GLOBALIZATION IN MATHEMATICS EDUCATION: INTEGRATING INDIGENOUS AND ACADEMIC KNOWLEDGE

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Abstract: A globalized world ask for mobilization of educators in view of the design of new curricula. Investigation and analysis of indigenous knowledge are needed in view of their possible integration with academic knowledge, to create an educational context where different cultures are equally valued and granted dignity. The investigation and the analysis are to refer also to mathematics, a discipline which cannot anymore be considered culture-free. In the paper, we try to give the readers the opportunity to ask themselves and answer the following questions: *What is mathematics? How look like today's classrooms? Which is the needed mathematics education?* The author suggests a possible answer to the last question by the proposal of three teaching units originated by the analysis of examples of the mathematics of professionals, the mathematics of indigenous people, the history of mathematics

Introduction

Teaching Mathematics is not an easy task… It is not just a matter of communicating notions and concepts! Before entering the classroom, mathematics teachers should ask themselves a few questions:

What contents to teach? [Curriculum]

How? [Methodology]

but also:

Why to teach a given piece of content? [Motivation]

What for? [Aims]

> *School textbooks or popularizing books are primarily engaged in the "what" to teach; literature on mathematics education mainly devotes the attention to the "how". Rather scarce are publications which aim to reflect on the big and little "why" of mathematics teaching.* (translated from Villani, 2003)

The paper aims to provide mathematics teachers with opportunities to give their own answer to the above questions. In view of this, we present three more questions and try to answer them:

What is mathematics?

How look like today's classrooms?

Which is the needed mathematics education?

What is mathematics?

In the attempt to answer the first question, we start quoting a UNESCO (1997) document:

> *Considering the central importance of mathematics and its applications in today's world with regard to science, technology, communications, economics and numerous other fields,*
>
> *Aware that mathematics has deep roots in many cultures and that the most outstanding thinkers over several thousand years contributed significantly to their development, and numerous other fields,*
>
> *Aware that the language and the values of mathematics are universal, thus encouraging and making it ideally suited for international cooperation,*
>
> *Stressing the key role of mathematics education, in particular at primary and secondary school level, both for the understanding of basic mathematical concepts and for the development of rational thinking,*
>
> *Welcomes the initiative of the International Mathematical Union (IMU) to declare the year 2000 the World Mathematical Year ...*

It is said: *mathematics has deep roots in many cultures*.

- If **M**athematics has deep roots in many cultures, can we assume that many cultures have produced mathematical ideas?
- If many cultures have produced mathematical ideas, can we assume that many cultures continue to produce mathematical ideas?
- If many cultures continue to produce mathematical ideas, can we say that mathematics is a cultural product?

- If mathematics is a cultural product, can we say that every culture has the ability to produce mathematical ideas?
- If every culture has the ability to produce mathematical ideas, can we assume that every society in which a given culture develops is also able to organize them and make them accessible to its members?
- If different societies are able to organize the mathematical ideas they produce, can we say that there are several different **m**athematics?
- If there are several different **m**athematics, what is **M**athematics that we started from?
- What are these **m**athematics?
- What is the relationship between these mathematics and the Mathematics?

Before trying to answer these questions, let us consider another excerpt from the UNESCO quotation: *the language and the values of mathematics are universal.*

- The language used by a society in the development and organization of its cultural products can be universal?
- For the **M**athematics and the several different **m**athematics are used the same languages?
- The values assigned to a given cultural product can be universal?
- The **M**athematics and the **m**athematics are assigned the same values?
- What they have in common, in terms of universality, the **M**athematics and the **m**athematics?

Mathematics is a tool created and used by humans to interact with the environment and with other men! But the environments and societies in which men live are different, they require the design and implementation of appropriate strategies and techniques of communication, which can not a priori be the same, independent of the context; on the contrary, in principle, they are different. It is in this way that, therefore, different mathematics are developed.

On this, albeit from different study experiences and lines of research, both D'Ambrosio and Bishop, substantially agree. The first, after having initially (D'Ambrosio, 1985) introduced the term *ethnomathematics* as

> *the mathematics which is practised among identifiable cultural groups such as national-tribal societies, labour groups, children of a certain age bracket, professional classes and so on*

broadens (D’Ambrosio, 1992) its meaning to

> *the arts or techniques developed by different cultures to explain, to understand, to cope with their environment.*

Bishop, for his part, notes that

> *mathematics must now be understood as a kind of cultural knowledge, which all cultures generate but which need not necessarily ‘look’ the same from one cultural group to another.*

In his researches and studies on different mathematics, Bishop believes, however, to grasp a common element, as necessary and sufficient condition for the emergence and development of each of them: the use of the following six categories of activities

Counting - Locating - Measuring - Designing - Playing - Explaining.

He shows in detail that almost all of the ideas around which **M**athematics has developed is based mainly on these cultural activities.

So, it is the six categories of mathematical activities that may be seen as *universal*; at the same time must be judged universal the validity of any mathematical theory, regardless of the society in which it is developed.

The foregoing leads to the need to reconsider one of the stereotypical qualities of mathematics, being *culture-free*, which is attributed to it as a result of being considered a universal knowledge. But we must also reconsider, consequently, its supposed to be *value-free*. How can we think in fact that a cultural product does not carry values? Similarly, as educating does not just mean teaching, even mathematics education cannot be separated from the consideration of the values that can be assigned to mathematical knowledge in different cultural contexts, mono- or multi-cultural.

The identification and definition of values is always difficult, as they carry strong elements of subjectivity. This difficulty is even more evident if we want to identify and define the values of cultural products that are foreign to the culture of the society in which we live and we were brought up. The same goes for mathematics or, better, to the body of knowledge that the use of the above six mathematical activities originate in different cultures. We are now aware of the risk of reading with 'Western' eyes knowledge and products of non-Western cultures, and of assigning values to them, even when it comes to mathematical knowledge. This risk was firstly highlighted by Vithal & Skovmose (1997), but more significantly, a little earlier, by

Zevenbergen (1995) that, with explicit reference to Bishop, criticizes the tendency of Western scholars to describe indigenous activities, objects and relationships in terms of Western mathematics (**M**athematics), stating that this compromises the inherent uniqueness of the indigenous culture.

What is, therefore, possible, it is only trying to identify the values that can be assigned to the mathematics used by us, by us as members of a particular society that has developed a specific culture, mathematics included, and that this culture, mathematics included, has drawn elements which have allowed and allow its development. Although aware of the fact that no society can be considered mono-cultural, despite the homogeneity of family and social roots, at the macroscopic level we can say that the mathematics of the 'Western' culture, that permeates our society, is essentially the **M**athematics, the academic mathematics.

In fact, even in 'our' society there are social groups for which the mathematical knowledge, that they also do have, do not coincide with the **M**athematics.

To sum up, a possible answer to the sequence of questions asked above could be: mathematics is not a unique set of contents; different cultures produced and keep producing different **m**athematics, which in turn contributed and keep contributing to form the academic mathematics, the **M**athematics, not necessarily being entirely a part of it.

It is of some interest to note that the choice of Bishop to indicate mathematics as the result of carrying out certain activities necessary to cope with the real life is opposed to what is suggested by several international and national educational bodies involved in the categorization of mathematical knowledge against which to evaluate the students' competence.

Here below a few classifications are shown:

- *Content categories (Overarching ideas)*: Space and Shape, Change and relationships, Quantity, Uncertainty and data [OECD/OCSE-PISA 2012]
- *Content domains*: Number – Algebra – Geometry –Data and chance [TIMSS 2011]
- *Strands of content*: Number and Operations – Algebra – Geometry – Measurement – Data Analysis and Probability [NCTM Standards 2000]
- *Nuclei Fondanti*: Numbers – Space and Figures – Relations and Functions – Data and Predictions [Italian MIUR: National Guidelines - Indicazioni Nazionali 2012]

How contrasting are these static categorizations to the active one proposed by Bishop! Contents vs Activities. Bishop's view of mathematics is of different cultural products *made for* ... rather than *made of* ...

How look like today's classrooms?

Multiculturalism represents the biggest change in our societies and schools, but some school systems are not yet ready to give immigrants and minority pupils the needed opportunities to develop their knowledge and abilities. A big effort still has to be made, both in terms of financial and educational resources. As far as mathematics is concerned, a significant attention should be paid to initial and in-service education of teaching staff; they should become aware of the fact that multicultural classrooms oblige them to find new teaching methodologies. The *universality* of mathematics should no longer be seen as a value, but as a limit to be overcome in the teaching practice, thus allowing pupils with different backgrounds (and carrying different values!) and the entire class to take real advantage from the new educational context.

When focusing their attention on the pupil's insertion into the class, teachers seem to be worried about the creation of a fair social setting, which should allow them to better act as educators; the risk is that the achievement of this fair social setting could be viewed, by a few teachers, as the single final aim, thus disregarding any additional attention to the methodologies to be used for an effective mathematics education of the whole class including minority pupils.

Developing different didactical activities just for immigrant or minority pupils could also be risky! This approach could result in their educational and, likely, social exclusion: we would say that too much care is as dangerous as too little concern. The definition of a *good* balance between the individuals' and the whole class educational needs is the real methodological challenge. Therefore, the promotion of actions aiming at creating an inclusive educational environment is the biggest challenge

> *Inclusive education is a process of strengthening the capacity of the education system to reach out to all learners... As an overall principle, it should guide all education policies and practices, starting from the fact that education is a basic human right and the foundation for a more just and equal society (UNESCO, 2009a).*

In accordance to what stated in the 2009 Report of the UNESCO Experts Group[1] on *Enhancing Quality Inclusive Education*, we can say that inclusive education (UNESCO, 2009b):

- is an evolving concept;
- is being broadened to refer to all marginalized groups, not just children with special needs;
- is not just about children being in or out of school, but about them receiving a quality education whilst in school;
- is based on the premise that all children can learn;
- requires that schools respond positively to diversity among learners;
- requires us to think about the question: 'inclusion into what?'

In the perspective of the social inclusion, we can say that

learning to live together begins with learning to learn together.

From the pedagogical point of view the pupil is to be considered a person, not just an individual in the classroom; the pupil is to be at the centre of the educational process.

Which is the needed mathematics education?

As far as mathematics education is concerned, the inclusive educational approach was already promoted in the American *NCTM Standards and Principles for School Mathematics – 2000*, where we can find *The Equity Principle*:

> *Excellence in mathematics education requires equity - high expectations and strong support for all students.*

> *All students, regardless of their personal characteristics, backgrounds, or physical challenges, must have opportunities to study - and support to learn - mathematics. This does not mean that every student should be treated the same. But all students need access each year they are in school to a coherent, challenging mathematics curriculum that is taught by competent and well-supported mathematics teachers.*

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[1] The author was a member in the Experts Group

Similarly, in the Australian *AAMT Standards for Excellence in Teaching Mathematics – 2002* we can read:

> *Excellent teachers of mathematics have a thorough knowledge of the students they teach. This includes knowledge of students' social and cultural contexts, the mathematics they know and use, their preferred ways of learning, and how confident they feel about learning mathematics.*

In the present globalized world everything is soaked in cultural products from diverse areas and societies. Therefore, inclusive education asks for integration of knowledge.

New curricula are to be designed and implemented, rooted on the pillars of the education for the XXI century:

Learning to know, Learning to do, Learning to be and live with others

What is a curriculum?

- The set of experiences that a school intentionally designs and implements for the pupil in order to achieve the set educational goals.
- The design of a curriculum is the process through which the educational research and innovation develop and organize.

Here below we suggest a few characteristics of and requirements for inclusive curricula:

- Tool to foster tolerance and promotion of human rights, to go beyond linguistic, cultural, religious, and gender differences.
- Break of gender stereotypes, not only in textbooks but also in teachers' attitudes and expectations.
- Design of educational and training modules adapted to the needs of pupils at risk of exclusion, but consistent with the formal education that the school system requires.
- Educational paths attentive to the individual, not for the individual.
- Attention to diversity, valuing diversity.
- Greater emphasis on practical, experience-based, active and cooperative learning.
- Participatory approaches that refer to traditional and indigenous knowledge.

In the next part of the paper we introduce examples of the way the last of the above mentioned qualities of an inclusive mathematics curriculum could be achieved.

Inclusive curricula for mathematics call for search and exploitation of mathematical activities in culturally different contexts. The search activities can be conducted under different strands: historical, ethnographical, anthropological. Some search results could represent the starting point for the design of a curriculum where **m**athematics are valued and granted the same dignity as **M**athematics. Integrating indigenous and academic mathematical knowledge is not a easy task, is a real challenge.

> *This is still an under-researched area compared to the above strands. Perhaps this is because it is in this area that ethnomathematics faces its most difficult challenge – that of impacting on the school mathematics curriculum.*

The above remark by Vithal and Skovmose (1997) about ethnomathematics and mathematics education seems to be still to date…

The main question is: How to harmonize the knowledge acquired from research about **m**athematics with **M**athematics curricula designed and implemented for decades, and often rigidly structured in the educational systems of different countries?

Hereafter, three proposals for teaching units to be used in primary and lower secondary mathematics classes are briefly presented.

The mathematics of professional groups: The zampoña micro-project

The research carried out in Italy as part of a project about teaching mathematics in multicultural contexts, funded by the European Union, led, inter alia, to the creation of a micro-project, *zampoña* (the Andean pipes), whose materials were collected in a multimedia CD-ROM (Favilli, 2004).

The term micro-project is introduced to refer to a teaching unit, whose main aims are to promote intercultural and interdisciplinary education, while offering pupils in multicultural classes the opportunity to appropriate and develop mathematical concepts and skills. The teaching unit has been developed from a craft activity typical of a specific culture: the construction of a *zampoña*.

As far as mathematics is concerned, the project is based on the coexistence of three kinds of mathematics "hidden" in the *zampoña*:

- the mathematics *implicit* in the very construction of the instrument, which the craftsman uses, in a more or less consciously, to create it;
- the *explicit* mathematics, embodied by the same zampoña;
- the *external* mathematics, proper to the person who observes and analyzes the craft, and associates to it his own mathematics, partly imposing it while he catches the craftsmanship implicit mathematical ideas and the instrument explicit ones.

It is therefore three different mathematical lectures, inextricably linked and intertwined with each other in the investigation, that, together, contribute to broaden the horizon of mathematics, freeing it from the idea that there is only one mathematics, and therefore that it is universal.

These considerations in mind, it is possible to extract several mathematical contents from the investigated activity, such as:

The methodology chosen and primarily suggested is *learning by discovery* and *group work*. Therefore, the teaching unit can be developed implementing a sequence of manual and reflective activities, followed by a formalization, divided into the following three main stages:

- *discovering the zampoña*: pupils are asked to emulate all phases of the work of the craftsman (as they are shown in the pictures below) and construct their own *zampoña*, thus appropriating some of the mathematical concepts embedded in them; at the same time, pupils acquire knowledge about the culture within which the craft is placed, thus making the activity an interdisciplinary one;





- *let’s know the zampoña better*: pupils should carry out an initial qualitative analysis of the instrument they have built, grasping, for example, the notion of function underlying the nature of the product and then extend it to other situations both real and inherent in the mathematical theory; other mathematical notions related to the *zampoña* and its construction that could emerge are: relations – sequences – order – classifying – measurement – mean average, mode and median – cylinder – circle;
- *what if we want to make a bigger zampoña?*: the problematic proposal aims to introduce the students to a quantitative analysis of the instrument, evoking the idea of measurement, and involving, during the analysis of data collected or provided by the teacher the concepts, among others, of ratio and proportionality.

In the construction of the *zampoña*, pupils are given two data tables that correspond to the measures carried out by the artisan to the end of its construction, the length and diameter of each of the (two sets of) rods he cut and used to construct the musical instrument.

In fact the craftsman measures the length of the rods to be cut through a wooden strip, of length and width compared to the size of the *zampoña* he wants to build; on this strip there are notches, corresponding to the musical notes. This way of measuring, which occurs to some sort of scale built on the basis of individual experience, is itself a reason for reflection and comment in the classroom, for the implicit knowledge and mathematical activities brought into play by the craftsman! In fact, the measurements were made by him using a linear meter only at the end of the construction and at the express request of the author. The comparison between mathematics used inside and outside the school and the different mathematical skills of students and illiterate workers were investigated, for example, by Nunes et alii (1993).

And what about the wealth of links to other disciplines starting from zampoña?

If we want just consider the experimental sciences, there is a clear chance to talk about:

- bamboo canes and soils and climates in which they grow better, their habitats,
- morphological characteristics and climatic conditions of the Andean region,
- flora and fauna characteristic of those places,
- the Amazon River that rises from the Andes and the vast Amazon region,
- the effect that this region has on the climate of the South American continent and the entire Earth planet, etc..

With regard to physics, albeit not in detail, can be introduced

- the laws governing the propagation of sound (in particular through a closed pipe),
- the concepts of wave length and frequency.

Regarding the human sciences it is possible to talk about

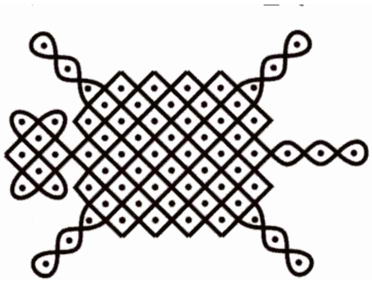
- the functionality of the ear,
- the physiology of the respiratory system.

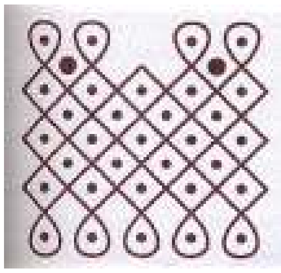
The mathematics of indigenous people: Sona sand drawings [2]

The story-tellers from the Tchockwe people, in Eastern Angola, and Tamil people, in South India, make use of the *sona* (singular: *lusona*), sand drawings, to give a better and more attractive description of their stories.

[2] The teaching unit is described also through quotations from Maffei and Favilli, 2006

 [3]





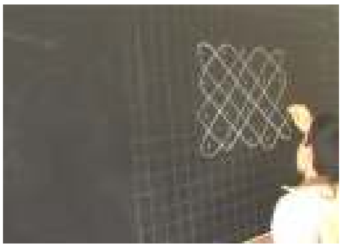
As in the case of many other findings from ethnographic researches (see, for example, Zaslavsky, 1973, for Africa), the *sona* drawing attracted the attention of a few mathematics educators. Their aim was not only to observe and describe the specific mathematical knowledge which the story-tellers where making use of, both explicitly and implicitly, but also to investigate the further mathematical notions and concepts that, in such activity, could be seen or deduced. Gerdes' investigations on the *sona* drawings (see, for example, Gerdes, 1999) represent the most significant and precious reference for anybody interested in the relationships between *sona* and mathematics.

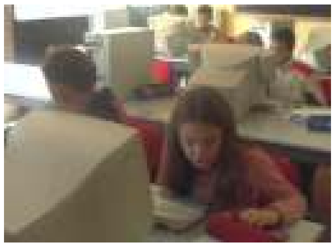
The possible use of this cultural activity, the *sona* drawing, as a didactical tool for the introduction of some of those mathematical notions and concepts in a school context have been and are still under investigation.

One of the first properties is that the number of lines (polygonals) necessary to complete each given *sona*, in accordance with a short list of drawing rules, exactly corresponds to the GCD of the two positive numbers representing the *sona* dimensions.

The didactic goal of the project is to engage pupils in reflective activities. In fact, it is well known that the students' main complaint about mathematics is that it is too mechanical and inflexible. Students very often try to solve problems by only applying memorized rules, with poor understanding of the concept involved. We believe that a more attractive approach to some mathematical concepts could be a solution.

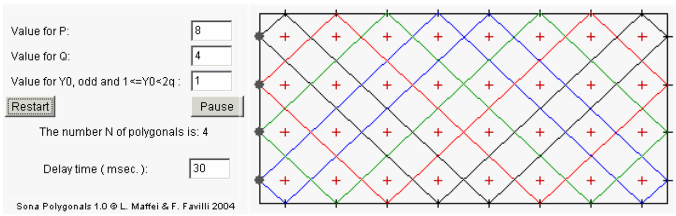
[3] http://ghezzig.wordpress.com/2009/10/22/il-cantastorie-sona/





In view of that, a non-standard didactic proposal for the introduction of the Greatest Common Divisor between positive natural numbers has been developed, under the assumption that GCD is too often introduced by teachers only in a technical and algorithmic way. Pupils, therefore, hardly realize the meaning and the potential of this concept, because GCD is usually associated just to fractions and their operations. The final result is poor attention and lack of interest which, in turn, cause hard comprehension of the concept.

As said, , using Java Programming Language, a graphical programme – *SonaPolygonals_1.0* – has been implemented (Maffei and Favilli, 2006) which draws, in movement, the *lusona* and computes the number N of lines necessary to complete a *sona* of PxQ points.



Further investigations and results about *sona* drawings and their relations to graph theory can be found in Pavanello (2013).

History of mathematics: Yupana, the Inka abacus[4]

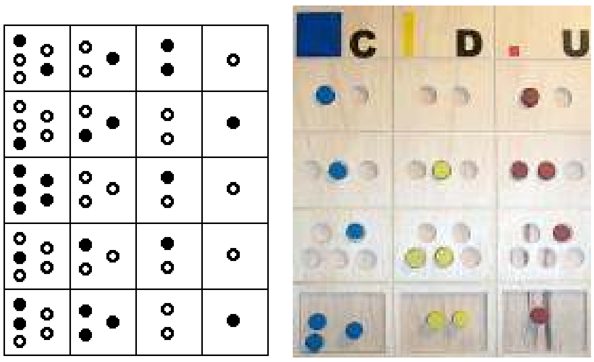
In the present paper, we introduce an electronic version of the *yupana*, the Inka abacus. One of our main aims is to show that it is possible to make attractive and usable ancient mathematical artefacts, which still clearly prove their didactic utility. The electronic *yupana*, in our view, represents an attempt to link tradition and modernity, indigenous and scientific knowledge, *poor* and *rich* cultures. It aims to represent an educational

[4] The teaching unit is described also through quotations from Fiorentino and Favilli, 2006

environment, where pupils and students can find a friendly tool throughout which they can achieve the notion of natural number, compute basic operations, familiarize with positional notation and base change and develop personal “computational algorithms”.

There is very little information about *yupana* and its use, mainly because the Spanish *conquistadores* destroyed most Inka cultural heritage. The only available representation of a *yupana* is part of a design drawn by the Spanish priest Guaman Poma de Ayala (1615) in his chronicle of the Inka empire submission.

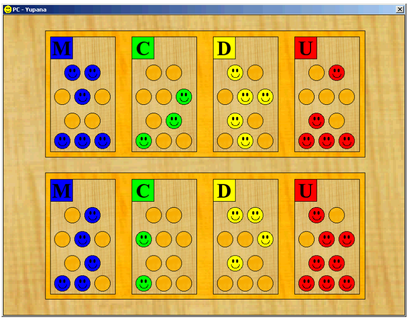
The *yupana* is represented together with the *quipu* (a statistical tool made by knotted strings). Only recently, mainly thanks to M. and R. Ascher (1980), mathematics researchers and historians have focused their attention into such mathematical instruments from the Inka culture. As far as we know, mathematics educators have paid poor attention to them so far…



Ancient and modern (wooden) yupana

In modern *yupana*, numbers are represented as configurations of wood pieces on the board, using different colours for units, tens and hundreds. In the lower part of the board, rectangular areas are used either as a pieces repository or as the starting place for the second operand in arithmetic operations. As far as didactics is concerned, the presence of these rectangular areas is a weak point of the *yupana*. In fact, these areas allow a different representation for the same entities (the numbers and the digits) and can be confusing for children whose concept of number is developing.

In the implementation of the electronic *yupana*, such difficulty is overcome by considering a *digit* only a whole column configuration. Consequently, the computer *yupana* is made of *two* complete traditional yupanas, as shown here below:



In this way, all operands are represented in the same manner and both are immediately visible in the *double yupana*. For instance, the double *yupana* above represents the numbers 6355 and 5248.

A different unifying approach arises from the only *modus operandi* that the computer *yupana* allows and enforces: the *drag-and-drop activity*. By dragging *one piece at a time*, the concept of number is induced by repetition of unitary increment/decrement steps. As in the wooden *yupana* shown above, the pupil is introduced to positional notation/arithmetic with the help of colour correspondences between the two *yupanas*: pieces with the same positional weight are given the same colour. This allows the user to perform positional arithmetic by moving pieces of the same colour to equivalent positions. In fact, the program allows drag-and-drops involving pieces/holes with the same positional weight **only**. In this way, positional arithmetic is actually a by-product! Moreover, the teacher can change the colour scheme to avoid unconscious colour/weight associations.

The drag-and-drop activity, coupled with the general statement "*operation is over when one of the two yupanas is empty*", provides another simple unifying framework for three basic mathematical operations: sum, subtraction and base change.

The sum is performed by dragging all the pieces from one *yupana* to the empty spaces in the other one. Whenever a *yupana* is empty the pieces in the other one represent the result.

Subtraction is accomplished by "eliminating" pieces with the same weight on both *yupanas*, i.e. by dragging pieces to equivalent pieces. It is worth noting that, in this case, the (upper or lower) position of the *yupana* that is empty at the end of the process gives appropriate information about the sign of the result!

Base changing is executed with the same rules as the sum, with two main differences:

- the number of positions (the holes) for each digit on the two yupanas is different (any base between 2 and 10 can be used);
- more colours are normally involved, leading to unitary operations only in the worst case (when the two bases are co-prime) but also to interesting "diagonal drags" (as in the case of bases 2 and 4, when a piece of weight 4 is moved).

These features make the electronic *yupana* a solid mathematical tool upon which a child may build his/her own mathematical foundations in his/her most appropriate and distinctive way: playing!

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