A MARKOV CHAIN MONTE CARLO ALGORITHM FOR ELASTIC AMPLITUDE VERSUS ANGLE INVERSION WITH NON-PARAMETRIC PRIORS AND NON-LINEAR FORWARD MODELLINGS

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Introduction. One of the main objectives of reservoir characterization is to exploit the acquired seismic and well log data to infer the distribution of elastic parameters and litho-fluid facies around the investigated area. From the mathematical point of view this process is an ill-conditioned inverse problem in which many models can fit the observed data equally well. For this reason, one goal of reservoir characterization studies is the quantification of the uncertainties affecting the recovered solution, which are expressed by the so-called posterior probability density function (*pdf;* Tarantola 2005). One challenge of this inversion process concerns the simultaneous estimation of discrete (i.e. litho-fluid facies) and continuous (elastic properties) model parameters from the observed data. Another challenge is related to the complexity of the property distribution and correlation. For example, the distribution of elastic properties is often multimodal due to the presence of multiple litho-fluid facies. An analytical and computationally fast derivation of the posterior model is only possible in cases of linear forward operators, Gaussian, Gaussian-mixture, or generalized Gaussian distributed model parameters and Gaussian errors in the seismic data. However, the validity of the Gaussian or Gaussian-mixture assumption is often case dependent because they could not be adequate to reliably capture the complex relations among elastic attributes and litho-fluid facies. At the same time, the linear forward model might not be sufficiently accurate to describe the relation between seismic data and elastic parameters in cases of strong elastic contrasts at the reflecting interface and far source-receiver offsets. For this reason, it is often advisable to numerically evaluate the posterior model through a Markov Chain Monte Carlo (MCMC) algorithm. From the one hand, MCMC methods have been successfully applied to solve many geophysical problems (Sambridge and Mosegaard, 2002) as they can theoretically assess the posterior uncertainties in cases of complex (i.e, non-parametric) prior distributions and non-linear forward modellings. From the other hand, these methods convert the inversion problem into a sampling problem and for this reason they require a much larger computational effort with respect to the analytical approach. Moreover, the use of non-parametric priors often complicates the inclusion of geostatistical a-priori information (e.g. a semivariogram model) into the inversion procedure and for this reason the use of non-parametric models is not so common in geophysical inversions. Finally, classical MCMC methods, such as the Metropolis-Hastings algorithm, are known to mix slowly between the modes if the target distribution is multimodal. To partially overcome this issue, multiple MCMC chains are usually employed so that the ability to exhaustively explore the high probability regions of the model space is enhanced.

We present an amplitude versus angle inversion algorithm for the joint estimation of elastic properties and litho-fluid facies from pre-stack seismic data in case of non-parametric mixture prior distributions and non-linear forward modellings. The algorithm inverts the pre-stack seismic responses along a given time interval using a 1-dimensional convolutional forward modelling based on the Zoeppritz equations. The distribution of the elastic properties at each time-sample position is assumed to be multimodal with as many modes as the number of litho-fluid facies considered. In this context, an analytical expression of the posterior model is no more available. For this reason, we adopt a MCMC algorithm to numerically evaluate the posterior uncertainties. With the aim of speeding up the convergence of the probabilistic sampling, we adopt a specific recipe that includes multiple chains, a parallel tempering strategy, a delayed rejection updating scheme and hybridizes the standard Metropolis-Hasting algorithm with the more advanced Differential Evolution Markov Chain method. For the lack of available field seismic data, we validate the implemented algorithm by inverting synthetic seismic data derived on the basis of actual well log data. The approach is also benchmarked against an analytical inversion approach (see de Figueiredo *et al*. 2018) that assume Gaussian-mixture distributed elastic parameters. The final predictions and the convergence analysis of the implemented method prove that our approach retrieve reliable estimations and accurate uncertainty quantifications with a reasonable computational effort.

The implemented MCMC algorithm. Let $\boldsymbol{\pi}$ and $\mathbf{e}$ be the facies and the elastic properties. In our case of a mixed discrete-continuous inverse problem, the posterior *pdf* can be written as:

$$p(\boldsymbol{\pi}, \mathbf{e} \,|\, \mathbf{d}) = \frac{p(\mathbf{d} \,|\, \mathbf{e}, \boldsymbol{\pi}) p(\mathbf{e} \,|\, \boldsymbol{\pi}) p(\boldsymbol{\pi})}{p(\mathbf{d})} = \frac{p(\mathbf{d} \,|\, \mathbf{m}) p(\mathbf{m})}{p(\mathbf{d})}, \quad (1)$$

where $\mathbf{m} = [\mathbf{e}, \boldsymbol{\pi}]$. Before the MCMC inversion, we exploit the available borehole data and/or the available geological information about the investigated area to define the $p(\boldsymbol{\pi})$ and $p(\mathbf{e}|\boldsymbol{\pi})$ distributions. In our implementation $p(\mathbf{e}|\boldsymbol{\pi})$ is a non-parametric mixture distribution that is directly derived from the available data (e.g. well log data) by means of the kernel density estimation algorithm. Then, we apply a normal score transformation to convert each non-parametric component of the prior to a Gaussian model, thus deriving the $p(\mathbf{z}|\boldsymbol{\pi})$ distribution where $\mathbf{z}$ represents the normal-score transformed elastic properties. After this transformation the conditional $p(\mathbf{z}|\boldsymbol{\pi})$ is a Gaussian mixture model from which we extract the mean vector and the covariance matrix of each component and the variogram model expressing the expected lateral or vertical variability of the elastic parameters. The transformation to a Gaussian mixture model allows for an easy inclusion of geostatistical constraints into the MCMC sampling in the form of a variogram model.

Being $\mathbf{m}$ the current model and $\mathbf{m}'$ the proposed (perturbed) model, the probability for the MCMC chain to move from $\mathbf{m}$ to $\mathbf{m}'$ can be computed from the Metropolis-Hasting rule:

$$\alpha = p(\mathbf{m}' \,|\, \mathbf{m}) = \min\left[1, \frac{p(\mathbf{m}')}{p(\mathbf{m})} \times \frac{p(\mathbf{d} \,|\, \mathbf{m}')}{p(\mathbf{d} \,|\, \mathbf{m})} \times \frac{q(\mathbf{m} \,|\, \mathbf{m}')}{q(\mathbf{m}' \,|\, \mathbf{m})}\right], \quad (2)$$

where $q()$ is the proposal distribution that defines the new model $\mathbf{m}'$ as a random deviate from a probability distribution $q(\mathbf{m}'|\mathbf{m})$ conditioned only on the current model $\mathbf{m}$. Note that the proposal ratio term in equation (2) vanishes if symmetric proposals (for example a Gaussian proposal) are employed. If $\mathbf{m}'$ is accepted $\mathbf{m} = \mathbf{m}'$ and another model is generated as a random perturbation of $\mathbf{m}$. The ensemble of accepted models after the burn-in period is used to numerically compute the posterior *pdf*.

To derive a reliable posterior model, we adopt multiple chains that start from different initial points defined on the basis of the a-priori information. To increase the computational efficiency of the algorithm we employ a parallel tempering strategy, in which multiple and interactive chains are simultaneously run at different temperature levels $T = [T_1, T_2, ..., T_{max}]$. According to stochastic criteria, swaps of models are allowed between chains at different temperatures, and in this context the high temperature chains ensure that low-temperature chains access all the high probability regions while maintaining an efficient exploitation capability. In the case of AVA inversion, we expect the posterior *pdf* having different spreads along the *Vp*, *Vs* and density directions due to the different resolvability of these parameters. For this reason, we increase the efficiency of the implemented MCMC sampling by using a delayed rejection scheme: This strategy automatically adapts the characteristics of the proposal distribution to the spread of the posterior *pdf* associated to different model parameters. Finally, we promote the mixing of the chains by including some principles coming from the Differential Evolution Markov Chain (DEMC) algorithm into our MCMC recipe. In brief our algorithm uses iterative perturbation of the facies, and of the elastic properties of the current model to sample the posterior *pdf*. A normal score transformation is used to easily include geostatistical constraint on the elastic properties, whereas a vertical transition matrix is used to constraints the facies model. A kriging interpolation is employed to preserve the a-priori vertical correlation in all the sampled models.

Results

The prior model and the vertical transition matrix are derived from actual well-log data pertaining to 5 wells investigating a gas-saturated clastic reservoir located in a shale-sand sequence. One of the two remaining wells is used as a blind test in the following inversion examples. Fig. 1 shows the a-priori non-parametric (Fig. 1a) and the Gaussian-mixture (Fig. 1b) marginal distributions for each elastic property derived for the reservoir interval. The non-parametric prior model is used by the MCMC algorithm, while the Gaussian-mixture is used by the analytical inversion approach. We note some important differences between the non-parametric and Gaussian-mixture distributions. In particular, the distributions are very similar for shale, but significatively different for brine sand and gas sand where the non-parametric distribution shows skewness or even multimodalities. Obviously, the Gaussian-mixture model does not capture these characteristics and for this reason it constitutes an oversimplified statistical model in this context. For this reason, we expect that the MCMC inversion outperforms the analytical approach. These considerations are confirmed by the normal probability plots of Fig. 1c where we observe significative deviations from the Gaussian model for *Vp* and *Vs* in the

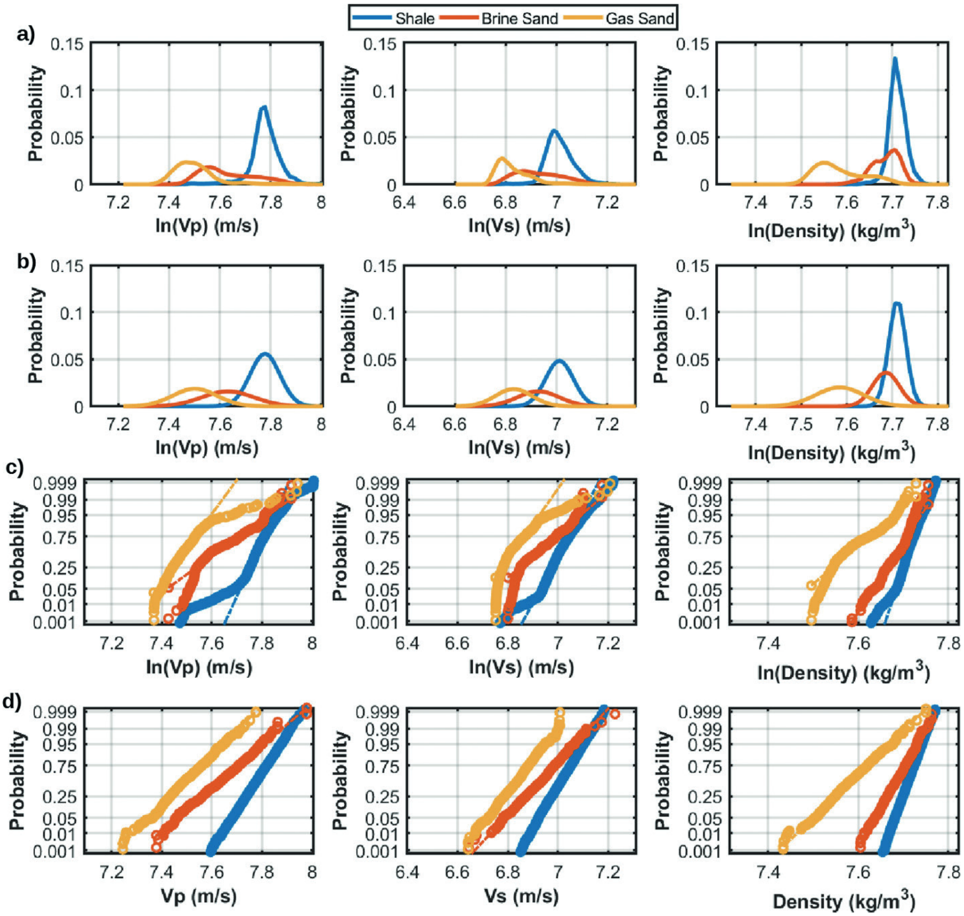

Fig. 1 - a) The a-priori non-parametric model derived through the kernel density estimation algorithm. b) The Gaussian-mixture prior model. c) Normal probability plot derived from the actual well log data. b) Normal probability plot derived on the normal score transformed actual well log data. In c) and d) the dotted lines represent the theoretical Gaussian distribution, whereas the circles represent the actual well log data.

brine sand and gas sand. These deviations disappear after the normal score transformation (Fig. 1d). In the synthetic seismic inversion example, we simulate a signal-to-noise ratio of 2 in the observed data, whereas an angle range of [0, 30] degrees and a 55-Hz Ricker wavelet are used to compute the seismic data. In the MCMC inversion we use 40 different chains running for 10000 iterations each and with a burn-in period of 5000: 20 chains run at T=1, while the remainder at logarithmically spaced temperature values. We consecutively perturb the elastic properties or the facies configuration at ten different time positions before the likelihood evaluation.

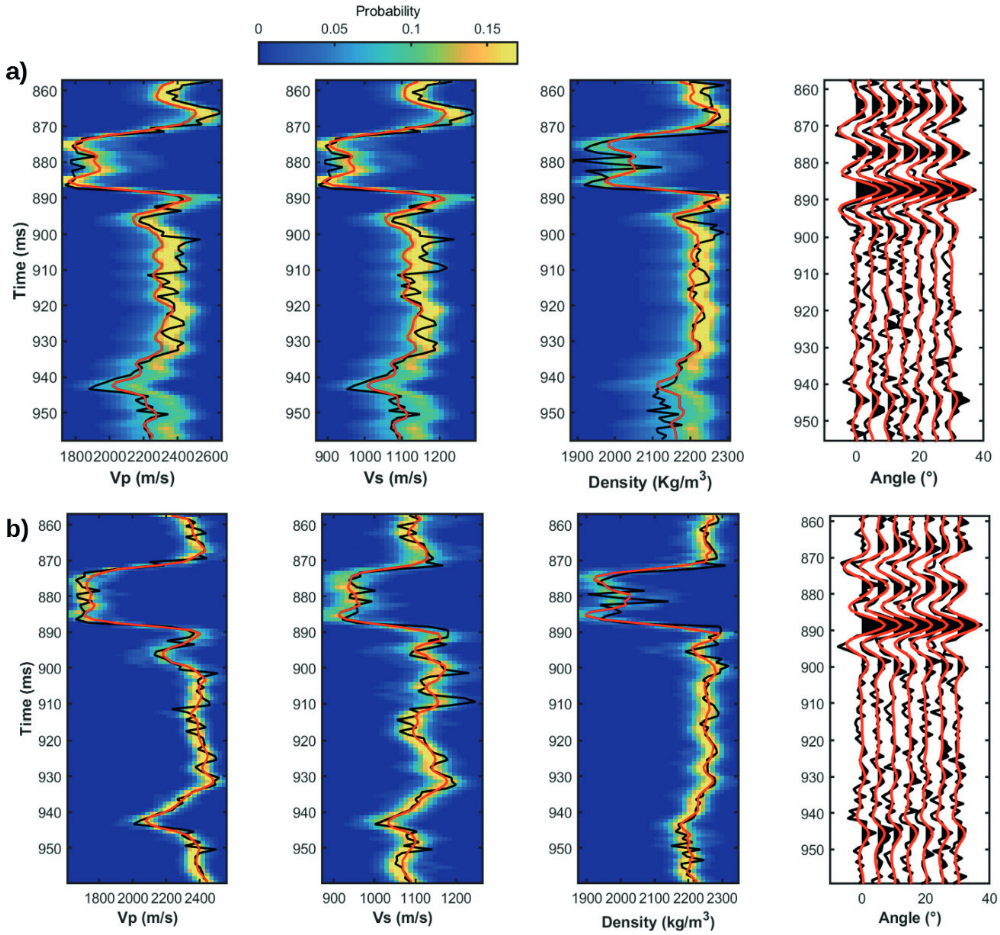

Fig. 2 - Estimated elastic properties provided by the analytical (a) and the MCMC (b) inversion for the synthetic inversion test. The black lines represent the true property values, the red lines are the estimated mean models, whereas the colormap codes the estimated posterior *pdf*. The rightmost plots show a comparison of the observed (black) and the predicted (red) seismic data computed on the estimated a-posteriori mean models.

The analytical and MCMC approach yield similar predicted elastic profiles (see Figs. 2a and 2b), although the MCMC inversion often provides slightly superior prediction intervals as demonstrated by the coverage probability values (not shown here for the lack of space). The differences between the outcomes of the two approaches can be clearly appreciated by comparing the facies classification results (Fig. 3). Indeed, just a visual inspection of the estimated facies models and the associated posterior *pdfs* confirms that the MCMC method (Fig. 3b) outperforms the analytical inversion (Fig. 3a) as it estimates a maximum-a-posteriori (MAP) facies solution with a closer match with the actual facies profile especially below 935

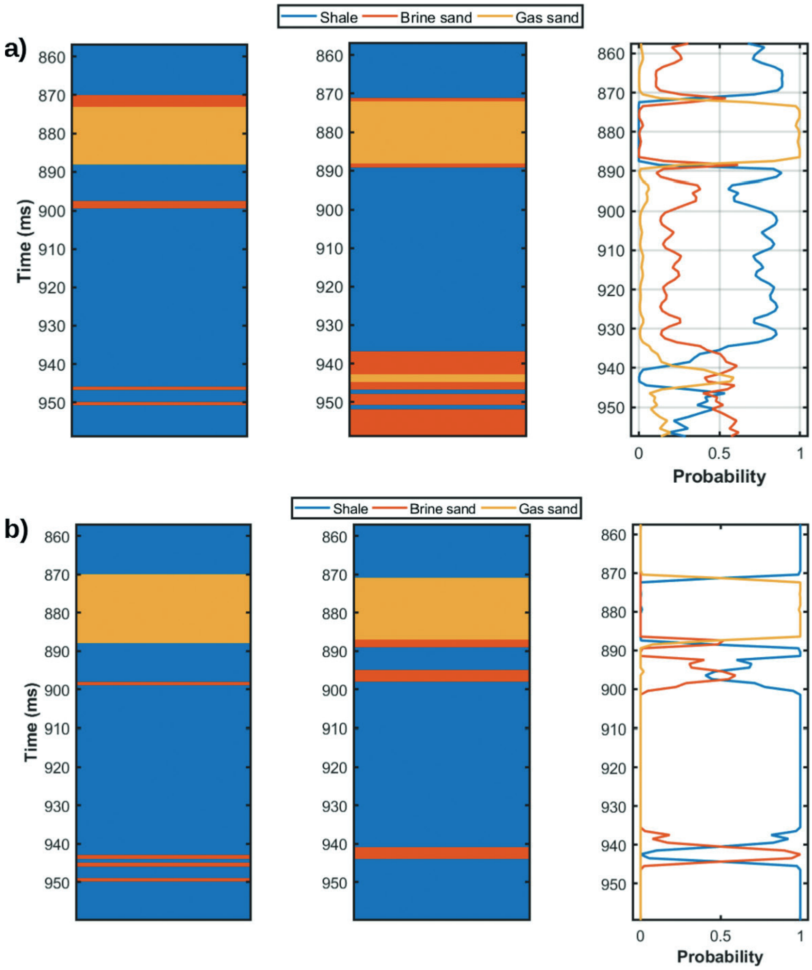

Fig. 3 - Facies classification results provided by the analytical (a) and the MCMC (b) inversion. From left to right we represent: True facies profile; MAP facies solution; Estimated posterior *pdf* of facies.

ms, where the analytical approach erroneously interprets a finely layered shale-brine sand sequence as a gas saturated layer enclosed in a thick brine sand sequence.

Conclusions. We presented a Markov Chain Monte Carlo (MCMC) inversion algorithm for elastic amplitude versus angle (AVA) inversion. The main advantage of the implemented MCMC recipe is that it is suitable for mixed discrete-continuous inverse problems, non-linear forward modellings and multimodal, non-parametric prior distributions. In other terms, our approach does not require any assumptions (i.e. Gaussianity) about the distribution of the continuous properties in a given facies. The method includes geostatistical constraints for the elastic parameters, a 1D Markov prior models for the facies distribution, and use the exact non-linear Zoeppritz equations as the forward modelling. Our implemented MCMC recipe is especially aimed at decreasing the computational effort and it includes multiple chains, a parallel tempering strategy, a delayed rejection updating scheme and hybridize the standard Metropolis-Hastings algorithm with the Differential Evolution Markov Chain method. Our inversion results and the convergence analysis (this analysis is not shown here for the lack of space) demonstrated that the implemented algorithm efficiently samples from a multimodal non-parametric mixture distribution with a reasonable computational effort. Our synthetic inversion experiments proved the importance of correctly modelling the multimodal behavior of the elastic properties to retrieve accurate predictions. Indeed, although the analytical inversion algorithm achieved satisfactory results, the non-parametric prior considered by the MCMC approaches guaranteed superior solutions and more accurate uncertainty quantifications. The MCMC inversion of field seismic data is the further step of this work.

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ANALISI CWT DI DATI GEOELETTRICI

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Introduzione. Il *metodo della resistività in corrente continua* è un metodo geoelettrico attivo che si basa sulla misura della differenza di potenziale elettrico in diversi punti della superficie del terreno, in seguito all'immissione di corrente. La ricostruzione della distribuzione delle resistività reali del sottosuolo avviene mediante il processo di inversione dei dati acquisiti, che prevede l'assunzione iniziale di un modello di resistività, anche corredato da informazioni esterne e da vincoli dettati dalla geologia locale. Il problema inverso relativo al metodo geoelettrico è di tipo non-lineare e questa è la principale caratteristica che lo rende differente dai campi di potenziale. Pérez-Flores *et al*. (2001) hanno sviluppato una semplice approssimazione lineare che svincola la soluzione dall'assumere valori di resistività di riferimento. La linearizzazione del problema ci permette di analizzare le misure del campo elettrico nello stesso modo dei campi di potenziale. Abbiamo così sviluppato un nuovo approccio metodologico per l'analisi dei dati geoelettrici, mediante l'applicazione del metodo di interpretazione *CWT* (*Continous Wavelet Transform*) alle differenze di potenziale elettrico misurate. Tale metodo è già utilizzato per l'analisi dei campi di potenziale (gravimetrico e magnetico) ed è in grado di definire profondità ed estensione areale dei corpi sepolti. Esso ha il vantaggio di prescindere dalla necessità del confronto della distribuzione del campo elettrico osservata con quella calcolata su un modello di sottosuolo ipotizzato.

Il dispositivo elettrodico scelto per la misura della differenza di potenziale (ddp) è il *dipolo-dipolo*, che risulta il più utilizzato in letteratura ed è particolarmente sensibile alle variazioni laterali di resistività. La lunghezza dei due dipoli (di corrente e di potenziale) è la stessa ed è data da a. La spaziatura invece tra i dipoli è pari ad $n*a$, aumenta secondo multipli interi di a. Il massimo valore assunto da n è 6: oltre tale soglia è difficile ottenere misure accurate del potenziale (Loke, 2004).

Metodo. L'analisi CWT applicata ad un segnale 1D, $f(x) \in L^2(R)$ attraverso una famiglia di funzioni a media nulla (wavelets) viene definita da Grossmann e Morlet (1984):

$$W(a,b) = k(a)\int_{-\infty}^{\infty} f(x)\overline{\psi}\left(\frac{b-x}{a}\right)dx = k(a)\int_{-\infty}^{\infty} f(x)\tilde{\psi}\left(\frac{x-b}{a}\right)dx,$$

dove $\tilde{\psi}(x) = \tilde{\psi}(x)$; $a \in \mathbb{R}^+$ è la scala (o dilatazione) dell'ondina ψ e $b \in \mathbb{R}$ è il parametro