem to be present here: the first, that nothing can be added to an actually infinite set; the second, that if all the negative integers have been assigned to past events then no new events can be added to the latter.

To the first assumption we can answer that to a set of cardinality \aleph_0 we can add not only any finite number of elements, as Smith (*op. cit.*) recognizes, but also any finite number of disjoint sets, each of cardinality \aleph_0 , without altering its cardinality. Assuming the axiom of countable choice, moreover, we have that even a countable union of countable sets remains countable. The second assumption is answered with the example of the famous so-called ‘Hilbert’s hotel’: it is a hotel with \aleph_0 rooms; even assuming that they are all already occupied, a new guest can be accommodated by moving the guest from the first room to the second, the guest from the second to the third, and so on, and assigning the first room, thus free, to the newcomer. In our case, every time a ‘new’ event becomes part of the set of past events, we can simply reassign the negative integers, ‘scaling’ them by one, as in the case of Hilbert’s hotel rooms. Basically, it

is a question of taking seriously the fact (which Dedekind even took as *defining* the notion of infinity) that a countably infinite set can be placed in one-to-one correspondence with an infinite proper subset of itself.

Dealing with Hilbert's hotel, Sorabji (*op. cit.*) simply emphasizes that this example is in no way a symptom of the absurdity of the notion of actual infinity, as Huby and Craig would like, but only an application of a true assertion about infinite sets, perhaps counter-intuitive at first sight, but certainly justifiable in the light of post-Cantorian mathematics. Sorabji also briefly discusses a very simple formulation of the argument that the past, if infinite, cannot be completed or 'accomplished': infinity, by definition, cannot come to an end, so it cannot be completed or 'accomplished' in any way. To this, he correctly replies that an infinite series may well have an end: in our case we consider the infinite set of past years, which ends in the present, and therefore has an end.

4. (Craig 2000, Whitrow 1966). Tristram Shandy's paradox would demonstrate the impossibility of an infinite past. This is a 'paradox' highlighted by Russell in the *Principles of mathematics* (1903, §340). Tristram Shandy is the well-known character of Laurence Sterne, who writes his autobiography so slowly that it takes him a year to describe the first day of his life. The argument is as follows: at every moment in the past Tristram Shandy was writing his autobiography, regularly taking a year to describe a day; therefore the distance between a past day and the time in which it will be described grows with time; therefore there is no day at a finite distance from any previous day in which all the previous days have already been described; now, the present day is at a finite distance from any past day; conclusion: in the present day not all past days have been described, and the autobiography is incomplete. However, if in relation to the present day there are an infinite number of past days and an infinite number of past days described, then in relation to (and with respect to) any present there are no days not described; but this contradicts the conclusion just obtained.

Smith's discussion of Tristram Shandy's paradox (*op. cit.*; see also Eells 1988) is fundamentally correct, but in our opinion it contains an example that is not entirely relevant. Smith asserts that what in the previous argument does not work is the transition from 'the number of past days described equals the number of past days' to 'there are no past days not described': the past days described constitute a proper subset of the set of past days, yet the two sets have the same cardinality. Smith suggests considering the set of even numbers and the set of natural numbers as a model, but this does not seem relevant here. Instead, we must take the set of positive integers multiples of 365 (ignoring leap years for simplicity) and the set of all positive integers: in fact, if one takes

a countably infinite set of days, the first day corresponds to one year, therefore to 365 days; the second to two years, or 2 times 365 days, etc. The two sets can be placed in one-to-one correspondence and therefore have the same cardinality; however one is a proper subset of the other. Smith correctly asserts that at no point in the past, and in no present, does Tristram Shandy complete his autobiography; however, in an infinite time in the direction of the future the autobiography will be completed, since, given \aleph_0 days, for each n , the n -th day will be described at the (n times 365)-th day, and the days needed to complete the work will never be missing.

Russell (1903, §340) already made a similar remark, and presented the argument in the following form: (1) Tristram Shandy writes down the events of a day in a year; (2) The series of days and years does not have an end; (3) The events of the n -th day are written down in the n -th year; (4) Each assigned day is the n -th, for a suitable value of n ; (5) Therefore each assigned day will have its own description; (6) Therefore no part of the biography will remain to be written; (7) Since there is a one-to-one correlation between the instants of happening and the instants of writing, and since the former constitute a proper part of the latter, the whole and the part have (in this case) the same number of elements.

Let us now see how the Tristram Shandy paradox is dealt with by Sorabji (*op. cit.*). He disputes the claim that the paradox can hold up as an argument against the possibility of an infinite past. Infinite time allows Tristram Shandy to describe an infinity of days, but not all; ‘infinitely many’ does not imply ‘all’; it follows, according to Sorabji, that Russell is wrong when he asserts that no part of the biography will remain unwritten; this holds, in particular, if we assume that Tristram Shandy’s life did not begin; nor will there come a day in which all days have been recorded.

While we do agree with Sorabji’s conclusion, we do not accept his argument. Indeed, he wants to refute the use of the paradox made by those who deny the possibility of an infinite past, yet he denies that the days are sooner or later all recorded. But this is precisely what cannot be denied: as we have seen, Russell is right in asserting that no part of the biography will remain unwritten; there are ‘enough’ years to ‘cover’ every day. It is clear that this does not mean that sooner or later there will be a year in which the work is completed: it is only in the infinite (past, present and future) totality of \aleph_0 years that the work will be completed. Even in the hypothesis, which Smith adopts from the start, that Tristram Shandy has lived eternally in the past, it is not clear why the lack of a beginning should change something: taken any day in the past, Tristram Shandy will describe it, sooner or later, taking one year; perhaps he has already described it at the present time, perhaps he has not yet, but this is not relevant with respect to the infinite totality of years that will in any case be needed to complete the autobiography (see, for further discussion, the exchange between Oderberg 2002 and Oppy 2002).

5. (Craig 2000). We can introduce infinite classes by means of the property satisfied by their members, without the need for ‘successive synthesis’ (in Kantian terms); but the events of the past are essentially given just in succession, so they cannot be actually infinite in number.

Here Smith’s objection (*op. cit.*) is very simple: the events of the past (discretely understood, as always in this discussion), if they are infinitely many, are given simultaneously in thought, and this does not prevent the fact that ‘in reality’ they are given in their normal succession, one by one. Under the same scope falls the ‘Kantian’ argument (on which see also Puryear 2014 and Morrison 2022) considered by Sorabji (*op. cit.*), echoing the thesis of Kant’s first antinomy: the universe must have had a beginning since an infinite series can never be completed by means of ‘successive synthesis’. It is clear, Sorabji replies, that this does not exclude an infinity of years, but rather a way of ‘reaching it’; no one argues that at a certain point the number of past years becomes, from finite that it was, infinite: it has, so to speak, always been infinite.

6. (Conway 1974, Whitrow 1980). Since it is admitted that in reality events are given in succession, how is it possible that in reality they form an infinite collection? Furthermore, it is not clear how it is possible to conceive someone who writes all negative integers from eternity (in the past) to end with the number -1; counting, by following the descending succession of negative integers, is certainly possible, but it is an inverse process with respect to the succession of events from the past to the present.

Smith objects to this argument in a rather articulate way (*op. cit.*). First of all, a discrete succession of events in time cannot form an infinite set in a finite time, but can do so in an infinite time; so the succession of negative integers has not actually been written, but could be written in an infinite time interval. We add that something stronger is valid: if we are willing to admit (just in the present connection) the continuity of the set of events, there is no reason why an infinite number of events cannot happen in a finite time. For example, a point that moves from the origin of the real line in the positive direction can of course travel the interval from 0 to 1 in a finite time while moving at finite speed. If we identify an event with a position of the point on the line, there are as many events as there are real numbers between 0 and 1, that is to say as many as there are real numbers themselves: these events all do occur, and in a finite time.

Furthermore, the fact that the counting processes with which we are familiar always have a beginning does not imply that one cannot imagine counting processes that do not have this property. If a counting process is simply, as Smith proposes, a synthetic series of counting acts, then nothing prohibits thinking of a one-to-one correspondence between past events and counting acts, such that

at present the series of such acts comes to an end. Therefore, if it is true that our own counting when referred to the past goes in the opposite direction with respect to the occurrence of events, we can well conceive a being who in every moment of the past was counting precisely in the order in which past events occurred.

Sorabji addresses the problems related to the counting of past years by first discussing the following objection (*op. cit.*): if the universe did not begin, the counting of years (assuming it has always been such as to assign greater numbers to successive years) should have already reached infinity at any time, however remote, in the past; but how can one conceive of completing a count of this type? Sorabji's answer is that there is a crucial difference between counting and crossing: the need to take a starting number in the case of counting. The absence of a starting point in the sequence of the past years results in the difficulty of imagining in a simple way any count (in a proper sense) of the years in the past: a count in fact always seems to require a first element. One could counter-object (the objection is reported by Sorabji, *op. cit.*, p. 219 and credited to N. Kretzmann) that a count can in fact be imagined, provided that it is 'backwards', i.e. such that one descends from numbers larger in modulus to numbers smaller in modulus, up to zero; and yet, if we are not prepared to say that whoever reaches zero in this counting has concluded to count infinity, we should not even be willing to admit that they have crossed an infinity of years. Sorabji invites, on the other hand, to imagine a beginningless measuring device embedded in a beginningless universe, such as to count how many years remain before a particularly important event, which will correspond to the year zero. It is certainly possible to imagine such a device, and therefore a sort of 'backward' counting. Note how this last counter-objection is similar to Smith's considerations above: in both cases it is admitted that the concept of 'counting' can be extended, without losing its essential properties, to include a sort of 'backward' counting (certainly different from any count we are used to). It is also quite curious that in the course of the same argument, as we have just seen, Sorabji at first asserts that what differentiates counting from crossing is the fact that the former must have a starting point, but then he concedes without problems that one can imagine a 'reversed' form of counting that has no such property. In our opinion, it is the first statement that should be given a provisional value, and then should be discarded: on second thoughts, Sorabji himself recognizes that the idea that counting necessarily presupposes a first element proves too restrictive.

We conclude by observing that a further demonstration of the pervasiveness of arguments based strictly on mathematical infinity in the discussion on the eternity of the world, already in the debate in the thirteenth century, is found in the short treatise by Boethius of Dacia *De aeternitate mundi* (ed. *Opera*, 1976).

Here we find at least three arguments that can be traced back to the patterns of reasoning on infinity that we have identified above. The first two are among the arguments against the eternity of the world (*op. cit.*, pp. 337f., arguments 6 and 10), and are given as follows.

(1) If something can be added to A, say B, then something can be greater than A; to all the time that preceded the present, one can add more time; therefore there may be something greater than all the time that preceded the present; but nothing can be greater than infinity; therefore all the time that preceded the present is not infinite, and therefore neither are motion nor the world.

(2) If the world were eternal, then it would have passed through an infinite motion and an infinite time, since if the world were eternal the time that preceded the present moment would be infinite; but that the infinite is crossed and taken as something determinate (in the text: *pertransitum et acceptum*) is impossible; therefore the world is not eternal.

The first of these arguments is basically on the line of the third of the arguments refuted above, the one concerning the library of \aleph_0 volumes to which, nevertheless, new volumes can always be added without altering the total number. Boethius correctly states that to all the time preceding the present, one can add still more time: the incorrect assumption is that this addition in itself determines an increase in the total number of temporal units, which instead, as we know, remain countably many. We could of course add, after Cantor, that it is not true either that there are no infinities 'greater' (in a precise sense) than countable infinity (which is the only infinity, of course, to which Boethius of Dacia could implicitly refer).

The second argument, on the other hand, is a classic example of application of the principle *impossibile est infinita pertransiri*, which we have already found in Bonaventure, discussing the second class of objections above, and falls under the counter-objections relevant to this principle, among which the distinction between the existence of an infinity of days, each finitely distant from the present, and the existence of a day infinitely distant from the present, remains fundamental.

Another argument in which, albeit not exclusively, considerations of a purely mathematical nature on infinity appear is the second of the series of arguments aimed at demonstrating the reality (not only the possibility, as we have been doing here) of the eternity of the world (Boethius of Dacia, *op. cit.*, p. 341), which is duly answered in the final part of the treatise (*ibid.*, p. 360). In the second part of the argument, in order to show that there was no eternity before the existence of the world, it is asserted that what is preceded by an eternal duration would never come into being; to this Boethius replies that, for example, what is done today, and which was not there before, has an eternal duration behind

it (that is, eternity itself, which has always been), and yet it undeniably comes to being. Now, this answer is too 'ostensive', so to speak, not to make one suspect an *ignoratio elenchi*; however, the argumentative technique at work here is none other than the one seen above (second class of objections) in the argument, discussed by Sorabji, similar but not identical to that of the 'non-traversability' of infinity: the crucial point is that there is no starting point in the past from which it would be necessary to cross an infinite number of temporal units to reach the present; to speak of the eternity of the world means precisely to deny the existence of such starting point.

Finally, it is interesting that in Boethius we explicitly find (*op. cit.*, pp. 353f.) the denial of the possibility for the mathematician (whether, according to the subdivision of the *Quadrivium*, arithmetician or geometer or astronomer or musician) to decide, starting from the principles of his science, in one sense or another the hypothesis that the world is eternal. The arguments we have reconstructed seem to some extent to support Boethius, at least on this point, and at least as regards the hypothesis that the world is not eternal: we have seen that the *a priori* arguments of mathematical nature aimed at proving this are not correct.

A fundamental problem, which in our opinion remains and which we have not addressed here as it is not directly relevant, is whether we can separate the aspect of pure conceivability in an abstract sense, in matters concerning time, from considerations of a different type, for example phenomenological (in a general sense), or cosmological, or, more generally, from considerations that philosophically take into account the concepts and results concerning the problem of time that have emerged in the last century in the physical sciences, mainly in the theory of relativity.

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