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Social discounting, migration and optimal taxation of savings

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Abstract

We study the problem of inheritance and capital income optimal taxation in an economy with migration. In particular, we analyze the role played by the weights attached to the individual utility functions in the social welfare function. If these weights correspond to the demographic weights, the disconnection brought in by migration is the underlying reason for non zero optimal tax rates.

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1 Introduction

The issue of inheritance taxation is very similar to that of capital income taxation, once they are analyzed within the optimal taxation framework: should one tax own future consumption and estate (i.e. perspective heirs' consumption) more than own present consumption?

As for capital income taxation, starting from the seminal works by Judd (1985) and Chamley (1986), the issue of dynamic optimal capital income taxation has been analyzed by a number of researchers. In particular, Judd (1999) has shown that the zero tax rate result stems from the fact that a tax

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on capital income is equivalent to a tax on future consumption: thus, capital income should not be taxed if the elasticity of consumption is constant over time. However, while in infinitely lived representative agent (ILRA) models[1] this condition is necessarily satisfied in the long run, along the transition path, instead, it holds only if the utility function is assumed to be (weakly) separable in consumption and leisure and homothetic in consumption. Another source of taxation can derive from the presence of externalities, which gives room to nonzero taxation as a Pigouvian correction device[2]. Abandoning the standard ILRA framework in favour of Overlapping Generation models with life cycle (OLG-LC)[3] has delivered another important case of nonzero capital income taxation. This outcome can be understood by reckoning that in such a setup optimal consumption and labor (or, more precisely, the general equilibrium elasticity of consumption) are generally not constant over life and even at the steady state, due to life-cycle behavior.

A similar reasoning can be applied to estate taxation. Note that this corresponds to a differential treatment of savings for own future consumption, on the one hand, and of savings for bequest, on the other hand. Thus, the first aspect to note is that the optimality of a nonzero tax on capital income does not necessarily imply the optimality of a nonzero tax on estates. In fact the latter can be justified on arguments analogous to those presented above: a nonzero estate tax could stem either from the violation of (weak) separability between "expenditure" on estate and (previous period) leisure or from a difference between the donor's and the donee's general equilibrium elasticities of consumption, according to the framework being analyzed. Another reason for levying a tax on inheritance could be correcting for an externality. Atkinson (1971) and Stiglitz (1987) consider the positive externality deriving from the fact that transfers benefit those who receive them. Holtz-Eakin et al. (1993), Imbens et al. (1999), Joulfaian et al. (1994) consider instead the negative externality deriving, in the presence of an income tax, from a fall in heirs' labor efforts. In the field of estate and transfers in general, the analysis of the motives for giving is another important aspect. In fact, different motives are associated to different forms of utility functions and, as a consequence, to different policy effects. Altruism, joy of giving, exchange related motives, accidental bequests have been widely studied in the literature (see Davies, 1996; Masson and Pestieau, 1997; Stark, 1999; Kaplow, 2001).

[1] See Atkeson et al. (1999) and Chari et al. (1999).

[2] See De Bonis and Spataro (2005), where the externality argument is applied to the case of differing social and private discounting in an ILRA framework and relatively to capital income taxation.

[3] See Atkinson and Sandmo (1980) and Erosa and Gervais (2002); for a review see Erosa and Gervais (2001).

In this paper we consider altruism motivated bequests. However, we introduce an element that is not considered in the existing models, i.e. the presence of migration. Moreover, we allow for a disconnection in the economy, in that we assume altruism to be limited to own descendants[4]. This element turns out to be a relevant determinant of taxation once it is embedded in the social welfare function, and precisely in the sense that the policymaker takes into account the demographic evolution of the population. In fact, the zero capital income and inheritance tax result applies only if the disconnection of the economy is disregarded. We identify instead a number of ways in which the demographic evolution of the population can be accounted for within the social welfare function via appropriate intergenerational weights, leading to different combinations of the inheritance and capital income tax rates, with at least one of them being nonzero.

The work proceeds as follows: in section 2 we present the model and derive the equilibrium conditions for the decentralized economy. Next, we characterize the Ramsey problem by adopting the primal approach. Finally, we present the results by focusing on the new ones. Concluding remarks and a technical appendix will end the work.

2 The model

We consider a neoclassical-production-closed economy in which there is a large number of agents and firms.

Private agents, who are endowed with identical preferences, differ as for their date of entry into the economy, s; natives are supposed to have entered the economy at time $s = 0$, when also the economy starts, while migrants start entering at time $s = 1$; agents live two periods: in the first period they receive an inheritance from their parents, work and consume; in the second period, they consume, give birth to $(1 + n)$ children and leave them a bequest upon dying; at the same time, in each period migrants enter the economy at a given rate α, proportional to the number of young individuals populating the economy: as a consequence, the population growth rate is equal to $(1 + \pi) = (1 + n)(1 + \alpha)$.

[4]The relevance of the "disconnectedness" of the economy has been firstly analyzed by Blanchard (1985), Buiter (1988) and Weil (1989). The latter shows, in a scenario with a continuous arrival in the economy of new individuals who are not linked to pre-existing cohorts, that finite horizons are not a necessary condition for the violation of the Ricardian equivalence. Analogously, in this work we show that finite horizons, or, more precisely, life cycle behavior, is not a necessary condition for the violation of the zero capital income and estate tax result in OLG models.

We normalize the size of the population at time 0 to unity, so that the whole population at time t has cardinality:

$$N_t = (1+\pi)^{t-1}(2+\pi)$$

and the size of each dynasty (started off with the entry of the founder) is:

$$D_{s,t} = \alpha(1+\pi)^{s-1}(2+n)(1+n)^{t-s},$$

with $s \leq t$.

Moreover, all individuals offer labor and capital services to firms by taking the net-of-tax factor prices, $\widetilde{w}_{s,t}$ and $\widetilde{r}_{s,t}$ as given. Firms, which are identical to each other, own a constant return to scale technology F satisfying the Inada conditions and which transforms the factors into production-consumption units. Finally, the government can finance an exogenous stream of public expenditure G_t, by issuing internal debt B_t and by raising proportional taxes both on interests, wages and inheritance, referred to as $\tau^k_{s,t}$, $\tau^l_{s,t}$ and $\tau^x_{s,t}$ respectively. Notice that taxes can be conditioned on the date of birth of both dynasties and individuals.

2.1 Private agents

Agents' preferences can be represented by the following instantaneous utility function:

$$U = U^1\left(c^1_{s,t}, l_{s,t}\right) + \beta U^2(c^2_{s,t+1}) = \sum_{i=1}^{2}\beta^{i-1}U^i,$$

where β is the intertemporal discount rate, $c^1_{s,t}$, $c^2_{s,t+1}$ and $l_{s,t}$ are consumption in the first (young) and second (old) period of life and labor supply, respectively, of the individual born in period t and belonging to the dynasty started in period s. Such a utility function is strictly increasing in consumption and decreasing in labor, strictly concave, and satisfies the standard Inada conditions. Since we assume that individuals care about the well being of their children, and defining $\gamma > 0$ the "degree of altruism" of parents towards descendants, agents maximize the following utility function:

$$\max_{\left\{c^1_{s,t}, l_{s,t}, c^2_{s,t+1}\right\}^{\infty}_{t}} \sum_{t=s}^{\infty}\delta^{t-s}\sum_{i=1}^{2}\beta^{i-1}U^i \tag{1}$$

$$sub\ \widetilde{w}_{s,t}l_{s,t} + x_{s,t}(1-\tau^x_{s,t}) = c^1_{s,t} + \frac{c^2_{s,t} + x_{s,t+1}(1+n)}{(1+\widetilde{r}_{s,t+1})} \tag{2}$$

$$\lim_{t\to\infty} \frac{x_{s,t}\left(1+n\right)^{t-s}}{\prod\limits_{i=s+1}^{t}\left(1+\widetilde{r}_{s,i}\right)} = 0, \quad x_{s,s} = 0,$$

where $\delta = \gamma\beta\left(1+n\right)$, $x_{s,t}$ the amount of the inheritance received by an individual born in period t, while $\widetilde{r}_{s,t+1} = r_{t+1}\left(1-\tau^{k}_{s,t+1}\right)$ and $\widetilde{w}_{s,t} = w_t\left(1-\tau^{l}_{s,t}\right)$ are the net-of-tax factor prices.

The FOCs of this problem imply:

$$\delta^{t-s}U_{c^1_{s,t}} = p_{s,t} \tag{3}$$

$$\delta^{t-s}U_{l_{s,t}} = -p_{s,t}\widetilde{w}_{s,t} \tag{4}$$

$$\delta^{t-s}\beta U_{c^2_{s,t+1}} = p_{s,t}\frac{1}{\left(1+\widetilde{r}_{s,t+1}\right)}$$

$$\left(1+\widetilde{r}_{s,t+1}\right)\left(1-\tau^{x}_{s,t}\right)p_{s,t+1} = p_{s,t}\left(1+n\right), \tag{5}$$

where the expression U_j is the partial derivative of the utility function with respect to argument $j = c^1_t, c^2_{t+1}, l_t$ and $p_{s,t}$ is the shadow price of wealth. These conditions yield:

$$\frac{U_{c^2_{s,t+1}}}{U_{c^1_{s,t}}} = \frac{\beta}{\left(1+\widetilde{r}_{s,t+1}\right)} \tag{6}$$

$$\frac{U_{c^2_{s,t+1}}}{U_{c^1_{s,t+1}}} = \gamma(1-\tau^{x}_{s,t})$$

$$\frac{U_{l_{s,t}}}{U_{c^1_{s,t}}} = -\widetilde{w}_{s,t}. \tag{7}$$

2.2 Firms

Under the assumption of perfect competition, in each period firms, supposed to be identical, hire capital, K, and labor, L, services according to their market prices (gross of taxes) and in order to maximize current period profits. This means that, for each firm i:

$$\frac{dF\left(K^{i}_{t-1}, L^{i}_{t}\right)}{dK^{i}_{t-1}} = r_t \tag{8}$$

$$\frac{dF\left(K_{t-1}^{i}, L_{t}^{i}\right)}{dL_{t}^{i}} = w_{t} \tag{9}$$

Note that capital is assumed to enter the production process with a one period lag.

Assuming a CRS technology for each firm i, the profit maximization conditions can be expressed, for the economy as a whole, in per worker terms, as:

$$f_{k_{t-1}} = \frac{r_t}{(1+\pi)} \tag{8'}$$

$$f_{l_t} = w_t, \tag{9'}$$

where $l_t = \sum_{s=0}^{t} \nu_{s,t}^{1} l_{s,t}$ and $k_t = \sum_{s=0}^{t} \nu_{s,t}^{1} k_{s,t}$ with $\nu_{s,t}^{1} = \alpha(1+\alpha)^{s-t-1}$ the weight of the young belonging to dynasty s in the whole young population in period t.

2.3 The government and market clearing conditions

The government is assumed to finance an amount of exogenous public expenditure by levying taxes on inheritance, capital and labor income and by issuing debt. In order to rule out the problem of time inconsistency, we suppose that the government has access to a commitment technology that ties it to the announced path of distortionary tax rates whenever the possibility of lump sum taxation arises[5]. The only constraints on the possibility of debt issuing are the usual no-Ponzi game condition and the initial condition $B_0 = \overline{B}$. Thus, one obtains the usual equation for the dynamics of aggregate debt:

$$B_t = (1+r)B_{t-1} + G_t - T_t, \tag{10}$$

where $T_t = \sum_{s=0}^{t} \left[\frac{1+n}{2+n} D_{s,t} \left(\tau_{s,t}^{l} w_t l_{s,t} + \tau_{s,t}^{x} x_{s,t} \right) + \frac{1}{2+n} D_{s,t} \left(\tau_{s,t}^{r} r_t \frac{c^2{}_{s,t}}{1+r_t} + \tau_{s,t}^{r} r_t \frac{x_{s,t}}{1+r_t} \right) \right]$, which can also be written, in per worker terms, as:

[5] In a dynamic setup, as far as capital income is concerned, there exists an incentive for the government to deviate from the announced (ex-ante) second best policy, upon achieving the instant in which it should be implemented; this is so because the stock of accumulated capital ex-post is perfectly rigid and now should be taxed more heavily than announced, since its taxation has a lump sum character. The commitment hypothesis implies also that the capital income tax at the beginning of the policy is given, that is, fixed exogenously at a level belonging to the $(0,1)$ interval.

$$b_t = \frac{(1+r_t)}{(1+\pi)} b_{t-1} + g_t - \tau_t. \tag{11}$$

Finally, the market clearing condition implies that, at each date, the sum of capital and debt equals aggregate private wealth in per worker terms (a_t), that is:

$$a_t = k_t + b_t. \tag{12}$$

3 The Ramsey problem

Since we adopt the primal approach to the Ramsey (1927) problem, a key point is restricting the set of solutions to those allocations that can be decentralized as a competitive equilibrium[6]. Thus, in this paragraph we define a competitive equilibrium and the constraints that must be imposed on the policymaker problem, in order to achieve such a competitive outcome.

The first constraint is the so-called "implementability constraint", i.e. the individual budget constraint with prices substituted for by the FOC's of the individual maximization problem (for the derivation see Appendix A.1):

$$\sum_{t=s}^{\infty} \delta^{t-s} \left(U_{c^1_{s,t}} c^1_{s,t} + U_{l_{s,t}} l_{s,t} + \beta U_{c^2_{s,t+1}} c^2_{s,t+1} \right) = 0, \ \forall s \tag{13}$$

which is referred to as the "implementability constraint".

As for the second constraint, summing eq. (2) over population to get aggregate wealth, subtracting eq. (10) and exploiting the market clearing condition, we get, in per worker terms:

$$y_t \geqslant c^1_t + c^2_t + k_t - \frac{k_{t-1}}{(1+\pi)} + g_t, \tag{14}$$

where $c^i_t = \sum_{s=0}^{t} \nu^i_{s,t} c^i_{s,t}$ and $\nu^i_{s,t} = \alpha(1+\alpha)^{s-t-i}$, $i = 1, 2$ is the ratio of the young ($i = 1$) and old ($i = 2$) belonging to dynasty s to the whole young population in period t.

Such expression is usually referred to as the "feasibility constraint" (see Appendix A.2 for a formal derivation).

We can now give the following definition:

[6]See Atkinson and Stiglitz (1980); on the other hand, the "dual" approach takes prices and tax rates as control variables (see, for instance, Chamley, 1986).

Definition 1 *A competitive equilibrium is: a) an infinite sequence of policies* $\pi = \left\{\tau_{s,t}^k, \tau_{s,t}^l, \tau_{s,t}^x, b_t\right\}_0^\infty$ *, b) allocations* $\left\{c_{s,t}^1, c_{t+1}^2, l_{s,t}, k_t\right\}_0^\infty$ *and c) prices* $\{w_t, r_t\}_0^\infty$ *such that, at each instant* t*: b) satisfies eq.* (1) *subject to eq.* (2)*, given a) and c); c) satisfies eq.* (8′) *and eq.* (9′)*; eqs.* (14) *and* (11) *are satisfied.*

Such allocations are often referred to as "implementable".

In the light of the definition given above, the following proposition holds:

Proposition 1 *An allocation is a competitive equilibrium if and only if it satisfies implementability and feasibility*[7].

3.1 Solution

Suppose that the policy is introduced at the end of period 0. In order to analyze the policymaker's problem, we introduce the following auxiliary function:

$$W_s = \sum_{t=s}^{\infty} \delta^{t-s} \left[\sum_{i=1}^{2} \mu_{s,t}^i \beta^{i-1} U^i + \lambda_s \left(U_{c_{s,t}^1} c_{s,t}^1 + U_{l_{s,t}} l_{s,t} + \beta U_{c_{s,t+1}^2} c_{s,t+1}^2 \right) \right].$$

The problem can thus be written as follows:

$$\max_{\left\{c_{s,t}^1, c_{s,t+1}^2, l_{s,t}, k_t\right\}_0^\infty} \sum_{s=0}^{\infty} W_s$$

sub

$$y_t \geqslant c_t^1 + c_t^2 + k_t - \frac{k_{t-1}}{(1+\pi)} + g_t, \ \forall t$$

$$\lim_{t \to \infty} \frac{k_t}{\prod_{i=1}^{t} (1 + f_{k_i})} = 0, \quad k_{t_0} = \overline{k},$$

where λ_s is the multiplier associated to the implementability constraint (and usually interpreted as the deadweight loss of distortionary taxation), $\mu_{s,t}^i$ is the weight that the government attaches to individuals belonging to dynasty

[7] The first part of the proposition is true by construction. The proof of the reverse (any allocation satisfying implementability and feasibility is a competitive equilibrium) follows a standard procedure and is available from the authors upon request.

s as for time t and period of life i[8]. Note that, contrary to previous studies, we allow $\mu^i_{s,t}$ to vary with time, age and dynasty[9].

The FOCs with respect to $c^1_{s,t}$, $c^2_{s,t+1}$, and k_t are, respectively[10]:

$$\frac{\partial W_s}{\partial c^1_{s,t}} = \phi_t \frac{\partial c^1_t}{\partial c^1_{s,t}} \Rightarrow \delta^{t-s} U_{c^1_{s,t}} \left[\mu^1_{s,t} + \lambda_s \left(1 + H_{c^1_{s,t}} \right) \right] = \phi_t \nu^1_{s,t} \tag{15}$$

$$\frac{\partial W_s}{\partial c^2_{s,t+1}} = \phi_{t+1} \frac{\partial c^2_{t+1}}{\partial c^2_{s,t+1}} \Rightarrow \delta^{t-s} U_{c^2_{s,t+1}} \left[\mu^2_{s,t} + \lambda_s \left(1 + H_{c^2_{s,t+1}} \right) \right] = \phi_{t+1} \nu^2_{s,t+1} \tag{16}$$

$$\phi_t = \left[\frac{1}{(1+\pi)} + f_{k_t} \right] \phi_{t+1}, \tag{17}$$

where $H_{c^1_{s,t}} = \frac{\left(U_{c^1_{s,t}} + U_{c^1 c^1_{s,t}} c^1_{s,t} + U_{l c^1_{s,t}} l_{s,t} \right)}{U_{c^1_{s,t}}}$, which is usually referred to as the "general equilibrium elasticity" of (first period) consumption, $H_{c^2_{s,t+1}} = \frac{\left(U_{c^2_{s,t}} + U_{c^2 c^2_{s,t+1}} c^2_{s,t+1} \right)}{U_{c^2_{s,t}}}$, the "general equilibrium elasticity" of (second period) consumption, and ϕ_t is the multiplier associated to the feasibility constraint.

Reiterating eq. (15) one period forward and dividing it by eq. (16), we get:

$$\frac{U_{c^1} \left[\mu^1_{s,t} + \lambda \left(1 + H_{c^1} \right) \right]}{\beta U_{c^2} \left[\mu^2_{s,t} + \lambda \left(1 + H_{c^2} \right) \right]} = \frac{\phi_t \nu^1_{s,t}}{\phi_{t+1} \nu^2_{s,t+1}}$$

and, by exploiting eqs. (6), (8') and (17) and by reckoning that $\frac{\nu^1_{s,t}}{\nu^2_{s,t+1}} = (1+\pi)$, we get:

$$\frac{1 + \widetilde{r}_{s,t+1}}{1 + r_{t+1}} = \frac{\mu^2_{s,t} + \lambda \left(1 + H_{c^2_{s,t+1}} \right)}{\mu^1_{s,t} + \lambda \left(1 + H_{c^1_{s,t}} \right)}, \tag{18}$$

---

[8] We omit the government budget constraint since, by Walras' law, it is satisfied if the implementability and feasibility constraints hold. Note that $\mu^i_{s,t}$ does not multiply the implementability constraint, otherwise $\sum_{t=d}^{\infty} \delta^{t-d} \left(U_{c^1_{s,t}} c^1_{s,t} + U_{l_{s,t}} l_{s,t} + \beta U_{c^2_{s,t}} c^2_{s,t} \right) = 0$ would not hold for all s.

[9] Farhi and Werning (2005) obtain a time varying social discount rate by assuming that the government values the welfare of future generations directly.

[10] We omit the solution for $l_{s,t}$ for the sake of brevity.

which provides the implicit expression for the optimal capital income tax[11].

By obtaining the FOC for $c^1_{s,t+1}$ and dividing it by eq. (16), we get:

$$\frac{(1+n)U_{c^1_{s,t+1}}\left[\mu^1_{s,t+1}+\lambda\left(1+H_{c^1_{s,t+1}}\right)\right]}{U_{c^2_{s,t+1}}\left[\mu^2_{s,t}+\lambda\left(1+H_{c^2_{s,t+1}}\right)\right]}=\frac{\nu^1_{s,t+1}}{\nu^2_{s,t+1}},$$

which yields:

$$\tau^x_{s,t}=1-\frac{\mu^1_{s,t+1}+\lambda\left(1+H_{c^1_{s,t+1}}\right)}{\mu^2_{s,t}+\lambda\left(1+H_{c^2_{s,t+1}}\right)}. \tag{19}$$

The implicit expression of the overall tax hitting savings for bequest is:

$$\frac{\left(1+\widetilde{r}_{s,t+1}\right)\left(1-\tau^x_{s,t}\right)}{1+r_{t+1}}=\frac{\mu^1_{s,t+1}+\lambda\left(1+H_{c^1_{s,t+1}}\right)}{\mu^1_{s,t}+\lambda\left(1+H_{c^1_{s,t+1}}\right)}. \tag{20}$$

4 Discussion of the results

Eqs. (18) and (19) show that there are two forces driving taxation as a whole: the first one depends on the evolution of the general equilibrium elasticity of consumption, the second one stems from the evolution of the social weights.

In particular, as far as capital income taxation is concerned, the first force arises from the difference between the general equilibrium elasticity of consumption in the first ($H_{c^1_{s,t}}$) and in the second period of life ($H_{c^2_{s,t+1}}$); the second reason for nonzero capital income taxation is due to the difference in the weights assigned by the government in the first ($\mu^1_{s,t}$) and in the second period of life ($\mu^2_{s,t+1}$).

As for the inheritance tax, the two forces are, respectively, the difference between the general equilibrium elasticity of consumption of young ($H_{c^1_{s,t+1}}$) and old ($H_{c^2_{s,t+1}}$) people in the same period of time; and the difference in the weights assigned by the government to individuals belonging to different generations ($\mu^1_{s,t+1}$ vs. $\mu^2_{s,t+1}$).

The first factor has been widely discussed in the literature. As for capital income taxation, $H_{c^1_{s,t}}=H_{c^2_{s,t+1}}$ obtains, for example, if one assumes

[11]Note that we do not have any condition ensuring that the tax rate will be in the $(0,1)$ interval, while it is possible that such a rate keeps sticking at the upper limit for a (finite) period of time since the introduction of the policy. However, in the rest of the work we maintain the assumption of interiority of the equilibrium tax rates for $t>0$.

that the utility function is homothetic in consumption and (weakly) separable in consumption and leisure. Otherwise, there is a force leading to taxing/subsidizing future consumption if consumption demand is getting more/less inelastic, respectively. Moreover, this factor marks the difference between the ILRA and the OLG-LC models as for the steady state result: in fact, in OLG ones, differently from the ILRA framework, H_c can vary with age even at the steady state. However, as shown in eq. (18) and eq. (19), even in the absence of a life cycle, in the present model the nonzero tax rule can still apply, because of the presence of a second factor. The same considerations apply as for the inheritance tax. Thus, the second force driving taxation can be isolated by supposing $H_{c^1_{s,t}} = H_{c^2_{s,t+1}} = H_{c^1_{s,t+1}} = H_c$[12].

Then, eq. (18) becomes:

$$\frac{1+\widetilde{r}_{s,t+1}}{1+r_{t+1}} = \frac{\mu^2_{s,t+1} + \lambda\left(1+H_c\right)}{\mu^1_{s,t} + \lambda\left(1+H_c\right)} \tag{21}$$

and eq. (19) becomes:

$$\tau^x_{s,t} = 1 - \frac{\mu^1_{s,t+1} + \lambda\left(1+H_c\right)}{\mu^2_{s,t+1} + \lambda\left(1+H_c\right)}. \tag{22}$$

As for the choice of the social weights, we can consider some exemplifying cases.

1) $\mu^i_{s,t} = \mu_s$. The weight is assigned to the dynasty by the government and is constant through time (and individual life). This implies the absence of both capital income and inheritance taxation: eq. (21) becomes $\frac{1+\widetilde{r}_{s,t+1}}{1+r_{t+1}} = \frac{\mu_s+\lambda(1+H_c)}{\mu_s+\lambda(1+H_c)} = 1$ and eq. (22) becomes $\tau^x_{s,t} = 1 - \frac{\mu_s+\lambda(1+H_c)}{\mu_s+\lambda(1+H_c)} = 0$. Since the weights assigned to individuals do not vary during their life and equal those assigned to their own children, there is no welfare gain in distorting individual choices.

2) $\mu^1_{s,t} = \mu^2_{s,t} \neq \mu^1_{s,t+1}$. The weight assigned by the government to a representative individual belonging to dynasty s is constant through life, though varying among generations of the same dynasty; in particular, let us suppose that it equals the share of the young people belonging to dynasty s within the whole young population when the individual is born $\left(\nu^1_{s,t}\right)$; by inspection of eq. (21) it is immediate to see that the capital income tax is zero, since there is no reason to discriminate against an individual's own future consumption; as for the inheritance tax, instead, in eq. (22) $\frac{\mu^1_{s,t+1}}{\mu^2_{s,t+1}} = \frac{\nu^1_{s,t+1}}{\nu^1_{s,t}} = \frac{1}{(1+\alpha)}$, indicating a positive rate: given migration, the children's weight within the

[12]This would be the case, for instance, in a CES function.

population is lower than their parents' one; as a consequence, the government discriminates in favor of an individual's own consumption and against descendants' one.

3) $\mu^2_{s,t+1} = \mu^1_{s,t+1} \neq \mu^1_{s,t}$. The weight is assigned by the government to the dynasty, so that the parents' weight equals their children's one; however, differently from case 1), the dynasty's weight varies through time; in this case the capital income tax is different from zero. In particular, let us suppose, in application of the Benthamite approach, that the social weight of each dynasty is equal to its actual demographic weight within the population, i.e. $\mu_{s,t} = \frac{D_{s,t}}{N_t} = \frac{\alpha(1+\alpha)^{s-t}(2+n)}{(2+\pi)}$. The relative size of each dynasty is decreasing through time, so that $\frac{\mu_{s,t+1}}{\mu_{s,t}} = \frac{\alpha}{(1+\pi)}$ and, hence, the tax rate is positive[13]. In other words, the government values future consumption more than the dynasty does and consequently uses the capital income tax to correct its consumption path. The inheritance tax is instead zero: since $\mu^2_{s,t+1} = \mu^1_{s,t+1}$, there is no reason to discriminate within a dynasty between donor's and donee's consumption.

4) $\mu^1_{s,t} \neq \mu^2_{s,t+1} \neq \mu^1_{s,t+1}$. The weight assigned in the first period of life is different from that assigned in the second period of life and from that assigned to one's heirs. In particular, suppose that $\mu^1_{s,t} = \nu^1_{s,t}$, $\mu^1_{s,t+1} = \nu^1_{s,t+1}$, $\mu^2_{s,t+1} = \nu^2_{s,t+1}$. Since the demographic weight is higher when young than when old, from eq. (21) we get that capital income is taxed; and, since the weight assigned to parents is lower than their children's one, estate is subsidized ($\frac{\nu^1_{s,t+1}}{\nu^2_{s,t+1}} = (1+\alpha)$). However, this subsidy does not completely outweigh the burden of interest income taxation, so that saving for bequeathing, though being discriminated against with respect to own present consumption, is favored over own future consumption (see eq. (20)).

5) $\mu^1_{s,t} = \mu^1_{s,t+1} = \mu^1 \neq \mu^2_{s,t} = \mu^2_{s,t+1} = \mu^2$. The government assigns a constant weight to the young in each period, equal to their share within the population ($\mu^1_{s,t} = \mu^1_{s,t+1} = \frac{(1+n)}{(2+n)}$), and a constant weight to the old, equal to their share within the population ($\mu^2_{s,t} = \mu^2_{s,t+1} = \frac{1}{(2+n)}$). Again, the capital income tax is positive, since consumption of the young is weighted more heavily than consumption of the old, given the higher demographic weight of the former with respect to the latter ($\frac{\mu^2_{s,t+1}}{\mu^1_{s,t}} = \frac{1}{1+n}$). As for the inheritance tax, since the ratio of the weights in eq. (22) equals $(1+n)$, inheritance is subsidized: children's consumption is favored with respect to their parents'

[13]See also De Bonis and Spataro (2004). Note that, if $(1+H_c) < 0$, the tax could be negative. In the CES case, as shown by de la Croix and Michel (2002), section 3.4, such a case is incompatible with the convexity of the implementability constraint. Hence, $(1+H_c) \geq 0$ and the tax rate is positive.

one, given the higher demographic weight of the young with respect to the old. This subsidy outweighs the effect of the capital income tax, so that savings aimed at bequeathing are tax exempt. This can also be seen from eq. (20), where $\mu^1_{s,t} = \mu^1_{s,t+1}$ (and $H_{c^1_{s,t}} = H_{c^1_{s,t+1}} = H_c$).

Note that in all cases the rationale for taxation derives from Pigouvian arguments. In fact, the results would apply even if lump sum taxes were available, i.e. for $\lambda = 0$. In fact, allowing the social weight to vary with time and age turns out to be equivalent to assuming a *constant intergenerational discount rate* and a *social intertemporal discount rate that differs from the individual one.* Thus, varying dynasty's weights lead the government to correct private accumulation of capital.

5 Conclusions

Analyzing the inheritance tax within the optimal taxation framework and in a parallel to the capital income tax produces the following results for an economy characterized by OLG and disconnection originating from migration.

First, in line with the traditional analysis, scope for a differential treatment of consumption in different life periods (capital income tax) and of own future and descendants' consumption (inheritance tax) arises if the general equilibrium elasticity of consumption varies between life periods and between generations, respectively.

Second, we find that an independent role is played by the weight attached to the individual utility functions by the government within the social welfare function. If these weights correspond to the actual demographic weights, the disconnection brought in by migration is the underlying reason for a nonzero optimal tax.

In all cases presented in this work the rationale for taxation can be reconducted to Pigouvian correction.

References

[1] Atkeson, A., Chari V. V., Kehoe, P. J. (1999): Taxing Capital Income: A Bad Idea. Federal Reserve Bank of Minneapolis Quarterly Review **23**, 3-17.

[2] Atkinson, A. B., Sandmo, A. (1980): Welfare Implications of the Taxation of Savings. The Economic Journal **90**, 529-549.

[3] Atkinson, A. B., Stiglitz, J. E.: Lectures on public economics. London: McGraw-Hill 1980.

[4] Barro, R. J., Sala-i-Martin, X. (1999): Economic growth. Cambridge, Massachusetts, London, England: The MIT Press.

[5] Blanchard, O. J. (1985): Debt, Deficits, and Finite Horizons. Journal of Political Economy **93**, 223-247.

[6] Buiter, W.H. (1988): Death, Birth, Productivity Growth and Debt Neutrality, Economic Journal **98** , 279-293.

[7] Chamley, C. (1986): Optimal taxation of capital income in general equilibrium with infinite lives. Econometrica **54**, 607-622.

[8] Chari, V. V., Kehoe, P. J. (1999): Optimal Fiscal and Monetary Policy. In J. B. Taylor and M. Woodford (Eds.), Handbook of Macroeconomics, Vol. 1, 1670-1745. Amsterdam: North-Holland.

[9] De Bonis, V. and Spataro, L. (2004): Recent Developments in Dynamic Optimal Capital Income Taxation: A Review, Economia politica, **21**, 269-297.

[10] De Bonis, V. and Spataro, L. (2005): Taxing capital income as Pigouvian correction: the role of discounting the future, Macroeconomic Dynamics, **9**, 469-477.

[11] de la Croix, D. and Michel, P. (2002): A Theory of Economic Growth. Cambridge University Press, Cambridge, UK.

[12] Erosa, A., Gervais, M. (2001): Optimal Taxation in Infinitely-Lived Agent and Overlapping Generations Models: A Review. Federal Reserve Bank of Richmond Economic Quarterly Review **87**, 23-44.

[13] Erosa, A., Gervais, M. (2002): Optimal Taxation in Life-Cycle Economies. Journal of Economic Theory **105**, 338-369.

[14] Farhi, E., Werning, I (2005): Inequality, Social Discounting and Estate Taxation. NBER Working Paper 1148.

[15] Judd, K. L. (1985): Redistributive Taxation in a Simple Perfect Foresight Model. Journal of Public Economics **28**, 59-83.

[16] Judd, K. L. (1999): Optimal Taxation and Spending in General Competitive Growth Models. Journal of Public Economics **71**, 1-26.

[17] Kamien, M. I., Schwartz, N. L. (1991): Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management. Second Edition. New York: North Holland.

[18] Ramsey, F. P. (1927): A contribution to the theory of taxation. Economic Journal **37**, 47-61.

[19] Weil, P. (1989): Overlapping families of infinitely lived agents, Journal of Public Economics **38**, 183-198.

6 Appendix

6.1 Derivation of the implementability constraint

In order to obtain the implementability constraint, write eq. (2) in its intertemporal form:

$$\sum_{t=s}^{\infty} \frac{\left(c_{s,t}^{1} + c_{s,t+1}^{2} - w_{s,t} l_{s,t}\right)}{\prod_{i=s+1}^{t} (1+\widetilde{r}_{s,i})(1-\tau_{s,t}^{x})} = 0. \tag{23}$$

Since $\frac{U_{c_{s,t}^1}}{U_{c_{s,t+1}^1}} \frac{(1+\beta)}{n} = \frac{p_{s,t}}{p_{s,t+1}} = \frac{(1+\widetilde{r}_{t+1})(1-\tau_{s,t}^{x})}{n}$, we have

$$\frac{1}{n^{t-s}} \prod_{i=s+1}^{t} (1+\widetilde{r}_{s,i})(1-\tau_{s,t}^{x}) = \frac{p_{s,s}}{p_{s,s+1}} \frac{p_{s,s+1}}{p_{s,s+2}} ... \frac{p_{s,t-1}}{p_{s,t}}.$$

By substituting into eq. (23), we obtain

$$\sum_{t=s}^{\infty} \frac{\left(c_{s,t}^{1} + c_{s,t+1}^{2} - w_{s,t} l_{s,t}\right) p_{s,t}}{p_{s,s}} = 0$$

and exploiting the FOCs from the individual maximization problem, we get

$$\sum_{t=s}^{\infty} \left(\frac{n}{1+\beta}\right)^{t-s} \left(U_{c_{s,t}^1} c_{s,t}^1 + U_{c_{s,t+1}^2} c_{s,t+1}^2 + U_{l_{s,t}} l_{s,t}\right) = 0,$$

which is eq. (13) in the text.

6.2 Derivation of the feasibility constraint

To derive the feasibility constraint, first aggregate eq. (2) over young population at time t:

$$\sum_{s=0}^{t} D_{s,t}^{1} a_{s,t} =$$

$$\sum_{s=0}^{t} D_{s,t}^{1} \left[(1+\widetilde{r})\,(1-\tau_{s,t}^{x}) a_{s,t-1} + \widetilde{w}_t l_{s,t} - c_{s,t}^{1} - c_{s,t}^{2} (1-\tau_{s,t}^{x})\,(1+n) \right], \quad (24)$$

where $D_{s,t}^{1}$ is the number of young people in dynasty s at time t, and by recalling that $A_t \equiv \sum_{s=0}^{t} D_{s,t}^{1} a_{s,t}$, and $\sum_{s=0}^{t} D_{s,t}^{1} \frac{a_{s,t-1}}{(1+n)} = A_{t-1}$, we can rewrite eq.(24) as follows

$$A_t = (1+r_t) A_{t-1} + w_t L_t - C_t^1 - C_t^2 - T_t,$$

where $C_t^1 = \sum_{s=0}^{t} D_{s,t}^{1} c_{s,t}^{1}$ and $C_t^2 = \sum_{s=0}^{t} D_{s,t}^{2} c_{s,t}^{2}$, where $D_{s,t}^{2}$ is the number of old people in dynasty s at time t,

Finally, by subtracting eq. (10) and exploiting the market clearing condition we obtain

$$K_t = (1+r_t) K_{t-1} + w_t L_t - C_t^1 - C_t^2 - G_t$$

which, in per worker terms, becomes

$$k_t = \frac{(1+r_t)}{(1+\pi)} k_{t-1} + w_t - c_t^1 - c_t^2 - g_t.$$