Design of a Test Setup for the Characterization of the Dynamic Transfer Matrix of Cavitating Inducers

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The paper describes a reduced-order analytical model for the characterization of the dynamic transfer matrix of complex test setups including cavitating pumps. The model, even if based on several simplifying assumptions (quasi 1-dimensional flow, small oscillations, incompressible working fluid, quasi-static response of all the components of the system), is able of providing good indications about the order of magnitude of the expected pressure and flow rate oscillations in the system under given flow conditions and, more in general, about the experiment design. The model has been applied to Alta's Cavitating Pump Rotordynamic Test Facility with the custom-designed DAPAMITO3 axial inducer, in order to start the design process of an experiment for the characterization of the inducer dynamic matrix. It has been found that a good mechanism for providing an external excitation to the facility can be represented by a device able of mechanically vibrating the water tank in a vertical direction, while the most suitable way for obtaining the second linearly independent test configuration, needed for the experimental characterization of the cavitating pump dynamic matrix, is represented by a variation of the suction line inertance. Finally, it has been shown that it is possible to measure the flow rate oscillations in the suction and discharge lines by means of the difference between the measurements taken by two pressure transducers, placed at two different sections of the relevant pipe line.

Nomenclature

A	=	pipe section
C	=	compliance
g	=	gravity acceleration
i	=	imaginary unit
L	=	inertance
l	=	pipe length
M	=	mass flow gain factor
p	=	pressure
H	=	generic dynamic matrix
h	=	height of the water column in the tank
Q	=	flow rate
R	=	resistance
r_T	=	inducer tip radius
S	=	real part of the pressure gain factor

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t	=	time
T_i	=	elements of the water tank dynamic matrix, $i = A, B, C, D$
V	=	volume
V_C	=	cavity volume
W	=	characteristic matrix of the linear system for the calculation of the pump dynamic matrix
X	=	imaginary part of the pressure gain factor
Y	=	element of the water tank dynamic matrix
y	=	amplitude of the externally imposed vertical oscillations
Δp_{loss}	=	total pressure loss in a given pipeline sector
Φ	=	nondimensional flow rate oscillation
ϕ	=	flow coefficient, $\phi = Q / \pi \Omega r_T{}^3$
γ	=	specific heat ratio
Ψ	=	nondimensional pressure oscillation
ψ	=	head coefficient, $\psi = \Delta p / \rho \Omega^2 r_T{}^2$
ρ	=	water density
σ	=	cavitation number, $\sigma = 2p / \rho \Omega^2 r_T{}^2$
ω	=	frequency of oscillations
Ω	=	inducer rotating speed

Superscripts

$\hat{}$	=	oscillating component of a given quantity
$\bar{}$	=	steady component of a given quantity
$\tilde{}$	=	nondimensional form of a given quantity

Subscripts

a,b	=	linearly independent experimental conditions
D	=	discharge line
d	=	downstream section (generic)
ind	=	inducer
S	=	suction line
T	=	water tank
u	=	upstream section (generic)
1	=	section at water tank outlet
2	=	section at test pump inlet
3	=	section at test pump outlet
4	=	section at water tank inlet

I. Introduction

THE dynamic transfer matrix of a component or device is usually defined as the matrix which relates some of the fluctuating quantities (generally pressure and flow rate) at the component discharge to the same fluctuating quantities at the inlet. It is well known that many of the flow instabilities acting on space rocket engines are significantly influenced by the dynamic matrix of the propulsion system turbopumps (Tsujimoto et al.[1,2]; Kawata et al.[3]); this is particularly true in presence of cavitation, which can provide the necessary flow excitation and compliance for triggering dangerous fluid mechanic instabilities of the turbopump or even, through the coupling with thrust generation, of the entire propulsion system (POGO auto-oscillations[4]). It is therefore clear that the study of the dynamic matrix of cavitating pumps is of primary importance for rocket engineers and turbopump designers.

Conventionally, the dynamics of hydraulic systems is treated in terms of "lumped parameter models", which assume that the distributed physical effects between two measuring stations can be represented by lumped constants. This assumption is usually considered valid when the geometrical dimensions of the system are significantly shorter than the acoustic wavelength at the considered frequency. As a direct consequence of this assumption, the dynamic matrix of a generic system can be written as:

$$\begin{bmatrix} \hat{p}_d \\ \hat{Q}_d \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \hat{p}_u \\ \hat{Q}_u \end{bmatrix} \quad (1)$$

where $\hat{p}$ and $\hat{Q}$ are, respectively, the pressure and flow rate oscillating components, and the subscripts u and d denote, respectively, the flow conditions upstream and downstream of the considered system. In analogy to electrical systems, the negative of the real part of H_{12} is usually denoted as the system "resistance", the negative of the imaginary part of H_{12} is referred as the system "inertance", the negative of the imaginary part of H_{21} is the system "compliance", and the negative of the imaginary part of H_{22} is the system "mass flow gain factor". In the dynamic matrix of a passive incompressible system (as simple duct lines filled with water or another liquid), as well as in a non cavitating pump, only resistance and inertance are present; conversely, for a cavitating pump, the compressibility of the cavitation region leads to a more complicated form of the transfer matrix, also including compliance and mass flow gain factor.

The first steps in the analytical and experimental characterization of the dynamic matrix of space rocket turbopumps date back to the work of Brennen, Acosta and their collaborators in the 70s[5,6,7]. However, more recent works have given important contributions by evaluating the previously obtained results through a careful analysis of the successive experimental and numerical data (Otsuka et al.[8]; Rubin[9]). A first, quite obvious, consideration about the influence of the dynamic matrix on the flow instabilities is related to the pump resistance, which has been widely reported to play a decisive role in its unstable behaviour[3]. Considering that at a frequency of 0 Hz the resistance has the same meaning of the slope of the pump performance curve, it is easy to understand that a positive value of the resistance is directly connected to surge-mode instabilities. Furthermore, it has been shown that other flow instabilities, like rotating cavitation, are promoted by particular combinations of the pump compliance and mass flow gain factor[1]. Various experimental activities have shown that, imposing a flow oscillation to the system, its behaviour can turn from stable to unstable depending on the value of the frequency of imposed oscillations[3,10].
Brennen & Acosta[5] presented a theoretical model for the analysis of the cavitation compliance in turbopumps, based on the consideration that the POGO instabilities in space rocket propulsion systems can be significantly affected and triggered by this parameter. The model is based on a quasi-static approach and predicts that the dimensionless compliance of the pump is typically decreasing with the cavitation number; however, comparison of the model results to the experimentally measured compliance of real turbopumps did not show a good matching. The authors suggest that this discrepancy could be principally caused by the quasi-static assumption, and speculate that the problem can be attenuated by the use of a "reduced frequency" of the oscillations instead of the actual one.
An extensive experimental activity for the characterization of the transfer matrix of axial inducers was presented by Ng & Brennen[7]. In their work, the transfer matrices of two different inducers were evaluated, under cavitating and noncavitating conditions, by providing given external fluctuations through devices acting like variable, oscillating resistances. The results for a scaled model of the axial inducer of the low pressure oxidizer pump of the Space Shuttle Main Engine showed that cavitation causes changes in all the elements of the transfer matrix, and these changes were appreciable even at cavitation numbers much higher than the head breakdown value. As expectable, the most important effects of cavitation were observed in the compliance and mass flow gain factor: these parameters, in particular, were not linear with oscillations frequency, thus confirming a deviation from the quasi-static assumption, especially at higher frequencies. The pump resistance remained close to its non cavitating value at low frequencies, but it became significantly smaller at higher frequencies and eventually, for particularly high frequencies and small cavitation numbers, could even change its sign.
Otsuka et al.[8] developed an analytical model in order to better characterize the compliance and mass flow gain factor of a cavitating pump, schematized as a plane cascade with fluctuating cavity regions in the blade passages. This characterization was needed in order to confirm the findings of Tsujimoto et al.[1], who reported a direct correlation between the two above parameters and the onset of rotating cavitation in a pump. Jun et al.[11] analyzed the dynamic response characteristics of the liquid hydrogen pump of the Japanese LE-7 engine by means of a one-dimensional nonlinear compressible flow model, finding that cavitation compliance has a dominant effect on the system response frequency, while the mass flow gain factor mainly influences the system stability. It was also found that the disturbance downstream of the pump is adsorbed by the pump itself and does not affect significantly the pump upstream flow.

Rubin[9] conducted an interesting analytical review by using some of the experimental data available in the previous open literature in order to estimate the behaviour of the transfer matrix for cavitating pumps. The main results of this analysis were: the pressure gain factor (element H_{11} of the pump transfer matrix) is higher under dynamic conditions than under quasi-static conditions, differently to what believed by many POGO instability analyzers who used the quasi-static value also for dynamic calculations; the pump resistance increases with frequency and can reach over twice its quasi-static value at higher frequencies; the pump inertance decreases with frequency and seems to be independent on the cavitation level if cavitation is present.

Recently Brennen[12], using an unsteady linear perturbations analytical model, postulated the existence of a new flow instability in cavitating impellers, partly triggered by asymmetry in the pump discharge that excites a surge mode in the blade passages. Nanri et al.[13] carried out another interesting analytical investigation, showing that the cavitation surge instability acting on a given pump, which is typically one of the most important causes of the POGO oscillations in a space rocket propulsion system, can be significantly damped or eventually suppressed when the resistance of the pump inlet line is increased.

The above summary literature review clearly shows the importance of the characterization of the dynamic transfer matrix of axial and centrifugal cavitating pumps, especially if their design and geometric features are similar to those of the typical turbopumps used in space rocket applications. Recognizing the criticality of this problem, the European Space Agency has granted a Technological Research Programme contract to Alta S.p.A., aimed at the experimental characterization of the dynamic matrix of three turbopumps of space interest (two axial inducers and one centrifugal impeller). The present paper will illustrate the first part of the work carried out in the framework of the above contract and, in particular: the description of a reduced-order analytical model (based on the improvement of a previous model developed by the authors[14]) for the characterization of the transfer matrix of complex systems; the application of the model to the analysis of the existing experimental facility and one candidate test inducer; the main choices made for the preliminary design of the test stand in view of the successive experimental campaign.

II. Experimental Facility Description

A. Test Facility

The experimental activity is intended to be performed in Alta's Cavitating Pump Rotordynamic Test Facility (CPRTF, Figure 1). The CPRTF is a low-cost, versatile and instrumentable cavitation test facility, operating in water at temperatures up to 90 °C (Rapposelli et al.[15]). The facility is designed as a flexible apparatus that can readily be adapted to conduct experimental investigations on virtually any kind of fluid dynamic phenomena relevant to high performance turbopumps in a wide variety of alternative configurations (axial, radial or mixed flow, with or without an inducer). The CPRTF has been especially designed for the analysis of unsteady flow phenomena and rotordynamic impeller forces in scaled cavitation tests under fluid dynamic and thermal cavitation similarity conditions. It can also be configured as a small water tunnel to be used for thermal cavitation tests for experimental validation of numerical tools and simulations.

The test section (Figure 1, right) is equipped with a rotating dynamometer, for the measurement of the instantaneous forces and moments acting on the impeller, and with a mechanism capable of adjusting and rotating the eccentricity of the impeller axis in the range 0÷2 mm and ±3000 rpm, for rotordynamic experiments. The inlet section, made in plexiglas, is transparent in order to allow for the optical visualization of cavitation in the inducer. The water pressure at the inlet of the test section can be adjusted by means of an air bag placed inside the main water tank, while the temperature regulation is obtained by a 5 kW electrical heater. A Silent Throttle Valve is used for the variation of the pump load. Two electromagnetic flowmeters, mounted on the suction and discharge lines of the water loop, provide the measurement of the inlet and outlet flow rates. The inlet pressure is monitored by an absolute transducer mounted immediately upstream of the test section, while a differential transducer measures the pump pressure rise. Photo cameras and high-speed video cameras are used to allow for optical visualization of the cavitating flow on the test article.

Pressure fluctuations in the different points of the facility can be measured by means of flush-mounted piezoelectric pressure transducers (PCB M112A22, ICP® voltage mode-type, 0.1% class), able of detecting the oscillations amplitude with a resolution of 7 Pa.

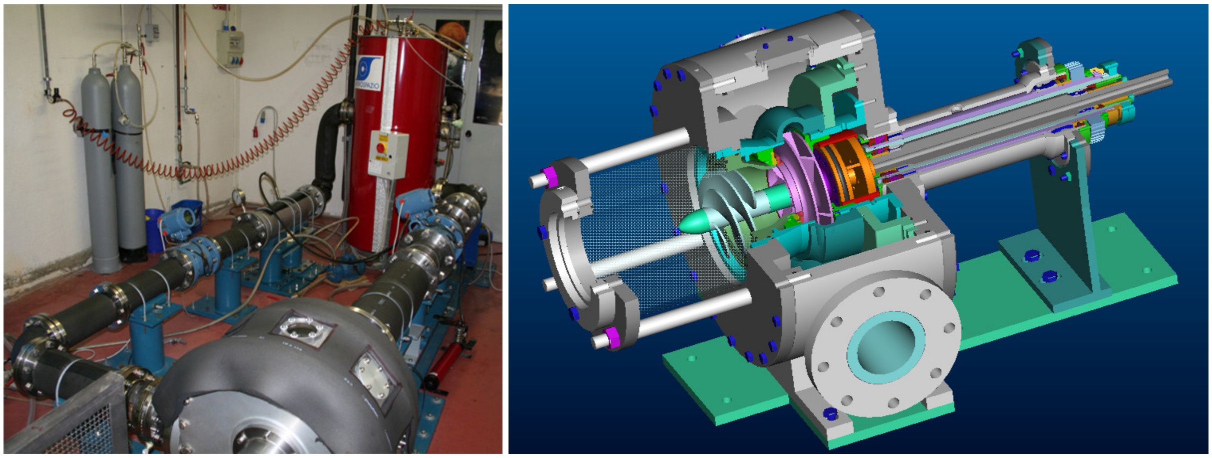


Figure 1. The Cavitating Pump Rotordynamic Test Facility (left) and cut-off drawing of the CPRTF test section (right).

A schematic of the present configuration of the facility is shown in Figure 2. The positions of the two flowmeters, the test section, the water tank and the flow control valve are shown in the Figure. The suction line has a pipe diameter of 6" and an approximate length (from the tank outlet to the pump inlet) of 2.8 m, including one 90° elbow; the discharge line, conversely, has a diameter of 4" and an approximate length of 4.8 m (from the pump outlet to the tank inlet), including three 90° elbows. This configuration has been used as a baseline for the calculations and the design considerations which will be described in the present paper.

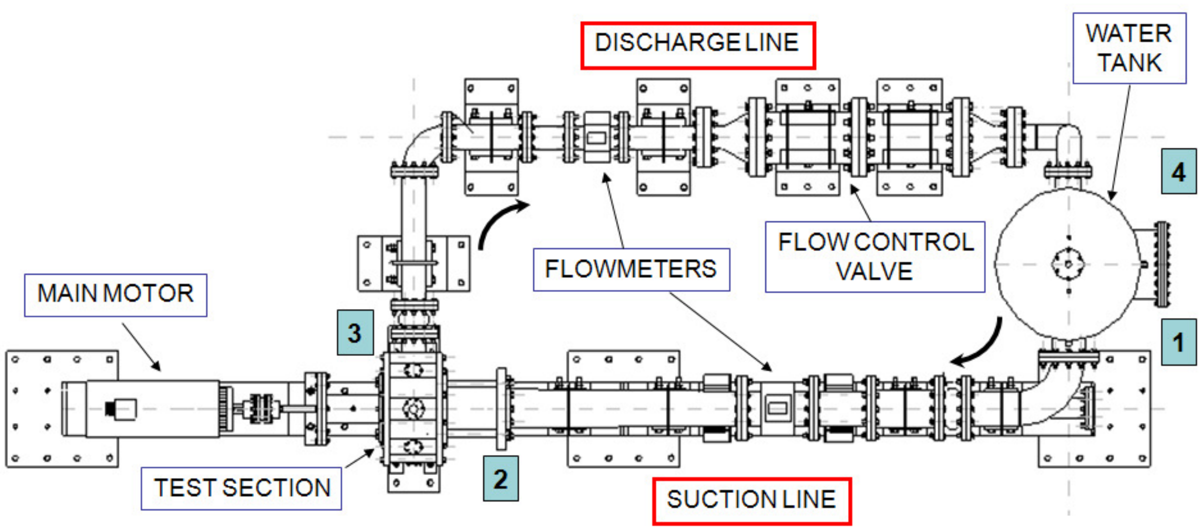
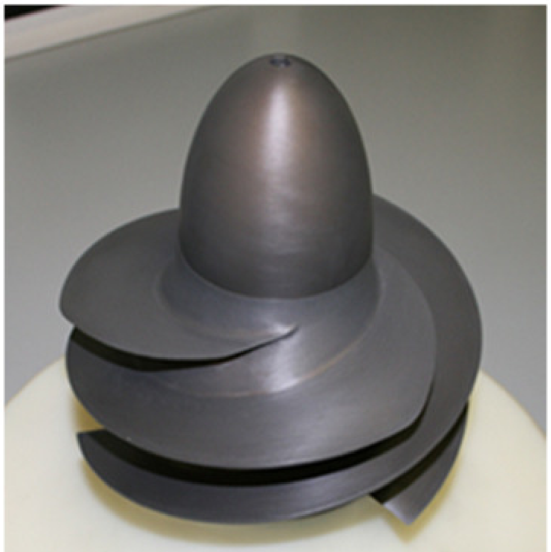


Figure 2. Top view of the Cavitating Pump Rotordynamic Test Facility (schematic).

B. Test Inducer

All the calculations presented in the following Sections are referred to a three-bladed, tapered-hub, variable-pitch axial inducer, named DAPAMITO3 (Figure 3). This inducer, whose main geometrical and operational parameters are reported in the Table on the right-hand side of Figure 3, is made in 7075-T6 aluminum alloy and has been designed by means of a reduced order model developed by Alta S.p.A. (d'Agostino et al.[16,17]) and able of reproducing blade and hub geometries similar to those typically used in space rockets turbopumps. The DAPAMITO3 inducer has been tested in the CPRTF in the framework of a previous test campaign (Torre et al.[18]), and the corresponding performance data have been used as inputs for the present calculations.



Design flow coefficient	[--]	0.059
Number of blades	[--]	3
Tip radius	mm	81.0
Inlet tip blade angle	deg	83.10
Inlet hub radius (fully-developed blade)	mm	44.5
Outlet hub radius	mm	58.5
Mean blade height	mm	29.5
Axial length (fully-developed blade)	mm	63.5
Inlet hub radius	mm	35.0
Axial length	mm	90.0
Ratio between tip incidence and blade angles	[--]	0.3
Tip solidity	[--]	2.03
Incidence tip angle @ design	deg	2.07
Outlet tip blade angle	deg	74.58

Figure 3. The DAPAMITO3 inducer (left) and its main geometrical and operational parameters (right).

III. Analytical Model

A. Introduction

As a first step for the design activity, a reduced-order model for the characterization of the pressure and flow rate oscillations in a given experimental facility has been developed, starting from the baseline characteristics of the previous model described in Cervone et al.[14]. The proposed model is based on the following initial assumptions:

- The flow is assumed unsteady, quasi 1-dimensional;
- All oscillations are assumed to be small (only 1st order terms are taken into account in the equations);
- The working fluid is assumed incompressible and its compliance is therefore considered negligible (as well as the compliance of pipelines and metallic components), except for regions where cavitation or air volumes are present;
- The response of all components of the system is assumed quasi-static. Even if it has been widely shown that this assumption is not valid in real pumps under cavitating conditions[5,7,8,9], it still represents a good approximation of their actual behaviour at lower values of the oscillation frequency. At the present first-order level of analysis, with the main task of providing a rough order of magnitude of the fluctuation levels in the facility components, a careful trade-off between simplicity of the equations and accuracy of the results has been carried out and led to the acceptance of the quasi-staticity assumption.

Under the above assumptions, the pressure and flow rate in a given point of the test facility can be written in complex form, as functions of time, as follows:

$$\begin{aligned} p(t) &= \overline{p} + \hat{p} \cdot e^{-i\omega t} \\ Q(t) &= \overline{Q} + \hat{Q} \cdot e^{-i\omega t} \end{aligned} \qquad (2)$$

where $\overline{p}$ and $\overline{Q}$ (usually real) are the pressure and flow rate steady values, $\widehat{p}$ and $\widehat{Q}$ (usually complex) are the pressure and flow rate oscillating components, ω is the frequency of the oscillations.
As pointed out before, the relevant components for the dynamic matrix are the oscillating ones; the set of equations used for the evaluation of the matrices of the components of the facility are presented in the following Sections. All the dynamic matrices are calculated by considering the nondimensional forms of the pressure and flow rate oscillations, $\hat{\Psi}$ and $\hat{\Phi}$, defined in the following way:

$$\begin{aligned} \hat{\Psi} &= \frac{\widehat{p}}{\rho \Omega^2 {r_T}^2} \\ \hat{\Phi} &= \frac{\widehat{Q}}{\pi \Omega {r_T}^3} \end{aligned} \tag{3}$$

where ρ is the water density, Ω is the inducer rotational speed and r_T is the inducer tip radius.
In the calculations presented in this paper, the mechanism for providing an external excitation to the facility has been assumed to be a device able of mechanically vibrating the water tank, in a vertical direction, with given frequency and amplitude of the oscillations. With respect to the more common solution consisting in the imposition of an external flow rate fluctuation at a given point of the facility[3,7], this choice has the clear advantage of simultaneously provide the same oscillations to both the suction and the discharge lines; moreover, the applicable level of oscillations is expected to be significantly higher with respect to the one obtainable by the direct imposition of flow fluctuations, typically very low and difficult to be measured especially at low frequencies.

B. Pipe Lines
Starting from the generic mass continuity and momentum equations and after simple manipulations, the dynamic matrix of a pipe line can be written in nondimensional form as follows:

$$\tilde{H} = \begin{bmatrix} 1 & -\tilde{R} - i\omega\tilde{L} \\ 0 & 1 \end{bmatrix} \tag{4}$$

where the resistance $\tilde{R}$ and the inertance $\tilde{L}$ are respectively equal to:

$$\begin{aligned} \tilde{R} &= \frac{2\Delta p_{loss}}{\overline{\phi} \cdot \rho \Omega^2 {r_T}^2} \\ \tilde{L} &= -\frac{\pi r_T}{\Omega} \frac{l}{A} \end{aligned} \tag{5}$$

in which $\overline{\phi}$ is the mean value of the flow coefficient at the given test conditions, A is the pipe section, l is the pipe length, and Δp_{loss} is the total pressure loss inside the pipe.

C. Water Tank
By suitable application of the mass continuity and momentum equations inside the tank, as well as the equations for the transformation of the tank air-bag volume (assumed isentropic), it is possible to find the following relationships between the nondimensional pressure and flow rate oscillations upstream and downstream of the tank:

$$\begin{aligned} \hat{\Psi}_d &= \tilde{Y}_T + \hat{\Phi}_u \left(\frac{i}{\omega}\tilde{T}_A - i\omega\tilde{T}_B \right) + \hat{\Phi}_d \left(-\frac{i}{\omega}\tilde{T}_A + i\omega\tilde{T}_B - \tilde{T}_D \right) \\ \hat{\Psi}_u &= \tilde{Y}_T + \hat{\Phi}_u \left(\frac{i}{\omega}\tilde{T}_A - i\omega\tilde{T}_B - \tilde{T}_C \right) + \hat{\Phi}_d \left(-\frac{i}{\omega}\tilde{T}_A + i\omega\tilde{T}_B \right) \end{aligned} \tag{6}$$

where the quantities $\tilde{Y}_T$, $\tilde{T}_A$, $\tilde{T}_B$, $\tilde{T}_C$ and $\tilde{T}_D$ are defined as follows:

$$\tilde{Y}_T = \frac{\hat{y}_T\left(g-\omega^2\bar{h}\right)}{\Omega^2 r_T^{\,2}}$$
$$\tilde{T}_A = \frac{\pi r_T}{\rho\Omega}\left(\frac{\rho g}{A_T}+\frac{1}{C_T}\right)$$
$$\tilde{T}_B = \frac{\pi r_T \bar{h}}{\Omega A_T} \qquad (7)$$
$$\tilde{T}_C = \frac{\pi^2 r_T^{\,4}\bar{\phi}}{A_u^{\,2}}$$
$$\tilde{T}_D = \frac{\pi^2 r_T^{\,4}\bar{\phi}}{A_d^{\,2}}$$

in which $\hat{y}_T$ is the amplitude of the vertical oscillations externally imposed to the tank, $\bar{h}$ is the mean value of the water column height inside the tank, g is the gravity acceleration, A_T is the tank cross section, and C_T is the tank air-bag compliance, defined as:

$$C_T = \frac{\bar{V}_T}{\gamma \bar{p}_T} \qquad (8)$$

where γ is the specific heat ratio of air, $\bar{V}_T$ and $\bar{p}_T$ are, respectively, the mean air-bag volume and pressure.

D. Cavitating Inducer

By elaborating the continuity and momentum equations for the case of a cavitating inducer, it can be shown that the corresponding dynamic matrix is:

$$\tilde{H} = \begin{bmatrix} 1-(\tilde{S}+i\omega\tilde{X}) & -\tilde{R}-i\omega\tilde{L} \\ -i\omega\tilde{C} & 1-i\omega\tilde{M} \end{bmatrix} \qquad (9)$$

The nondimensional elements of the above dynamic matrix can be defined as follows: $\tilde{S}+i\omega\tilde{X}$ is the pressure gain factor, $\tilde{R}+i\omega\tilde{L}$ is the pump impedance, $\tilde{C}$ is the cavitation compliance and $\tilde{M}$ is the mass flow gain factor. These parameters are calculated by means of the following relationships:

$$\tilde{S}+i\omega\tilde{X} = -2\left.\frac{\partial\psi}{\partial\sigma}\right|_{\bar{\phi}} - \frac{2L_{ind}}{\rho\Omega^2 r_T^{\,2}}\left.\frac{\partial V_C}{\partial\sigma}\right|_{\bar{\phi}}\omega^2 + \left(\frac{2}{\pi\Omega r_T^{\,3}}\left.\frac{\partial\psi}{\partial\phi}\right|_{\bar{\sigma}}\left.\frac{\partial V_C}{\partial\sigma}\right|_{\bar{\phi}}\right)i\omega$$
$$\tilde{R}+i\omega\tilde{L} = -\left.\frac{\partial\psi}{\partial\phi}\right|_{\bar{\sigma}} - \frac{L_{ind}}{\rho\Omega^2 r_T^{\,2}}\left.\frac{\partial V_C}{\partial\phi}\right|_{\bar{\sigma}}\omega^2 + \frac{1}{\pi\Omega r_T^{\,3}}\left.\frac{\partial V_C}{\partial\phi}\right|_{\bar{\sigma}}\left.\frac{\partial\psi}{\partial\phi}\right|_{\bar{\sigma}} i\omega - \frac{\pi r_T L_{ind}}{\rho\Omega} i\omega \qquad (10)$$
$$\tilde{C} = \frac{2}{\pi\Omega r_T^{\,3}}\left.\frac{\partial V_C}{\partial\sigma}\right|_{\bar{\phi}}$$
$$\tilde{M} = \frac{1}{\pi\Omega r_T^{\,3}}\left.\frac{\partial V_C}{\partial\phi}\right|_{\bar{\sigma}}$$

in which ϕ, ψ and σ are respectively the flow coefficient, the head coefficient and the cavitation number at the given flow conditions, V_C is the cavity volume and L_{ind} is the inertance of the inducer blade passages.
The driving parameter in the above equations is the presence of a vapor region between the inducer blades, the oscillations of which generate a shift between the inlet and outlet flow oscillations. The evaluation of the volume of cavity formed inside the pump, as a function of the flow coefficient ϕ and the cavitation number σ, is therefore needed. For the present calculations, the values of $\frac{\partial V_C}{\partial\sigma}$ and $\frac{\partial V_C}{\partial\phi}$ have been obtained by suitably generalizing to the case of the DAPAMITO3 inducer the experimental data presented by Brennen[19] for a 10.2 cm diameter axial

inducer; the corresponding estimated values are shown, as functions of the cavitation number, in Figure 4. The complete elements of the dynamic transfer matrix of the DAPAMITO3 inducer, as estimated by Eqs. (9) and (10), are presented in Figure 5 as functions of the ω/Ω ratio, for the design flow coefficient and a cavitation number $\sigma = 0.1$.

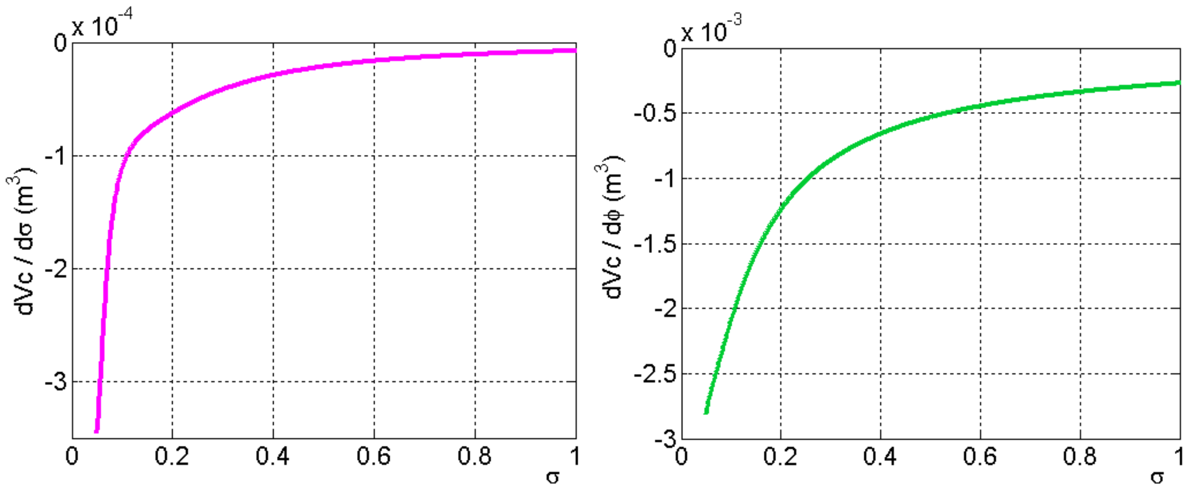


Figure 4. Estimated derivative of the cavitating volume with respect to the cavitation number (left) and the flow coefficient (right) for the DAPAMITO3 inducer, as functions of σ, at $\phi = 0.059$ and $\Omega = 3000$ rpm.

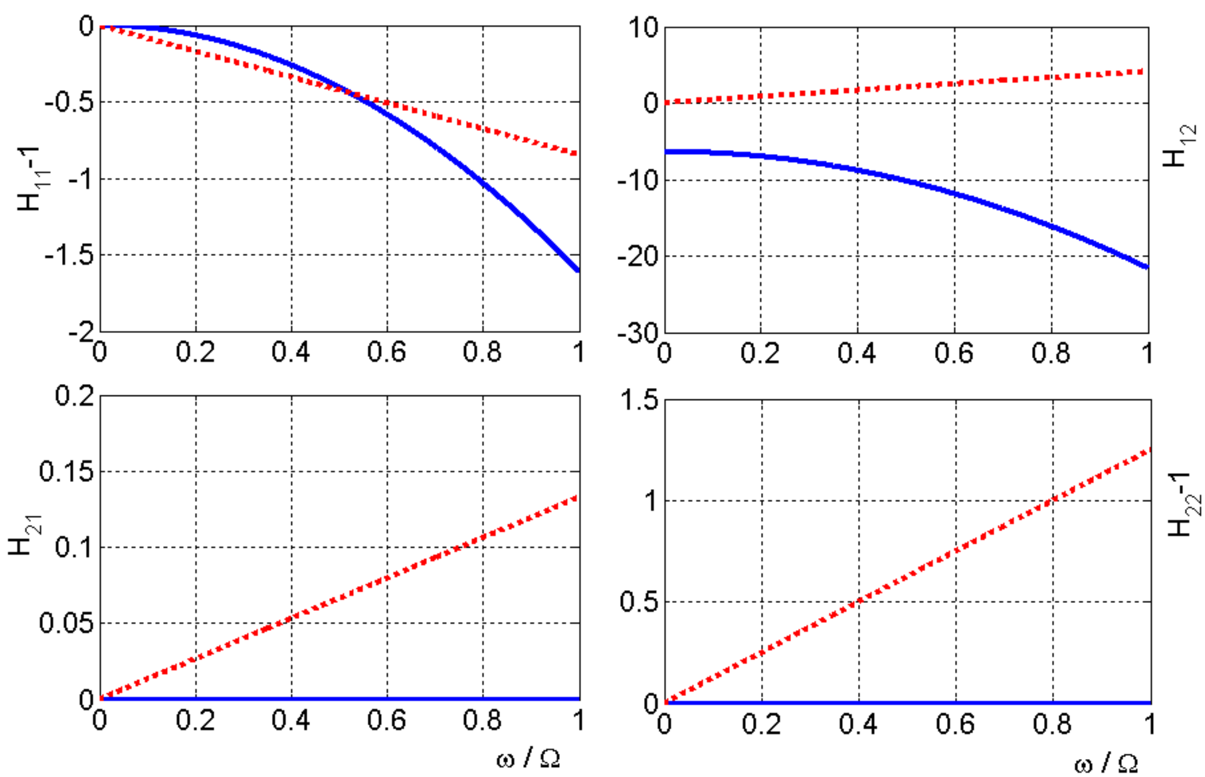


Figure 5. Estimated dynamic matrix of the DAPAMITO3 inducer, for $\phi = 0.059$ and $\sigma = 0.1$. Dynamic matrix components are shown as functions of the ω/Ω ratio (solid blue lines: real parts; dotted red lines: imaginary parts).

E. Characterization of the Complete Facility

The test facility can be easily schematized by considering four significant points of it (indicated by light blue boxes in Figure 2), namely: the water tank outlet, denoted by the subscript 1; the test pump inlet (subscript 2); the test pump outlet (subscript 3); the water tank inlet (subscript 4). With these notations, the suction line is included between sections 1 and 2, while the discharge line is included between sections 3 and 4; the measurements of the pump fluctuating quantities are therefore expected to be taken at sections 2 and 3.
By using the above defined relationships for all the components of the facility, the following set of equations can be written for defining the nondimensional fluctuating quantities (pressure and flow rate) at each one of the four relevant sections:

$$\begin{cases} \hat{\Psi}_2 = \hat{\Psi}_1 + \left(-\tilde{R}_S - i\omega\tilde{L}_S\right)\hat{\Phi}_1 \\ \hat{\Phi}_2 = \hat{\Phi}_1 \\ \hat{\Psi}_3 = \left[1-(\tilde{S}+i\omega\tilde{X})\right]\hat{\Psi}_2 + \left(-\tilde{R}-i\omega\tilde{L}\right)\hat{\Phi}_2 \\ \hat{\Phi}_3 = -i\omega\tilde{C}\hat{\Psi}_2 + \left(1-i\omega\tilde{M}\right)\hat{\Phi}_2 \\ \hat{\Psi}_4 = \hat{\Psi}_3 + \left(-\tilde{R}_D - i\omega\tilde{L}_D\right)\hat{\Phi}_3 \\ \hat{\Phi}_4 = \hat{\Phi}_3 \\ \hat{\Psi}_1 = \tilde{Y}_T + \hat{\Phi}_4\left(\frac{i}{\omega}\tilde{T}_A - i\omega\tilde{T}_B\right) + \hat{\Phi}_1\left(-\frac{i}{\omega}\tilde{T}_A + i\omega\tilde{T}_B - \tilde{T}_D\right) \\ \hat{\Psi}_4 = \tilde{Y}_T + \hat{\Phi}_4\left(\frac{i}{\omega}\tilde{T}_A - i\omega\tilde{T}_B - \tilde{T}_C\right) + \hat{\Phi}_1\left(-\frac{i}{\omega}\tilde{T}_A + i\omega\tilde{T}_B\right) \end{cases} \quad (11)$$

where the resistance and the inertance of the suction line are denoted by R_S and L_S, while the resistance and inertance of the discharge line are denoted by R_D and L_D.
Equations (11) represent a set of 8 equations in 8 unknowns, which can be easily solved to find the fluctuating quantities for a particular combination of facility design and test pump, given the amplitude and frequency of the externally imposed oscillations.

In actual tests, in which four fluctuating quantities are measured (pressure and flow oscillations at pump inlet and pump outlet), at least two experimental results obtained under linearly independent conditions are needed to find the four unknown elements of the pump dynamic transfer matrix. Indicating by subscripts *a* and *b* the two experimental results used for the calculation, it is possible to write:

$$\begin{bmatrix} \hat{\Psi}_{3a} \\ \hat{\Phi}_{3a} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \hat{\Psi}_{2a} \\ \hat{\Phi}_{2a} \end{bmatrix}$$
$$\begin{bmatrix} \hat{\Psi}_{3b} \\ \hat{\Phi}_{3b} \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \hat{\Psi}_{2b} \\ \hat{\Phi}_{2b} \end{bmatrix} \quad (12)$$

and the following system of four equations in four unknowns is obtained:

$$\underbrace{\begin{bmatrix} \hat{\Psi}_{2a} & \hat{\Phi}_{2a} & 0 & 0 \\ \hat{\Psi}_{2b} & \hat{\Phi}_{2b} & 0 & 0 \\ 0 & 0 & \hat{\Psi}_{2a} & \hat{\Phi}_{2a} \\ 0 & 0 & \hat{\Psi}_{2b} & \hat{\Phi}_{2b} \end{bmatrix}}_{W} \begin{bmatrix} H_{11} \\ H_{12} \\ H_{21} \\ H_{22} \end{bmatrix} = \begin{bmatrix} \hat{\Psi}_{3a} \\ \hat{\Psi}_{3b} \\ \hat{\Phi}_{3a} \\ \hat{\Phi}_{3b} \end{bmatrix} \quad (13)$$

It is clear that, for a better accuracy in the evaluation of the elements of the pump dynamic matrix H, the determinant of the matrix denoted as W in Eq. (13) should be as far as possible from zero or, alternatively, the condition number of W should be as small as possible. This is obtained by taking two experimental results under sufficiently independent conditions, as it will be shown in the following Section.

IV. Results and Discussion

A. Pressure and Flow Rate Oscillations

The amplitude and phase of the pressure and flow rate oscillations at pump inlet and outlet, evaluated by applying Eqs. (11) to the present configuration of the CPRTF facility with the DAPAMITO3 inducer, are shown in the Figures 6, 7, 8 and 9. The results presented in the Figures have been obtained for a pump rotational speed of 3000 rpm and an amplitude of the externally imposed vertical vibration of the water tank equal to 1 mm. The phase of oscillations is assumed to be zero when they have the same phase of the tank vertical vibration.

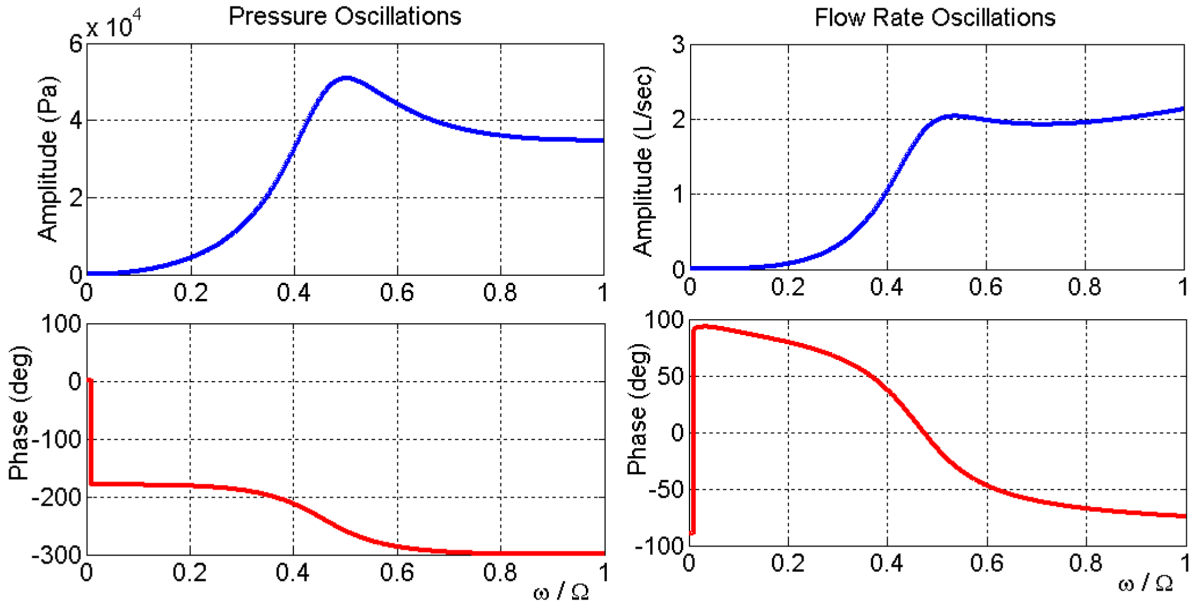


Figure 6. Estimated pressure and flow rate oscillations at the pump inlet, as functions of ω/Ω, for the CPRTF with the DAPAMITO3 inducer (Ω = 3000 rpm, ϕ = 0.059, σ = 0.1, $\hat{y}_T$ = 1 mm).

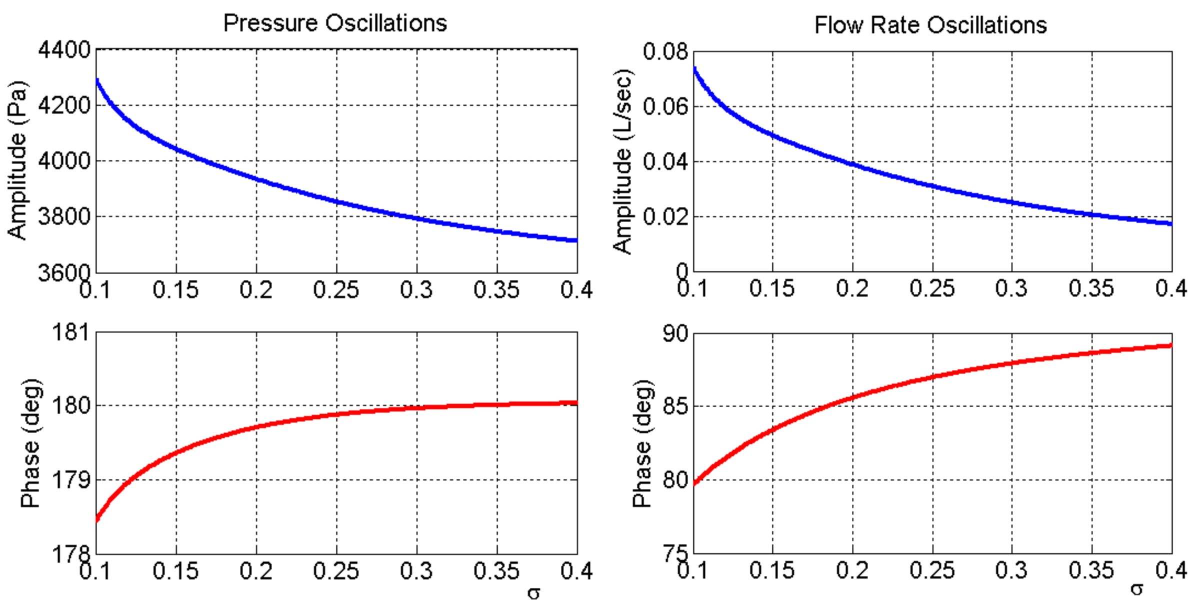


Figure 7. Estimated pressure and flow rate oscillations at the pump inlet, as functions of σ, for the CPRTF with the DAPAMITO3 inducer (Ω = 3000 rpm, ϕ = 0.059, ω/Ω = 0.2, $\hat{y}_T$ = 1 mm).

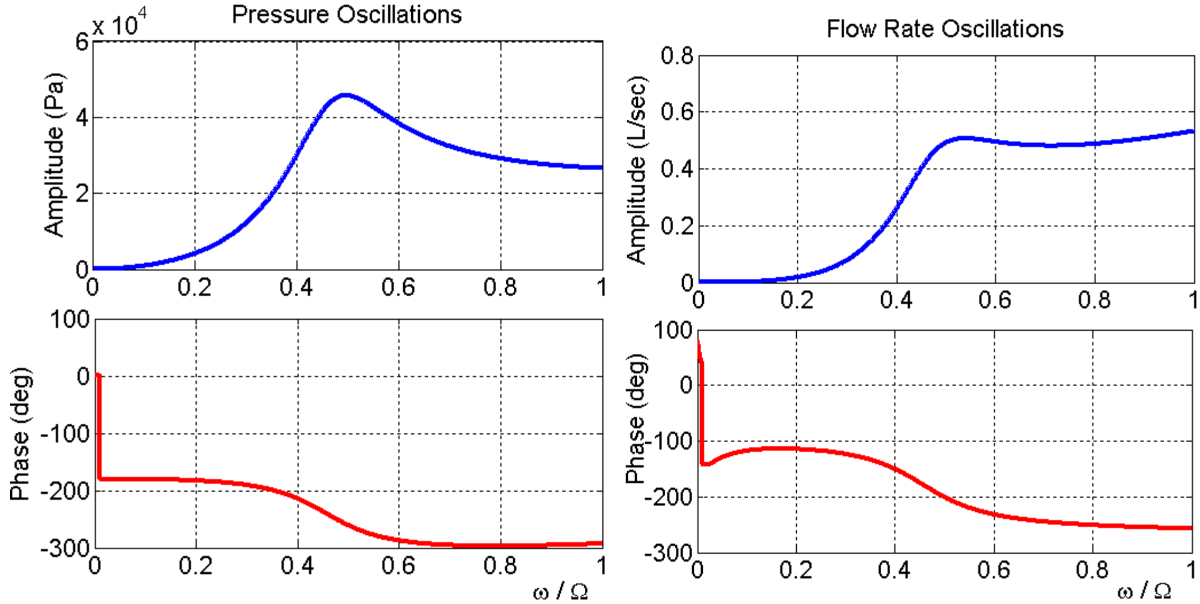


Figure 8. Estimated pressure and flow rate oscillations at the pump outlet, as functions of ω/Ω, for the CPRTF with the DAPAMITO3 inducer (Ω = 3000 rpm, ϕ = 0.059, σ = 0.1, $\hat{y}_T$ = 1 mm).

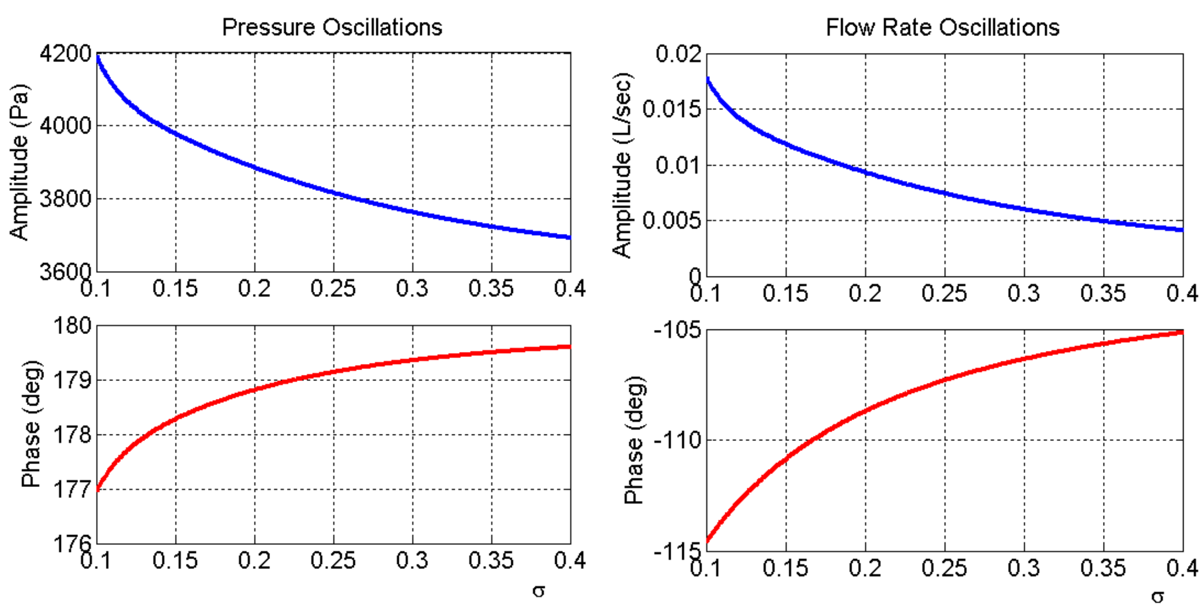


Figure 9. Estimated pressure and flow rate oscillations at the pump outlet, as functions of σ, for the CPRTF with the DAPAMITO3 inducer (Ω = 3000 rpm, ϕ = 0.059, ω/Ω = 0.2, $\hat{y}_T$ = 1 mm).

The results seem to show that a peak of the oscillations amplitude occurs for a value of ω/Ω equal to about 0.5, which for the considered case (Ω = 3000 rpm) corresponds to a nominal frequency of the facility equal to about 25 Hz. The amplitude of the pressure oscillations is well above 1000 Pa, and therefore easily measurable by the piezoelectric transducers presently installed in the facility in all the examined cases, except for lower values of the oscillation frequency ($\omega/\Omega < 0.1$).

B. Measurement of the Flow Rate Oscillations

As previously described, the flow rate in the suction and discharge lines of the CPRTF is presently measured by means of two electromagnetic flowmeters. These instruments, even if able of providing a good measurement accuracy in terms of the mean flow rate, are characterized by a very poor frequency response and are therefore not suitable for the measurement of the flow rate oscillations. One possible way for overcoming this drawback is represented by the use of a couple of pressure transducers installed in two different sections of the pipeline for which the flow rate oscillation needs to be measured. Subsequently, by knowing the resistance and inertance of the pipe section between the two pressure transducers (easily measurable and/or valuable), it is possible to convert the two measured pressure fluctuations into the corresponding flow rate oscillation in the pipeline.

As an example, Figures 10 and 11 show the estimated amplitude difference between the pressure oscillations measured by two transducers placed at a distance of 1 m in the suction line and at the same distance in the discharge line of the CPRTF with the DAPAMITO3 inducer, for a pump rotational speed of 3000 rpm and an amplitude of the externally imposed vertical vibration of the water tank equal to 1 mm.

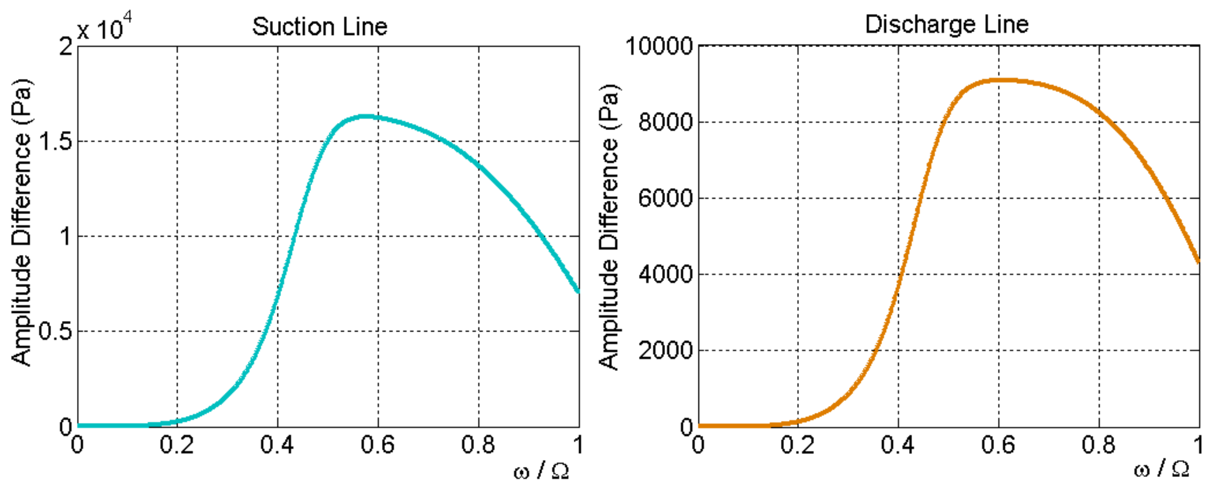


Figure 10. Amplitude difference between the pressure oscillations measured by two transducers placed at a distance of 1 m in the suction line (left) and in the discharge line (right), as functions of ω/Ω, for the CPRTF with the DAPAMITO3 inducer (Ω = 3000 rpm, ϕ = 0.059, σ = 0.1, $\hat{y}_T$ = 1 mm).

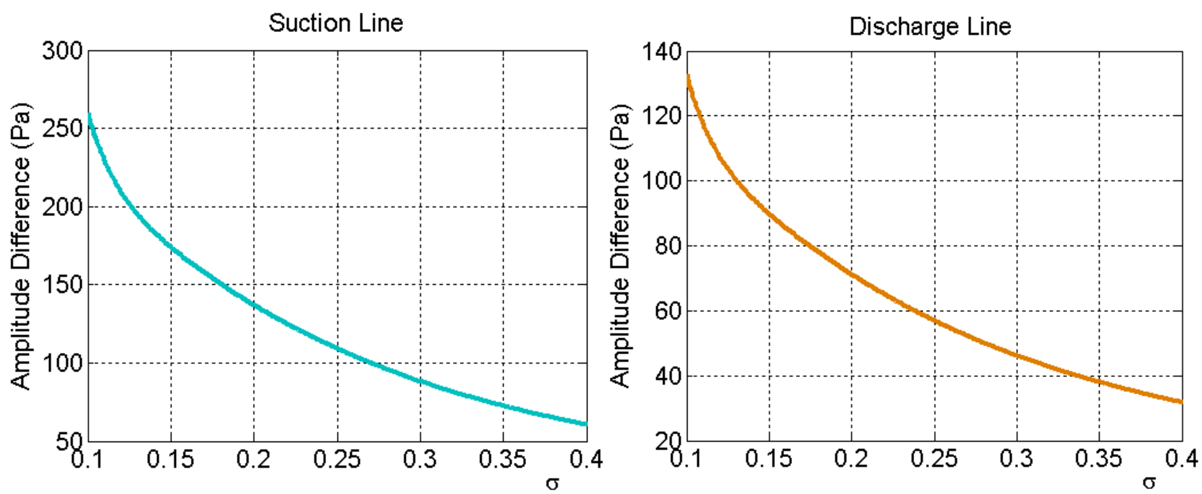


Figure 11. Amplitude difference between the pressure oscillations measured by two transducers placed at a distance of 1 m in the suction line (left) and in the discharge line (right), as functions of σ, for the CPRTF with the DAPAMITO3 inducer (Ω = 3000 rpm, ϕ = 0.059, ω/Ω = 0.2, $\hat{y}_T$ = 1 mm).

It is possible to observe that the amplitude difference between the pressure oscillations measured by the two transducers is slightly higher in the suction line than in the discharge line; therefore, the two transducers could in principle be installed at a closer distance in the suction line. The estimated amplitude difference is higher than 100 Pa, and therefore measurable with a good accuracy by the available piezoelectric pressure transducers, for values of ω/Ω higher than 0.15. If lower values of the oscillation frequency need to be investigated, two possible solutions are available: increasing the distance between the two pressure transducers or increasing the amplitude of the externally imposed vertical vibration of the water tank (the amplitude difference is approximately doubled when the vertical vibration amplitude is doubled).

C. Possible Facility Modifications for the Second Linearly Independent Test Condition

As pointed out in the previous Section, the most important design choice for the implementation of a test setup able of measuring the dynamic matrix of cavitating pumps is represented by the second linearly independent test condition needed for the definition of the complete set of matrix components. Given a particular design choice, its figure of merit to this respect can be represented by the condition number of the characteristic matrix W, as defined in Eq. (13), which needs to be as small as possible for a better accuracy in the evaluation of the dynamic matrix.
Two different design options for the second linearly independent condition will be analyzed here:

Option 1. The first test configuration is represented by the original CPRTF facility setup (Figure 2), while the second test configuration is obtained by simply moving the flow control valve from the discharge line to the suction line. The pressure losses provided by the valve and needed for adjusting the desired operational point of the test pump are therefore entirely moved to the suction line. Inertances of the suction and discharge lines remain unchanged in this option.

Option 2. The first configuration is still represented by the original facility setup, while the second configuration is obtained, in this case, by changing the inertance of the suction line in a similar manner to what shown by Figure 12. A pipeline having a total length of about 3.8 m (highlighted in red in the Figure) is added to the suction line, consequently moving the position of the water tank outlet section. The inertance of the suction line is therefore significantly increased; the suction line resistance is also slightly increased, as a consequence of the longer pipe line and the presence of two additional 90° elbows.

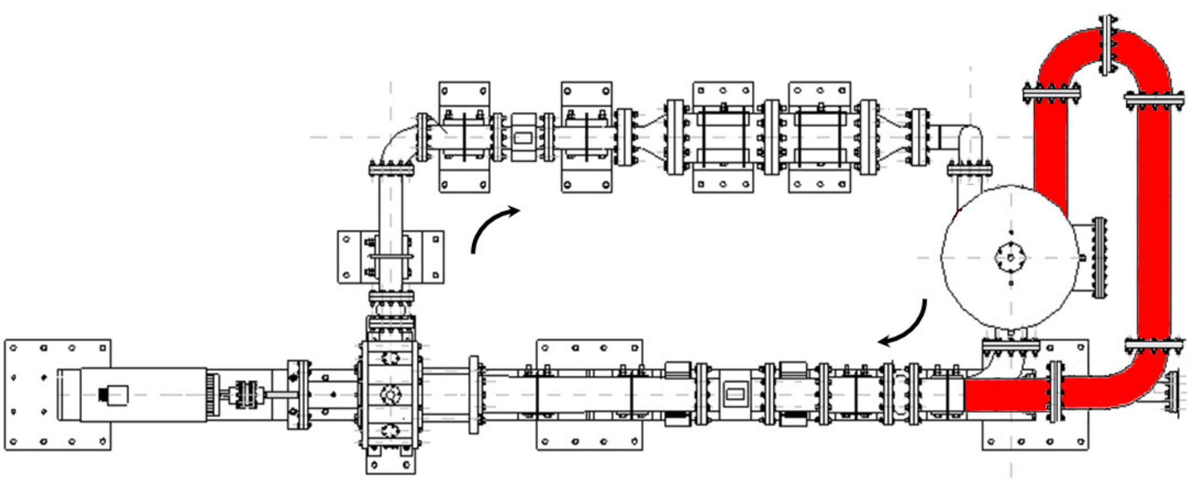


Figure 12. Suggested modification to the original facility setup for obtaining the second linearly independent test configuration (added pipe lines are coloured in red).

Figures 13 and 14 report the ratio of the condition number of the matrix W for Option 1 to the same quantity for Option 2, as a function respectively of ω/Ω and σ, at a pump rotational speed of 3000 rpm and an amplitude of the externally imposed vertical vibration of the water tank equal to 1 mm. It is clearly shown that the ratio of the condition numbers ranges between 2 and 15 in all the cases of interest. This confirms some of the findings of the previously published paper by Cervone et al.[14] and, more in detail:

– For an accurate characterization of the dynamic matrix of a cavitating pump, tested in a facility with characteristics similar to the CPRTF, a change of the suction line inertance is significantly more effective than a change of its resistance;

– The effectiveness of a change of the suction line inertance typically increases with the frequency of oscillation, due to the fact that the imaginary part of a pipe line impedance is linearly dependent on ω as shown by Eq. (4).

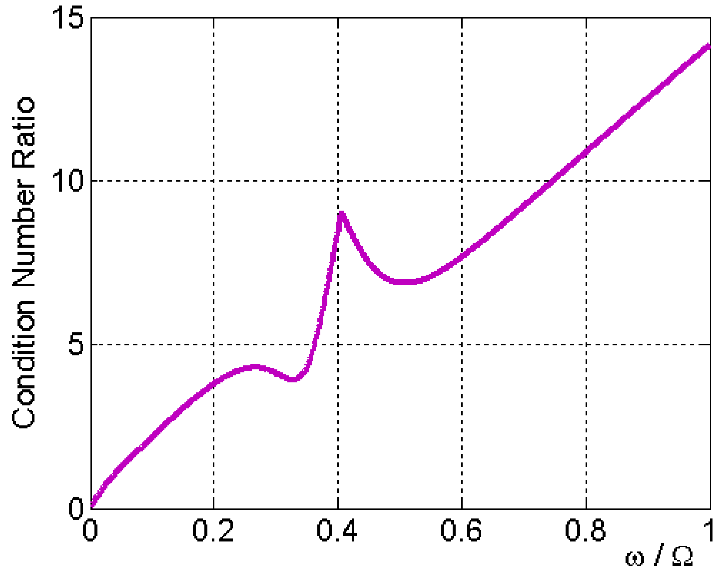


Figure 13. Ratio of the condition number of the matrix *W* for design Option 1 to the same quantity for Option 2, as a function of ω/Ω, for the CPRTF with the DAPAMITO3 inducer (Ω = 3000 rpm, ϕ = 0.059, σ = 0.1, $\hat{y}_T$ = 1 mm).

V. Conclusions

A reduced-order analytical model has been developed for the characterization of the dynamic transfer matrix of complete test setups, as well as the pressure and flow rate oscillations at each point of the facility under a given externally imposed fluctuation, as functions of the operational conditions.

The model, even if extremely simple and based on a significant number of assumptions and approximations, has proven to be able of providing important indications for the experiment design. In particular, the model has been applied to the specific case of Alta's CPRTF facility with the DAPAMITO3 axial inducer, showing that the expected amplitude of oscillations at pump inlet and outlet should be easily measurable by means of the already available piezoelectric pressure transducers. Furthermore, the following design considerations have been drawn by the results of the analysis:

– The best solution for obtaining the second linearly independent test configuration, needed for the experimental characterization of the cavitating pump dynamic matrix, is represented by a variation of the suction line inertance, which shows to be particularly effective at higher frequencies of oscillation.

– The mechanism for providing an external excitation to the facility can be represented by a device able of mechanically vibrating the water tank, in a vertical direction, with given frequency and amplitude of the oscillations. This solution has been preferred to the more common one, consisting in the imposition of an external flow rate fluctuation at a given point of the facility, due to the possibility of simultaneously provide the same oscillations to both the suction and the discharge lines.

– The flow rate oscillations in the suction and discharge lines can be measured by means of the difference

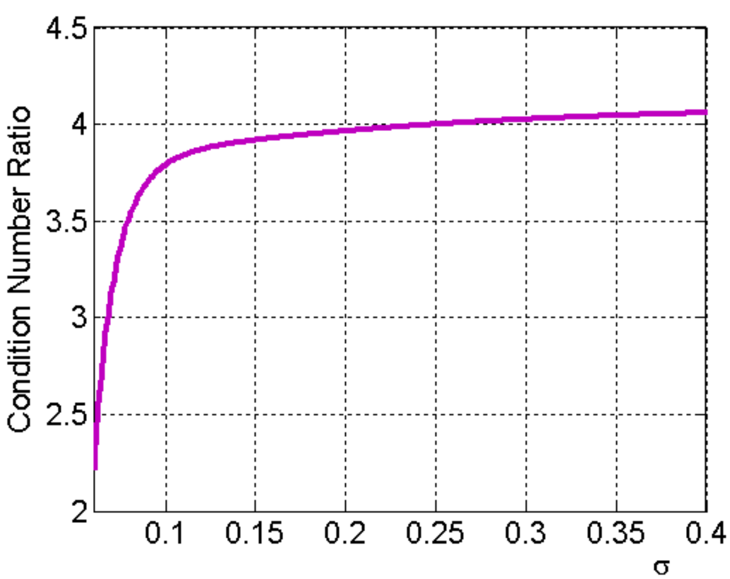


Figure 14. Ratio of the condition number of the matrix *W* for design Option 1 to the same quantity for Option 2, as a function of σ, for the CPRTF with the DAPAMITO3 inducer (Ω = 3000 rpm, ϕ = 0.059, ω/Ω = 0.2, $\hat{y}_T$ = 1 mm).

between the measurements taken by two pressure transducers, placed at two different sections of the pipe line. Evaluations carried out by means of the analytical model have shown that two transducers placed at a distance of 1 m are able of measuring the estimated level of flow rate oscillations with a good accuracy, except for lower values of the oscillation frequency. Oscillations of lower frequencies can be measured by increasing the amplitude of the externally imposed vertical vibration of the water tank or by increasing the distance between the two pressure transducers.

Acknowledgments

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