



Comments on high-energy total cross sections in QCD



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ARTICLE INFO

Article history:

Received 26 January 2015

Received in revised form 25 March 2015

Accepted 26 March 2015

Available online 31 March 2015

Editor: A. Ringwald

Keywords:

Total cross sections

QCD

Nonperturbative approach

ABSTRACT

We discuss how hadronic total cross sections at high energy depend on the details of QCD, namely on the number of colours N_c and the quark masses. We find that while a “Froissart”-type behaviour $\sigma_{\text{tot}} \sim B \log^2 s$ is rather general, relying only on the presence of higher-spin stable particles in the spectrum, the value of B depends quite strongly on the quark masses. Moreover, we argue that B is of order $\mathcal{O}(N_c^0)$ at large N_c , and we discuss a bound for B which does not become singular in the $N_f = 2$ chiral limit, unlike the Froissart–Łukaszuk–Martin bound.

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1. Introduction

The behaviour of hadronic total cross sections at high energy is one of the oldest puzzles of strong interactions. Experimental results, up to the largest energies available at hadronic colliders [1–4], show a steady rise of total cross sections for $\sqrt{s} \gtrsim 5$ GeV [5], where s is the total center-of-mass energy squared. The theoretical challenge is to explain the observed behaviour starting from the first principles of QCD, which is believed to be the fundamental theory describing strong interactions. So far, most of the efforts have been focused on phenomenological approaches, aimed at finding the appropriate parameterisation of experimental data, usually taking inspiration from the Regge–Gribov theory. To date, the majority of the parameterisations agree on the leading energy dependence being of the “Froissart”-type $\sigma_{\text{tot}} \sim B_{\text{exp}} \log^2 s$ with universal B_{exp} [5–11], although alternative behaviours are also considered [12,13]. A universal $\log^2 s$ rise, first proposed by Heisenberg [14], has been supported by several theoretical arguments [15–21], and recently also by numerical results in lattice QCD [22].

A correct prediction (from first principles) of the high-energy behaviour of total cross sections would nontrivially confirm the validity of QCD as the fundamental description of strong interactions, in a largely untested energy–momentum regime. In fact,

the main difficulty in attacking this problem in the framework of QCD is its nonperturbative nature, as it is part of the more general problem of *soft* high-energy scattering, characterised by small transferred momentum squared t ($|t| \lesssim 1$ GeV²) and large s . To avoid the shortcomings of perturbation theory in the presence of the soft scale t , a nonperturbative approach to these processes has been developed [23–30], which relates the relevant scattering amplitudes to the correlation functions of certain nonlocal operators, the so-called Wilson loops, in the fundamental theory. To our knowledge, this approach is so far the closest to a systematic derivation from first principles.

In a recent paper [31] we have argued, within the above-mentioned nonperturbative approach [23–30] in Euclidean space [32,33], that hadron–hadron total cross sections at high energy behave like

$$\sigma_{\text{tot}} \sim B(1 - \kappa) \log^2 s \leq 2B \log^2 s. \quad (1)$$

The prefactor B is determined from the stable asymptotic hadronic spectrum, considering strong interactions *in isolation*, by maximising the following ratio,

$$B = \max_{a, j^{(a)} > 1} B^{(a)}, \quad B^{(a)} = \left(\frac{j^{(a)} - 1}{M^{(a)}} \right)^2, \quad (2)$$

where a runs over the particle species, and $j^{(a)}$ and $M^{(a)}$ are the spin and mass of particle a , respectively. Only higher-spin particles ($j^{(a)} > 1$) have to be considered: if they were absent, then σ_{tot} would be at most a constant at high energy, and $B = 0$. The parameter κ is bounded by unitarity to be $|\kappa| \leq 1$, but is otherwise

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undetermined at this stage. In Ref. [31] we remarked that the most natural value yielding the universality observed in experiments is $\kappa = 0$, corresponding to a black-disk-like behaviour at high energy. However, we do not have a purely theoretical argument to show that this is actually the case. Furthermore, the phenomenological analyses in the literature give different estimates of the “blackness” of the scatterers in the high-energy limit, see, e.g., Refs. [8,10,11,34,35]. In Ref. [31] we also gave a numerical estimate of B using experimental data for (QCD-)stable mesons, baryons and nuclear states. The “dominant” particle, i.e., the one which maximises $B^{(a)}$, turns out to be the Ω baryon, and yields $B_{\text{QCD}} \simeq 0.56 \text{ GeV}^{-2}$, which compares well to the experimental value $B_{\text{exp}} \simeq 0.69\text{--}0.73 \text{ GeV}^{-2}$ [5]. Interestingly enough, our value for $2B_{\text{QCD}}$ is about two orders of magnitude smaller than the analogous prefactor $B_{\text{FLM}} = \frac{\pi}{M_\pi^2}$ appearing in the Froissart–Łukaszk–Martin bound [36–38], and only about 50–60% larger than the experimental value, resulting in a much more restrictive “Froissart-like” bound (which is satisfied by B_{exp}).

It is part of the standard lore that hadronic total cross sections should be mostly governed by the “gluonic sector” of the theory, and this leads to expect that they could be described fairly accurately using the *quenched* approximation of QCD, i.e., pure $SU(3)$ gauge theory. In this case, and in the framework of the nonperturbative approach discussed above, the relevant spectrum for the computation of the prefactor B would be the stable, higher-spin part of the *glueball* spectrum. However, using data from Ref. [39], the resulting value of B turns out to be 2–3.5 times the one obtained using the physical, *unquenched* spectrum, suggesting the presence of unexpectedly large unquenching effects [31].

The high sensitivity of B to the presence or not of dynamical quarks raises an interesting question: how much does the actual value of B depend on the details of QCD? More precisely, how much does it depend on the values of its parameters, i.e., the number of colours N_c and the quark masses? Since only the stable spectrum enters the maximisation (2), the crucial point is to understand how the stability of hadrons changes as the parameters are varied, and how this affects the overall scale of total cross sections. This is precisely the purpose of this paper. In Section 2 we discuss the large- N_c limit. In Section 3 we discuss the chiral limit. In Section 4 we discuss the regime of large quark masses, making contact with the quenched approximation. Finally, in Section 5 we draw our conclusions.

2. Large N_c

We begin by discussing the behaviour of high-energy total cross sections in the ’t Hooft large- N_c limit [40–42]. The first point we want to clarify is precisely how this limit has to be taken. Eq. (1) describes the asymptotic high-energy behaviour of σ_{tot} , i.e., σ_{tot} for center-of-mass energies much larger than any other mass/energy scale in the problem. Formally, Eq. (1) has to be written as

$$\lim_{s \rightarrow \infty} \frac{\sigma_{\text{tot}}}{\log^2 s} = B(1 - \kappa) \leq 2B. \quad (3)$$

The quantity B is well defined for every finite N_c , as the number of stable states is finite, and so it is sensible to consider its large- N_c limit.¹ It is therefore clear that we take *first* the large- s limit, and *then* the large- N_c limit (differently, for example, from what is done in Refs. [43–45]). Taking, instead, *first* the large- N_c limit, and *then* the large- s limit, the leading contribution to σ_{tot}

comes from “Pomeron exchange” (understood here as the exchange of gluons between the colliding mesons), and is of order $\mathcal{O}(1/N_c^2)$, according to the usual counting rules.² In Ref. [45] the two limits $s \rightarrow \infty$ and $N_c \rightarrow \infty$ are taken together, as the particles’ momenta are scaled proportionally to $\sqrt{N_c}$ as N_c is increased. The resulting total cross section is proportional to $\log^2 N_c$. In our approach we do not have to scale the momenta, since they are formally taken to infinity before taking the large- N_c limit; all that matters is the large- N_c behaviour of the spectrum.

The large- N_c behaviour of meson and baryon masses is well known [40–42]: meson masses are of order $\mathcal{O}(N_c^0)$, while baryon masses are of order $\mathcal{O}(N_c)$. Roughly speaking, this is due to the fact that while mesons are always $q\bar{q}$ states, independently of N_c , baryons are made of N_c quarks. Concerning higher-spin states, no higher-spin QCD-stable meson is known in the “real world”, i.e., for $N_c = 3$, and unless this is a subtle consequence of $\mathcal{O}(1/N_c)$ corrections to the meson masses at $N_c = \infty$, there is no reason to expect the situation to change when N_c is large (but finite). On the other hand, a QCD-stable higher-spin baryon exists for $N_c = 3$, namely the Ω baryon ($j^{(\Omega)} = \frac{3}{2}$). In the baryon sector, large- N_c QCD possesses an effective light quark spin-flavour contracted symmetry $SU(2N_f)$ for N_f degenerate light quark flavours [46,47]. Real-world QCD is close to have an exact $N_f = 2$ isospin symmetry, so for the physically most interesting case, at large N_c the contracted symmetry is $SU(4)$. Here we work with $2+1$ light flavours (up/down + strange), neglecting isospin breaking effects. Furthermore, the large- N_c limit is taken keeping N_c odd, so that baryons are fermions as in the real, $N_c = 3$ case.

Dashen, Jenkins and Manohar argued in Refs. [46,47] that in terms of this emergent, large- N_c symmetry, baryons can be classified in multiplets corresponding to the irreducible representations of the contracted spin-flavour symmetry. These representations are labelled by the isospin i , the spin j , and a further quantum number k , related to the number N_s of strange quarks as $N_s = 2k$. The allowed values of k for given i, j are obtained via the usual composition rule for angular momenta, so that $|i - j| \leq k \leq i + j$. Large- N_c consistency conditions, obtained by imposing unitarity in pion-baryon and kaon-baryon scattering processes, constrain the form of the baryon masses as follows [46,47],

$$\begin{aligned} M &= N_c m_0 + m_1 k + \frac{1}{N_c} \left[m_2 i(i+1) + m_3 j(j+1) + m_4 k^2 \right] \\ &\quad + \mathcal{O}(1/N_c^2) \\ &\equiv M_1(i, j, k) + \mathcal{O}(1/N_c^2), \end{aligned} \quad (4)$$

with mass parameters $m_i = m_i(N_c)$ which possess a $1/N_c$ expansion. This formula is valid for $j = \mathcal{O}(N_c^0)$, i.e., fixed spin as N_c becomes large.

The mass formula (4) is the starting point for the study of the large- N_c behaviour of the prefactor B defined in Eq. (2). Low-lying higher-spin states have masses differing from the lightest baryon mass by terms of order $\mathcal{O}(j(j+1)/N_c)$, so for $j = \mathcal{O}(N_c^0)$ they will become stable at large enough N_c , since meson masses are $\mathcal{O}(N_c^0)$ and so the available phase space for decays shrinks to zero. The corresponding $B^{(a)}$ is of order $B^{(a)} = \mathcal{O}(1/N_c^2)$, which leads to σ_{tot} behaving as expected according to the naive large- N_c counting rules. However, it is also possible that states with even higher spin

¹ Although exactly at $N_c = \infty$ there is an infinite tower of stable mesons with unbounded spin, so that $\max_a B^{(a)}$ may not exist there, this does not affect our limiting procedure.

² The leading contributions to meson-meson elastic scattering amplitudes are actually expected to be of order $\mathcal{O}(1/N_c)$, and correspond to the tree-level amplitudes of the large- N_c mesonic effective Lagrangian [42]. However, being real, forward tree-level amplitudes do *not* give a contribution of order $\mathcal{O}(1/N_c)$ to σ_{tot} via the optical theorem. A nonzero contribution of order $\mathcal{O}(1/N_c^2)$ to σ_{tot} is obtained, instead, from the imaginary part of the one-loop meson-meson elastic scattering amplitudes. (We thank the referee for clarifying this point.)

Table 1

Results for the mass parameters m_i and for $\delta m'$ from the fit of the (isospin averaged) masses of the octet and decuplet baryons with the mass formula $M^{(a)} = M_1(i^{(a)}, j^{(a)}, k^{(a)})$, Eq. (4) (first column), and with the formula $M^{(a)} = M_1(i^{(a)}, j^{(a)}, k^{(a)}) + \tilde{m}^{(a)}$, with constrained $\tilde{m}^{(a)}$ (third column). Statistical errors on the simple fit (second column) and uncertainties due to higher-order terms (fourth column) are also reported. Masses are in MeV.

	M_1	$\varepsilon_{\text{stat}}$	$M_1 + \tilde{m}$	ε_{N_c}
m_0	287.73	0.27	287.4	0.3
m_1	429.3	2.7	432	3
m_2	101.8	2.5	97	5
m_3	198.2	1.9	202	4
m_4	-109.6	5.4	-125	15
$\delta m'$	-67	10	-55	12

are stable at large N_c , which could change the large- N_c behaviour of B . To see this, recall that a state with a given value of k is possible only if $k \leq i + j$. Furthermore, if $2k = N_s$ out of N_c quarks are strange quarks, one has $i \leq N_c/2 - k$, and so also $j \geq 2k - N_c/2$, which is effective if $N_s \geq (N_c + 1)/2$ (as $j \geq 1/2$). Consider now the Ω baryon, defined for arbitrary N_c as the baryon made of N_c strange quarks, therefore having $j^{(\Omega)} = k^{(\Omega)} = N_c/2$ and $i^{(\Omega)} = 0$. In a hypothetical decay of Ω into a baryon with $N_s \geq (N_c + 1)/2$ strange quarks and spin j , one has from the bound above

$$\Delta j \equiv \frac{N_c}{2} - j \leq N_c - N_s \equiv 2\Delta k; \quad (5)$$

in a hypothetical decay to a state with $N_s < (N_c + 1)/2$, since $2\Delta k > (N_c - 1)/2$ and $\Delta j < (N_c - 1)/2$, the bound (5) still holds. As a consequence, a decay to a baryon with a decrease of Δj in spin has to come with at least a decrease of Δj in (the absolute value of) strangeness, which requires the emission of Δj kaons.³ It is therefore possible that the mass balance between initial and final states remains negative, as it is for $N_c = 3$, therefore making the Ω stable also at large N_c .

To make this statement quantitative one should know the exact mass formula, rather than its approximation (4), which in principle is valid only for $j = \mathcal{O}(N_c^0)$. However, numerical studies on the lattice [48] (up to $N_c = 7$) find good agreement with the mass formula (4) also for states with $j = \mathcal{O}(N_c)$. This indicates that higher-order terms in Eq. (4) give small contributions even for $j = \mathcal{O}(N_c)$, so that they can be neglected (in a first approximation), and Eq. (4) can be used to give a sensible quantitative estimate of the stability of the Ω baryon at large N_c .

Working in the isospin limit, one can estimate the mass parameters m_i at $N_c = 3$ by fitting the (isospin averaged) masses of the physical octet and decuplet baryons with the mass formula (4). The error on the masses is taken as the sum (in quadrature) of the experimental error and of an extra uncertainty, accounting for isospin breaking and electromagnetic effects. This uncertainty is estimated as the standard deviation of the masses in an isomultiplet, and set to 1 MeV for isosinglets (raising this to 2–3 MeV yields similar results). The fit of the baryon masses with Eq. (4) yields effective parameters, which include contributions from higher-order terms neglected in Eq. (4). To estimate the corresponding uncertainty ε_{N_c} , we have repeated the fit including an extra term $\tilde{m}^{(a)}$ in the mass of each baryon, i.e., using the expression $M^{(a)} = M_1(i^{(a)}, j^{(a)}, k^{(a)}) + \tilde{m}^{(a)}$ to fit the mass of baryon a . The parameters $\tilde{m}^{(a)}$ were constrained to be “small” by means of the usual constrained-fit techniques [49]. In particular, we took these extra parameters to be normally distributed around

zero with standard deviation $\sigma = 10$ MeV. This choice is motivated by the fact that they are of order $\mathcal{O}(1/N_c^2)$, and that the simple fit indicates that m_i are of order $\mathcal{O}(100$ MeV). The results are reported in Table 1. Variations of the resulting parameters between the two fits give an estimate of ε_{N_c} , and are at most of 15%.

All the parameters but m_4 are positive at $N_c = 3$; if their sign remains the same at large N_c (which is supported by lattice results [48]), it is easy to obtain a bound on the mass balance ΔM_1 , plugging Eq. (5) into Eq. (4),

$$\begin{aligned} \Delta M_1 &\equiv m_1 \Delta k + \frac{1}{N_c} \left[-m_2 i(i+1) + m_3 \Delta j \left(\frac{N_c}{2} + j + 1 \right) \right. \\ &\quad \left. + m_4 \Delta k \left(\frac{N_c}{2} + k \right) \right] \\ &\leq \left[m_1 + 2m_3 \left(1 + \frac{1}{N_c} \right) \right] \Delta k \xrightarrow{N_c \rightarrow \infty} [m_1 + 2m_3] |_{N_c=\infty} \Delta k. \end{aligned} \quad (6)$$

Within our approximations, stability of the Ω baryon at large N_c is ensured if $(\Delta M_1 - 2\Delta k \cdot M_K)|_{N_c=\infty} < 0$ for all possible channels. Using the bound (6), this is certainly the case if $\delta m \equiv (m_1 + 2m_3 - 2M_K)|_{N_c=\infty} < 0$. The numerical results of Ref. [48] indicate that $|m_i|$ decreases as N_c is increased, so using $m_i(N_c = 3)$ instead of $m_i(N_c = \infty)$ should make the bound even more conservative. On the other hand, $\mathcal{O}(1/N_c)$ corrections to the meson masses have not been measured in lattice simulations of the full theory. Numerical results for the quenched theory [50] suggest that the variation of meson masses between $N_c = 3$ and $N_c = \infty$ is of the order of 10%. A reasonable upper bound on δm is therefore $\delta m \leq m_1(3) + 2m_3(3) - 2M_K(3) \cdot 0.9 \equiv \delta m'$. Our final result is

$$\delta m' = -67 \pm 10_{\text{stat}} \pm 12_{N_c} \text{ MeV}. \quad (7)$$

We remind the reader that this bound is rather loose, since it does not include the negative contribution of m_4 , and it overestimates m_1 and m_3 . Moreover, $\delta m'$ remains negative up to a reduction of around 15% of M_K .

Our conclusion is that stability of the Ω baryon at large- N_c is at least plausible. If it is indeed so, since the corresponding $B^{(\Omega)}$ is of order $B^{(\Omega)} = \mathcal{O}(N_c^0)$, then one would necessarily have $B = \mathcal{O}(N_c^0)$. This is in contrast with the expected $\mathcal{O}(1/N_c^2)$ from the naive large- N_c counting rules, but not in contradiction, as that expectation holds in the limit $N_c \rightarrow \infty$ at large but fixed s .

3. Chiral limit

We now turn to the chiral limit. More precisely, we consider the $N_f = 2$ chiral limit, with only the up and down quark masses set to zero. In this case the spectrum of the theory contains three massless pseudoscalar Goldstone bosons (the pions) due to the spontaneous breaking of $SU(2)$ chiral symmetry. Note that massless particles of spin 0 leave Eqs. (1) and (2) unchanged [31].

Generally speaking, the chiral limit can only turn stable states into unstable states, and not vice versa, due to the possibility of decaying through the emission of massless pions. This possibility however does not concern the Ω baryon. Whether or not the Ω remains stable depends on how much its mass, and the masses of the other strange baryons and of the kaon, change as the chiral limit is approached. It is likely that the difference between the physical masses of these particles and the corresponding masses in the chiral limit is of the order of the current light-quark masses, i.e., a few MeV. On the other hand, $M_\Omega - M_X - M_K \Delta N_s$ is negative and of magnitude $\mathcal{O}(0.1\text{--}1$ GeV) for all baryons X in the octet and in the decuplet, i.e., at least two orders of magnitude larger than the expected effect of the chiral limit on the kaon and

³ We keep assuming that no higher-spin meson becomes stable for $N_c < \infty$. Notice that decays into more baryons/antibaryons are forbidden at large N_c by a negative mass difference of order $\mathcal{O}(N_c)$ between initial and final states.

strange baryons masses. The effect of this limit on the masses of nuclei is again expected to be a few MeV, so we expect that the Ω remains the dominant particle. An interesting consequence of this result is that our “Froissart-like” bound, Eqs. (1) and (3), is not singular in the $N_f = 2$ chiral limit. The Froissart–Łukaszuk–Martin bound, on the other hand, is singular in this limit since the prefactor $B_{\text{FLM}} = \frac{\pi}{M_\pi^2}$ diverges for massless pions.⁴ Concerning the double chiral/large- N_c limit, it is likely that the prefactor B comes out to be of order $\mathcal{O}(N_c^0)$. Indeed, the estimate given in Section 2 is likely to remain valid due to the small effect of having massless up and down quarks on the masses of non-Goldstone particles.⁵

4. Large quark masses

Let us finally discuss the limit of large quark masses. For quark masses larger than some critical value, purely gluonic states (*glueballs*) will become stable, and will enter the set over which $B^{(a)}$ has to be maximised. Eventually, as the quark masses are further increased, at most only a finite number of higher-spin mesons and baryons will remain stable against decays, which can now take place through the emission of glueballs, since these have finite masses in the limit $m_q \rightarrow \infty$. Of course, the values of $B^{(a)}$ corresponding to mesons and baryons keep decreasing as the quark masses increase. The bottom line is that for large enough quark masses, the relevant part of the spectrum over which one has to maximise $B^{(a)}$ will consist only of stable higher-spin glueballs. Eventually, as $m_q \rightarrow \infty$, one will end up with the quenched theory, where B has been shown to be at least larger than $B_Q \gtrsim 1.09 \text{ GeV}^{-2}$ [31].

It is interesting to remark that, according to our results, in the problem at hand the full and the quenched theory are not equivalent in the large- N_c limit. This is essentially due to the fact that while baryon masses grow like N_c , the stability of glueballs is not improved as N_c grows, since they can always decay into light mesons (at least for physical quark masses), whose masses are essentially unaffected by the large- N_c limit. Therefore, glueballs do not enter the game, while baryons still play an important role even though they become heavier and heavier. It is however worth noticing that both the quenched and the unquenched theory are expected to have $B = \mathcal{O}(N_c^0)$ at large N_c . Indeed, we have argued above that the full theory is likely to show this behaviour due to the stability of the N_c -quark Ω baryon. In the quenched theory, glueball masses are $\mathcal{O}(N_c^0)$, and a few higher-spin stable states exist at $N_c = \infty$ according to lattice results [51], and so $B_Q = \mathcal{O}(N_c^0)$.

5. Conclusions

In this paper we have discussed how hadronic total cross sections at high energy depend on the details of QCD, namely on the number of colours and the quark masses. The starting point is the relation between the overall scale of total cross sections and the hadronic spectrum found in Ref. [31], in the framework of the nonperturbative approach to soft high-energy scattering [23–30] in Euclidean space [32,33].

Our results indicate that while a “Froissart”-type behaviour $\sigma_{\text{tot}} \sim B \log^2 s$ is rather general, relying only on the presence of higher-spin stable particles in the spectrum, the value of B can depend quite strongly on the details of the theory, and particularly on the quark masses. (For example, it is likely to be discontinuous as the $N_f = 3$ chiral limit or the limit $m_q \rightarrow \infty$ are approached.) On the other hand, we expect that B behaves smoothly as the large- N_c or the $N_f = 2$ chiral limits are approached. There are three results that we want to highlight in particular.

- In the large- N_c limit, B is likely to be of order $\mathcal{O}(N_c^0)$, due to the stability of the Ω baryon, in contrast with the expectation based on the naive counting rules.
- The more restrictive “Froissart-like” bound of Eqs. (1) and (3) is not singular in the $N_f = 2$ chiral limit, again due to the stability of the Ω baryon.
- In the large- N_c limit, the full and the *quenched* theory are not equivalent for what concerns total cross sections.

Acknowledgement

M.G. is supported by the Hungarian Academy of Sciences under “Lendület” grant No. LP2011-011.

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⁴ The non-optimality of B_{FLM} had been already pointed out in Ref. [44].

⁵ The $N_f = 3$ chiral limit (with also the strange quark mass set to zero) is more problematic: in this case also kaons become massless, and the Ω baryon is no longer stable. The role of dominant particle will presumably be taken by some stable, higher-spin nuclear state, but we cannot make any definite statement. In the double chiral/large- N_c limit, as the masses of nuclei are likely to be of order $\mathcal{O}(N_c)$, one would still have $B = \mathcal{O}(N_c^0)$ if there were stable nuclei with spin $\mathcal{O}(N_c)$ (but not larger), but again we cannot make any definite statement.

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