Multiculturalism, Migration, Mathematics Education and Language

Teachers' Needs and Teaching Materials

Franco Favilli (ed.),
on behalf of the Project partners

M³EaL Project

Lifelong Learning Programme

Education and Culture

Multiculturalism, Migration, Mathematics Education and Language
– **M³EaL** – is a project funded with support from the European Commission under its *Longlife Learning Programme* (Project No. 52633-LLP-1-2012-1-IT-COMENIUS-CMP — Grant Agreement: 2012-3837/001-001) and coordinated by C.A.F.R.E. of the University of Pisa (Italy).

The other project partners are: University of Vienna (A), Charles University in Prague (CZ), University of Paris Est Créteil (F), University of Thessaly (GR), University of Siena (IT) and University of Agder (N).

By the training of school education staff, the project aims at enhancing the quality of mathematics teaching in multicultural / multilingual classrooms.

Project life was October 2012 – September 2015.

Table of Contents

INTRODUCTION

Multiculturalism, Migration, Mathematics Education and Language – M³EaL – is a project funded with support from the European Commission under its *Longlife Learning Programme* (Project No. 52633-LLP-1-2012-1-IT-COMENIUS-CMP — Grant Agreement: 2012-3837/001-001).

The project partners are:

- C.A.F.R.E. of the University of Pisa (Italy), *Coordinator Institution*
- University of Vienna (A)
- Charles University in Prague (CZ)
- University of Paris Est Créteil (F)
- University of Thessaly (GR)
- University of Siena (I)
- University of Agder (N).

By the training of school education staff, the project aims at enhancing the quality of mathematics teaching in multicultural / multilingual classrooms.

Project life was October 2012 – September 2015.

Rationale of and background to the project

The multicultural nature of modern society constitutes one of the most significant changes to have influenced schools in many European countries, especially at primary and middle school level. The teacher's job is all the more difficult because he/she is usually not sufficiently prepared to deal with the new classroom context with pupils having a migrant background, coming from countries with different cultures and different languages. The teacher is seldom aware of the need to rethink and if necessary modify his/her methodological and pedagogical approach. This attitude is even more evident in maths teachers who often consider their subject universal and culture-free.

Little has been done in Europe as far as maths teaching in multicultural contexts is concerned. The different languages and cultures present in the classroom make the teaching/learning process even more arduous than it already is, especially for pupils from minority cultures and/or with a migrant background or for Roma pupils.

This project envisages the design and piloting of materials for both the initial and in-service training of middle school maths teachers who constitute the project's primary target group. The secondary target group is their pupils, in particular those from other cultures.

The materials have been produced after careful analysis of the video recordings of teaching activities. Their focus was also on the role of language in the communication of mathematical concepts and their aim was to stimulate the maths teacher's awareness of the need to find a satisfactory balance between mathematical language and classroom language, especially when dealing with pupils with a different culture and language.

The project's training proposals aimed at promoting maths teaching strategies which are relevant to activities and problems taken from everyday life (addressing Comenius specific objective b) including that of different cultures in order to highlight their positive aspects (addressing Comenius specific objective a). The project furthermore addresses several of the LLP programmes specific objectives, namely objective a (to contribute to the development of quality lifelong learning and to promote high performance, innovation and a European perspective […]), objective d (to reinforce the contribution of lifelong learning to social cohesion, […] intercultural dialogue, gender equality[…]), and objective k (to encourage the best use of results, […] and to exchange good practice […] to improve the quality of education and training). We also want to refer to the ET 2020 strategic framework, which states "promoting equity and social cohesion" and the need to "acquire key competences" (one of which is mathematical competence, another one is cultural awareness) for all citizens. It specifically mentions as one of its goals "to ensure […] that all learners – including those from disadvantaged backgrounds, and […] migrants – complete their education […]. Education should promote intercultural competences […]".

Most of the consortium members have co-operated in several previous projects (such as the Comenius 2.1 projects: "IDMAMIM: Mathematics and Multiculturalism", "Lower Secondary School Teacher Training in Mathematics", "Making Mathematics Teachers Mobile" and "CLIL across Contexts: a scaffolding framework for the CLIL teacher education") and in several meetings. During these co-operations the need for addressing multicultural issues in mathematics teaching has been mentioned. This led to the idea of preparing a project proposal dealing specifically with this issue.

Rationale for the setting-up of the consortium

The consortium includes scholars whose expertise and competences in the fields of mathematics or language education and teacher training are recognised at national and international level. They are used to co-operate in project consortia. The interests of the persons involved in the project range from linguistics to language acquisition methodologies (including CLIL and L2) and language teacher training, from mathematics education (in multicultural contexts and with Roma pupils included) to ethnomathematics and mathematics teacher training, from inclusive education to teachers–minority parents educational partnership.

All partners involved have contributed equally to most of the project activities and outcomes, under the continuous co-ordination of the co-ordinator Institution. They therefore acted as a transnational team.

The consortium has benefited from the continuous collaboration with schools and institutions active with minorities that have been associated to the project.

This book is printed and freely distributed among teachers, trainers, members of associations working with minorities, local authorities, during national and international meetings, seminars and conferences. The production of the DVD, the design and production of the project web-site, and the organisation of the project final international conference has been under the direct responsibility of the co-ordinator, after a common discussion on its structure.

The production of the DVD, the design and production of the project web-site was sub-contracted to an ICT expert.

Investigation of the field (state of the art) and innovative character

Mathematicss teachers, especially those working in secondary schools, feel the necessity for training and materials which reflect the needs of their classes in terms of linguistic and cultural differences. Their pupils from minority cultures and/or those with a migrant background encounter even more difficulties than their native classmates in acquiring fundamental maths skills. It is a matter of fact that, in general, the rate of early school leavers among these pupils is significantly higher than that among native ones.

The above mentioned needs have been identified in several research studies carried out by scholars such as: i) Abreu, D'Ambrosio and Skovmose for multicultural and inclusive education; ii) Barton, Barwell and Clarkson for the role of the foreign language in mathematics learning and iii) Bishop, Favilli and Gerdes as to the educational approach and methodologies for mathematics.

The project aims to identify teaching strategies for teachers and activities for pupils who allow both to approach the challenges and facing them satisfactorily. The methodological tools used, to be considered innovative compared with the standard routine of the maths classroom, are the following:

A) Great attention to the language used in order to provide suitable compromise between the simplicity of classroom language and the complexity of maths language, bearing in mind, however, that the language used in class is an element of further complexity for pupils from minority cultures with a different mother tongue;

B) Proposals for didactic units for the maths classroom which facilitate interdisciplinary extensions and which are inspired, above all, by practical problems and situations from everyday life and from different cultures.

These methodological tools should, in general, help to make all pupils more interested and motivated to learn maths; in particular, enable pupils with different cultures and languages to overcome some of the difficulties they encounter in maths due to these very differences: the teaching of maths by using aids to activate different thought processes and skills which otherwise risk remaining latent because of language shortcomings.

Moreover, the above-mentioned methodological tools facilitate the appreciation of the positive aspects of different cultures and create favourable conditions for intercultural dialogue in the classroom, thus creating an inclusive educational setting.

A further innovative aspect is the contribution from language specialists to the communication and intercultural issues of the teacher training activities.

Aims and objectives

The projects' main aim is to provide teacher trainers and teachers with materials and teaching modules that allow them to adequately address the needs of their classes in terms of multicultural and language aspects in the teaching and learning of mathematics. This is particularly important for (but not restricted to) students from minority cultures (such as Roma pupils) and students with a migrant background. These materials will both be useful in raising the awareness of teachers and students to the importance of culture and language in the teaching/learning process, as well as in practical use in modules in the classroom. So the materials will have a positive effect on the teachers' attitude towards cultural differences, and they can be used to support the work of the teacher in the classroom – and not only in classrooms with minority or migrant students, but in all classrooms.

The specific objectives of the project are:

- To collect, compare and analyse data about pupils of minority cultures and/or migrant background in the partner countries, and of mathematics teachers' needs in classrooms with minority and migrant pupils.
- To develop materials for teaching in schools, particularly in multicultural classroom settings. Each partner will develop materials that mostly refer to situations or activities typical for different specific cultural areas.
- To pilot these teaching materials in associated schools of at least two partner countries, to improve the materials using the feedback data from piloting, and to pilot them again in associated schools of at least another different partner country.
- After a further improvement, to have these materials reviewed by an expert external evaluator, before their presentation to and discussion with the participants of the intermediate conference.
- After final improvement, to translate all the materials and make them available in the partner languages and in EN, both in printed form as a book, and in electronic form as a DVD and on the webpage.
- To have materials used in all the associated schools and, also through the collaboration of local educational offices/authorities and associations working with minorities, promote their dissemination in other schools of the partner countries.
- To organise and execute a final international conference to disseminate the materials also outside the partner countries.
- To create and maintain a webpage for informing about the project and disseminating the outcomes.
- To sustain the use of the project outputs by continuing their use in teacher training modules at the partner institutions, as well as continuing the dissemination activities, e.g. at teacher training events or education conferences.

Methodology

The partners collected, compared and analysed data about students with minority and migrant background in the partner countries, and about the requirements of mathematics teachers teaching in classrooms with these students (milestone). This data were used to get a realistic picture of the situation and to be better able to address the needs of the teachers and the students. Based on these needs, the partnership produced teaching materials addressing these issues.

The teaching materials were developed referring to and analysing situations or activities typical for specific cultural areas. The analyses focused on the mathematics that is both explicitly and implicitly present in the considered situations or activities. The analyses also suggested to develop links to other subject contents (such as history, geography, language, literature…), thus giving the pupils opportunity to get better knowledge of the concerned cultures and creating a real intercultural and interdisciplinary perspective in the classroom.

The teaching materials were circulated, discussed, agreed upon within the whole partnership, with respect to their validity and usability in the different partner countries. They were then piloted in the associated schools of the partner countries and in workshops and courses in the partner institutions. The teaching materials were also reviewed by an external evaluator, Leo Rogers, former professor of Mathematics education at Oxford University (UK).

An intermediate international conference was organised and executed, where the draft teaching materials were presented, both to inform about the project and its results, and to gain further feedback data. Based on this data, the teaching materials were developed, piloted and improved based on the piloting feedback from teachers and students. They were translated and published. The piloting activities were partly video recorded. The records were used for feedback analysis. A final conference was organised and executed to make participants aware of the project and to disseminate the outcomes. The outcomes are published on three media: the website, the DVD and in book format.

European added value

Although multiculturalism in schools is a widespread phenomenon in all European countries, much still needs to be done to identify teaching strategies which enable different disciplines to be taught effectively to pupils from minority cultures as well as to the rest of the class. It is of vital necessity to define and experiment teaching materials which do not discriminate against pupils from different cultures and languages. This is particularly true for Mathematics, a subject which – according to comparison results of several international studies – proves by far more difficult to learn than any other.

This project aimed to collect information about experiments carried out in the partner countries and teaching materials for Mathematics in multicultural contexts which have proved effective or worthy of further study and piloting. The project aimed also to produce teaching materials that take into consideration specific local factors such as the cultural and linguistic context of the classroom, the social environment outside the school, the school system and tradition, the majority culture of the country in question as well as several minority cultures present in the European schools, etc. One of the aims of the project was to create, on the basis of national and international experiences, examples of teaching materials to be adapted to the different contexts and cultural-linguistic factors present in European schools. These materials, piloted in different partner countries, represent a synthesis of various national experiences and a tool for reflection on teaching methods which secondary school maths teachers will be able to use throughout Europe.

Expected impact of the project

The main target groups are mathematics in-service teachers and student teachers (pre-service teachers) in secondary schools. The main dissemination tools was represented by continuous contacts with and information to teacher professional organisations, associations working with minorities, local authorities that were invited to attend and contribute meetings, seminars and conferences.

The dissemination of the experiences obtained through the piloting of the teaching materials will increase teachers' confidence in their ability to cope with the multiculturalism in the classroom. The courses and workshops for teachers and trainees organised, in each partner country, to pilot the teaching materials will allow teachers with pupils from different cultures to pay greater attention to communication in maths classes. The careful and precise use of the language, and better acknowledgment and valuation of the cultural differences in classrooms represent key factors for mathematics teachers' development. The consequent new approach to mathematics teaching should represent the methodological change which minority pupils and the whole class could benefit from in their learning process.

Pupils – specifically, but not only, those from different cultures and with different languages – will benefit from the possible changes in the teachers' didactical approach.

Video recordings, teaching materials, reports, etc. will be an additional resource for teacher trainers and represent possibilities for change in their educational approach.

To measure the impact and the results of the project activities on the different target groups, specific evaluation and assessment instruments were produced, making mainly use of questionnaires, individual reports and classroom observation sheets.

The partner institutions will continue to offer national and international courses and workshops for pre- and in-service teachers using the project outcomes. The partners also plan to organize Comenius courses in the years after the project. They will also continue distributing the DVD and the book at teacher training events and at international education conferences. The outputs will be also freely distributed to teachers and schools, to other institutions involved in pre- and in-service teacher training, and through educational authorities and policy makers. The website will be hosted by the coordinator institution, which makes it easy to maintain it, update it, and keep it online.

Dissemination and exploitation strategy

Dissemination

Continuous information was and will be given to the educational community, local, regional and national educational authorities, associations working with minorities about the most significant provisional and final outcomes. Results of the activities were and will be presented at national and international conferences.

The focus of the dissemination is based on the following aims: a) to diffuse knowledge about the content of the project, b) to inform people of the experiences gained throughout the development of the project, c) to provide a forum for the exchange of experiences inside as well outside the project partner institutions.

In view of this, a web site devoted to the project was designed and produced and the final report and final materials of the project have been published and are being distributed.

Making available the main project outputs (website and DVD) in the languages of the partner countries and in EN will ensure the opportunity for the educational community to easily access the project results.

The final international conference was open to the participation of educators and teacher trainers from all over Europe interested in the project topic. Internationally known experts were invited to contribute the scientific programme of the conference.

Comenius courses will be organized in the following years.

Exploitation

The project web-site makes the project results, experiences and outputs easily available to the secondary school mathematics teachers and teacher educators across Europe. Besides the languages of the partner countries, EN is used.

The co-ordinator and the project partners will continue the dissemination of all the project activities and the final results by their presentations to the national and international community of teacher educators and scholars in Mathematics and Language Education during Conferences and Workshops, which they are used to attend since many years.

The above measures should ensure a long-term exploitation of the project results.

Sustainability

As the teaching materials will be used in courses that are organised within the regular pre- and in-service teacher training framework of the partner institutions, the use of the materials will be sustained beyond the project lifetime. Making the materials available to other teacher training institutions, to resource centres and educational authorities, as well as to organisations working with migrants, within and after the project lifetime, allows for an effective distribution and multiplication. Also the continued promotion of the materials at teacher training activities and mathematics education conferences will help sustain the impact of the project by providing information and materials to target group members and other stakeholders. The hosting of the webpage at the coordinator institution assures the continued availability of the materials after the project lifetime.

Part I

Survey on teaching Mathematics in multicultural classrooms

1. INTRODUCTION

The survey was carried out in six European countries (Austria, Czech Republic, France, Greece, Italy, Norway).
A questionnaire was developed by the project team, based on data about multicultural backgrounds of pupils in the participating countries and about available resources for teaching in multicultural classrooms, and distributed among teachers with different experience and length of practice. The questionnaire is divided into four parts.

The purpose of the first part is to find out the basic information about the respondent.

The second part asks for information about the school where the teacher is teaching and the social background.

The third and fourth parts focus on the respondent's prior experience and the support available for teachers working in multicultural settings. Attention is paid to the situations specific for working in such conditions as well as to support of any kind that such a teacher has and/or would like to have. The main questions (about prior experiences and available support) are primarily yes/no-questions with an opportunity to elaborate on the answers in an open-ended format

2. THE QUESTIONNAIRE

About you

1. What is the age range of your students?
2. Do you teach other subjects? Which ones?
3 **a.** What is the length of your teaching experience?
b. What is the length of your mathematics teaching practice?
4. **a.** Did you receive any training for teaching in multicultural classrooms in your initial teacher training? If yes, which type? In what extent did this training have a positive impact on your teaching?
b. Did you receive any training for teaching in multicultural classrooms in your in-service teacher training during the last 12 months? If yes, which type? In what extent did this training have a positive impact on your teaching?
c. Did you receive any training for teaching in multicultural classrooms in your in-service teacher training before the last 12 months? If yes, which type? In what extent did this training have a positive impact on your teaching?

About your school

5. What is the size of population of the village/town your school is situated in?
6. What region is your school located in?
7. What is the (average) percentage of [migrant students] in your class (es)?
8. Does your school have an official program for supporting [migrant students]?

Prior experiences

9. Have you ever taught mathematics to [migrant students]? If not, please go to question 13.
10. Did you encounter any specific issues while teaching maths in these classrooms? If not, continue at question 12.
11. How did you try to handle the situation?
a. Did you find suitable materials? Please describe them.
b. Did you share the encountered difficulties with other colleagues?
c. Did you receive any support from school management? Please describe them.
d. Do you see any advantages of having [migrant students] in your classroom? Please describe them.
12. Did you change your teaching strategies when you had [migrant students] in your classroom? Please describe how.

Supporting materials and environments

13. What kind of materials would you need for teaching in multicultural classrooms?

a. Supporting pedagogical documents?

b. Information about the cultural backgrounds of minority groups?

c. Concrete didactic units from various cultural backgrounds?

d. What else?

3. SYNTHESIS OF THE QUESTIONNAIRE ANALYSIS

The survey was carried out in six European countries (Austria, Czech Republic, France, Greece, Italy, Norway). A questionnaire was developed by the project team, based on data about multicultural backgrounds of pupils in the participating countries and about available resources for teaching in multicultural classrooms, and distributed among teachers with different experience and length of practice.

The questionnaire is divided into four parts. The purpose of the first part is to find out the basic information about the respondent. The second part asks for information about the school where the teacher is teaching and the social background. The third and fourth parts focus on the respondent's prior experience and the support available for teachers working in multicultural settings. Attention is paid to the situations specific for working in such conditions as well as to support of any kind that such a teacher has and/or would like to have. The main questions (about prior experiences and available support) are primarily yes/no-questions with an opportunity to elaborate on the answers in an open-ended format.

The questionnaires were collected from 154 in-service teachers in different parts of the countries, teaching at lower secondary level. This research sample was not representative, researchers in most teams found it difficult to motivate teachers to fill in and submit completed questionnaires. For the aim of this survey, however, representativeness of the sample was not important. What the researchers were looking for were illustrative data collected from practitioners who have, or do not yet have, experience from multicultural classrooms. The findings from the survey are, of course, influenced by the differences in the situation with migrants and minorities in the different countries. While respondents in France and Italy have rich experience with teaching migrant students and thus have elaborated discourse on the topic, teachers in other countries found it harder to formulate their ideas due to lack of hands on experience.

31 respondents came from Austria, 12 from the Czech Republic, 35 from France, 25 from Greece, 79 from Italy and 22 from Norway. The Czech Republic collected their data during in-service teacher training seminars. In Norway questionnaires were sent to teachers in schools in different parts of the country. These were teachers the researchers had collaborated with in different connections (when they came to courses, seminars or workshops, or were doctoral students in their program). They were asked to distribute the questionnaire to some of their colleagues who could be willing to reply. The questionnaires were also sent to all mathematics responsible teachers in one city in southern Norway and asked for their support to distribute to all math teachers. The team were more successful in getting replies from teachers whom they had good contact with. The many teachers in the city produced only very few replies. One of the headmasters told the team that he would get at least one such request each week which means he would throw most of them in the paper basket in

order not to burden the teachers with extra work. In Austria, the questionnaires were handed out to teachers who a) attended in-service teacher training courses at the partner university, or b) who did their pre-service training at partner university, or c) were colleagues at schools where teachers from groups a) or b) teach. This amounted to schools from three (out of nine) provinces of Austria. 53 questionnaires were distributed, of which 36 were given back (31 of which were from lower secondary teachers). In France, the questionnaires were sent to teachers in the region of Créteil by mail, the research team had met these teachers in in-service training, 18 out of these questionnaires were filled, and other 17 questionnaires were handed out to attendants of currently running teacher training course. In Italy, the questionnaire was sent to a mailing list of around 100 math teachers who are the members of a professional association in Tuscany. Most of them are among the respondents. To get, at least, a small piece of information about other Italian regions, a few questionnaires were also emailed to individual teachers who are used to co-operate to the Italian team for national projects.

Qualification of respondents differed in the countries depending on the system of pre-service teacher training. While all teachers in Austria and France and majority of Greek teachers (68% of respondents) teach only Mathematics (with the exception of one French teacher who is also in charge of a training for students discovering the professional world at the end of lower secondary school), respondents from other countries have qualification to teach other subjects as well. In Italy, all teachers teach Mathematics and Science at lower secondary school level; other countries support two or more subject qualification. The most frequent combination in Greece is Mathematics with Geography, in the Czech Republic Mathematics with one of the natural sciences (biology, chemistry or physics). Norway reports to have the highest proportion of multi-subject qualified respondents (14 out of 22 teach more than 4 subjects and mathematics, 18 out of 22 respondents teach more than 3 subjects and mathematics).

In all countries, teachers with a wide variety of teaching experience answered the questionnaires. Out of the respondents, Norway had teachers with the longest average length of practice of 21 years, while France with 10 years the shortest teaching practice. The Norwegian team confirmed that the average age of Norwegian teachers was high and the country was at the brink of enter of new generation into the teaching profession.

Out of the respondents, only Italian teachers reported to have training in multicultural issues in greater numbers (19% of respondents during their undergraduate studies, 14% as part of their in-service training within last year and 24% before that). For the rest of the countries, usually just one or two of the respondents in the country gave affirmative answer.

Though the average percentage of students with a migration background varies in the participating countries, depending on school size, school location, socio-economic or cultural background etc., an overall majority of teachers have already encountered such students in their classrooms. The majority of respondents from all countries come from urban areas (with more than 10000 inhabitants). Considerable number of respondents come from the country capital cities (Vienna – 84%, Prague – 42%, Créteil, Paris – 100%, Athens – 56%) while most Italian respondents come from Tuscany – 76%.

The average number of migrant students in schools for the Czech Republic, Greece and Austria is about 12%, for Italy 13% and Norway 8%. In France, it is not possible to ask about pupils' mother tongue or origin. The system speaks of pupils newly arrived to France (ENAF). The questionnaire for French teachers was therefore adapted, using the French term instead of the term migrant pupils.

The average of pupils newly arrived to France in the classes of French respondents was 3%.

In general, there are big differences among individual schools in each of the countries. In all of them some respondents report to come from school with no migrant student at all (the Czech Republic, Austria, France, Italy, Norway) but at the same time from schools with almost half (or even more than half) of their pupils migrant or cultural minority (Austria 45%, Italy 65%, Norway 40%).

With the exception of the Czech Republic and France where just one third of teachers (CR) or less than half (FR) have prior experience with teaching mathematics to minority students, all countries have a majority of respondent teachers who have this experience (ranging from 83% in Austria to about two thirds in Italy, Greece and Norway). The majority of these teachers report to have encountered problems while teaching these students (all in Norway and Czech Republic, about two thirds in the other countries).

When trying to overcome these difficulties, teachers from Greece report they used ICT (GeoGebra), worksheets and visual aids, Italian respondents used books and papers, teaching units (developed by mathematics education researchers), math teacher training programs offered by the Ministry and local entities, ICTs, vocabulary of symbol names, textbooks. Austrian teachers looked for help on internet websites, especially those focusing on CLIL. French teachers also used Internet resources for primary school, textbooks, ICTs, teaching units (given by colleagues), dictionaries, or asked another pupil to translate. Internet and ICT seem to play an important role when dealing with minority pupils in lessons in all countries.

Sharing of the encountered problems was reported only by a minority of the respondents: in Austria only 1 respondent shared their experience, one half of teachers in Norway and about one third in the other countries. This suggests that more cooperation and communication might be needed. Austrian teachers report to have been most supported by their authorities (more than half of the teachers who had encountered difficulties when teaching migrant pupils). Only about one quarter of Greek teachers report to have received this kind of support. In other countries the same help was mentioned by about one third of the respondents. The form of this support was different in the countries. Austrian teachers were offered in-service training for work with migrant students, Czech and French authorities offered help of psychologist or social worker. Italy offers L2 (curricular and extra-curricular) classes, intercultural board for school inclusion, additional funds, cultural mediators, literacy courses, school facilities, additional teaching hours. French teachers speak of help of colleagues teaching FLS (French as a second language). One Czech and one French teachers report the student was moved to another class: the Czech teacher does not specify, the French teacher speaks of a special structure class.

Teachers named a variety of issues that they encountered while teaching maths in multicultural classrooms. The benefits reported by the teachers are cultural enrichment for everybody, overcoming of barriers and prejudices, growth of tolerance, growing awareness of differences between countries as far as teaching materials, punishments, interpersonal relations at schools are concerned, general improvement of all in mathematics (as the teacher had to pay more attention to how they explain things, used new didactic approaches and different cultural activities for the advantage of the whole class), knowledge interchange and discovery of new methods for calculation.

Teachers speak of the following changes in their teaching strategies: inclusion of more individual and group work and less frontal teaching, more learner centred approaches, more collaboration and peer support, more homework and generally slower pace of instruction, simplification of language and grammar (clearer instructions and more explanations), use of more visual material and aids, graphs, diagrams, schemes, illustrative examples. The Italian respondents also focus on etymology, history and translation of mathematical terms to make it more comprehensible. Teachers speak of careful analysis of materials to be used in their lessons and more monitoring and analysis of pupils' difficulties. The teachers speak of more differentiation in their instruction, e.g. Italian teachers report on using simplified worksheets and tasks for minority pupils. The description of changes in teaching strategies strongly suggest that these changes were helpful also for special needs pupils and low achievers.

As far as supporting materials are concerned, 85% of all respondents would appreciate information on cultural backgrounds of their minority pupils and concrete didactic units from various cultural backgrounds. There are big differences in preferences of respondents in different countries, information about cultural background seems to be very important for Greek respondents (88%) but also Italian teachers (with 70% of yes answers is the most asked for material support in Italy) and little important for Austrian respondents (only two out of 31 respondents).What the Austrians ask for most are concrete didactic units (58%). Overall, a slightly lower but still considerable proportion of all respondents (75%) would find also supporting pedagogical documents useful. Supporting pedagogical documents are most asked for by the French teachers (86%), while Czech and Austrian teachers are sceptical to their usefulness (asked for only by 33% of Czech respondents and 26% of Austrian respondents).

The respondents also proposed other materials that they would find useful in these situations. Teachers asked for bilingual maths and science dictionaries, language support materials and L2 courses for minority pupils, adapted textbooks (easily readable, generally accessible, with a lot of images on fundamental disciplinary concepts, presenting daily life contextualized activities), visual aids, presence of assistants in the classrooms, ICT in the classroom, interactive boards and internet access, e.g. to access websites with mathematics in pupils' native language, vocal translator software and video-cameras. They also asked for a reduced number of pupils in the classroom, for multicultural training and information not only about the culture but also about background education of the pupils, information on syllabi, national programmes and standards and learning/teaching styles in pupils' native countries.

4. THE QUESTIONNAIRE ANALYSIS

QUESTIONNAIRE ANALYSIS: AUSTRIA

The Austrian team received 36 responses, 31 of which are from lower secondary teachers (students' age 10 – 14), on which this analysis is based. The average teaching experience is 14 years. Only 2 teachers received any training about teaching in multicultural classrooms, by attending professional development courses specialising in this area. On average, the percentage of students with a migration background in the classes of these teachers is 12%, with a wide range between 0% and 45%. About 83% of the teachers taught mathematics to migrant students at some point in time. Almost none of their schools (only 1 case) offer a special programme for pupils with a migration background. Typical issues that have been reported by teachers were difficulties of such students in understanding complex word problems (9 teachers) and in expressing mathematical connections with a satisfying degree of exactness (7 teachers). Advantages seen by teachers were addition of multicultural contexts (8 teachers), other calculation methods (5 teachers), and the need for teachers to express things in several different ways and by that also helping non-migrant students understanding things better (5 teachers). 3 teachers reported finding suitable materials in the internet (mainly on CLIL sites or on sites dealing with word problems or contexts from numerous cultures). Only 1 teacher connected with colleagues to seek support, and 2 connected with school management (and were subsequently sent to the professional development courses mentioned above).

The majority of teachers with migrant students (18 teachers) wished for concrete didactic units from various cultural backgrounds. 8 teachers asked for a pedagogical support, mainly in expressing mathematical content in a suitable verbal and non-verbal way, so that non-L1-learners have better opportunities to understand explanations. Only 2 considered additional background information about students' cultures as a suitable tool.

QUESTIONNAIRE ANALYSIS: CZECH REPUBLIC

The questionnaire was answered by 14 in-service teachers, two primary school teachers (teaching 6-11 year old pupils) and twelve lower secondary school teachers of mathematics (teaching 11-15 year old pupils). They came from different part of the Czech Republic, from villages, towns and cities. Among the respondents there were teachers with very short teaching practice as well as very experienced practitioners (3 with 1-5 years of teaching practice; 3 with 6-10 years: 2 with 11-12 years: 2 with 20-25 years and 2 with 26-30 years of teaching practice).

The percentage of immigrant and minority pupils in classes was influenced by the size of the population in the town or village. While village and small town teachers claim to have no immigrants in their classes, in towns with population higher than 5000 the average percentage of immigrant and minority pupils was 1.5% and in towns and cities with population higher than 10 000, this proportion was 6.7% on average. Thus multicultural classes seem to be the phenomenon of larger towns and cities.

As the Czech Republic is not a significantly multicultural country yet in comparison to some other European countries(though the proportion of foreigners has been constantly growing since the political changes in 1989, the number of foreigners in the Czech Republic about 70 000 in 1993 and 436 000 in 2011, i.e. about 4.3% of population of the Czech Republic (https://www.czso.cz/csu/cizinci.nsf/kapitola/ciz_pocet_cizincu), it does not have a long tradition and much experience of integration of pupils with different cultural and national backgrounds into schools. Teachers who graduated before 1989 had no opportunity to attend any courses of multicultural issues in education or intercultural psychology during their undergraduate studies. However, the questionnaire showed that “fresh” university graduates (teachers with teaching practice shorter than 1 year) also had no undergraduate training in multicultural issues, which is very unsatisfactory. Out of the 14 respondents only one states to have attended a course focusing on teaching in multicultural classrooms. Considering that 12 out of the 14 respondents have some experience with presence of a child of different culture in their class, it is more than necessary to introduce courses in multicultural and cross cultural issue into pre-service but also in-service training.

The analysis of the questionnaire suggests that teachers with shorter teaching practice, i.e. probably younger teachers, seem to have more experience with teaching in multicultural classes, have been forced to deal with the situation, to think of materials they could use and also of benefits of the situation. Only two respondents speak of support of their school management – 1 respondent refers to a very complicated problem of a pupil who she could not manage, the headmaster decided to move the pupil to another class (the respondent does not mention how the situation changed after the pupil had been moved) and 1 respondent speaks of being offered by school management the chance to attend voluntary training for work with minority pupils. The benefit of having a child with different cultural background in the class is that all pupils get to know other cultures and learn about differences between people

(3 teachers). With respect to teaching strategies that had to be adapted to the new situation in the class there was more use of individual work (1 teacher), more control of the class (one teacher) and very clear instructions (1 teacher). Among the materials they would find really useful and supportive they most often mentioned a dictionary of mathematics terminology in the pupil's language, one teacher also mentioned the benefits of having pictures and one believed support o an assistant in the class would be very helpful.

QUESTIONNAIRE ANALYSIS: FRANCE

The questionnaire was answered by 19 in-service teachers in lower secondary school of region of Créteil (one has left last year the region). Their pupils are 10-17 years old and all of the respondents teach pupils 12 years old or less.

Among the respondents, there were teachers with very short teaching practice as well as very experienced professionals: 6 teachers with 2-5 years of teaching practice; 7 with 6-10 years; 4 with 11-20 years; 2 with more than 20 years of teaching practice. All of them teach only mathematics and their experience as teachers is only as mathematic teachers, excepted for one of them who is also in charge, at the end of lower secondary school, of a training for pupils discovering the professional world.

The towns where they teach have more than 10000 inhabitants (14 teachers) or between 5001 and 10000 (5 teachers).

In France there are some special classes, named UPE2A (Pedagogical Unit for Pupils Arriving and Allophone), for pupils newly arrived in France, and we can't ask any question about minorities, language spoken at home and so on. So we had to adapt some questions:

- the questions 9,10,11,12 concern pupils newly arrived in France (ENAF) (and not all the migrant pupils)
- there is no different program for migrant pupils, so the 8th question has been changed, in: "Does your school have any particular structure for supporting pupils newly arrived in France?"

47% of the respondents are teaching in a school in which there is a particular structure to help pupils newly arrived in France: a UPE2A (4 teachers), a structure to teach French language as a foreign language or secondary language (2 teachers), a structure to help pupils with difficulties and pupils newly arrived in France can be helped in this structure among other pupils (1 teacher).

The average of the percentages of pupils newly arrived in France in the classes of the respondents is 4% (min = 0%, max = 15%).

58 % of the respondents (11 teachers) have already taught to pupils newly arrived in France in their school or in a previous school. It is sometimes an old experience or a new experience this year. One of them has taught in a special class for pupils newly arrived in France (UPE2A), another has taught FLE (French language as a foreign language).

Only one of the respondents has received an initial training about multicultural classes (initial training in the region of Créteil): 3 hours about allophone pupils with some ideas to adapt mathematics teaching to this public, and three hours about the management of conflicts connected to the multiculturalism.

The 10 other teachers who have taught pupils newly arrived in France, have not received any training for teaching in multicultural classrooms (initial training or in-service teacher training) and they have been forced to deal with the situation. Only two of them consider that they have found some suitable materials. The 10 teachers have searched special materials on Internet and in primary textbooks and almost all of them have discussed with their colleagues, sometimes with colleagues teaching FLE (French language as a foreign language). However one teacher remarks that his colleagues don't manage to adapt their teaching strategies to classroom with many pupils having difficulties with the school language and his opinion is that they really teach only to five pupils per classroom...

Teachers generally don't receive any help from school management. Only two of them have received an help, in one case to move a pupil newly arrived in France to a special class and in the another case to call together different professionals (headmaster, teachers, educational psychologist and social assistant) to welcome the pupils the best.

Some of them have changed their teaching strategies, for example:

- using more pictures, diagrams, colours and less text (5 teachers),
- working in differentiated groups with many different activities, even with individual activities (3 teachers),
- adapting assessment like MCQ,
- introducing for all the pupils, at the beginning of every activity, five minutes for individual search long enough to be able to help the pupils newly arrived in France (1 teacher).
- using the results of the test which is proposed to every pupil arriving in France (level in French language, school skills and familiarization to write in his previous school language, other skills and interests) as a diagnostic, establishing some individual discussions with the pupil and a guidance by another pupil (1 teacher).

All the respondents think that it would be interesting for them to have some specific resources to teach in multicultural classes:

- supporting pedagogical documents (11 teachers),
- information about teaching mathematics in primary school (contents, methods) (2 teachers),
- information about the cultural backgrounds of minority groups, about geopolitical reasons for their migration (in the aim to avoid some contexts in the situations proposed to pupils, to understand some ethnic conflicts) (10 teachers) but two teachers think that the cultural backgrounds are not the main problem,
- concrete didactic units from various cultural backgrounds (8 teachers),
- information about curricula in the countries of origin (1 teacher),
- one teacher suggests that the new technologies could be used efficiently: pupils could use an individual computer with Internet (or their i-phone which is in their pockets...) in the classroom to look at mathematics websites in different languages.

One teacher suggests also that the exams have to evaluate mathematic knowledge and skills rather than the adaptation to the school mathematics French context.

6 teachers think that there could be some advantages of having pupils newly arrived in France in a classroom if their level in French language is good enough to an “ordinary classroom” and if they went to school before their arrival in France: cultural exchange, opening up to mathematics teaching in other countries, motivation of foreign pupils, motivation of pupils (French or not) to help the newly arrived pupils even if they have themselves some difficulties, exchanges about the material conditions in school, about school system (punishment for example). In other cases, the respondents consider that a special classroom is better for these pupils in a first transitional phase.

In this French questionnaire, some questions concerned only pupils newly arrived in France. However, many teachers in the region of Créteil have to teach in multicultural classes.

QUESTIONNAIRE ANALYSIS: GREECE

Answers to the questionnaire were received from 44 in-service teachers, of whom 25 are teachers in the lower secondary school (teaching 12-15 years old pupils), 8 teachers are in the upper secondary school (teaching 15-18 years old pupils) and 11 are in the primary school (teaching 6-11 year old pupils).

The teachers came from the Regions of Attica (Athens, outskirts) and of Thessaly (Volos and outskirts). 10 teachers are working in towns, 2 teachers in villages and 32 teachers in places with more than 10000 inhabitants. The average length of teaching is 19 years and the length of teaching mathematics 18 years, varying from 5 to 30 years.

In the lower secondary classes 8 teachers are teaching some other subjects: geography, professional orientation and environmental education. In the upper secondary classes there are 3 teachers who are also teaching astronomy. In the primary classes it is common for teachers to teach most if not all the subjects of the school programme.

Almost all the lower secondary teachers claim that they got no training or education for teaching in multicultural classes. One teacher just mentions a presentation about the theme of multiculturalism he attended once during his undergraduate studies and how useful it became for him when he had to teach in a multicultural class. There is no significant difference in the others two groups of teachers (upper and primary teachers). One teacher of each group claims that the preparatory training program he attended just before starting teaching in school had multiculturalism as one of its themes and two upper-secondary teachers mention that they have recently attended (before /during the last 12 months) an in service seminar about multiculturalism which they evaluate positively as it was associated with applications in the classroom.

In the lower secondary classes on average 12 % are immigrant/ minority pupils, varying from 4% (1 class) to 25 %. Classes in towns with a population higher than 5000 and in cities with a population larger than 10 000 appear to have similar numbers of immigrant / minority pupils, while in small towns the proportion of pupils from different culture drops to 8%. As expected, primary school classes have more immigrant pupils (19%), whereas in the upper secondary classes this proportion drops to 6%, varying from 0% to 5% (3 classes). Several teachers from all the three levels of education mentioned the reduction in the percentage of immigrant pupils in recent years (families leave the country).

Only 5 teachers of the total sample state that the school has an official programme for support of immigrant children, mostly programmes of teaching Greek as second language (referred as supplementary teaching about the requirements of language classes).

While all of the teachers claimed that there are minority/ immigrant students (at least one student) in their classes, when asked about their experience in teaching in multicultural classes only 61% of the teachers respond positively. In our opinion, this "contradiction" could be interpreted by the fact that in many classes- as noticed by teachers- pupils of different cultural origins have been born in Greece, so they already had sufficient communicative skills in the Greek language. In the same line perhaps we could also interpret the low percentage of teachers (25%) who claim they have not met special problems when teaching pupils with immigrant/minority background.

Concerning environment and support, 2 teachers claim that they received support from school management and 13 teachers (30%) shared the problems with other teachers. Of all teachers respondent, 25% of the teachers saw advantages in teaching in multicultural classes. According to them, the benefits are associated only with the teaching of others subjects and not with math (1 teacher) and two teachers state that having pupils from other cultures in the class make students of the dominant culture develop empathy towards diversity; the rest of the teachers who answer positively did not specify their answer.

From all teachers, 22% (10 teachers) felt forced to look for appropriate materials they could use for teaching in multicultural classes. As suitable material, teachers mention the use of software and geometric instruments, worksheets, visual materials and two teachers answered that they used mathematics history and a cross-disciplinary approach. One teacher answered that he/she made material by him/herself without describing the type of the material he/she made. A teacher that taught in a multicultural type school, many years ago, wrote: *the students' level was not suitable to deal with this situation, because there were students that couldn't count or read.*

With respect to teaching strategies, about 50% of all teachers in our sample and 56% of lower secondary teachers respond positively. Simplification was the more common strategy appeared. Some teachers don't clarify what kind of simplification they used while others speak about math terminology or vocabulary in general. A second common strategy was spending more time for these students. The use of more visual material, more homework, collaborative teaching, more examples and slow pacing during the teaching were also referred as preferred strategies. One teacher answered: *I used mathematical problems adapted in their interests and their cultural background,* while another: *no change of strategies, because I didn't have the appropriate conditions.*

The kind of materials that respondents think that they would need for teaching in multicultural classrooms are: a) Supporting pedagogical documents (64%), b) Information about the cultural backgrounds of minority groups (82%), and c) Concrete didactical units from various cultural backgrounds (77 %). When few teachers became more specific about the kind of material needed, they most often mentioned a dictionary of mathematics terminology in the pupil's language, and two teachers suggest that new technologies could be used efficiently (internet connection, interactive blackboard).

QUESTIONNAIRE ANALYSIS: ITALY

The questionnaire was answered by 79 in-service lower secondary school teachers of mathematics (teaching 11-14 year old pupils), mainly from Tuscany Region. The majority (48%) of the schools are in towns and cities with more than 10.000 inhabitants, whereas 28% of the schools are in villages and towns with 1.001 to 5.000 inhabitants. Among the respondents there were teachers with very short teaching practice as well as very experienced practitioners (3 with 1-5 years of teaching practice; 17 with 6-10 years; 24 with 11-20 years; 35 with more than 20 years of teaching practice). All of them teach both mathematics and sciences, as in the Italian lower secondary schools the two subjects are taught by the same teacher.

Italy is a significantly multicultural country in comparison to some other European countries, with the percentage of foreigners increasingly growing in the two last decades. As to the education system, the number of foreign (i.e. with non-Italian citizenship) pupils in the Italian schools raised from about 307000 (3.5% of the school population) in the 1993/94 school year to 756000 (8.4%) in 2011/12. The figures and the percentages are even greater if all pupils with minority cultural background are considered.

The percentage of minority pupils in the respondents' schools is 13%, ranging from 1% to 65%.

In Italy, multicultural class is an actual, but also a varied phenomenon, due to its

- *geographical distribution*: foreign pupils mostly concentrate in Northern and Central Italian schools, only a small percentage being enrolled in Southern ones;
- *citizenship variety*: in a town or a region foreign pupils can represent a high number of citizenships (at present, about 200) with significantly different distribution of presences (about 141 000 from Romania, 103 000 from Albania, 96 000 from Morocco, 34 000 from China, …);
- *flow change*: the presence of foreign pupils is a direct consequence of the migratory flows from different areas and countries, that is, in turn, caused by different and, sometimes, unpredictable events, such as job search, poverty, wars …

Despite inclusive education is, according to the *The Italian Way to Intercultural Education and the Integration of Foreign Pupils* white paper (Italian Ministry of Instruction - 2007), the pedagogical choice for the multicultural classes, the answers to a questionnaire item clearly show for Mathematics, that in Italy very little has been done until now, as far as subjects teaching is concerned. Only the teaching of the Italian language as a second language has been paid adequate attention: teacher training courses are being organised, and foreign pupils are provided with special assistance and extra classes for Italian language learning.

Only in the last decade, the initial secondary teacher training courses had a module on intercultural education under a pedagogical perspective. The questionnaire analysis showed that very few other training opportunities were made available (1 respondent refers to a two-years project held by an Albanian cultural mediator). Only 8 out of 79 teachers benefited from these opportunities.

The recent in-service training referred to by 4 respondents in the questionnaires were represented by the attendance to meetings and two training courses on mathematics education in multicultural classrooms.

Prior in-service training activities were mentioned by 11 teachers: seminars and conferences, refreshing courses organised by INDIRE (the Ministry of Education Institute for the Training and the Educational Research), Schools, Universities, Associations and Local Authorities. The impact of the training is variously referred to: the individual enrichment and position towards the different educational context is clearly and frequently outlined. A change in the way the lesson and the material (including the production of a textbook) are prepared and organised, are also mentioned by 4 respondents.

The large majority (72%) of the respondents' schools have an official programme to support minority pupils educational inclusion, whereas 61% of the teachers already experienced multicultural classrooms.

In particular, answers refer to intercultural boards and teachers in charge of multicultural issues, intercultural projects, linguistic laboratories (L1 and L2), and individually centred programming.

The educational context and the difficulties met by teachers (49%) were tackled in different ways:

- *looking for adequate materials* (25%) : books and papers, teaching units (by mathematics education researchers, math teacher training programmes by the Ministry and local entities), ICTs, vocabularies of symbols names, textbook,...
- *searching for collaboration from other teachers* (35%);
- *asking for support from the school management* (27%): L2 (curricular and extra-curricular) classes, intercultural board for the school inclusion, additional funds, cultural mediators, literacy courses, school facilities, additional teaching hours;
- *identifying advantages from the educational context* (37%):
    - *The teacher*
      Professional development: identification of new didactic approaches and different cultural activities to be introduced in mathematics classes for the advantage of the whole class;
      Change of attitude.
    - *The pupil*
      Interpersonal relations and communication;
      Cultural enrichment;
      Inclusive behaviour;
      Overcoming prejudices and cultural barriers;

Knowledge interchange.

- *Both teachers and pupils*
  Wider way of thinking and respect of different cultures.

Nevertheless, a few difficulties emerged from the analysis of the answers by the respondents:

- Large number of pupils in the classroom impacts negatively on the socialization process;
- Language related issues can slow the teaching activity;
- Ordinary teaching hours are not sufficient to put into practice the necessary didactical changes;
- Theory – Practice possible conflicts.

Respondents explicitly observe that most educative actions are based on personal research, as searching on web-sites for educative models or suggestions to teach foreign pupils (e.g. Education 2.0,...) and participating in forum; studying relevant publications. They comment that, despite the large amount of materials available, only a small part had an adequate experimentation. Other respondents say to have experienced additional activities out of the class-group and peer-to-peer support to minority pupils by native class-mates with good communication skills.

Nearly half of the respondents (42%) claimed that a methodological change was necessary; in particular:

- *About language*
  Simple, clear and basic language;
  Small use of words and definitions, large use of icons, schemes, examples…;
  Slow speech, capital letters;
  Critical reading and rewriting of the textbooks;
  Smooth introduction of concepts and terminology;
  Etymology, history and translation of the mathematical terms.

- *About general methodology*
  Careful analysis of features of the class-group;
  Systematic control and interpretation of difficulties (due to the language understanding or to misconceptions/cognitive blanks);
  Individually centred teaching, even by using simplified work-sheets or summary forms;
  Group and workshop activities, cooperative learning, concepts mapping;
  Narrative methodology;
  Historical references;
  Greater care in topics selection and methodological choices (e.g. practical activities, use of structured material,...);

Knowledge socialization activity: valuing minority pupil's knowledge about a given topic, and identifying and discussing differences and similarities to the mainstream knowledge;
Introduction and comparison of arithmetic algorithms from different countries;
Slowing teaching flow.

- *About assessment methods*
  Choice of assessment methods relating results with actual individual possibilities;
  Simplified classroom tasks.

The need for supporting pedagogical documents was identified by 59% of the teachers, information about the cultural backgrounds of minority groups by 71% and concrete didactic units from various cultural backgrounds by 66% of the respondents.

In addition, 37% of the teachers claimed to need also:

- Specific training courses and meetings for experiences sharing ;
- Reduced number of pupils in the classroom;
- Language support materials and L2 courses;
- ICTs;
- Textbooks:
  - easily readable and generally accessible
  - including a lot of images on fundamental disciplinary concepts
  - presenting daily life contextualized activities
  - bilingual
  - digital;
- Technical scientific dictionaries;
- Vocal translator software;
- Video-cameras.

Other requests concern:

- structures for the development of basic abilities in Italian language and mathematics to be attended before the inclusion in the classroom;
- information about the different cultural and educational context;
- information about standard learning/teaching styles in students' native countries.

The respondents outline the practice of "age-related inclusion" that implies a relevant discomfort concerning communication both for teachers both for students. Moreover, they claim the lack of a system to collect reliable information about individual previous curricula: generally very few of these can be found and they are not trustworthy in spite of the usefulness. Moreover they ask for official disciplinary programmes in use in the origin countries.

QUESTIONNAIRE ANALYSIS: NORWAY

The quantitative part:

Answers were received from 26 teachers from 7 different regions of Norway. Of these teachers 12 teachers are primary teachers, 10 lower secondary and 4 upper secondary teachers.

All of them are teaching other subjects besides mathematics, such as Norwegian, English, religion, art, sports, music, social science, natural science, chemistry , physics, economics, justice, special pedagogy, media, ICT and so on.

The average length of teaching is 19.1 years varying from 2 months to 36 years and the average length of teaching mathematics is 17.3 years varying from 2 months to 36 years.

Almost all teachers claim that they got no training or education for teaching in multicultural classes.

Four teachers are working in places with 1001-5000 inhabitants, 3 teachers in places with 5001-10000 inhabitants and 19 teachers in places with more than 10000 inhabitants.

The regions (fylke) represented are Troms, Oslo, Narvik, Vestfold, Vest-Agder, Aust-Agder and Rogaland, representing areas from the far north to the south of Norway.

On average 8.3 % of pupils are immigrants, varying from 0 (in 11 cases) to 40 %. 10 teachers reply that the school has an official programme for support of immigrant children, mostly a programme for teaching Norwegian as a second language. 62 % of the teachers have taught immigrant pupils at some occasion.

58 % of the teachers have met special problems when teaching immigrant pupils.

12 teachers found appropriate material for teaching in multicultural classes; 11 teachers shared the problems with other teachers; 8 teachers received support from the school management; and 11 teachers saw advantages in teaching immigrant pupils.

9 teachers claimed that they changed their teaching methods when teaching a multicultural class.

Concerning support and environment the number of answers with yes from teachers were in 13 a) 15, 13 b) 17, 13 c) 13 and 13 d) 16.

From the qualitative part of the questions:

Concerning question 4 the following comments are given:

There were hardly any multicultural pupils when I took my education (32 years ago). I am currently taking part in in-service education but this has not been a theme. I have taught multicultural classes when I had pupils from other cultures, 1-3 per class. (Answer 11)

I read a paper about ethno-mathematics. No, but I generally search for how all pupils think. (Ans 18)

No education but we are a receiving school so we get training every day. Minor courses arranged by the school were offered. (Ans 26)

Question 8: Norwegian as second language (3). We have one class where all pupils are accepted on special conditions because they have lived a short time in Norway. There are specific resources for these pupils (6). We have a special class where these pupils are taught. When they are ready for it they come into normal classes, first in practical subjects (7). Yes, in Norwegian (12). Norwegian-2 offer, but my impression is that it is up to the teacher who gets these lessons to choose content (which is normal in Norwegian-2) (16).

Question 10: Language problems and cultural and other differences (1).Lack of language and concepts. Another algorithm for multiplication (4). It varies how well the pupils know Norwegian and how good they are in the studies. Some are weak in Norwegian, but strong in subjects. Others are good in Norwegian (maybe born in Norway) but weak in the subjects. As ethnic Norwegian pupils. There are a few pupils who are weak in both Norwegian and the subject and it is problematic to teach them because they are often far behind (6). Difficult to know what is lacking conceptual learning from before or if it is language problems. To discern specific mathematical difficulties or are they to be blamed lack of understanding because of limited language acquisition? (11). Language becomes a barrier (12). Language problems – the pupil came directly from a refugee camp in Etiopia (13). Difficult because the pupils do now know Norwegian (14). Do you mean own classes with immigrants? Or classes containing immigrants? My problems are to help immigrant children to develop good concepts for use in mathematics. What concepts are central and how can we train them without working concretely, which is time-consuming and binds much teacher resources. I wish an overview with concepts grouped after how basic they are for example the number names be more basic than more, fewer, as many as. Especially hard is it with pupils who speak good Norwegian and seems to be as good as the others. Then they fall through in single subjects because they lack the basic concepts, which all take for granted that they can. New pupils in Norway and the youngest pupils in first and second class will first of all demand the oral concept. They also need to say the words themselves and use them in practical situations. Pupils who learn other algorithms at home than in school. This creates frustration for both pupils and parents. It would be good to know a little about other algorithms and ask the pupil to choose (usually they want to count as other do) (16).

Descriptions

a) of materials: I found literature and concrete material based on values (3). Focus on conceptual learning (4). I take longer time to explain the content and ask them to formulate the question in such a way that they understand what they are asking for (6). I talked to the first language teacher and investigated if he could check the concepts and understanding in the first language, possibly assist with the learning of mathematics for this pupil in the same way as I did in the common class. I found suitable material but cannot remember what it was (11). The pupils got textbooks for primary school pupils (12). The pupil worked initially much with the four operations- as he had some knowledge about that (13). Difficult to find textbooks that do now demand good knowledge of Norwegian (14). Now I assume you mean groups with pupils with 2 languages. I have only had lessons with one pupil alone. Then I did not search for material but had lots of different things. For example blocks which we talked about with concepts linked to size, big, small, bigger, broader, thinner, longer, higher and so on. Or other times words for places: in front of, behind between …(16).

b) shared problems: I talked a lot with colleagues with more experience and math colleagues. Also got help to map difficulties andfind material. (3). Learnt about the algorithm and talked with the pupil and agreed about the most adequate one (4). Yes, we do exchange experiences a lot at our school (11). Yes, with a special teacher who worked with other children in the class (13). Partly, but since there are so few immigrant pupils in our school there are few teachers in the same situation (14). Yes, the conceptual structure is linked to first language learning. The teacher who is main responsible and the Norwegian teacher can naturally talke about these things (16).

c) support from school management: They helped me with support from the resource team which has been created at the school. They were interested in how things developed (3). The management was not directly involved (4). I did not talk to the school management as I solved the problem with the first language teacher (11). Later the first language teacher also entered the scene, and took some teaching in mathematics (13). Little support (14). No, but I think it is because of lack of trust (16).

d) advantages: Norwegian students learnt about other cultures (1). What she was helped from, several other students had help from. Not least in relation to concepts and concretisation. It helped to focus on concepts and understanding of content and weak concepts/experiences with several pupils (3). Contributes to create tolerance, respect and understanding for other cultures. Created concrete insight in another algorithm (4). It is helpful with variation because then different solution methods can be presented (6). The advantage is to get new approaches in ways of thinking which we can share. Additionally this is a richness in other subjects also (social science, religion…) (11). No, they are as pupils commonly, but not special for immigrants (12). With mathematics glasses on? No. With contact teacher’s glasses on, yes. Pupils get insight into other cultures, religions (16).

Question 12: You have to be more distinct (1). I became more aware of pupils’ background of experiences in relation to mathematical concepts. Quicker to create experiences for pupils and work with language (3). Had to be more awake especially in relation to language and the concepts you use. Was careful to see the pupil after instruction of new material in order to be sure that she understood what we had went through and talked about. Had to be more aware and careful about how I presented about Islam (4). Not especially because of immigrant pupils. I use many strategies because pupils think in different ways. We talk much mathematics in lessons where pupils explain to each other how they think… also immigrant pupils (11). Yes, more clear instruction, more repetitions (12). I tried to find suitable approaches for these pupils. Difficult to get them along in ordinary teaching in the class (14).Tried to have more focus on concepts with the concepts of the week in first class. I cooperated with the Norwegian teacher. Think it was good for all pupils, maybe except for the pupil who had 2 languages with non-Norwegian speaking parents and got no help at home (16).

Question 13

a) Maybe. Have not met such things yet so it is difficult to say (6). Mathematical approaches that do now assume good knowledge in Norwegian (14). As mentioned overview of basic concepts. Preferably with visual picture cards (16).

b) All pupils come from different families with different cultural backgrounds and are raised differently. Could rather have focused on each individual pupil's background than the whole minority-group (6). You get that little by little (14).c) Maybe. Had been interesting to see how different culture look upon education (6). I do not think the cultural background is the most important (14). It is useful to know how many years of schooling the pupil has got in the home-country. And a little about what is emphasised in mathematics teaching there. It was useful for me to read Hvenekilde, it has some overview of different algorithms used in other countries. It is useful in the meeting with both pupils and parents (Anne Hvenekilde, 1988, "Mathematics in a language we understand!": pupils from language minorities and the mathematics. Oslo: Cappelen) (16).

d) I have little experience from this and thus it is difficult for me to answer, but it would probably be most helpful with information (4). More careful follow-up of each pupil demands making free time for this (6). As I am not teaching these pupils I have not thought about the problems (7). Have not worked with the problems (9). I think I do not need special mathematic material for immigrant pupils, but use the material we have in the school, and this has been complemented much in later years in addition to visualisation with Smartboard. We have material pupils themselves can manipulate in order to train practically and transpose this to abstract thinking later on. To get information about the cultural background of the pupil is useful in order to easier help the pupil. Parents are part of the child's tradition and can maybe have more difficulties in helping the child. It is important for the teacher to understand their background and possibly draw their culture in where it can make the teaching richer and put the immigrant pupil in focus (11). Easier access to interpreter (12).Information about the background of the pupil both socially and about subjects. Get education or courses about how to teach multicultural pupils (13).Could use diagnostic tests in order to find the level. They should rather not assume special knowledge in Norwegian. A special textbook with pictures, numbers and exploring tasks. Teaching approaches on the internet. Learning of concepts and basic skills in mathematics (14). No need as I am not teaching in multicultural classes (15).

Preferably app’s or computer-programs where pupils interactively can train concepts. Linked to mass, time, number, position , ... In this way pupils can work at home or independently in school without needing a teacher resource all the time. I mean it should be technically possible that the pupil himself writes the word and then can hear it or the teacher can collect the iPad and control that the word is correctly pronounced. Parents to bilingual pupils often wish to help. Then it could be fine to show our standard-algorithms and explain them visually on the internet with supporting concrete material or where one could choose explaining voice in different languages (16).

Part II

Teaching materials for Mathematics in multicultural classrooms

WHO DID WHAT IN MATHEMATICS IN MY COUNTRY?

by Andreas Ulovec*

INTRODUCTION

The idea of this unit is to use history of mathematics to demonstrate that many cultures have contributed to the development of mathematics as a science. Students, in small group work, find out about mathematics that has been developed in their country of origin or in their culture, and/or about famous mathematicians of their culture. These small groups then design posters with their results and present them in class. By this, the students will recognize that the contributions of each individual in their classroom, particularly those from students with a minority or migrant background, can offer new perspectives and insights, and are therefore to be welcomed and indeed encouraged. It will show that mathematics is a truly intercultural subject, and that today's mathematics would not exist without the inputs from many cultures. It will also allow students with a minority or migrant background to actively present a small part of their cultures' achievements, and by that to see their background as an asset, and not a burden or an obstacle. To the teacher, the work and the findings of the students can be used as examples for the interculturality of mathematics in later lessons and with other students, even without repeating the unit itself. It may also lead to the teachers' increasing awareness and sensitivity with respect to intercultural aspects.

Main piloting

by Andreas Ulovec and Therese Tomiska

The proposal

This unit consists of 5 lessons, with 45-50 minutes per lesson.

In *Lesson 1*, the teacher gives a short general introduction into the history of mathematics (short overview of timeline, including times of development of most important concepts [of school mathematics' topics], and most important names, e.g. Pythagoras, Newton, Leibniz …, depending on which names the students are familiar with. It is important to choose names from different cultural backgrounds).

* Faculty of Mathematics - University of Vienna, Austria.

After this, students are divided in small groups (approx. 3 students each). If the classroom situation allows this, each group should have at least 1 student with a migrant background. Ideally, these students should be of a different migrant background for each group. Here, it is not important whether the students have a recent migrant background, just that there is some connection to another culture and/or country. Should this not be possible, each group is either picking a country, or is assigned a country by the teacher. In any case, each group is linked to a specific country at the end of this part.

The students receive the following instructions: "Find either one mathematician of the country, or a topic of mathematics that has been invented, developed or is otherwise closely connected with the country. Then design a poster presenting the most important information about this person or topic, and prepare a short (5 min) presentation about it. The poster should also contain basic information about the country."

In *Lessons 2-3*, the actual group work, eventually outside of the classroom (library, computer room), takes place. Students are to commence their search, by using the school library, internet resources, or other materials that might be provided by the teacher. Teacher is to check on the group work, particularly seeing to it that the timeframe is kept.

In *Lesson 4*, students design the poster and prepare the presentation. The teacher is to provide guidance, particularly as to poster content, which information is provided there, and how it is presented.

In *Lesson 5*, students show their posters to the whole class and give a 5 minute presentation about their most important findings. Each presentation ends with a short question-and-answer session. The eventual assessment of this activity should not concentrate about the perfect design of the poster, or the quality of the presentation, or the choice of the mathematician or topic. It should provide feedback about the readiness of the students to approach mathematics as an international subject, to see the importance of achievements made by other cultures, and – particularly for the majority students – to accept the input of fellow students who come from a minority background.

The piloting

General Information

The unit was piloted in grade 7 (age of students: 16-17) of a secondary school in a suburb of Vienna. The teacher is a female mathematics teacher with 5 years teaching experience. She occasionally talks about history of mathematics in her regular teaching activities. The piloting took place during regular lessons, and 10 students participated. The unit was video recorded, and feedback has been collected after the unit by an interview with the teacher and a video analysis by the unit author.

Classroom piloting

The teacher introduced the topic and gave the instructions as stated in the proposal, i.e. the students are to form small groups of 2-3 students, then find a mathematician from one of the students' countries and prepare a poster and a short presentation about this mathematician." The whole introduction as described in the proposal was not done in a separate lesson, since students were already aware of mathematics' timelines and important historical figures in mathematics.

The students chose the following mathematicians: Olga Taussky-Todd (then Moravia, today Czech Republic), Gottfried Leibniz (Germany), Archimedes von Syrakus (then Greece, today Italy), Kurt Goedel (Austria, later USA).

Lessons 2-3 took place outside the classroom. The students had one week time to prepare the presentations. They mainly used internet resources, and also the local library. It was not necessary for the teacher to provide additional materials. Since this was a major exam time, the teacher and students decided not to realize the posters, but have an oral presentation of the chosen mathematicians' biographies. In any case, one of the groups decided to make a poster anyway.

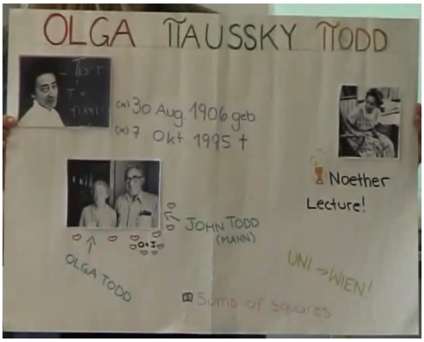


Photo 1. Poster of Olga Taussky-Todd (note the π symbols in place of the T's)

The general absence of posters made Lesson 4 unnecessary, so the piloting continued in the classroom with Lesson 5.In this lesson, the students were called to choose one representative of their group and conduct the presentations.

The first group (2 students) decided to have a presentation with both students participating. One student was responsible for the poster presentation, the other orally presented the biography of Olga Taussky-Todd, using a note sheet.

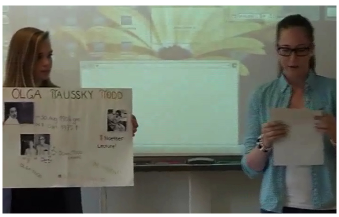


Photo 2. Presentation of group 1 regarding Olga Taussky-Todd

This choice of mathematician was interesting for several reasons. First, it was the only female mathematician chosen. Second, it was an almost contemporary mathematician who is fairly unknown to the public. And third, it was a mathematician who had a migration background herself.

The second group consisted of three students. They chose one representative to present the biography of Gottfried Leibniz. The student used note cards and displayed a classic drawing of Leibniz with a projector.

Photo 3. Presentation of group 2 regarding Gottfried Leibniz

This group noted that it was difficult for them to find a topic of mathematics that Leibniz developed and that could be explained with a certain degree of understanding to school students.

Group 3 also consisted of three students who chose the classic Greek mathematician Archimedes. They used the same approach as group 2, i.e. an oral presentation with note sheets and a projection of an image of Archimedes.

Photo 4. Presentation of group 3 regarding Archimedes of Syracuse

This group particularly noted the interdisciplinary nature of mathematics and physics, and also the fact that research about people from a more distant past is fairly difficult, as facts are hard to find and also hard to discern from legend.

The fourth group, two students, chose the Austrian-American mathematician Kurt Goedel. They were the only group using a PowerPoint-Presentation.

Photo 5. Presentation of group 4 regarding Kurt Goedel

The group also mentioned the forced emigration of Goedel, as well as his acceptance and scientific establishment in the USA.

Since no questions came after the end of the first presentation, the teacher decided to have the question-and-answer session for all groups together at the end of the lesson. Students particularly were interested in the story of Goedels forced emigration due to his Jewish heritage, as well as why Olga Taussky-Todd decided to emigrate. The teacher used this opportunity to speak about reasons of migration in general, as well as the importance of the acceptance that both Taussky-Todd and Goedel experienced in their country of immigration.

Interview with the teacher

The day after the teaching unit was piloted, an interview was conducted with the piloting teacher. This interview took place in the conference room of the piloting school and lasted approximately 30 minutes. The teacher appreciated the possibility that this unit gave her with respect to thematising the issues of interculturality and students with migration background. She mentioned that as far as general society aspects are concerned, in the past she already used several opportunities that arose from teaching to talk about gender issues and gender stereotypes, but until this unit she never saw a good opportunity to discuss cultural, migration and minority issues. From her observations of the small group work, she saw that the students with migrant background were very eager to contribute the story of “their” mathematician (particularly those who chose Taussky-Todd and Leibniz), and this input was very well received by the other group members. She also mentioned that the group who decided to present an Austrian mathematician (without having an idea of who this might be) had a hard time to find an “appropriate” figure, mainly because at the beginning they restricted their search to the Classic and Middle Age time periods, as well as to the most “famous” names. Only after the teacher told them that it was not necessary for the chosen person to be famous in the sense that everyone knows their name, and that they might as well look to more modern times, were they able to proceed. In one of this groups’ discussions (after choosing Goedel) they also wondered why – despite Goedels contribution to mathematics being one of the most fundamental – practically no one knows about him. The teacher also mentioned the fact that in this particular class she never had any issues with or about students with a migrant background, and that they are very well integrated into the class community.

Second piloting

by Barbro Grevholm**, Kari-Sofie Holvik and Camilla Norman Justnes

The piloting

General Information

The teaching unit was piloted by two female mathematics teachers with several years teaching experience working in one lower secondary school in Kristiansand and one lower secondary school in Trondheim, respectively. The Norwegian project team sent the material to the teachers approximately 3 months before the planned piloting activity. The teachers had 5th (11-12) and 8th (14-15) grade available for piloting. After a meeting with the project team, the first teacher chose to conduct the piloting during a regular mathematics classes (40 minutes) in the 8th grade. Several students in the class are migrant students. The second teacher carried out the unit in a 5th grade class. After the piloting, the teachers produced written reports and evaluations of their work. These reports are the basis for the summary here.

Classroom piloting

From Karuss school in Kristiansand

The teacher conducted session 1 in the form of a short introduction into the history of mathematics (supported by a PowerPoint presentation) and especially about Fibonacci.

After this the students were organized in groups of three, where at least one student had a background from another country, for example from Turkey, Eritrea, India, The Czech Republic, Vietnam, former Yugoslavia and China. They were given the task to find out something about the history of mathematics from these countries.

In session 2 the students continued to work in order to find material on the internet. For several of the groups it was difficult to find important mathematicians or important history of mathematics for the intended country. In such cases they were given permission to search relevant material also from neighboring countries.

In session three the students selected the most important parts of their findings and prepared a presentation for which they wrote down main points on posters.

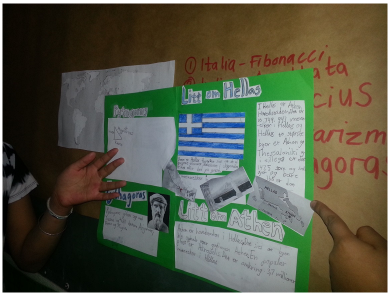
In session four the groups performed their presentations in front of the class.

From Saupstad school in Trondheim

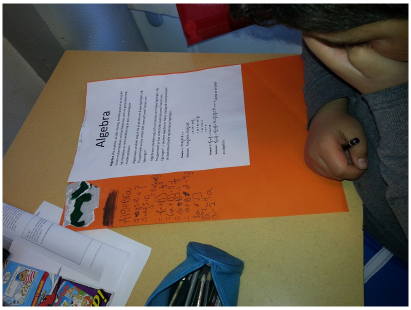
The teachers used a week for this teaching unit and both the lessons in mathematics and Norwegian. Session 1 started with showing the TV-programme Siffer, which is available on the web-site of nrk (main Norwegian TV-channel). In addition the teachers ordered books from the main library in the city. They received about 20

** Faculty of Engineering and Science - Department of Mathematical Sciences - University of Agder, Norway.

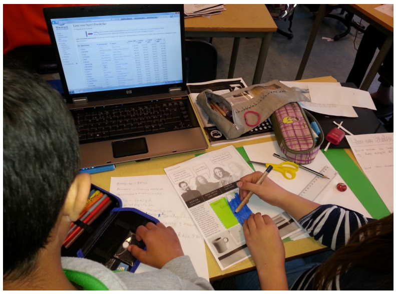
books, mainly intended for adults. The pupils worked in the ordinary classroom and computers were available on rolling tables. The pupils worked in pairs, where one pupil had a foreign background. As only few countries were represented among the pupils, like Turkey (40% of pupils) and Ghana, the teachers wrote the names of some other countries on papers and the pupils could draw another country from them. The sessions otherwise followed the outline in the teaching unit. The instructions in the unit for the pupils were handed out in written form to the pupils also given to parents attached to the weekly programme. After the pupils had chosen subject or mathematician, teachers helped them with copies of relevant parts from the books. They also used the web-site www.matematikk.org (otherwise pupils have a tendency to use Wikipedia as main source). Presentations were made in the form of exposition of posters. See photos below. A timeline was used and along it different texts were attached. The teachers prepared a historical presentation and it was exposed in connection to the pupils' presentation of their findings.



The poster presenting Greece and Pythagoras' theorem



Pupils preparing the text about Algebra



Pupils working with the findings about Pascal

Abacus/kuleramMe!

Kuleramme kalles abacus på matematikkspråket.
Kulerammen er et hjelpemiddel for å regne.
For lenge siden brukte man kulerammen.
Den vi bruker i dag er kinesisk og er fra år 200.
Slik kan en kuleramme se ut

What Sofie wrote on the poster about the abacus

The timeline and reports from many different cultures.

Written reports and evaluations from the teachers

Karuss school: The pupils seemed very interested and motivated in the beginning. It was fun to do something else in the mathematics lesson and interesting to find out something about your home countries. The pupils lost courage a little when they did not find anything, but were engaged again when they were allowed to include other countries. The teacher often had to assist in understanding what they read about, for example analytical mathematics, vector calculations and so on. It was great that one group of pupils explained Pythagoras' theorem for right angled triangles, which we will study later this year.

The pupils found most interesting and important discoveries in Greece, Italy, Egypt and China. The teacher writes: "I had thought that pupils earlier thought that the discoveries of mathematics were done in Norway, but that was not the case for many, maybe just because many of them have backgrounds from other cultures. I think it is difficult to say if they have experienced mathematics as an international subject area in a more obvious way in this project, especially history of mathematics.

We are a class (and school) with many pupils with minority languages, but in the everyday work neither adults nor children think about that. It is as common or rare that someone tells us about something from Eritrea as from Vennesla (small place in Norway). The challenges these pupils have are about understanding pieces of text in mathematics with many, for them, unknown words".

Saupstad School: The pupils were very much engaged in this project. One problem was the fact that although there are many immigrant pupils they come from few countries. Thus teachers had to find a method of including more countries. This was done with drawing from a lottery of countries that the teachers had decided about. Teachers helped the pupils with sources of information in order to limit the common use of Wikipedia. They involved TV-programmes, books, library, and quality web-sites. From the photos made by the teachers it is visible how much effort the pupils have made in presenting their findings. The time-line adds a possibility to experience the historical development of mathematics.

Third piloting

by Charoula Stathopoulou[***] and Ioannis Fovos

The piloting

General Information

This activity was implemented in the school that is inside the Special Juvenile Detention Centre of Volos, with students being young detainees from all classes of Junior High School. The age of the students was from 17 to 21 years old. In the class there were a significant number of students who were from countries of Asia, Africa and Europe (Albania, Romania, Morocco and Pakistan). The activity was designed by the teacher of mathematics who has 25 years teaching experience in secondary education, 12 out of which in the school environment of a detention center. The activity was implemented in cooperation with the Greek teacher of the school, who had previously been informed and had prepared for this teaching intervention.

Classroom piloting

In the 1st lesson

Firstly, the students are informed of the thematic area and purpose of the class, the basic procedures of teaching and learning that would follow, as well as the way it was going to be carried out and the assessment process.

The teacher started the conversation about the international and intercultural character of the development of scientific thought, and especially of mathematics, and invited the students to contribute to the conversation by mentioning a famous mathematician, a mathematical area or a mathematical theoretical construction that they possibly knew was related to their country.

The students showed a lot of interest in the subject and willingness to engage in the class interaction, but it was noted by everyone involved that only one student was in the position to contribute to the conversation, mentioning a mathematical invention related to his country of origin.

1st activity: *the inquiry-thinking orientation*

The teacher introduced to the class an inquiry-thinking orientation, projecting a worksheet with questions regarding the historical development of mathematical concepts. The class talked, formulated hypotheses and noted the need for information resources that would confirm or disprove the hypotheses in the questions talked about. This activity functioned as a pre-organizer of the activities that would follow.

2nd activity: *Mathematical inventions in different continents*

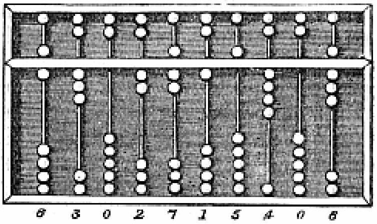
Since students came from three neighboring continents, Europe, Asia and Africa, a power-point was projected, with a historical review of the contribution of different

[***] Department of Special Education - University of Thessaly, Greece

civilizations to the development of mathematics, depending on the continent they belong to.

The presentation included:

About Asia: historical facts from Babylonian and Chinese Mathematics. The students were involved in the interaction, very few things about the existence of the Babylonian civilization were mentioned, whereas they did not know anything about Chinese mathematics.

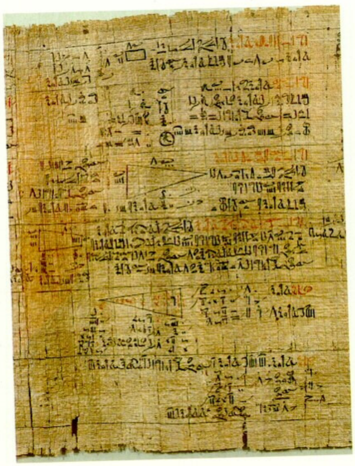


Example of Chinese mathematics achievements from Presentation

<u>In the 2nd lesson</u> (continuation of the 2nd activity)

Focus on Indian and Islamic Mathematics, as well as the Persian Mathematician Al Khwarizmi. The students knew very little about Islamic Mathematics. Only one student from Morocco recognized the Persian Mathematician Al Khwarizmi.

<u>About Africa</u>: Egyptian Mathematics was mentioned, the existence of which the students knew, without, however, being able to provide specific references to fields of its development (e.g. The Rhind Papyrus).

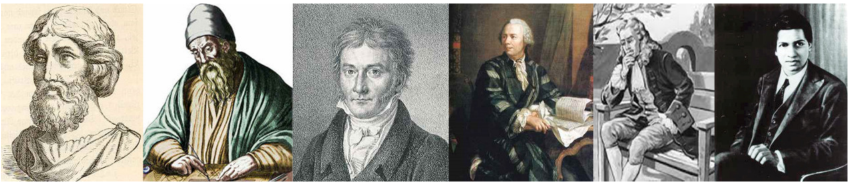


Example of Egyptian mathematics achievements from Presentation

<u>About Europe</u>: Greek Mathematics was recognized by all students as a milestone in the future development of mathematical science, without, however, the pupils being able to pinpoint mathematical fields where it is distinguished (e.g. mathematical

proof, logic and accuracy). The tour of the contribution of the continents was completed with a reference to the era of the Renaissance.

After that, the class focused on 6 great mathematicians of all time: Pythagoras, Euclid, Gauss, Euler, Newton and Ramanujan. Some facts about their lives were presented, as well as their contribution to mathematical science. In addition, a video about Pythagoras was shown. The students actively contributed to the discussion, recognizing the individuals and trying to connect those names to their personal experience, which are the circumstances in which they first heard of some of the names or mathematical concepts. The students had heard of all the aforementioned mathematicians, except for Euler, without, however, having any specific knowledge, with the exception of the Pythagorean Theorem.



The six chosen mathematicians

3rd activity: *Who did what in mathematics in my country?*

Due to the prohibition of Internet use by students inside the detention centre, the teacher undertook the search for information, based, however, on their own guidance.

So, after being divided into 4 groups, each one gave instructions and key words, which were going to be used to do the search. The head of the 1st group was a Romanian student, who asked for information about a specific Romanian scientist, a pioneer in aerodynamics. The 2nd group, which consisted of Albanian students, since they did not know any Albanian mathematician, asked for a search for great mathematicians of Albanian descent. For the 3rd group, which consisted of Pakistani students, since they also did not know any Pakistani mathematician, what we did for the 2nd group would be done. Finally, the 4th group, headed by a student from Morocco, asked for some great Moroccan mathematicians to be found, as well as facts about the Persian mathematician Al Khwarizmi. The common characteristic of all four groups was the fact that the information that would be brought in the next class had to be in their native languages.

In the 3rd lesson: "Studying information about mathematics in our country of origin"

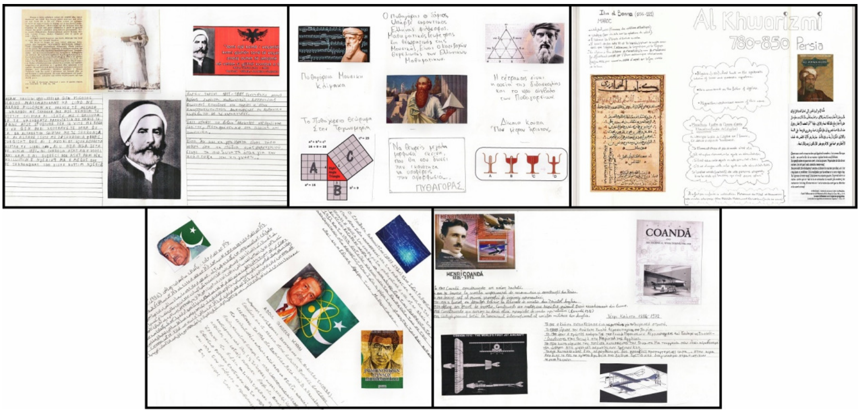
Based on the instructions of the students, the teachers had searched and stored web pages containing the relevant information in the mother tongue of the students and they presented it with the help of a projector. Each group chose the information they thought was useful, the material was printed and distributed to the members of the group, so that they could study it and record what they would put in the poster they would make.

At the end of the 3rd teaching hour, the same worksheet that had been given in the 1st hour was given, which included questions regarding everything that had been presented about the history of mathematics and the mathematicians. The students, in total, answered 14 out of 15 questions correctly, except for 4 students, who, in the 6th question about the use of the Decimal Classification System, answered "Greek mathematics" instead of "Chinese mathematics".

In the 4th lesson: "Designing-implementation of poster and whole class presentation"

All students got their paperboards and pencils and started designing the way they would place the photographs and pictures they had chosen in a suitable way and talked about the text they were going to write. The classroom was turned into a laboratory, where the 4 groups worked feverishly and the teachers moved from one group to the next, supervising their progress. Due to the special circumstances that prevail in correctional institutions, the use of some materials is prohibited, so the students had to make do with what was provided. The group with the Albanian students, because it was the biggest, made two posters, one about an Albanian mathematician and the other one about the Greek mathematician Pythagoras.

Due to the fact that the time in our disposal was not enough for them to complete their projects and because it is forbidden to take the specific materials to their cells, we continued in the 5th lesson, when, after finishing its construction, each group presented its poster, saying a few words about its content.



The finished posters

In general, those students had never before learnt anything about the history of mathematics, which they found so interesting that all of them focused, paid a lot of attention during the lessons and worked very eagerly to make the posters, with the aim of showing the importance of their country's contribution to the development of science.

Conclusions from the three piloting

by Andreas Ulovec

The piloting clearly showed that students are interested in the history of mathematics and mathematicians and mathematics content from different cultures and time-periods. The active participation of migrant students and the introduction of their cultural backgrounds can certainly enrich the learning situation. The teaching unit was well adapted to be used by teachers with pupils in different age-groups and easy to follow and engaging for both teachers and pupils. The suggested activities open the opportunities to experience learning of mathematics from a variety of sources outside the classroom.

ORNAMENTS IN TEACHING SYMMETRY

by Hana Moraová* and Jarmila Novotná*

INTRODUCTION

The following unit looks at the potential of multicultural content of ornaments of different culture and their potential use in mathematics classroom. What mathematical structures can be practiced using the cultural content of ornaments? What cross-curricular links does the unit bear? How can it help integration of migrant pupils into the classroom?

Piloting with trainees

The unit was first piloted on a workshop with Czech pre- and in-service teachers. In the workshop the participants were introduced to the issue of teaching mathematics in multicultural classrooms. The aim of this piloting was 1) to show pre- and in-service teachers how easy it is to add multicultural content in to lessons of mathematics, 2) to get more ideas for what mathematics is hidden in ornaments.

Consequently, the trainers presented a number of ornaments from different cultures and asked the trainees to pose as many mathematical problems working with these ornaments as possible.

Anticipated mathematical topics for development

- Symmetry, rotation, translation, plane geometry, tessellation.
- Other topics developed by the trainees: proportionality, linear functions, ratio, combinatory, least common multiple.

Aims of the workshop

***For trainees*:**

- Investigating solving/learning strategies
- Posing problems
- Discussing these problems in groups

For trainers:

- Enrichment of mathematical content that can be used with ornaments
- Enrichment of repertoire of possible multicultural problems for mathematics lessons

* Faculty of Education - Charles University in Prague, Czech Republic.

Main piloting

by Hana Moraová and Jarmila Novotná

1. Description of the activity

The activity was based on the concept of substantial learning environments – SLE developed by Erich Wittmann (1995), namely the concept that “A good teaching material for teachers and pupils should be the one which has a simple starting point, and a lot of possible investigations or extensions.” The simple starting point in this case was a number of ornaments whose origin was in different cultures (with the intention of allowing minority pupils to be heard, to present ornaments typical for their culture or home, to break the wall between home and school culture between mathematics naturally used at home and mathematics used at school – Meany, Lange, 2013). The trainees were invited to pose as many problems with the content as possible. Problem posing is an important component of the mathematics curriculum, and is considered to be an essential part of mathematical doing (NCTM, 2000; Tichá, Hošpesová,2010). It is an activity a mathematics teacher does almost on everyday basis when they need to supplement problems from the textbook.

Stage 1 The trainees

- Introduction to multicultural issues and intercultural psychology and their implications for mathematics classrooms
- Discussion of traditional way and typical tasks in teaching symmetry
- Activity – symmetries in letters in different alphabets, small and capital letters, symmetries in words

Stage 2 The trainees

- Activity –ornaments of different cultures
- Types of ornaments – symmetrical vs. asymmetrical, nature, geometry, line, tessellation, rosette
- Task: Pose a problem and/or develop a lesson plan and activities using your (or other selected ornament). What mathematical content is there?
- Present your problem/lesson plan to other trainees
- Discussion of the plans, selection of best activities

Stage 3a The trainees

- Prepare the final draft of the lesson plan to be piloted, prepare the needed teaching materials and aids

or

Stage 3b The trainers

- Choose one of the proposed activities
- Draw the final draft of a larger didactical unit (several lessons) which is flexible and can be adapted for use at different levels and in different grades

- Adapt the teaching unit to meet the needs of the selected classroom, prepare the needed teaching materials and aids

(This is the ideal scenario in case of pre-service and in-service training. In case of this piloting, the final draft was made by the research team/trainers as there was too long time between the in-service training session and the piloting at school, see the following text.)

Stage 4 Piloting at school

- The final draft is taught at a selected lower secondary school
- Immediate feedback from the learners (about 5 minutes)
- Post-lesson interview with the teacher
- Teacher's written reflection on the activity

Stage 5 Piloting in a selected school abroad

2. Assignments

a) Problems posed by trainees

- The Pythagoras theorem: measure and calculate with the triangles in the presented ornaments.
- Compare line symmetry, rotation and translation. What is typical for which ornaments?
- Find all the different geometrical shapes you can find in one ornament; name them and describe them.
- Study the concept of tessellation; find which ornament can make tessellation.
- Copy the ornaments on a square grid, look at their area. Use square grids of different scale, study proportionality.
- Calculate proportion of area of one colour.
- How much fabric with this ornament would you need to make one e.g. kilt (about tartan)?
- Find the generating element.
- How many lines of symmetry are there in a specific ornament?
- The least common multiple (in case of Indian line ornaments).
- How many beads are needed to make one segment of Native American ornament?
- How much band is needed to decorate a wall of certain dimensions?
- Patchwork and ornaments, what geometrical shapes are possible for production of patchwork?
- How many threads of each colour do we need to make one segment of tartan?
- Draw symmetrical ornaments, copy them from the original or create pupils' own ornaments.

b) Lesson plan

The trainees studied the ideas and agreed to build the following didactical unit. The unit was developed and elaborated in detail but for the needs of piloting was then adapted by the teacher to suit the needs of the School Education Programme, mathematics curriculum of the particular group and the needs of the children.

Note: For materials used during the unit see e.g. www.googleimages.com.

Lesson 1

- Title of the lesson: ORNAMENTS
- Revision of symmetry: look for lines of symmetries in different types of letters (appendix 1) – 10 minutes
- Lead-in: presentation on types of ornaments in different cultures (10 minutes)
- Main activity
    - show ornaments from different cultures
    - on one or two show the different types of symmetry and transformations
    - give each student one ornament and ask them to find all lines of symmetry
    - ask students to name and copy all symmetrical geometrical figures in the ornament
    - ask students to formulate conclusion about typical ornaments of a particular culture
- Homework: bring an ornament decorated object from your home, bring pictures of various ornaments from your holidays.

Lesson 2

- Lead in: present your ornaments, what types of ornaments are they, what line symmetries did you find?
- Main activity:
    - Give each student one of the three ornaments (Celtic, Native American, Arabic rosette) and a square grid with different scales
    - Ask students to find all lines of symmetry in their ornament
    - Ask students to copy the ornament into the square grid
    - Ask students to count the number of at least partially coloured squares
    - Ask students to calculate the area of the ornament (taking partially coloured squares as covered squares)
- Follow-up: Copy the following chart in the whiteboard

scale	0.5 cm	0.75 cm	1 cm	1.25 cm	1.5 cm	2 cm
area						

What proportionality is there between the scale and the area?

Piloting at school

The Czech activity was piloted at ZŠ Fr. Plamínkové s RVJ in Prague in the 5th grade.

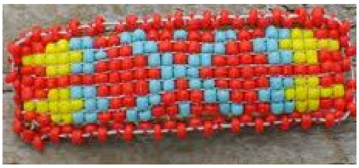
The research team closely inspected the Framework and School Education Programmes for Primary School Education in the Czech Republic (MŠMT 2013, http://www.plaminkova.cz/skolni-vzdelavaci-program) to see which of the topics listed in the above described proposal are suitable for this age group. Czech 5th graders do not yet have knowledge of symmetries and do not work with them explicitly, but are likely to have intuitive knowledge of them. In the 5th grade they learn to work in square grid and can build their preconcepts of plane geometry (area and perimeter). They have not been introduced to the concept of proportionality.

The decision was made by the team to adapt the teaching unit and to pilot two teaching units:

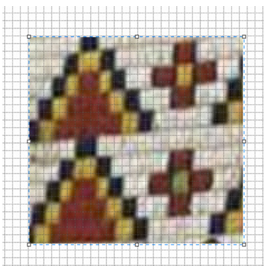
Lesson 1

Introduction to ornaments, discussion of ornaments, types, shape, differences in cultures, basic elements; discussion of Native Indian ornaments (made of beads).

Children were given square grid (0.5 cm) and a Native Indian ornament and were asked to copy it square by square (accurately), then they were asked to calculate the number of blue squares, size of blue cross, blue and yellow figure; however this could not be used as introduction to area because of the scale.



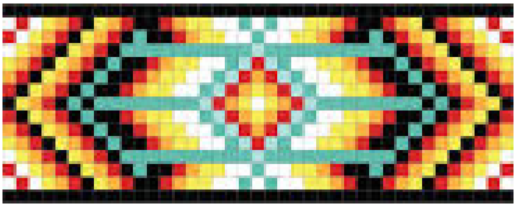
Children were given 1cm square grid and another example of a Native American ornament that had been embroidered not made of beads, i.e. it was made of rectangular, not square elements. Two part of the ornament had been copied on a square grid (one rectangle made of three times three squares). The children were asked to copy these figures into 1x1 cm square grid.



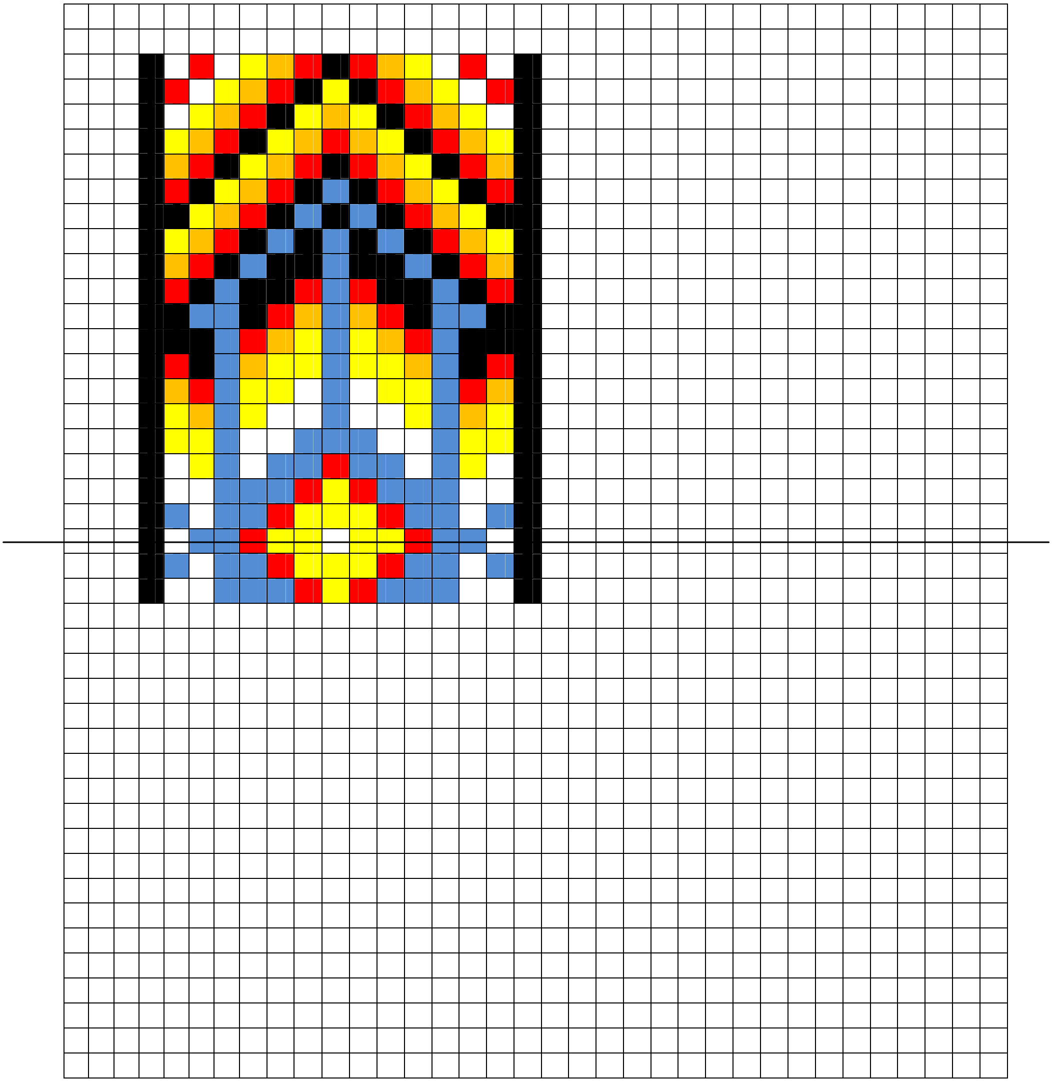
As the area of each square was $1cm^2$, the pupils could then easily state the area of different geometrical figures they had drawn (rectangle, 2 rectangles, cross, pyramid etc.). The same was done with perimeter.

Lesson 2 – work on symmetry (a combined Math and Art lesson)

Children were shown two original Native American ornaments. They spoke about the figures they can see there.



They were given a model of an ornament with line of symmetry indicated and asked to finish the ornament. An example follows. More models are available on the project website or on the DVD with the full text.



They used crayons, beads, threads etc. to create models of ornaments.

Task for homework: Look at ornaments you have at home. Where are they? What types? What shapes, colours, materials etc.? Copy them or take a picture of them. We will be working with them in the next lesson.

Comments by the piloting teacher

The teacher who was piloting this activity in general evaluated the teaching unit as motivating to her pupils. The pupils were occupied and working hard most of the two lessons. The materials allowed differentiation in the lesson (selection of less and more difficult figures, number of figures to state the area and perimeter).

The teacher suggested that with this age group only 1cm square grid be used in all activities as it makes counting different squares more mathematically more meaningful from the very beginning. She warned that in the first task pupils need to be alerted to the fact if they do not begin with the central blue figure but start from counting the number of red squares and drawing the outlines of the red rectangle, they often find out very late that their original outline had been incorrect and does not leave space for the central blue cross. If this happened pupils were not much motivated to start from the very beginning.

Second piloting

by Antonella Castellini, Lucia Alfia Fazzino and Franco Favilli[**]

The a priori analysis

The context

The activity was both designed for and developed by a group of students from two different classes of the "*Istituto Comprensivo 1*" in Poggibonsi (Province of Siena) during the weeks of flexible teaching. In short, during these weeks the classes are open to carry out various disciplinary or interdisciplinary activities and to develop projects outside the school. The group consisted of 15 to 18 pupils of two second year classes of Lower Secondary School.

The aims

The topics dealt with in the teaching unit allowed to retrieve previous pieces of knowledge, but also to see them in a different and certainly more creative way. Significant added value was represented by affectivity, since the activity urged the reference to the typical cultural values of the students' native country. The teaching unit topics allowed the introduction of new teaching methods, such as problem solving, which so far had not been used for the development of mathematical skills.

The reasons for this choice were basically four:

- to look at reality with mathematical eyes;
- to develop intercultural education, accompanied by the desire to let students know about other cultural roots;
- to develop a positive attitude to mathematics, through meaningful experiences (as suggested by the Italian National Guidelines 2012);
- to describe, name and classify geometric figures, identifying their relevant elements and symmetries, also in order to make all students able to reproduce the figures (as prescribed by the Italian National Guidelines 2012).

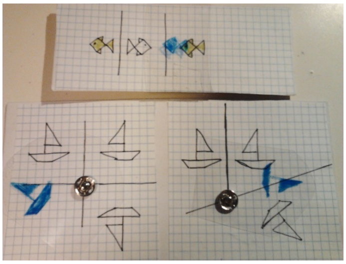
[**] CAFRE – University of Pisa, Italy.

The design

The methodology adopted was that of the workshop, intended not just as a physical place, but as a classroom activity where doing and thinking are closely related. The workshop is understood as a teaching situation where the meaning of mathematical objects are constructed through experiences that are rich and stimulating for the students themselves.

All the activities of the teaching unit were designed with reference to the isometric transformations, a topic partially introduced in the previous school year. The topic was addressed by the use of a mirror: by putting a drawing or an object in front of the mirror and by the observation of the reflected image, the students were able to discover the key features of the axial symmetry.

The next step was to build (as the composition of symmetries with parallel or perpendicular axes) the other isometric transformations – translations and rotations – and identify their basic features. Afterwards, all the students had to build some dynamic models that allowed them to represent these geometric transformations.



The object-action connection allowed the students to be free to design and to interpret. Indeed, it is for this reason that it becomes important to see, observe, interact with a dynamic, non static object.

The static nature limits the student, forcing them to look at one aspect and does not help them to analyze the situation from different points of view. Besides, it does not stimulate students' curiosity, and above all, it does not allow them to speculate, or even less, to argue, thus excluding a substantial part of those processes that are fundamental in the formation of mathematical thinking. Therefore, besides the use of the dynamic models already known by students, a new activity with the use of the room of mirrors was planned.

The teaching unit

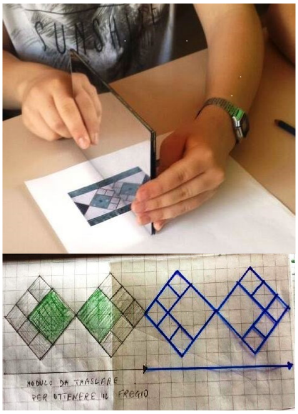
<u>*Lesson 1*</u>

In the first lesson two ornaments - a frieze and a rosette - are delivered to the students, who are divided into 4 groups of 4-5 students each. Students are then asked to analyze them by using the flat mirror in order to identify what kind of isometric transformations allowed the creation of the ornaments.

After a brief plenary examination, the students have to choose a spokesperson who will present to the other groups the geometric transformations identified in their own group. The purpose of this activity is to reflect on previous knowledge and to compare the results of the different groups, with the main objective to make them infer conjectures and to learn how to argue with other about them.

<u>Frieze group</u>

Students begin to observe the frieze using the flat mirror: "At first we put the mirror horizontally towards the decor, but while observing, we realized that such a symmetry was not possible because a frieze is usually very long. So, we put the mirror vertically to the drawing and we observed that there is an axial symmetry when considering the square as its module. If we consider the squares a, b and c, it can be clearly seen that there is a double axial symmetry with parallel axes. Then, looking more closely, we realized that if we consider figure a and b as a whole then there is also a translation to the right."

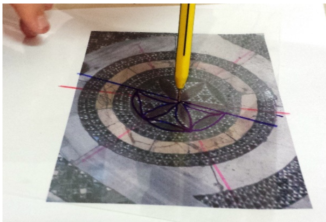


At this point they asked themselves the question "but how long is the translation vector?"

With the help of the dynamic model they have verified that the length of the translation vector is exactly twice the distance between the two parallel axes that gave rise to the movement.

<u>Décor group</u>

Pupils' comments are directly quoted: "In this pattern we saw immediately that there are axial symmetry with incident axes. Then A. pointed out to us so that there is also a rotation, because it is produced by the composition of two symmetries with incident axes. So we decided to draw the axes of symmetry and we found the centre of rotation. To check the rotation we took the acetate and had our dynamic model.



Interesting is the fact that the pupils show on the decor the module and the axes of symmetry to explain their reasoning and resort again to the dynamic model for dissolving any possible doubt.

In this first lesson, to get confirmation of what had been observed, it has been very helpful the use of the mirror and the dynamic models that the pupils had built.

Once reviewing the various isometric transformations is finished, students are asked to do at home, individually, a written summary of the work done in the class (log book), and to search for and bring the day of the next lesson, objects and / or fabric available in the house that contain decorations and are proper to their country of origin or taken from visited countries.

Lesson 2

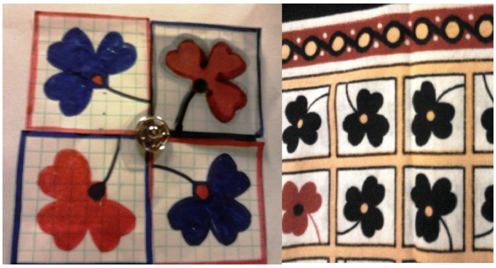
Both the objects and the fabrics are made available to students. Each group selects the item it prefers. A fabric from Senegal (the most successful in students' eyes, perhaps because very colourful and with different types of decorations) and another fabric that a female student uses at home to cover the sofa, this also pretty colourful and with very regular decor. Students discarded different laces (many of which crocheted by grandmothers...) as well as ceramic plates and boxes, actually less numerous and not very colourful.

This task is assigned to each group:

1. motivate the choice of the object;
2. identify the isometric transformations with the help of the mirror plane;
3. reproduce the chosen ornament on two squared sheets that you have been given;
4. identify the generating pattern of the ornament;
5. present to other groups the chosen decor, providing each of them with the generating pattern and the instructions to create it.

Clovers group

The members of the group that has chosen the ornament with clovers motivate their choice with the fact that it was "easy and pretty." In fact, as they write in their report, they were initially mistaken: "We thought only of simple symmetries in both in the petal and in the ornament, from square to square, but then, looking better with the mirror, we realized that there was the small flower stem and that it was not an axial symmetry, then, but a central symmetry. (....) For this we used the acetate with the popper, to get it better understood."

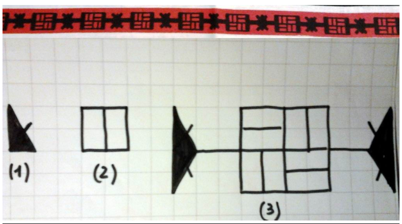


The work done provides clear evidence of their research and shows how the mirror helped them to better identify the isometric transformations in the decor, but also how much the use of dynamic patterns was useful to view the changes.

From the base pattern, with successive rotations of 90 degrees, they were able to represent the decor of the fabric. It is interesting to point out the choice of the colour that the group made to highlight the two figures matching in the central symmetry.

<u>Small frame group</u>

The group that chose the decoration with small frames states it did it because "it looked like one of the small frames that were made during the elementary school years". Pupils have reproduced easily the pattern highlighting immediately the translation which allowed them to reconstruct the entire decor. But also on this occasion, after a moment of euphoria for the speed of their execution, students have paid attention to the central decor that is "what is made of small rectangles." The pupils have realized that there were other symmetries: in particular, a pair of central symmetries or a series of four 90° rotations. They were then sure that there was nothing else and started to write a sequence of instructions to make their mates able to reproduce the little frame. While writing, however, they realized that also the other decor - "the black star type one" – appeared actually to have the same property of double central symmetry. There have been, therefore, many discussions: "How do we do it? Have we to give three different instructions? One for each of the two patterns, and one for the small frame at the same time? And what then is the generating pattern?". Pupils agree to proceed this way and give three different instructions by providing classmates with the three patterns shown in the photo: two patterns to recreate the central motifs with the rotations and the other to make the translation.



<u>Lozenge group</u>

Again the choice is motivated by the pupils saying that "they are very colourful, but there are curves and straight lines, that is two different ways of decorating". But it is this situation that makes it difficult for the group to represent on the paper the fabric design and necessary a little help from the teacher. The pupils then realize once again that only four rotations of 90° are needed to recreate the ornament and easily identify and compose the basic pattern.

Rosette group

The group motivated the choice of the rosette by saying that it reminded them of some Christmas drawings. The ornament undermined the students: they failed to identify the starting pattern. They could see symmetries, but only in two pairs of the ornament elements and they recognized the rotations, but could not explain how to make use of them. They discussed a lot about "separating the two designs, the one with a small square from the other. They tried to make use of a pattern, then they tried with another and finally decided that "we can work with three patterns to reproduce the entire rosette: two of them must rotate four times by 90°, the third one must rotate 8 times by 45°".

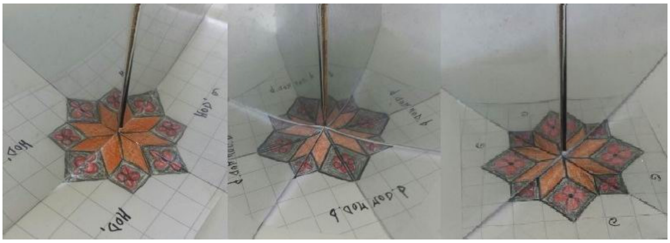
Lesson 3

A new activity was introduced, which could be titled:

From the room of mirrors to proportionality

Consider the patterns you have made in the previous lesson and which originated the rosette ornament.

Place the patterns, one at a time, in-the mirror room and tell how many of them are needed to complete the ornament.



Pupils begin to put the mirrors over the pattern at an angle of 90°. In such case they see four images (three are reflected and one is the real one) forming a flooring, that is, an entire decor. Having then put the other pattern, they realise that “with the 45° angle, however, we get 8 images, so they doubled. And yet the angle is decreased, more precisely it's halved!”.

This remark stimulates the curiosity of the students who begin to try other pieces of the ornament. "Let's try with half rosette, we certainly get two images only". Then they come to say that if the angle of the mirror room decreases, the number of images increases.

The teachers therefore to get deeper into the topic and suggest pupils to use the protractor to find the angle between the mirrors and to place in the room of mirrors a thin object such as, for example, a pencil. They also suggest to build a table with the angle between the mirrors and the corresponding number of images obtained. The angle will have a measure equals to a sub-multiple of the full angle. In the table, then, there are going to be not only the pairs (90, 4), (180, 2), (45, 8), but also, for example, (30,12), (40,9). The table will clearly show the relationship *angle width* ←→ *images number* because it is easy to see that if the angle halves, becomes a third ..., the number of images doubles, triples .... Pupils can, therefore, observe that the product of the angle width by the number of images is constant and equal to 360°, the full angle.

In this way the students have then discovered intuitively, but at the same time rigorously, the law of inverse proportionality!

Afterwards, the teachers decide to ask pupils to represent the data in the table by points on a Cartesian plane and connect them. Pupils can, then, realise, that the points can be seen as elements of a curve that they do not know yet, a branch of hyperbole.

It is here that the student B. asks: "Why does this graph start at 10°? If I close the room, that is if the angle is zero, what happens? I do not see anything so I don't get any figure, the images are zero ... but then it does not work ... there's something wrong". Here it is how the one student's doubt becomes a resource for everyone! The teachers suggest then pupils to enter a piece of string in the room of mirrors and look attentively what happens when closing mirrors slowly. The closure action enables students to understand that the images are not zero but infinite "in fact, we do not see them because they are inside!". Once again the dynamism of an object leads to examine an important limit case it would not be easy to deal with and understand just by arithmetic, since the division by zero is impossible. In this way, on the contrary, pupils, through this operation that have verified to be impossible, can grasp the idea of infinity.

Lesson 4

Art tessellations

The students have already worked on the tessellation of the plan and know what are the regular polygons that make it possible and the reason why. A slightly modified version of this activity is then proposed to the students, with the aim to unleash their "artistic creativity".

Students are asked to cut out a part of a square and place it on the opposite side. In this way they get a pattern that, by subsequent translations, creates a flooring. The same activity can be proposed using other regular polygons, such as the equilateral triangle or the parallelogram. The creativity of each student will transform the pattern he/she has into a subject that will be the "hero" of this new and very personal flooring.

The activity was very much enjoyed by the students who, after a temporary confusion due to the actual construction of the pattern, had fun creating beautiful floors, while showing imagination and artistic sense.

Unlike what happened with the square and the parallelogram, the use of the triangle as the polygon to start from proved to be difficult. "Where was the cut part to be placed in order to obtain a flooring? Is it ok to put it on any of the other two sides? Or, is it necessary to place it on the same side where it has been cut from?" These questions arose spontaneously and led to a good discussion that was developed with good arguments. Once again a context of emotional and meaningful reflection encouraged spontaneous and interesting questions that, properly managed by the teacher, can give the opportunity to break new grounds or retrace paths already experienced, but from a different perspective, thus developing a continuous reconstruction of knowledge.

Conclusions

We are firmly convinced that there is a strong need to change attitudes towards mathematics and that as the Italian National Guidelines 2012 say, in the classroom "a positive attitude towards mathematics through meaningful experiences" should be promoted. Therefore, more than accumulating knowledge by just passing a number of notions and information often not interlinked or interrelated, teachers should try to stimulate students' aptitude to pose problems that can increase their motivation and foster discoveries.

The teaching unit described above falls within this framework, making use of workshop activities in such a way that learning is really centred on students, on their needs and on their characteristics. The student is the investigator and as such, acquires the ability to identify, accept, confront and solve new problems, both individually and in groups.

The unit development is based on three methodological cornerstones:

1. to set context problems;
2. to foster questions;
3. to work in groups so that the heterogeneity of the students is a resource for the entire class with a view to get more and more inclusive learning.

Many are the problems teachers have to face such as dealing with unmotivated students, working in socially non-homogeneous and multicultural classes and with students of different cultures… It is therefore necessary to design educational courses that allow students to see the reality from different perspectives and also to develop greater self-awareness.

The teaching unit allowed teachers to meet today's students needs without sacrificing the teaching of the basic concepts of the discipline. Even though mathematics is often seen as an abstract subject, it could instead become somewhat closer to the students and to their reality. The use of everyday objects, also linked to different cultures as the decorated fabrics from Africa, gives the subject an affective meaning which should not be underestimated. Even the realization of ornaments, an activity that makes the student free to experiment and indulge within its fantasy, offers an emotional dimension that is important because learning is difficult if the sphere of emotions is not positively affected. On the other hand, teamwork allows students to learn how to defend their conjectures and at the same time, to accept changes when their classmates' arguments are clear and justified.

The whole activity, therefore, is based on fundamental aspects of the learning process; in fact, it requires students to be active, constructive, collaborative, contextual and process reflective. This way, it provides their students with excellent cognitive skills.

Third piloting

by Andreas Ulovec[***] and Therese Tomiska

General Information

The teaching unit was piloted by a female mathematics teacher with five years teaching experience working in an upper secondary school near Vienna. The Austrian project team sent the material to the teacher approximately 3 weeks before the planned piloting activity. The teacher had a 5th (age 14-15 years), 6th (15-16) and 8th (17-18) grade available for piloting. After a meeting with the project team, she chose to conduct the piloting of lesson 1 during a regular mathematics class (50 minutes) in the 6th grade, and lesson 2 during a 50 minute class using field work as a teaching method. Eight students (age 17-18), three of which are migrant students, attended the class, of which lesson 1 was observed, and lesson 2 was video recorded and observed by a member of the Austrian project team.

Classroom piloting

The teacher introduced the topic by bringing several objects with Japanese, South African, as well as US motives from her private possession into the classroom for lesson 1. The students formed groups of two and were asked to look for symmetries as well as for different geometrical figures, and finally compare the different kinds of figures and symmetries that they found in the objects from the different cultures. Each group then shortly presented their findings in front of the whole class, and the other groups wrote them down into their notebooks. At the end of the lesson, the teacher asked the students to bring ornaments or pictures of different cultures' ornaments into the next lesson, as suggested in the proposal. The students argued however that only very few of them (or their families) actually had suitable ornaments and pictures at home. Repeating the lesson with more objects from the teachers' collection was seen as not very interesting by both the teacher and the students. The students then came up with the idea to go out into nature and bring pictures of symmetries or geometric figures that are found on flowers or plants instead. The teacher argued that if symmetry in nature would be interesting for the students, it would be better to actually make a field work session out of lesson 2, instead of just looking at the pictures. It therefore was decided that lesson 2 will be modified, and students will go out together with the teacher, look for symmetries in nature, and take photos for later discussion of symmetry and scale in class (this last part, i.e. back in classroom, was not part of the piloting).

[***] Faculty of Mathematics - University of Vienna, Austria.

Photos 1-3. Patterns from Japan, South Africa, and USA

Lesson 2 started with the teacher reminding the students on the different kinds of symmetries and figures, as well as special angles (e.g. from Fibonacci numbers). Then the teacher and the students went out into a field near the school to look for the occurrence of symmetries and geometric figures in both natural and artificial objects. Students first looked for the occurrence of certain angles on plants. Very soon they realized that 137.5° was a very frequent angle on a number of species of plants, a fact that impressed the students very much. Students took pictures of the objects to use in the next lesson.

Photo 4. Looking for certain angles on a thistle

The unit continued with the students looking for symmetries, particularly for mirror-symmetry. There, the students were mostly able to state that the object does actually show some kind of symmetry, but were not always able to name the kind of symmetry concerned. So fairly often, the students pointed out symmetries and the teacher explained the particular symmetry on this object.

Photo 5. Mirror-symmetric blade of grass

Students then started a discussion about how exact these symmetries actually are. The teacher used this opportunity to point out that real objects (regardless of whether they are artificial objects like the ones she brought into the classroom in lesson 1, or whether they are natural objects like grass) are never exactly symmetric in a mathematical way, and that this is where modelling comes into play.

At the end of the lesson, also artificial objects (advertising pillar, patterns on t-shirts) were checked out, and students and the teacher discussed whether the patterns or the form of the pillar have cultural and/or practical reasons. Several of the shirt patterns were photographed; they came from different cultural backgrounds without the students (according to their own statements) having known this fact when they bought the shirts. The teacher gave as homework for all students to find out what cultural backgrounds the photographed patterns have and what their cultural relevance is.

Photos 6-7. Cultural patterns on students' t-shirts

The lesson ended with the class being back in the school building, where the homework assignment was repeated.

Conclusions

The piloting showed that even if the unit is modified and – at least superficially seen – moves away from the intercultural aspects, these aspects can be easily brought back into the minds of the students by referring to everyday life objects and their cultural connections.

Conclusions from the three piloting

by Hana Moraová and Jarmila Novotná

The proposed and piloted activities are of strongly multicultural nature. The teachers may use printed materials, materials downloaded from the internet or use everyday objects and motifs that surround us. Whatever the form, the use of these materials activates the pupils, motivates them to creative thinking and to looking for relations, it broadens their perspectives. The fact that very different ornaments are typical for different cultures and that these ornaments are often used to decorate everyday objects allows minority pupils be heard, bring contents from their own culture and motifs from their own homes into classrooms, it allows the teacher to show that mathematics is universal.

The experience from the three piloting shows that the materials can be used in a very flexible way. They can be adapted to the needs and knowledge of different groups and individual pupils. They can be used directly in the classroom, in the form of various projects, individual work outside of school. They are of strongly cross-curricular nature and can be used simultaneously in several subjects. The proposed environment and activity meet the criteria of Wittmann's substantial learning environment (1995).

The piloting show that if the activities are well planned and developed, they allow inclusion of pupils with different cultural backgrounds and traditions but also with very different interests. If they are given the chance by the teacher, each pupil will find their "cup of tea" and moreover can bring their own experience for the benefit of all. It is up to the teacher how the activity will be presented to pupils and how much freedom they will be given while working on it.

References

Meany, T. and Lange, T. (2013). Learners in Transition between Contexts. In Clements, M.A., Bishop, A.J., Keitel, C., Kilpatrick, J., & Leung, F.K.S. (Eds.), *The Third International Handbook of Mathematics Education*, Vol. 27 (pp. 169-202). Springer.

NCTM – National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics*. Reston, VA: NCTM

Tichá, M. and Hošpesová, A. (2010). Tvoření úloh jako cesta k matematické gramotnosti[Problem posing as a way to mathematical literacy, in Czech]. In *Jak učit matematice žáky ve věku 11 – 15let; sborník příspěvků celostátní konference*(pp. 133-145). Plzeň: Vydavatelský servis.

Wittmann, E.Ch. (1995). Mathematics education as a "Design Science", *Educational Studies in Mathematics*, *29*, 355-374.

Framework Education Programme for Elementary Education (2013). Prague: MŠMT.

Appendix 1

Czech **Matematika, MATEMATIKA**,

Hebrew **הקיטמיתמ**

Chinese **數學**

Japanese **数学**

Russian **Математика, МАТЕМАТИКА**

Greek **Μαθηματικά, ΜΑΘΗΜΑΤΙΚΆ**

Persian تایضایر

INTRODUCTION TO AN ANCIENT MAGIC SQUARE

by Marie-Hélène Le Yaouanq* and Brigitte Marin*

INTRODUCTION

The aim of this unit is to have the students work at the same time on decimal numeration and on the use of the French language, in writing and speaking in mathematics both in terms of vocabulary and explaining one’s reasoning. It is also to allow verbal exchanges about written and oral numeration used yesterday and today in various countries and to highlight the input of other civilizations to the construction of mathematics in Europe.

This unit is based on this ancient magic square discovered in 1956.

http://home.nordnet.fr/~ajuhel/Grenier/car_mag.html

*UPEC - ESPE, Créteil (France).

Main piloting

by Marie-Hélène Le Yaouanq and Brigitte Marin

Presentation of the piloting with teachers

The first step of the project took place during an in-service teachers training aiming at using the history of mathematics in class. The document representing the ancient magic square was given as such to the teachers. They had to discover the numbers in their current writing hidden behind the symbols.

As most of the teachers did not know the Arabic numerals appearing in the square, they had to look for a strategy to decrypt it. Three teachers whose mother tongue was Arabic directly transcribed the square and found that it was not magical! They were asked to set up a different approach. They rather quickly gave up trying with the algebraic approach and they set up reasoning on units and tens and on the number of time they appeared.

Once the square was decrypted, the trainers gave information about the used symbols, which were the ancestors of the current Arabic numerals. Some figures writing had changed so it led some teachers to be wrong. Then the trainers asked the teachers to find a use for this document in class.

So this first part had a double goal for the trainers: on the one hand to put the teachers in a search situation which they would have to reproduce with their own students in class, exchanging with teachers on their practices, and on the other hand to make the teachers aware of the search difficulties on the given document to enable them to adapt the situation.

A priori analysis

1. Mathematics concepts involved and curricula

The students' activities rely on numeration and on additive connections. During elementary schooling, work is done on decimal writing, first on whole numbers and then on decimal fractions, but numerous difficulties still remain in Year 6.In the Year 6curriculum, the students are explicitly expected to be able "to know and to use the value of figures according to their place in the writing of a whole or decimal number".

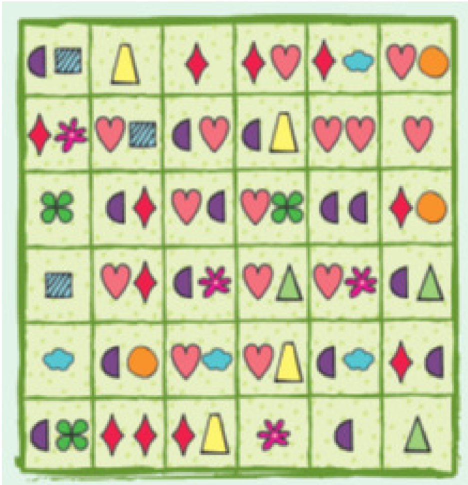
So the integration of the proposed work in Year 6 is normal.

This work also relies on the taste for searching, on the ability to hold a series of reasoning steps of potentially different natures, and to use orders of magnitudes.

2. Predictable difficulties and suggested adaptations

The teachers who studied the initial magic square pointed out some difficulties:

- The difficulty to name the symbols appearing in the square, which makes the slightest communication of reasoning difficult. Then they suggested that for the students the ancient Arabic numerals should be replaced by drawings of mathematical figures or daily life objects. Thus mathematical and daily life terms will be required to name them, so the student will have to practice mathematical or common vocabulary.



Adapted square (source: Hélice, 6th, Didier; see Annex 1)

- The difficulty to make sure that the students really understand what a magic square is, led to provide a first work step to discover magic squares with smaller sizes (3× 3 or 4 × 4).
- It will be necessary to have a thorough comprehension of some terms that will be needed, such as figures, numbers, lines, columns, diagonals, addition and total. The work done in the discovery of the magic square will allow to introduce or reactivate these words.
- The search of the magic constant of the 6x6 square implies to add successive whole numbers from 1 to 36 and it was thought more relevant to give it so as not to increase difficulties at the beginning of the activity.

3. Description of the training sequence in class

a. Predicted proceedings

The training sequence in class was adapted with the teacher who implemented it and anticipated a three -session organization, that is to say four hours.

<u>Session 1</u>: *Magic square discovery* (1 hour)

The teacher gives a mission to his students on a collaborative software (Framapad[1]): they have to discover a legend, the one of the Lo river tortoise.

Each student has a laptop connected to the Internet and launches research on the legend.

[1] http://framapad.org/

They have to discover what is hidden behind number 15, a crucial element in the legend, which is nothing but the constant of a 3x3 magic square. They then have to check that a given square is magical and to fill in three squares in order to make them magical.

To conclude, they must write a summary of what they have learnt on the collaborative tool. The session is filmed.

Session 2: Group work on the adapted square (2h)

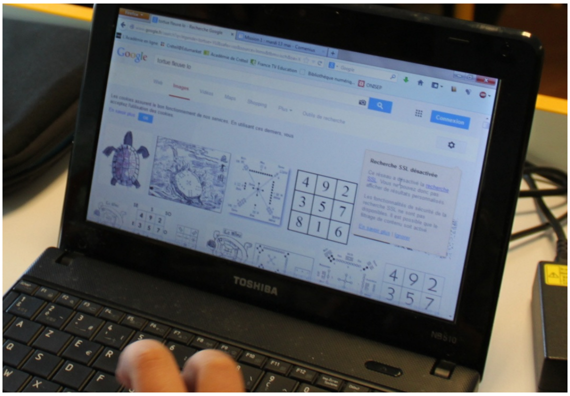
It's a two consecutive hour session. The adapted square is handed out to the students, with questions enabling them to decrypt it. The students work in groups of three or four in order to promote exchanges and reasoning formulation. The session is filmed and dialogues between the students are recorded.

Session 3: *Collective summary* (1h)

This session will be dedicated to the review of the work done, to the study of the initial square, to exchanges and to cultural and historical inputs.

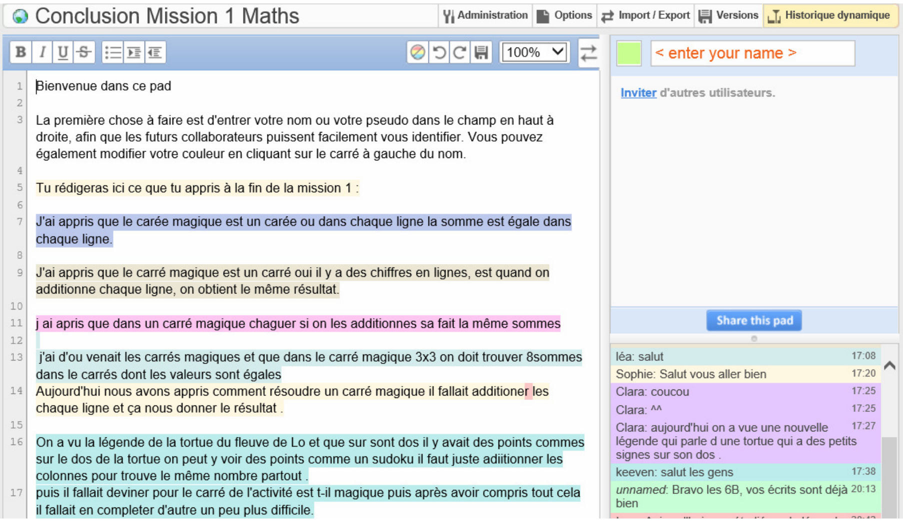
b. Real proceedings and analysis

Session 1: The "mission" aspect highly motivated the students during the first session. Big differences in the mastering of the online search for information could be observed, for example in using or not keywords in a search engine, and then while making a selection among given websites, in choosing to ask for text or pictures, between rewriting or copying and pasting to answer questions in the teacher's document.

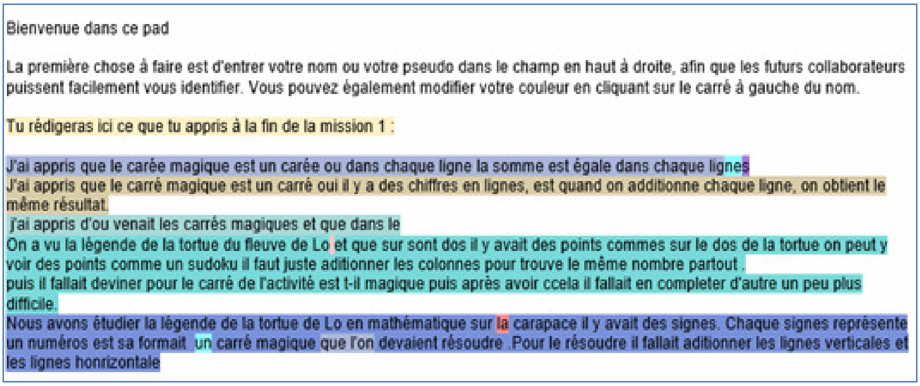


The skills to search and select the requested information, then to work together thanks to collaborative tools are part of the competences that must be worked on in lower secondary school.

The fastest students started the collective writing of the summary at the end of the session, and the other ones finished it at home the day after. One could notice that the fastest students only based their summary on the magic square, whereas the students who encountered more difficulties during the session, remained on the legend and dedicated their summary to a description of the tortoise.



Some students reread and corrected mistakes in sentences that had already been written by others (change of colour in the line). Some activities of writing, of reading and correcting, and of reviewing of transitional writings are at stake, on a differentiated basis, during a short time, and well included in the work in mathematics.



They write a final summary of what they have learnt on the collaborative tool, with the teacher's help.

The students newly arrived in France only had to write a sentence on a piece of paper describing what they had seen (the tortoise, the points…).

Session 2

Session 2 will begin with a review of the collective summary on the collaborative tool written by some students between the two sessions, in order to simplify andcomplete it.

Then the search on the adapted square can start.(Annex 1, questions 2, 3, 4)

An important difficulty appears at the beginning of the activity: the students understand well that each symbol hides a figure, but for some of them the ten symbols consequently represent 1,2,3,4,5,6,7,8,9,10.

But interpreting that this square contains consecutive whole numbers from 1 also represents a problem, because, for most of the students, numbers start from 10 or even 11. It is an unexpected difficulty: for these students, 0,1,2,3,4,5,6,7,8,9 are not numbers but digits.

Another difficulty consists in differentiating the number of occurrences of a symbol from the figure it represents. Discovering the first two figures takes a long time, searching is easier afterwards but four symbols are still to be decrypted at the end of the session.

Session 3

The search is soon over. Some pupils realize that one diagonal was not used in the reasoning and ask to check the total of the numbers written in this diagonal. The initial square is then given to the students as well as explanation on its discovery in China. A student makes analogy with the decrypted square.

Another student newly arrived in France reads and writes some of the Arabic numerals that he recognizes, which generates a very strong interest in the class. The students then show a great deal of imagination to try to explain the reason why the square was found in China. Then the teacher provides information on the history of the Arabic numerals.

A posteriorianalysis

First it can be noticed that this training sequence raised the students' interest. Skills in reading, searching and selecting information have been worked on. The use of a collaborative tool enabled to have some students take part in written work in various ways: by writing, rewriting, revising, correcting...

We can also notice that some mathematical terms are not spontaneously used at the beginning of the session, such as "lines" and "columns", often replaced by "horizontal lines" and "vertical lines", but that they are correctly used at the end of the session.

However, the designation of the symbols showed a real difference between the "EANA"[2] students and the others.

The EANA students[3] rely on school language, using vocabulary they have learnt since their arrival (from 1 to 5 months). They quote the circle, the half circle, the rhombus, the triangle and the rectangle. French speaking students havemore recourse to daily life things (moon, apple turnover). Thus they had difficulties in naming the red shape and none of them suggested the rhombus. Finally they all mentioned the square instead of the rectangle, and therefore seemed to be using a generic word in reference

2 "Elèves allophones nouvellement arrives" en France

[3]Three students whose language is Portuguese; one student from Bangladesh; one from Pakistan and one from Sri-Lanka.

to a daily concept instead of the scientific concept studied during the mathematics course. That did not cause any problem of communication between them.

The main difficulties clearly appeared in the understanding of the questions and in the explanation of arguments. Complex syntactic constructions are sometimes useful and require reformulating.

But, above all, the language used must exactly express the notions at stake. The meaning of the words “figure” and “number” varies according to the context, in the mother tongue or in mathematics, and it appeared that these notions were not fully mastered by the large majority of the students. However they performed without any difficulty isolated school tasks that were repeated and classic in the work on numeration, such as giving the tens figures of a number.

The framework of solving a problem, in communicative situation and interplay involves another level of availability of mathematical knowledge and of mastery of the language.

Bibliography, sitography:

Chabanne J.-C., Bucheton D.(2002) *Écrire en ZEP : un autre regard sur les écrits des élèves*, Delagrave édition - CRDP Versailles http://www.cndp.fr/bienlire/04-media/b-biblio03.asp?prodid=42772 (availableaddress on 2015 May 01)

Vygotski, L.S. (1934). *Pensée et Langage,* Editions sociales (Traduction de F.Sève, 1985).

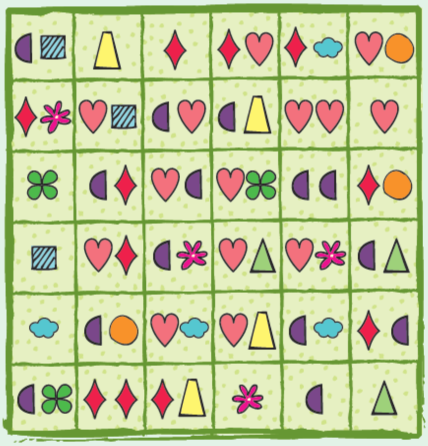
Marin B. (2011-2012), La reformulation en classe: un discours équivoque*, La construction des inégalités scolaires,*sous la direction de Rochex J.-Y., Crinon J., Presses Universitaires de Rennes.

IfrahG. (1994), *Histoire universelle des chiffres*, édition Robert Laffont.

Saint-Andrews University, *The Arabic numeral system*, http://www-history.mcs.st-and.ac.uk/Indexes/Arabs.html (available address on 2015 May 01)

Annex 1: (source: Hélice, 6th, Didier)

@ Un grand carré magique



Ce carré est magique mais chaque chiffre est remplacé par un symbole. Cherchons à découvrir tous les nombres contenus dans les cases !

1 Préparation du travail

a/ Reproduire ce carré avec des cases vides.

b/ Dessiner sous ce carré les symboles utilisés dans le carré magique ci-contre.

2 La constante magique

La constante magique de ce carré est 111. Expliquer ce que cela signifie.

3 Les chiffres des dizaines

a/ Ce carré contient les premiers nombres entiers à partir de 1.

Quels sont les nombres qu'il contient ? Quels sont les chiffres des dizaines possibles ?

b/ Compter le nombre d'apparitions de chaque symbole comme chiffre des dizaines et démasquer un premier chiffre.

c/ Écrire le chiffre démasqué dans le carré vide partout où il se trouve.

d/ Dans la 1re colonne, quel doit être le chiffre des dizaines inconnu pour pouvoir obtenir la constante magique ? En déduire le 3e chiffre des dizaines à démasquer. Compléter le carré.

4 Les chiffres des unités

a/ Quel symbole des chiffres des unités n'est jamais seul dans une case ? Quel chiffre cache-t-il ?

b/ Observer la dernière colonne : quel chiffre permet-elle de découvrir ?

c/ Quel chiffre la 3e ligne permet-elle de découvrir ?

d/ Quel autre chiffre peut-on alors connaître ? (Penser aux diagonales.)

e/ Finir de remplir le carré. Vérifier que le carré obtenu est bien magique !

5 Un carré bien ancien

Observer un très vieux carré magique conservé au musée de Xian en Chine, sur le site : http://home.nordnet.fr/~ajuhel/Grenier/car_mag.html. Traduire chacun des chiffres inscrits.

Second piloting

by Maria Piccione[**]

Introduction

The proposal concerns arithmetical concepts: in particular, numeration (decimal and positional writing system), additive relations and, more in general, symbolic representations. It offers an adequate context to deal with fundamental curricular contents of Lower Secondary School, as it can be found in the Italian national guidelines.

The proposal was firstly described by the trainer and discussed with the two teachers of the classrooms where it was going to be piloted.

The proposal

The work context refers to the decrypting of an old magic square discovered in China.

The activity aims at revisiting already known concepts, in order to improve their knowledge and to explicitly express certain fundamental properties, as well as at developing symbol sense. In other words, it promotes a metacognitive analysis of *sign-sign meaning* concepts in Arithmetic to clarify the relation "natural number - symbolic representation", and therefore the relation "digit-position-value" and finally the "order of magnitude" concept. Hence it faces the cognitive obstacle corresponding to the epistemological obstacle when passing from number concept to numeral concept.

In addition, the proposal gives rise to significant opportunities for introducing both the demonstration activity and the algebraic thought, and for thinking on geometrical questions. The learning context is suitable to promote the development of language skills in understanding and writing a text and in discussing by explaining plans-strategies-solutions.

As regards the *affect* domain, the approach uses two methodologies, game and narrative, which are not only useful to stimulate curiosity, fantasy, creativity, discovery, role assumption but also to prevent states of anxiety, frustration or feelings of inadequacy.

A historical and multicultural perspective can be used to outline the developments of written and oral numeration from the past to current days, in various countries and cultures.

In particular, it allows to show the gap between the instinctive human activity of "counting" and the slow process which brought to elaborate a writing system using a few signs to represent even large numbers.

[**] Department of Information Engineering and Mathematics - University of Siena, Italy

The piloting

The teaching unit was piloted in two classes of the second year (7th grade) of the Lower Secondary School “G. Papini” (Castelnuovo Berardenga, Siena), involving 42 pupils (11 immigrant and 10 with cognitive problems), by two teachers (V. La Grotteria & P. Sabatini) with the collaboration of the trainer. The activity was carried out by following the general design provided by the French project team, structured in four sessions, with two arguments in addition, the former being a geometrical digression and the latter an arithmetical scheme construction. These two steps were not scheduled in the plan, but they emerged during the class-work. They corresponded respectively to a *metacognitive-linguistic aim* and a *cognitive aim*, that is:

- to let the pupils reflect on the difficulty of explaining in technical terms even an apparently simple procedure (namely the construction of a square subdivided in congruent cells);
- to allow the pupils to construct a clear *mental image* about the way natural numbers can be arranged ten by ten, which highlights on digits recurrence in the role of units or tens from 0 to 99.

We could not use Framapad software, due to management issues of the computer lab: nevertheless this difference, with respect to the original direction, did not compromise the work from the point of view of cognitive aspects. Otherwise, it gave everybody the possibility to contribute to it in some way (at least just in the task of writing numbers).

<u>Session 1</u>: *Approaching the magic squares* (3 hours)

<u>Presentation of the task</u>. The work began with a short story told by the trainer (see Appendix A.1), and the display of an image of the old square of Xian:

The word “square” was investigated to verify the ability to explain the meaning of a very familiar geometrical term.

Some unexpected difficulties emerged related to the concepts of “side” and “right angle”, contrarily to what resulted from the conditions of “equality between segments” and “equality between angles”, which were immediately understood in terms of “overlapping”.

More precisely, the word “side”, which was referred, at first, to the word “boundary”, gradually improved the sentences formulation referred to proper definition, according to the following steps:

- *“the part of the boundary”*
- *“the part of the boundary between two vertices”*
- *“segment joining two vertices”.*

The word “right angle” caused difficulties both in formulating complete sentences and in explaining the paper folding construction as exemplified below:

- *“the space comprised between two segments which are each other …”*
- *“I fold once a part of the sheet and then I fold the other part inward, being careful not to put it either before or after”;*
- *“ I fold once a part of the sheet and then I fold the second crease over the first one”.*

In some cases, it was related to “measure”, and to “incidence between a horizontal and a vertical straight line”.

<u>Internet searching and resuming results</u>. By this time, the expressions “magic square”and “square of order 6” needed to be investigated. A research on the internet started in order to answer this issue.

The students, in groups of 2-3 for each computer table in the computers’ lab, began searching on the Internet.

After that, each group wrote a text resuming the information they collected and deemed relevant, also relatively to the magic squares history.

Students of each group read the text aloud. Some specific terms emerged, such as “array”, “order”, “magic sum” and were outlined while reading; moreover some properties were underlined, in particular, the existence and uniqueness of magic squares depending on order. It generated a feeling of wonder for the big amount of them, with the surprising jump of cardinality from the third to the fourth order (from 1 to 880), and from the fourth to the fifth one (from 880 to 275.305.224),reachingthe level of the billions of billions to the next step.

Working in a collaborative way, the pieces of information gathered by the various groups were summarized together in a single text, written down by a student, with the aim of being included in a final poster exhibition (Photo 1).

Training in a simple case. This activity was addressed to consolidate the defining properties of a magic square. Every group was given a sheet of paper containing a colored picture of a 3x3 magic square and some questions on the types of numbers appearing in it and on the addition properties of the numbers which are satisfied in each row, column and diagonal. As this work pointed out, the majority of the students deal with the terms "row" and "column" in the natural language sense, and with the term "diagonal" in the geometrical interpretation; whilst they do not use the word "constant", which they paraphrase as reported below:

- *"it always comes the same result".*

Some students could indicate the objects by gestures. (Photo 2)

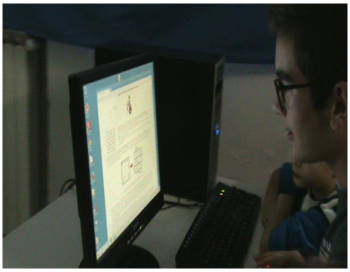


Photo 1

Photo 2

Session 2: Discussing words "order of a square" and "magic sum" (2 hours)

The students had been intrigued by the new concepts of "order of a square" and "magic sum".

Working as a whole class, with the guidance of questions posed by the teacher, the following activities was carried out to construct the order of a square concept:

- drawing on the board the square corresponding to each order (from 1 to 6)
- explaining operations to be performed
- relating the order of a square with the number of its cells
- noting the banality of the magic square of order 1 and the non-existence (impossibility) of models for the order 2.

As the attempts in verbal explanations have shown, transition from the intuitive to the rigorous idea of order has not been easy. In order to support this conceptual construction, the students performed a drawing activity, which actually resulted to be helpful: subsequently, they were able to say that the order is *"the cells number along a side"* or *"the cells number of each row", "the subdivisions number of a side".* Difficulties emerged both in realizing and in describing the steps of the sequential instruction, which denoted a low level of procedural thought development among students.

The magic sum concept has not risen problems; the discussion about it has led a (clever) student even to explicit the calculation rule of the magic sum depending on the order, arousing the enthusiasm of the classmates:

"by the order I can find the total number of the cells; then I have to add all the numbers from 1 until this found number and finally I have to divide this new one by the rows number".

<u>Recalling current numeration</u>. Still working as a whole, the class recalled the properties of current numeration, and they were asked to explain the sense of digit and the sense of digit depending on position.

A useful instrument to perform the work has been the drawing on the blackboard of a table as in the figure below:

0	10	20	30					
1	11	21	31					
2	12	22	32					
3	13	23	33					
4	14	24	34					
5	15	25	35					
6	16	26	36					
7	17	27	…					
8	18	28						
9	19	29						

Photo 3

This scheme let the pupils able to get a visualization of the regular distribution of unit digits and tens digits in the written succession and to discuss about the observed occurrences.

<u>Session 3:</u>. Decrypting the magic square (1 hour)

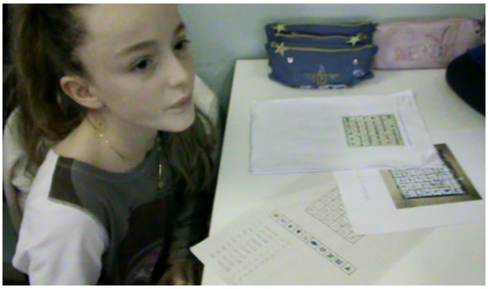
<u>Solving a pretty magic square</u>**.** Once gathered in small groups, the pupils were committed to the solution of a magic square of order 6 where symbols of the original old square were replaced by other familiar figures.

They began by attempts to try the coherence of supposed numerical value of some figures; then they proceeded in a more systematic way following a few hints given by the teacher and using the "guide scheme" extended until number 36. They could carry out the strategy of counting both digits and figures recurrences and could compare the results in order to get conclusions (first discovering the symbol of number 3, then the one of 0 …).

Photo 4a

Photo 4b

<u>Decrypting the magic square of Xian</u>**.** During this session, the teacher provided each groups with a copy of the ancient Chinese magic square with the task to decrypt the Arabic numerals available. The correspondence between the symbols of the two squares was immediate. (Photo 5)



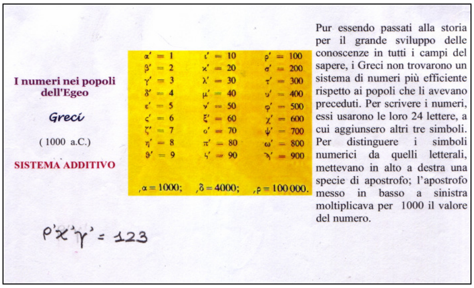


Photo 5 **Photo 6**

<u>Session 4</u>: Come back to cultural angle (1 hour)

An historical view about the evolution of Arabic numerals was done through reading and summarizing in class group-work nine tables prepared by the teachers. The material referred to numeration systems in Sumerian, Babylonian, Egyptian, Indo-Arabian, Mayan, Greek, Chinese and Roman cultures.

Two additional tables were concerning the "Numbers of computer" and the "History of zero".

Each group tried to write a number in the system that was given. (Photo 6)

<u>Final test</u>. (1 hour)

A final questionnaire was submitted to the students to control the traces of the work done, about five months after the end of the task. The answers provided many educational guides. The most important and general guidelines refer to the density and depth of the proposal contents which requires *time* for performing, in order to use its full learning potential.

A posteriori analysis.

The proposal appeared to be productive in gaining interest under various aspects, both on methodological and on topic related levels: historical approach, cooperative working, used tools, role interpretation, discovering and decrypting activity.

In *affect domain*, the experience, in spite of mental application, can be lived as a game, contextually to a feeling of good competition among groups. The new problem of decoding results generates moments of surprise and satisfaction, as for finding the solution by reasoning and logic efforts.

In *cognitive domain,* the activity offers a context in which the students may apply Arithmetical concepts and operations, revisiting them and increasing awareness of numeric properties and regularities, getting general formulas (the first n natural numbers sum, the "magic key" value) and improving the symbol sense.

The work requires and develops ability of

- getting information in a text in order to re-writing a text
- understanding the instructions
- explaining, defining, argueing.

The activity highlighted difficultiesreferable (in several cases) to a mechanical teaching/learning of Arithmetic. The exposure language in teaching seems to lead to generate linguistic stereotypes that give a "dress" to a weak conceptual body.

It revealed even an unexpected difficulty in the drawing procedure of a square divided in a given number of congruent cells: many pupils could not do it in an ordered way: by simply recalling a mental image, they could not explain the steps of the geometric construction of the structured figure.

The activity is adequate for training (pre-service and in-service) Mathematics teachers.

Suggested links

http://www.lannaronca.it/programmazione/quadrati%20magici.htm
http://it.wikipedia.org/wiki/Quadrato_magico
http://areeweb.polito.it/didattica/polymath/htmlS/Interventi/Articoli/AvventuraCubi/AvventuraCubi.htm

APPENDIX

A.1

The told story. *An archaeologist friend of ours went to work in China, she visited the Museum of Xian, where she had been intrigued by an archaeological finding: a metal plate with the image of a square pattern containing strange signs. The caption underneath reported the following phrase "Magic square of order6". Back to Italy, she asked us about the mathematical meaning of the plaque. Our amusement in solving the enigma is why we propose to you the same discovery game...*

Third piloting

by Hana Moraová[***]

The teaching material was piloted in the 5th (primary) and 7th (secondary) grades in ZŠ Fr. Plamínkové, a Prague school. As there are almost no migrant pupils in the school, the material was piloted using the CLIL methodology i.e. the lessons had two objectives – mathematical and linguistic. This was meant to simulate the situation in which pupils struggle to understand the language of instruction in a mathematics lesson, as the language of instruction is not their mother tongue.

The lessons were video recorded.

The piloting in both classes had two primary objectives – to develop language skills of the pupils, introduction (revision) of key concepts in English (row, column, diagonal, add, multiply, sum, product etc.). The unit was meant to develop the pupils' receptive and productive language skills: the piloting began by conversation about magic, secret, superstition and legends, by narrating the legend of Lo-Shu. The pupils were also trying to answer the teacher's questions in English. The mathematical objective was search for the magic number, mathematical reasoning, formulation of arguments, discovery of properties of numerical operations. The teaching experiment was concluded by having the pupils work following a basic algorithm (how to make an odd order magic square).2 different classes, 7th and 5th graders, CLIL lesson, i.e. language and mathematics goals.

1st piloting: 7th grade, CLIL lesson, use of smart board – using the possibility to write on an existing picture of the turtle shell, have experience with mathematics in English lessons, language less of an obstacle

Language aims: speaking activity – discussing magic and miracles, listening – the Lo Shu legend

Mathematics goals: looking for patterns and regularities, discovering (re-discovering) properties of arithmetic operations, patterns in a square grid

Problems encountered – the pattern on the turtle in the presentation had a mistake, i.e. the numbers gained did not make a magic square

When asked to multiply each number by the same number, some pupils multiplied 1x1, 2x2, 3x3 etc., which does not make a magic number

Warm up: discussion – what are legends, what is the difference between a legend and a fairy tale, give examples of legends, do you like legends and magic?

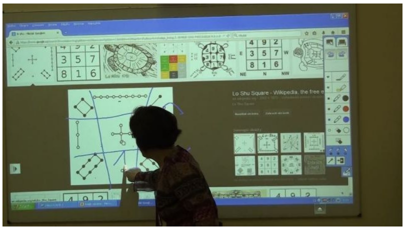
Lead-in: Telling the legend of Lo-Shu.

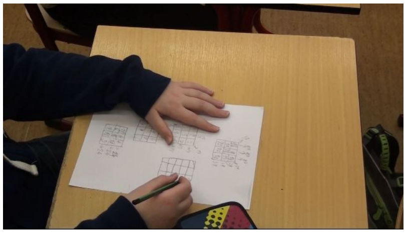
[***] Faculty of Education - Charles University in Prague, Czech Republic.

Main activity:

1. discovering the numbers in a magic square and what makes a magic square magic (failed because of a wrong picture)
2. discovering what happens if the same number is added to each number, will it still be magic? Why?
3. discovering what happens if each number is multiplied by the same number (misunderstood by 2 pupils)
4. discovering what happens if we swap rows or columns equidistant from the centre
5. showing the principle of putting numbers in a magic square
6. pupils work on a 5 times 5 square, trying to put the numbers, a few finish in time and successfully, the teacher monitors and tries to help if there are problems

The end of the lesson. The teacher decides to pilot the same unit again, trying to eliminate the problems from this lesson.





2nd piloting: 5th grade, CLIL lesson, use of smart board – using the possibility to write on an existing picture of the turtle shell, the first lesson of mathematics in English

Language aims: speaking activity- discussing magic and miracles, listening – the Lo Shu legend, introduction of basic mathematics vocabulary in English (odd, even, multiply, add, subtract, diagonal, line, column)

Mathematics goals: looking for patterns and regularities, discovering properties of arithmetic operations

The original lesson plan was modified to avoid some of the problems from the previous lessons (the selected image had the right pattern so the pupils could really discover the numbers on the shelf)

Warm up: What is magic? Language speaking activity, eliciting ideas from the children

Lead-in: The teacher tells the story of Lo-Shu monitoring understanding (more difficult vocabulary items like floods, turtle, sacrifice)

Main activity:

1. the picture of the pattern on the shell projected on smart board, a grid made and pupils asked to think why this is magic

The pupils very active looking for tens of different regularities (pattern of odd, even numbers, the sum of numbers in a triangle, the sum of numbers in corners etc.), after about 7 minutes of trying out the idea of 15 discovered (through the idea that the opposite numbers have the sum of ten).

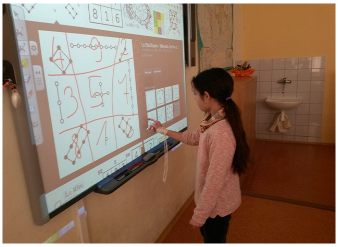
2. pupils asked to add the same number to each number, elicited one number that everybody would be working with (6); pupils work and find out that the square is still magic, the teacher asks why?

   After some proposals the pupils see that the difference between the original sum 15 and the new sum 33 is 18. With the teacher realize that 18 is three times six, e.g. the new number added three time in each row, column and diagonal.

3. pupils are asked to multiply each number with the same number (taking into account experience from 7th grade number 3 elicited and first three numbers are done together as an example to avoid ambiguity]; pupils work and find out the square is still magic, the sum being 45; again asked to look for the reason why.

   They experiment, propose different ideas of why it works but need considerable help of the teacher to see that 15 times 3 is 45, i.e. that multiplying each summand, or the whole sum by the same number brings the same result.

The lesson ended. As more time was left for discovery of the magic, fewer activities were done. But the teacher evaluated this lesson as the best one and more beneficial for the pupils who had discovered more of the things on their own.



Conclusions from the three piloting

by Marie-Hélène Le Yaouanq and Brigitte Marin

These experiments carried out tend to confirm the interest of the “magic square” activity and its adaptability to different class levels (from CM2 to 5è, that is to say from the fifth year of primary school to Year 7).

First and foremost, the students are actively placed in a position of research. The presentation of the legend, the “magical” aspect of the situation, allows an involvement of the students who are receptive to the playful dimension and to a presentation that gives them the impression of taking up a challenge. This involvement even remains throughout a sequence of three or four sessions.

In France and in Italy, starting the activity by the means of a web quest followed by exchanges and by the writing of a summary, develops competences related to searching, classifying and exploitation of information, as well as competences in the mastering of either written or spoken language. In the Czech Republic, the discovery of the situation, led in English, is rather close to the discovery of the work done in France with the allophone students.

However, differences show up in the implementations. Differences in length from 1 to 4 sessions can be observed, which are linked to different objectives in Mathematics. The mathematical work includes the discovery of the magic square but then, it focuses on numeration in the case of the decoding of the ancient magic square or the discovery of certain mathematical properties of a magic square.
The multicultural and historical aspect is managed differently in Italy, with a group work on the different numerations and in France, with live exchanges in the class, based on the presence of foreign students.

The decoding of the ancient magic square puts the emphasis on the difficulty of symbolism, but also on real reasoning difficulties concerning numeration and the position of a figure in a context that is different from “classical” exercises, which may lead to stereotyped answers without constructing the concept strongly enough. The historical aspect and the study of different numerations is consequently an interesting extension from a mathematical point of view.

The decoding of the magic square confirms that the linguistic and the cognitive aspects are intimately linked. Therefore, a difficulty in the mathematical reasoning forces the students to exchange in a precise manner in order to be able to move forward, and, at other times, difficulties to express what they have perfectly understood from a mathematical point of view make them aware of the necessity to work on the language. The case that was explored seems to provide a context allowing the development of both language and mathematical competences, while remaining motivating for the students and adaptable to different class levels and to different objectives.

PUTTING BINS IN OUR SCHOOL'S YARD

by Charoula Stathopoulou* and Eleni Gana*

INTRODUCTION

This activity, based on a needs analysis, which was undertaken in an earlier phase of the project, intends to respond to the teachers' explicit request for teaching resources and provides an example of a good practice when teaching mathematics in multicultural/multilingual classes in secondary education.

The activity is informed by the broader research regarding language's role in mathematics teaching in multicultural/multilingual classrooms as well as by the recent literature for inclusive pedagogical initiatives regarding students with different cultural and linguistic backgrounds.

Specifically, the design of the activity adopts the methodological framework of a task-based approach (Willis 1996, Leaver and Willis 2004, Puren 2004). Through the activity **"Putting bins in our School's yard"** students were asked to answer to an existing, real life problem that emerged in their school environment. Finding a suitable space in the school yard to place recycling bins constitutes a real and recognizable issue for students' involvement in mathematics and provides them with the motivation to invest in the teaching and learning processes. Therefore, the mathematical situation is expected to make sense for all students; it does not also rely on well-defined mathematical procedures but instead allows for students' agency and collective negotiation of the mathematical notions (e.g. proportionality) and techniques (e.g. measuring). The very design of the activity sustains rich opportunities of communication between classmates and between students and the

*Department of Special Education - University of Thessaly, Greece

teacher in order for the class to reach a reliable solution. Students from different cultural and linguistic backgrounds could contribute with their own mathematical and discursive resources and make connections between informal and formal mathematics. In addition, in the context of students' joint action and synergy foreseen in varied phases of the activity, the different experiences and ways of dealing which the students bring to the group could be appreciated, and more equal relationships among peers could eventually emerge.

The technique of measuring and the notion of proportionality constituted the main mathematical content of the activity. The measurement – a procedure for the determination of an object's/ phenomenon's size – is an important mathematical process for determining the world around us, and helps to better control one's environment. The concept of proportionality is another basic mathematical concept, which runs through all matter of mathematics in compulsory education in various versions: scales, similarity, linear equation etc.

Workshops: Piloting with trainees

Before the final planning and the implementation of the activity (main piloting), there were three workshops, in which teachers of mathematics in secondary education (10 teachers) and one regional consultant for pedagogical issues in the lower secondary education took part.

The first workshop focused on language and cultural issues involved when teaching mathematics in multicultural/multilingual classrooms. The results of the needs analysis which preceded it showed that most teachers, although they taught mathematics in multicultural/multilingual classrooms, had not been trained accordingly. Therefore, in the first meeting there was a discussion of strategies which promote the language origin and the cultural experiences of students as teaching resources, as well as strategies that support the transition from everyday speech to mathematical speech. In the second meeting, the teachers were introduced to the main idea and the basic procedural phases of the activity. We discussed aspects of the mathematical content, its relation to the curriculum and the challenges that could present for the students of different linguistic backgrounds. During the third workshop we reached the final decisions of the activity's design. The teachers contributed to the decisions about: the phases of the project's implementation, time estimation in each phase, students' expected reaction on the activity and possible strategies suitable for the activity's implementation.

The design of the activity: Brief Description

- ***The Rationale***

We selected to use a task-based activity, since through a task-based instruction students are expected to make sense of a mathematical situation for which no well-defined procedures exist. It is expected to also create rich opportunities for mathematical communication where probably language's issues could emerge.

The present activity is based on the existent ongoing discussion among students and teachers about the lack of efficient recycling policy in their school and how they could respond to this situation. So, it is about a real life problem that demands mathematical concepts and techniques, thus it is expected of all the students - since it is challenging for them - to be engaged looking for solutions and developing a suitable model.

♦ ***The task***

We present to the students a formal letter send by the Municipality concerning the official intent to provide rubbish bin/recycling bins to the school: students must respond through considering the space possibilities of their school yard, reasoning for the number of bins as well as for their choices concerning the bins' positions in the school yard.

♦ ***The students and the school***

The activity was designed for piloting - 1^{st} piloting –in the 6^{th} Gymnasium in Volos in the 7th grade (1st grade of Gymnasium) classroom, by the mathematics educator Ioannis Fovos. Out of a total of 22 students of the class, the 6 are Roma students and were bilingual using Romani, an oral language without written code, at home.

♦ ***The objectives***

- To connect mathematical notions and techniques in order to explore social issues and provide arguments for real life situations. Students will have the opportunities to use multiple solution strategies. For the implementation of the above, students were expected to use: space notions, measuring, similarity, proportion etc. Furthermore, it was expected that students would use propositions for locating, such as: down, up, above etc., probably in different ways, since, through previous research in the spot we have noticed that Roma people have a different space coding connected to their cultural peculiarities that is also depicted in the propositions they use.
- To communicate verbally and/or in other modes their ideas and their own ways of acting mathematically and to negotiate their reasoning socially, that is in the group, in the entire class, when addressing a formal agent, using different register and multimodal ways.
- To be actively involved in exploring different discursive patterns (genre, vocabulary and grammatical structures) when using and communicating mathematics in different contexts.

Main piloting

by Charoula Stathopoulou, Eleni Gana and Ioannis Fovos

The implementation of the activity

The project was first piloted in a lower secondary school (6th Gymnasium) in Volos city.

1st session: initial information *(10 minutes)*

The teacher informed the students that their class was selected for the implementation of a new type of mathematical activity and that the whole procedure was going to be videotaped. Almost all the students were challenged by this perspective, with the exception of one Roma student (a girl) who declared that she did not want to take part.

2nd session.

a. Reading the letter *(20 minutes)*

The activity started with the teacher reading a letter sent (or supposed to be sent) by the administration and according to this, the students had to locate the suitable places for putting the bins in the school yard and to justify their choices. After finishing the reading, the teacher asked: *what do you think could be our first step for solving this problem*?

b. How to put the bins? *(20 minutes)*

Some students' characteristic answers on the above question:

- to take a picture from the roof of the building;
- to make a plan like this, showing the plan of the first floor that was next to the door of the classroom.

Students agreed with this suggestion and started to think how to do it. They realized that measuring the sides of the schoolyard was needed.

The teacher asked: since the measuring is going to give the real dimensions how could these be depicted on paper? Students answered that they knew how to do it although they could not remember the term 'scale'; a term that was reminded by the teacher. Then the teacher informed the students that they would move to the schoolyard to measure its sides and each group would make their own plan. At the end of this meeting, the Roma student who in the very initial stages of the activity was negative towards the activity, approached the teacher to say that she would like to contribute and was provided with the students' names in her group. She probably found the pragmatically oriented character of the activity interesting and/or challenging for her.

3rd session: Measuring the schoolyard's sides *(30 minutes)*

The main objective in this session was for students to conduct the measurements of the sides of the school yard and to prepare a draft plan of it.

The first measurement happened to be launched by the above referred Roma student. Then one student of each one of the other groups measured the rest of the sides of the yard. A student from every group had the responsibility to keep notices of the measurements. For the measurement, the odometer was used; a tool that seemed to be very attractive for the students:

A student measuring the yard

4th session: designing the plan *(in the classroom, 2hours)*

The objective in this session was for students to find the right scale, even though they would not express their results in a mathematical register. They also had to design the plan of the schoolyard on a paper.

The students worked in groups. Specifically, 5 groups were formed, 3 of which Roma students participated in. Classroom desks were arranged for students to sit around, thus facilitating the processes of collaboration.

<u>The students' difficulties in constructions and strategies:</u>

The initial strategy of all the students in order to depict the plan of the yard in their papers was to transfer meters to cm using in this way unconsciously the scale of 1:100. In some cases, when they realized that paper (A4 size) was not adequate, they changed the arrangement of length-width and marginally managed to draw the floor plan. After realizing that the paper was not enough, one group just enlarged the paper fixing one more sheet to their original one, while the other groups divided each size by two making the scale 1: 200. As we could notice, students did not immediately apply the rule of thirds to find the right dimensions for the plan on the paper but they used their informal strategies that led to a satisfactory depiction of the yard. However, using these strategies, apart from one group, the other groups were not able to express which the final scale they used was.

It is interesting to see the answers of the students regarding both the process and the final result:

> Student: …..*then, because it didnot fit we divided it by two.*
>
> Teacher: *only the big ones? Did you leave the small ones as they were?*

Student: *In the beginning, yes, we did only the big ones, but then we thought to divide them all so that we get the correct shape.*

The students, in their justification, although they do not directly refer to scale, they suggest it, saying: “so that the shape would be right.” They comprehend, that is, that the shape they have to make is the same as the shape of the schoolyard; they understand the concept of similarity and the notion of scale. The fact that it is an authentic activity dictates the need for a working solution in the particular context.

An additional difficulty that students faced in designing the floor plan of the schoolyard was that their final shape had to be a closed polygonal line. Having overlooked the need to measure the angles so that they had an exact mapping, the students were often led to arbitrary adjustments regarding some sides of the schoolyard so that they got a closed polygonal line.

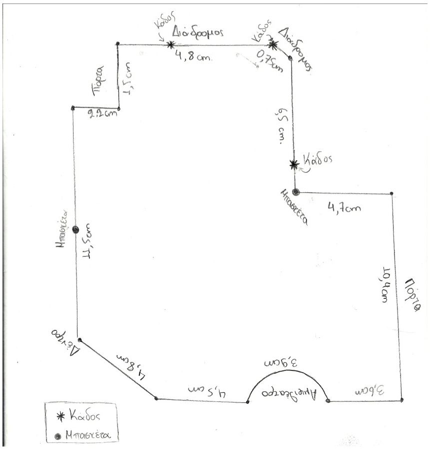
Teacher: *did your shape close properly?*

Student 1: *no, as it did not close, we made the hallway a bit bigger.*

Student 2: *in order to close the design, we made 22, 4 cm more crooked, so that it could fit.*

Students working in groups in the classroom

Below, one of the designs (floor plans) that the students created in their groups is presented.

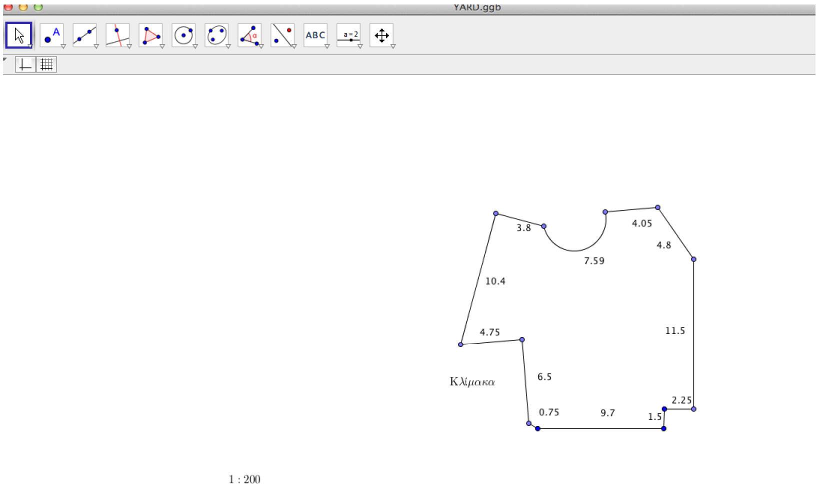


5th- 6th session: collaborative exploration of the groups' plans (1h+1h)

The teacher scanned all the groups' drawings and displayed them on a projector; each group had the chance to present the procedure of their design focusing on difficulties, strategies they had used etc. Furthermore, seeing their designs projected, students used the wireless mouse to describe and to show points that caused them difficulty as well as correct possible mistakes.

Then, the teacher introduced the GeoGebra displaying the screen on a projector so that all the students had access and so that after that they could make their own designs in their group, also showing and justifying the positions they selected for the bins to be placed at, with the use of icons. The teacher first constructed three consecutive rectilinear line segments as sides of the schoolyard. Since the shape of the yard was not a typical rectangle they faced the problem of making the figure a closed polygonal line. The need to measure angles surfaced through the teacher's suitable questions in order to determine the problem. This need was connected to the specific micro-framework, which is determined by the characteristics of the software and which had not appeared in the previous micro-framework -where the students worked with pencil, paper and geometry instruments- and in which they thought they could make arbitrary adjustments so that the polygonal line depicting the schoolyard closed. So, the teacher then, together with a few students, using the big protractor of the school, measured the non-right angles. After that, they put the values on their drawings so that they could construct the drawing more accurately with the software.

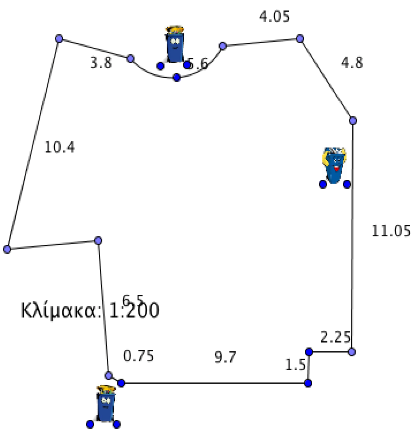
Finally, they jointly constructed the following drawing using GeoGebra:



7th -8th session: the response letter to the municipality (1h+1h)

In these last sessions the teacher asked the students to justify the places they selected to put the bins in. Some characteristic answers of the students:

- the first bin should be in the amphitheatre, in front of the column, because many



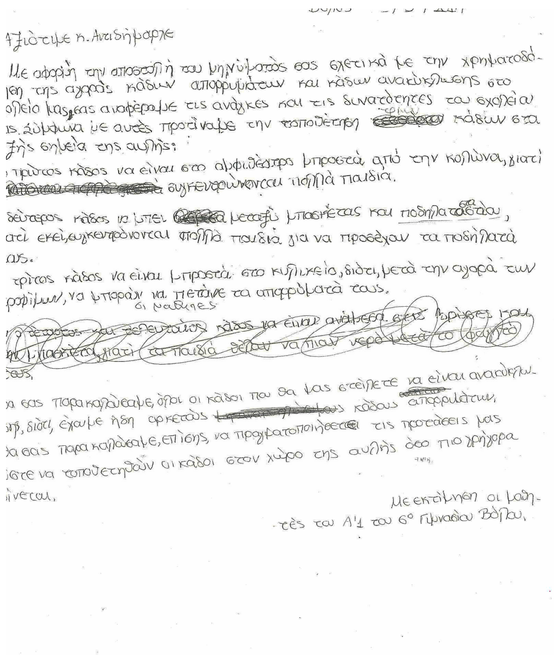
children assemble there during the break;

- the second bin should be placed between the basketball hoop and the bike racks because many children gather there in order to keep an eye on their bikes;
- the third bin we think should be placed in front of the school canteen so that the students throw their rubbish there after buying food.

After plenary discussion for the best plan, the students resulted in one of 1:100 scale and they put the bins as it is shown in the drawing below:
It should be noted that the use of the software tool added a dynamic dimension to the drawing of the schoolyard and gave the students the possibility of experimenting in placing the bins, as well as showing the limitations of the previous framework, where the students drew the schoolyard using tactile materials.

In the end, the students wrote down an official letter for the administration of the municipality: dialogic construction of the text, teacher's scaffolding in the use of the mathematical register.

<u>Letter to municipality:</u>



Αξιότιμε κ. Αντιδήμαρχε

Με αφορμή την αποστολή του μηνύματός σας σχετικά με την χρηματοδότηση της αγοράς κάδων απορριμμάτων και κάδων ανακύκλωσης στο σχολείο μας, σας αναφέρουμε τις ανάγκες και τις δυνατότητες του σχολείου μας. Σύμφωνα με αυτές προτείναμε την τοποθέτηση τριών κάδων στα εξής σημεία της αυλής:

ο πρώτος κάδος να είναι στο αμφιθέατρο μπροστά από την κολώνα, γιατί συγκεντρώνονται πολλά παιδιά.

ο δεύτερος κάδος να μπει μεταξύ μπασκέτας και ποδηλατοστασίου, γιατί εκεί, συγκεντρώνονται πολλά παιδιά για να προσέχουν τα ποδήλατά τους.

ο τρίτος κάδος να είναι μπροστά στο κυλικείο, διότι, μετά την αγορά των τροφίμων, να μπορούν να πετάνε τα απορρίμματά τους.

~~ο τέταρτος και τελευταίος κάδος να είναι ανάμεσα στις βρύσες και την μπασκέτα, γιατί τα παιδιά θέλουν να πιουν νερό μετά το φαγητό τους.~~

Θα σας παρακαλούσαμε, όλοι οι κάδοι που θα μας στείλετε να είναι ανακύκλωσης, διότι, έχουμε ήδη αρκετούς κάδους απορριμμάτων. Θα σας παρακαλούσαμε, επίσης, να πραγματοποιηθούν τις προτάσεις μας ώστε να τοποθετηθούν οι κάδοι στον χώρο της αυλής όσο πιο γρήγορα γίνεται.

Με εκτίμηση οι μαθητές του Α'1 του 6° Γυμνασίου Βόλου.

Conclusions

The teacher, who piloted the activity, as he participated in the workshops and had substantial contribution to its formulation, faced all the issues that appeared during the implementation of the activity creatively. The students, in turn, engaged with interest in a task-based activity, which regarded real problems in the school context and created conditions for an empirical approach to mathematics. The various micro-frameworks of cooperation among the students that developed gave the potential of reversing the traditional relations that prevailed in the class among students and between the students and the teacher. The students with a different linguistic and cultural origin (Roma) were dynamically involved in the cooperative performance of the mathematical procedure that their group had to perform; they made use of the linguistic experience of everyday speech in order to comprehend mathematical speech and negotiated mathematical concepts and techniques in a framework that was meaningful to them.

Finally, it has to be noted that although in the design of the activity the use of the language of home/community had been a possibility, the Roma students systematically avoided to use Romani, that is the language they used in their everyday transactions, in the class context. This choice of theirs is obviously connected to all the previous experience of those students and could not easily be changed in the context of only one activity.

Second piloting

by Maria Piccione**

TEACHING UNIT IMPLEMENTATION

The proposal refers to a *mapping* work, which typically involves basic geometrical concepts of school curriculum. It can be classified as modelling real space activities and then having the relevant feature to require the management of the relation *space size* - *sheet of paper size*. For this reason it represents a conceptual field that supports the development of methods and reasoning underlying geometrical competencies.

The activity is aimed to offer the students the opportunity to face a real life problem, without any indication of solution, exactly requiring tackling the construction of a *mathematical model*. By the given task, we expect they are able to identify and to enforce mathematical notions and techniques they already have (space notions, such as alignment, angle and length, shape, measure) and to get an approach to other ones not yet constructed (scale, proportion).

The activity provides a context for mathematical peers communication, which requires decision-making and negotiation of reasoning.

Description of the activity

This piloting took place in lower secondary school "Paolo Uccello", a school located in a neighbourhood on the northwest suburbs of Florence, with a group of 7 pupils from a second class (degree) under the guidance of the trainer and the mathematics teacher (Prof. A. Scialpi).

The original proposal has been implemented with some changes. The first one concerns the choice of the space to be depicted, also to comply with school safety

**Department of Information Engineering and Mathematics - University of Siena, Italy

rules: a large hallway, well shaped for the relief work, inside the school, rather than the space surrounding the school - too wide on the whole and regular shaped in more limited areas. The conceptual domain remained unchanged, while the modelling work changed for what concerned certain realization features, being simplified. The environment taken into account could anyway be considered as a "*meso-space*" according to the ratio between physical space and subject perceptive space sizes.

The students were given the task to solve a practical issue by an official invitation letter on behalf of the School Director.

In addition, working with a group of students (and not with the whole class) other changes occurred.

The planned activity followed three phases within the times specified by side.

Session 1(1 hour)

*Presentation of the task***.** By reading the official letter, the students understood the request to collaborate in implementing a plan for the waste separation; for this task they had

- to make maps of certain building local places;
- to indicate appropriate settings for the rubbish bins and determining the number of these ones.

They also seemed to share the educative reason of their involvement aimed at developing a respectful attitude towards the environment.

Work planning **.**The teacher did not explain the word map; she asked the small group of students how they would organize the work, by performing

- *what to do*
- *how to split tasks*
- *what tools they needed*
- *how to register the collected data*

It could be noticed that they easily realized they have:

- **to measure** the boundaries of the corridor taking into account the various openings (doors, stairs, windows) and obstacles (columns, radiators)
- **to adopt tools** to take measurements.

They had already experience with a special material consisting in rods (Photos 1a and 1b) and they spontaneously thought that the long rods were adequate to operate on big distances, while, on the smallest ones, the shorter rods could be used to complete the covering of the boundary lines of the floor.

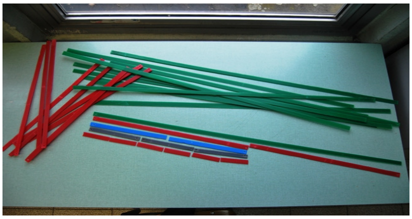


Photo 1a

Photo 1b

Furthermore, they agreed they had:

- **to record** the obtained measurements back on paper, and
- **to depict shape** to develop a model.

Common squared sheets of paper were taken for this purpose.

Session 2 (2 hours in two times).

*Taking the measurements***.** The students cooperated together at the new activity by placing the rods on the floor, along the walls, to take measurements; the need to split up some rods occurred to measure small remaining parts of the boundary. The trainer proposed to divide them into equal parts and the pupils agreed that this would have simplified the operation of setting length-relations between a single part and the one of origin. Then, 3/4 of them began mapping the results on the chosen sheets, without performing any choice of scale.

*Developing the model***.** At the beginning, the "mappers" were just "drafters": they made a drawing taking in account the shape of single parts of the hallway, labeling the segment by the corresponding measures expressed in *rods-units* nominated by color. The first attempt of mapping was at a very elementary level with components of tridimensional representation. (Photos 2a and 2b).

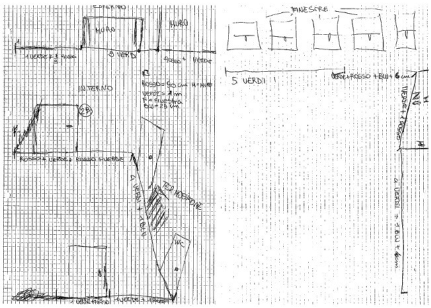


Photo 2a

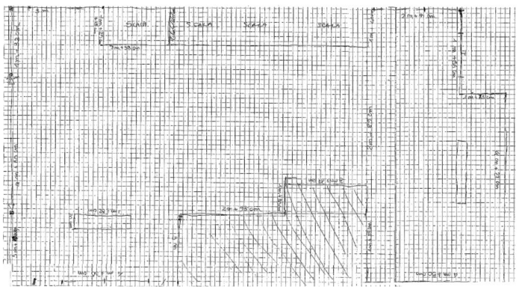


Photo 2b

The trainer suggested that the pupils carry out a global display instead of local ones: they stopped to analyze the whole structure and discussed about it.

Then she aroused the curiosity among the pupils to know the length of a long rod with respect to the standard units; and they made control: *just 1 meter*!

It is relevant to outline that the group depicted to the paper the representation of the hall, without considering at all a *scale-problem*, but only roughly respecting the ratio between object measures and corresponding segments' lengths measures.

In the end they recognized and agreed that they would have to decide in advance *how many squares* on the squared paper it was appropriate to let correspond to 1 meter. They became aware of the need of this convention and they would implement it for the final model.

It was clear that telling "square" they meant "sides of squares" but they found it natural to use more speedy expressions.

This shows the very didactical value of such an activity to approach the fundamental scale-concept itself.

Various logistic comments and suggestions regarding the positions of bins were expressed during the activity.

Session 3 (2 hours)**.**

*Working in the computer laboratory***.** The students of the two classes had never used a software of dynamic geometry before. This situation put them in a new work context. They made use just of basic rules of GeoGebra, shown by the teacher, and were able to apply them to draw the official map.

As a first level of application, the work was made on the squared screen: this modality implies a reduced use of the software with respect to its real potentiality and aims, but at the same time it leaves open the crucial problem to set a relation among lengths.

They tried several choices and finally they found out an adequate correspondence leading to a ratio. Then a very careful transfer of data was made from the map drawn on the sheet to the rising one on the screen. By themselves they gave roles of "performers to the keyboard" and "prompters of data". (Photos 3a and 3b)

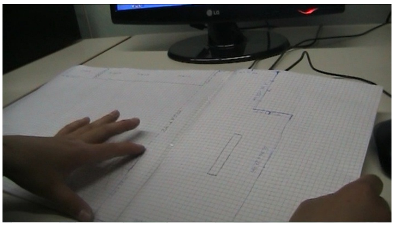


Photo 3a

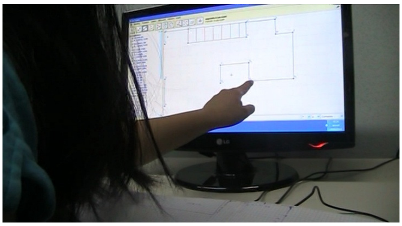


Photo 3b

Comments

*Affective domain***.**

The students took on the task with pride (having a social commitment) and seriousness.

While they were working in the corridor, curious people (students or adults) asked what they were doing: with enthusiasm they responded that they were carrying out a "relief" aimed at a School project, giving explanations.

The novelty of structure and typology of the activity (request, location, methods of implementation) let the students applying themselves to the work with interest, pleasure and sustained attention.

*Cognitive domain***.**

The activity offered a context in which the students had to apply mathematical concepts and procedures, revisiting them and increasing awareness of their theoretical and practical importance (measure as a procedure, units, unit conversion, results of a measurement, scales, proportions, model developing).

It created opportunities for mathematical communication, which allowed pupils:

- comparison/discussion of ideas
- common decision making
- switching from common language to specific language
- verifying the difference of the formulation of verbal messages in both previous registers.

*Difficulties***.**

The work gave the students the opportunity to face difficulties they did not expect in the execution of the model, mainly related to managing a non-standard shape and solving the problem of choice of the scale.

This aspect underlined a methodological problem: the teacher could realize the importance of setting up a problematic situation where the students find the conditions, have time and autonomy to outline the need of a new conceptual object.

Results.

*Interview***.** An interview to the participants, a year after the activity, points out the presence of a clear trace of the work in long-term memory. Namely, the answers show that the activity allowed them to rethink

- the meaning of the measurement procedure;
- the role of the unit and sub-units choice;
- the effects of this choice.

We suppose that this awareness improvement is linkable just to the use of unofficial tools, what was non-planned a priori. Measuring with non-standard units seems to be a key-element that remained firmly acquired. All cite the measure with the rods as something that has affected them: "*the thing I was intrigued most was when we measured with the plastic bars*"; someone said that had always measured by the meter and were surprised to discover that it could be possible use rods: "*I never took measures by colored rods before that day*". A girl speaks explicitly about the process of transition to the submultiples: "*and when we divided to measure the smaller parts*".

Moreover, the answers outline that the learning context fostered the recognition of

- properties of the measurement procedure aimed at the realization of a map;
- actions ("*not simple*") necessary to transfer data collected from reality to the scale graphic representation.

The use of expressions such as: *"to draw back the perimeter of the area", "to depict a map of the area"*, "*to measure the perimeter of a big area*" show that the relation boundary-interior of a figure is clear so as the operations to make, i.e. the meaning of the involved concepts exists but the corresponding terminology is improper and "fuzzy" and needs to be settled and well defined. The words "map" and "relief" should be emphasized, for it happened that they didn't become part of the language spontaneously (except in a case).

It is possible to notice that, the pupils recognized to be facing a true problem: transforming linear information (measures) in a two-dimensional shape. "*it seemed to be difficult, in the beginning, but we could speedily manage!"*

On the contrary, no reference has been made to geometrical properties they found in the space, in particular, the perpendicularity/parallelism between the walls and the rectangular shape of the columns in the hall: a posteriori, we relate this fact to the use of both the squared paper, and the grid on the screen of GeoGebra. We think that this choice facilitated the task to the point that the students were not aware of the whole system of underlying geometrical relations among the different elements of the figure. Undoubtedly this suggests the need of avoiding this facility and using a blank sheet of paper to bring out the problem during the relief/mapping activity and after using computer. Actually, under these conditions, the transition from paper to the computer screen seems to have been experienced only as a repeat of the same problem, only requiring more precision; except for the work necessary for the reproduction on scale that took place during this activity.

They also seemed to remember and pointed out the difficulties they met dealing with the particular space shape and wide size. This fact led us to make an experiment of visual exploration of a large double glass door standing to about 3 meters by it: they could express differences between managing the representation of a small or large space: the first one is easier because it allows "*to see the full form"*, "*to have fewer elements in the middle", "to do fewer calculations", "to spend less time", ...*

Finally, on meta-cognitive point of view, other aspects emerged, mainly concerning the control of personal strategies of action (active listening of the teacher's advice and suggestions, being calm, confident, patient and precise, paying attention ...).

Third piloting

by Pier Giuseppe Vilardo and Franco Favilli[***]

INTRODUCTION

In this activity students are expected to solve a practical problem through mathematical tools without knowing a well defined procedure. Definitely one of the key issues that will be introduced with this activity is *proportionality*, mainly through a geometrical approach.

The following task is requested to the students: they must identify the gathering points for classes outside of the school in case of evacuation; after identifying these points, they must provide unique written instructions to the municipality workers that will place poles in the collection points reported by students.

First, they will think over the task individually for about fifteen minutes, and then all the activities will be conducted in 6 groups of 3-4 students.

AIMS

Aims for students

General aims

- To connect mathematical notions and techniques to solve real social issues.
- To verbally communicate or by other means their ideas and their ways of mathematical action.
- To discuss their solutions.
- To be actively involved in exploring different exhibition paths for communication of mathematical procedures in different contexts.

Specific aims

- To measure.
- To use semantic structures to locate positions in space.
- To represent real objects in scale.
- To use proportions.
- To use geometric software to draw maps.
- To develop models.
- To use an appropriate mathematical language.
- To strengthen the knowledge of geometrical language.

[***]CAFRE – University of Pisa, Italy

THE CLASSROOM ACTIVITIES

Second Year students of Lower Secondary School; 12 activity hours (10 hours initially expected), 22 students.

The activity was carried out in a second class of the lower secondary school of the Istituto Comprensivo di Castelnuovo Magra (Province of La Spezia) and was used by the teacher as an introduction to the ratio, reduction scales and proportions.

Students were asked to identify, outside of the school parking lot (Photo 1), the gathering points for the 7 classes (out of 11 of the same school), coming out in the parking lot in case of evacuation. Earlier, another class had prepared the poles to be planted in the grass in the identified collecting points.

Photo 1: The school parking lot

After identifying the areas the students had to find a way to provide the municipality workers with understandable and unambiguous instructions so that workers could plant the poles exactly where the students had decided. This request was made orally by the teacher.

After making the request, the students were given about 15 minutes for individual thinking and then they were joined in 6 groups, (four groups of four students and two groups of three students).

Initially, the groups made plans and maps of the school and the parking lot; the maps were more or less similar to the actual situation of the school and the parking lot, and the students were concerned mainly with identifying the gathering points, basically according to their visual memories of the parking lot itself.

Then each group reported the solutions found. The groups were asked to report on the basis of their level of "progress", starting from "the most simple" solutions to the "most complex" ones.

The first group wrote verbal instructions in which they reported generically where the classes were to be located. Nevertheless, on the basis of these instructions it was impossible, for those in charge of placing the poles, to identify the right places.

Even the second group wrote verbal instructions, but they also added the distance from one pole to another.

The third group wrote verbal instructions, but less generic than the first two, because they used existing buildings (greenhouse) as landmarks and they also drew a simple map. However, they did not realize the importance of the map they had made, because they showed it only after the teacher asked them about it. Finally, after several questions, they realized that what they were doing was a map of the school, and that it would be very useful to indicate the position of the poles.

The following three groups all made a map: the map of the fourth group was the most simple, while, on the other hand, the maps of the fifth and the sixth groups were more elaborated. The fifth group also thought to measure the parking lot and to use these measurements to draw a map using a software that, according to the measurements, is able to draw maps. The fifth group pointed out that the map was the easiest and fastest way to solve the proposed problem.

After reviewing all the proposed solutions, the students decided that the map was the best solution. Then they started debating on which group had drawn the best map. The teacher then asked them what was necessary to draw a good map; all of them agreed that it was necessary to measure the parking utilizing suitable tools, such as 10-20 meters long measuring tapes. All in all, this activity lasted two hours.

During the following lesson the students measured the parking lot. Initially some of them were perplexed so they asked the teacher for help in order to start measuring accurately. A group utilized a laser pointer (they were asked how this tool worked; they knew how to use it but they did not know how it worked). After two hours all of them had completed the measurements.

After the measuring, the students started drawing the maps (they needed additionally two hours to complete this task). All the groups chose different ways to draw the school parking map. In particular, two groups completed their work at home, without being asked by the teacher, and in order to do so they used dynamic geometry software, such as GeoGebra, and “planner 5d”, available on line; they had already thought about using this software. The other groups instead drew the map using graph paper. Only two out of six groups drew the map using a reduction scale.

When the maps were ready (Photo2), all the groups showed and explained their works.

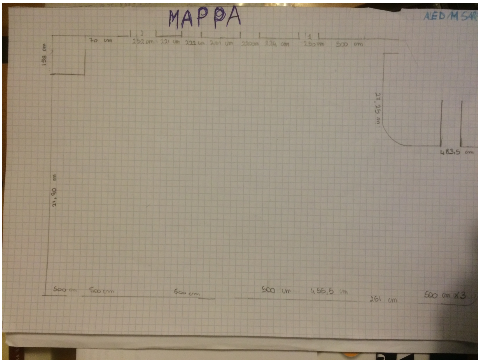


Photo 2: The map drawn by a group

From the explanations it was clear that four out of six maps were not useful because, when using them, it was impossible to allocate the poles in an univocal place (in two maps the poles were not even represented, but rather only the parking lot was drawn). When the students finished showing their works, there was a little discussion about what they did and most of them did not notice that four maps were not useful; so, many of the students' questions focused on issues of secondary relevance (in order to complete this activity we needed two hours).

For this reason the teacher decided to let the student verify on the field if the maps were correct (two hours activity). In order to do that he gave the poles to the students divided equally in the same six groups, and he asked each group to place the pole according to the map of another group. In this way they discovered that only two maps out of six allowed to place the poles (Photo 3).

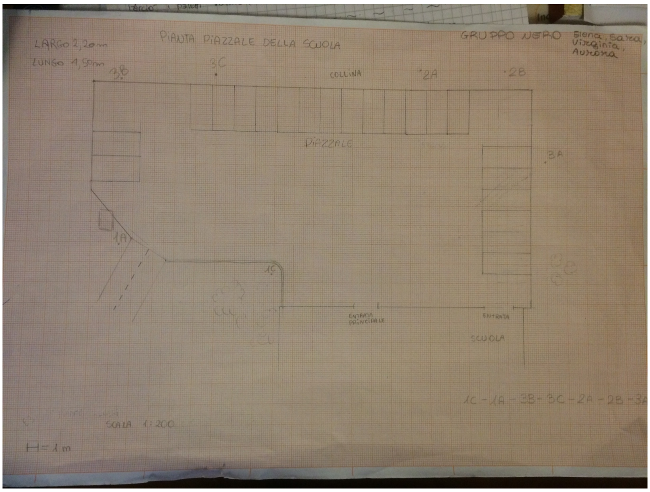


Photo 3: One of the maps with the poles location

In fact, some of them were not drawn with a reduction scale and others had not had the instructions about where to place the poles, because the poles were not drawn in the map (Photo 4).

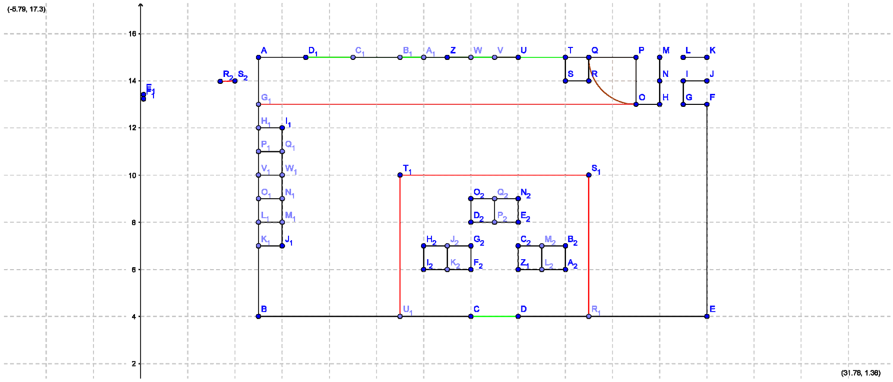


Photo 4: A GeoGebra map without the poles

So, the students decided that only two maps were correct. Finally, all the students wrote a report to describe what they had done.

CONCLUSION

First of all, we have to say that all the students participated enthusiastically and with great interest in the activity proposed, even if in most cases the results were not satisfactory, basically because most of the maps drawn (4 out of 6) were not suitable to answer the initial task. Nevertheless, all the students were able to recognize their mistakes and choose the correct maps. So, only in a few cases did the initial answer led to a spontaneous use of a graduation scale to draw the map in order to complete the task; however the work on the field showed the students the utility of such a mathematical instrument. In order to answer the initial question the students were free to explore their own path and only on rare occasions, especially during the talking among themselves, were they directed towards some kind of solution, such as drawing the map. However, also in this case they were not asked to draw the map in a particular way, but they were free to do it as they wanted. Thus, some of them utilized a line and a square and other geometrical software. Finally, they were not asked to make a scale reduction. So, the activity allowed to discover those students that had already used the proportions and that were able to use them correctly before they were formalized by the teacher. Obviously, only a few of them were able to do that. For the others, the activity was equally useful because it showed them how necessary this mathematical instrument was, thanks to the real-based task. Finally, in order to formalize the proportions, other real-based tasks were chosen to be done by these students, such as the preparations of recipes.

Fourth piloting

by Andreas Ulovec[****] and Therese Tomiska

The piloting

General Information

The teaching unit was piloted by a female mathematics teacher with five years teaching experience working in an upper secondary school near Vienna. The Austrian project team sent the material to the teacher approximately 3 weeks before the planned piloting activity. The teacher had a 5^{th} (age 14-15 years), 6^{th} (15-16) and 8^{th} (17-18) grade available for piloting. After a meeting with the project team, she chose to conduct the piloting during a regular mathematics class (50 minutes) in the 6^{th} grade. Eight students (age 17-18), three of which are migrant students, attended the class, which was video recorded and observed by a member of the Austrian project team.

Classroom piloting

The teacher modified the teaching unit by not having bins situated in the school yard, but by having the bins situated in the music room during the school ball (which is where the buffet tables are situated during the ball) – a situation that was of interest for the students, since they will have to organize the ball and buffet in the next year. Also, the teaching unit was shortened to one lesson of 50 minutes. The lesson took place in the schools' computer laboratory, as suggested in the proposal.

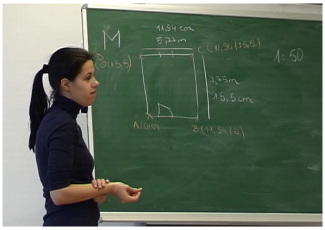
The unit started with an introduction of the topic by the teacher (5 minutes) and continued with a brainstorming session about prerequisites. Very soon students realized that it would be convenient to have a map of the classroom to be able to try out the various situations and make an informed decision. The teacher had prepared such a map on paper and drew the schematics of it on the blackboard.

Why is this map not good enough?

This was followed by a session in which students discussed about whether more information on this schematic map would be needed. Students found out that without a scale it would be impossible to use this map for planning, and they selected a suitable scale.

[****]Faculty of Mathematics - University of Vienna, Austria.

Since the students were already in the computer laboratory, it was suggested that it would be much easier to have the map in an electronic form, so that it would be easier to try out several possible positions of the bins. Another discussion took place clarifying the prerequisites of transferring the map from the blackboard to the computer screen. The students found out that coordinates needed to be introduced and calculated, using the maps' scale.



Introduction of coordinates

After the basic room construction with GeoGebra, students found out that the size of the door was not obvious from the map. They started to measure the width of the door, using math text books as reference objects. After this, the text book itself was measured with a ruler to convert the measurement results in metric units.

Measuring the door

Students then continued the discussion about reasonable positions for the bins and concluded that it would make sense to have the bins near the eating and buffet tables. They realized that they have to revise their strategy from positioning (small) bins to positioning (rather large) tables.

At the end of the session, students discussed what other constraints are given when they now need to position tables instead of bins, and weight, larger measures, as well as required minimum space between tables (for people to pass or stand and eat) were mentioned.

Conclusions

The piloting demonstrated that the objectives set in the proposal were definitely met in the implementation of this unit. The exploration of real life situations, the use of space notion, measurements, and scale, as well as the verbal communication on different levels all did occur.

Conclusions from the three piloting

by Charoula Stathopoulou and Eleni Gana

The 4 piloting in 4 different schools (3 in lower secondary and one in upper secondary) showed that the activity was challenging for the students and they were actively involved in the process of teaching and learning.

The original design of the activity aimed to create rich opportunities of communication between students during searching for solutions and to facilitate students from different cultural and linguistic backgrounds to contribute through their own mathematical and discursive resources to the negotiation of mathematics notions and techniques.

A task-based activity was chosen, since: a) the pragmatic orientation of the activity was expected to cause the interest of the students, as mathematics corresponded to an empirical framework that was meaningful to them; b) the formulation of the problem did not provide predefined processes of solving the problem; c) it validated and built on the informal mathematical experiences and ways of expression that the students had (everyday language) to comprehend mathematical speech; d) allowed for various micro-frameworks of cooperation and communication among the students to be restored and for traditional relations among them to be undermined, mainly regarding students linguistically and culturally different.

In every school there was a different formulation of the problem, as the schools had a different formation of space but also different issues to address. However, the conceptual domain was the same as well as the mathematical content under question.

The fact that it was not a problem that demanded a simple algorithmic application but, on the contrary, the students were asked to choose strategies in order to solve it, activated the students to use exploratory processes and at the same time encouraged cooperation among them.

In all schools it was treated as a problem requiring modeling and the need to make decisions. Software of dynamic geometry were utilized so that they could enhance the exploratory dimension and the communication in the classroom through mathematics.

As was shown from the piloting, the students, having to face a problem in a realistic framework, activated self-regulation mechanisms when their original solutions were not effective. In addition, it was evident that the specific framework encouraged empirical and informal solutions and the teachers, through scaffolding processes, led the students to formal mathematical solutions.

Utilizing task based activities, we create opportunities of including students of different cultural and linguistic backgrounds and furthermore to challenge dichotomies that appear in mathematic teaching as, of:

- out of school and classroom mathematics,
- students that are listening and students that are doing,

- cognitive and emotional processing.

All in all, the activity proved challenging, interesting and meaningful to the students in all contexts, mobilizing them to address realistic problems in a creative way, while cooperating and utilizing everyday knowledge coming from different linguistic and cultural backgrounds to consolidate more formal mathematical concepts.

References

Brousseau, G. (1983) : Etudes de questions d'enseignement. Un exemple : la géométrie",*Séminaire de didactique des mathématiques et de l'informatique*, LSD IMAG, Université J. Fourier, Grenoble (1982-1983)

Hiebert, J., Carpenter, T. P., Fennema, E., Fuson, K., Human, P., Murray, H., ...& Wearne, D. (1996). Problem solving as a basis for reform in curriculum and instruction: The case of mathematics. *Educational researcher, 25(4)*, 12-21.

Hmelo-Silver, C. E., Duncan, R. G., & Chinn, C. A. (2007). Scaffolding and achievement in problem-based and inquiry learning: A response to Kirschner, Sweller, and Clark (2006). *Educational Psychologist, 42(2)*, 99-107.

Leaver, B. L. and Willis, J., (2004).*Task-Based Instruction In Foreign Language Education: Practices and Programs.* Washington, D.C.:Georgetown UniversityPress.

Puren, Ch. (2004). De l'approche par les tâches à la perspective co-actionnelle, Cahiers de l'APLIUT [En ligne], vol. XXIII N. 1 |, URL : http://apliut.revues.org/3416 ; DOI : 10.4000/apliut.3416 (retrieved 15/7/2015)

Willis, J. (1996). *A Framework for Task-Based Learning*. Harlow, U.K.: Longman.

MASTERING MATHEMATICS, MAINSTREAM AND MINORITY LANGUAGES

by Franco Favilli*

INTRODUCTION

Little has been done in Europe as far as maths teaching in multicultural contexts is concerned. The different languages and cultures which may be present in the classroom make the teaching/learning process even more arduous than it already is, especially for pupils from minority cultures and/or with a migrant background or for Gipsy pupils.

A teaching unit is described below. Its aim is to provide teachers with a tool to help their pupils overcome the learning barrier represented by the contrast between the simplicity of classroom language and the complexity of mathematics language. Indeed, teachers have to bear in mind that the language used in class is an element of further complexity for pupils from minority cultures with a different native language.

The primary target group are mathematics teachers of primary and lower secondary schools in socio-culturally diverse areas, while the secondary target group consequently consists of students from cultural minorities and/or culturally deprived groups.

The educational aims

The educational aims of the teaching unit can be roughly divided into general and mathematical aims.

Among the ***general aims*** the following may be considered:

- awareness of the positive values of cultures different from our own;
- creation of favourable conditions for intercultural dialogue in the classroom, and an inclusive educational setting by the use of different languages and pedagogical tools;
- development of awareness and critical attitudes towards the use of language and its interpretation;

* CAFRE – University of Pisa, Italy

- awareness of the role played by the use of a specific and unambiguous language in subject teaching;
- capacity to express the reason for the choices made and used during the activity;
- acknowledgment of the need to reflect on the texts and the role played by words;
- increase of students' ability to understand and process texts;
- deeper comprehension by foreign students of a written text;
- students' respect of the different working times of their classmates;
- fostering social relationships in the teamwork;
- development of students' autonomy.

Among the ***mathematical aims*** the following may be considered:

- increase of learners' capacity to understand and to elaborate the mathematical discourse;
- improvement of the ability of reading and understanding mathematics textbooks and word problems;
- improved usage of mathematical language;
- reinforcement of mathematical glossary knowledge;
- development of the ability to find a proper balance between natural language and mathematical language;
- identification of pre-knowledge and attitudes towards mathematics by foreign students.

The teaching unit should lead teachers to identify and reflect upon ***students*' *potential needs***, such as:

- difficulty in using mathematical language correctly: uncertainties, doubts and mistakes shown in understanding the written texts express the need to favour the communicative process, when teaching, in order to help students to express themselves clearly and accurately;
- the need to use the linguistic competence appropriately since its use represents a fundamental step towards the construction of knowledge despite the time-consuming effect;
- the need to develop activities such as this one, because they offer information about pupils' knowledge, their conceptualisation level, their potential gaps, and misconceptions. This information is fundamental to be able to intervene in the classroom with appropriate and well planned teaching approaches.

Activities

The teaching unit consists of five main activities. All activities should be carried out in small groups, each of which including a minority pupil at least.

- **Analysis of a textbook** (*Reading* and *Writing*)

Students are asked to read a chapter of their textbook and then to search for and make a list of "difficult" words and verbs in the vehicular language, to discuss about their meaning and translate them into the foreign languages spoken in the classroom, thus producing a *micro-dictionary*.

Students are then asked to search for and make a list of words and verbs that are relevant to the mathematical language, compare them to the same words and verbs in the natural language, discuss about and write their potential different meanings and translate the words and verbs into the foreign languages spoken in the classroom, thus producing a *mathematics glossary* and a *mathematics dictionary*.

All groups are asked to re-write the analysed pages of the textbook in the vehicular language and minority pupils are asked to translate the most significant sentences into their own mother tongue language.

- **Analysis of a "word problem" from a National standard assessment test** (*Reading* and *Writing*)

The teacher chooses a "word problem" from a National standard assessment test that is meaningful as to the language used. Students are then given the same tasks as in the first activity.

- **Natural language and mathematics language**

Students are asked to identify potential conflicts originated by different meaning of words and verbs that are common to both the natural and mathematical languages, and to write the two different meanings in their own mother tongue language.

- **Writing a "word problem"**

Students, still working in groups, are asked to write a word problem in the vehicular language. The problems are presented to the whole class for discussion about the linguistic clarity and the mathematical notions required. Greater attention is paid to minority students.

- **"Writing a textbook"**

Students, still working in groups, are asked to write in the vehicular language a "page of a textbook" on a mathematical topic chosen by the teacher. The "pages" are presented to the whole class for discussion about their linguistic clarity and the mathematical notions involved. Greater attention is paid to minority students.

Main piloting

by Francesca Colzi, Stefania Massai and Franco Favilli

General information

School: Istituto Comprensivo "Don Lorenzo Milani" – Viareggio (Province of Lucca)
School level: Lower Secondary School
Number of teachers: 2
Number of classes: 6 (school year 2013/2014: two second and two third lower secondary school classes – school year 2014/2015: two second year classes)
Students age: 11 to 16.
Number of students: 63
Origin of foreign students: Albania, Georgia, India, Morocco, Romania, Russia.
Number of teachers in the classroom during the piloting: 1 or 2 (the subject teacher and the support teacher for special needs students).

Assignments for students

In order to foster a critical reading of the text, teachers can ask the students questions such as the following ones:

- *did you find it difficult to understand the meaning of some words?*
- *did you find most difficult the words in the common language or those of mathematical language?*
- *did you understand the overall meaning of the text?*
- *within the text did you get any link with your experience?*
- *does the text make you think of something meaningful?*
- *did the use of the Italian language dictionary help you?*
- *did the use of the language translator help you?*
- *what are the advantages / disadvantages of the use of a dictionary?*
- *what are the advantages / disadvantages of the use of a translator?*
- *did you already know the meaning of the words you searched on the translator in your native language?*
- *did the text written by your schoolmates make it easier for you to understand?*
- *how did you choose the words to use, when you rewrote the text?*
- *was it difficult to rewrite the text?*
- *did you find any words or phrases with ambiguous or unclear meaning?*
- *did the use of images facilitate your task?*
- *was it more difficult to rewrite the text taken from the textbook or to tackle the problem?*
- *how could the text be made more attractive and familiar?*
- *was it difficult to write the check test for your schoolmates*
- *were you able to carry out the check test prepared by your schoolmates?*

The piloting

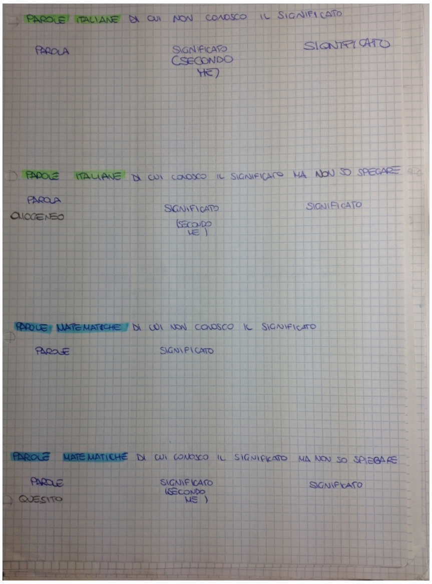
<u>Activity 1</u> (school year 2013/2014 – grade 8 – 2 hours)

The chosen mathematical topic – probability – is not introduced yet.

The students' task is to read the pages about the topic in their textbook and to make lists of the words:

- they do not know the meaning of in the Italian language;
- they do not know the meaning of in the mathematical context;
- they know – but not clearly – the meaning of in the Italian language;
- they know – but not clearly – the mathematical meaning of.

After the reading, the students search for the meaning of unclear words in the Italian language.



<u>Activity 2</u> (school year 2013/2014 – grade 7 – 3 hours: 2 hours by small groups of students; 1 hour by the whole class).

An "expert" group of 5/6 students read a topic about the "circle" in their textbook.

The group, purposely without foreign students, is asked to write a new text about the same topic so that it is clear to all their classmates. The group has also to prepare a test to check their classmates' understanding of the topic.

<u>Activity 3</u> (school year 2014/2015 – grade 7 – 2 hours)

The activity refers to the teaching unit "Geometrical puzzles" in the LOSTT-IN-MATH project[4] (in which students are asked to work in pairs, one giving instruction

[4] http://losstt-in-math.dm.unipi.it/

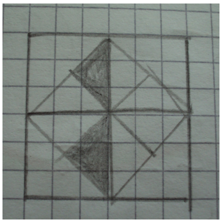
to the classmate who has to accordingly draw the geometrical figure the "instructor" has chosen without revealing its name.

Unlike in "Geometrical puzzles",

- students do not work in pairs, one student giving the rest of the class the instructions to draw – step by step, instruction by instruction – the figure he/she only knows;
- students are asked to draw compound geometrical figures and allowed to include the name of the "partial" geometrical figures in the instructions the "instructor" gives.

At the end of the instructions delivery, the resulting figures are compared and the given instructions are analysed and discussed to understand the potential reasons of the wrong drawings: ambiguous instruction or misunderstanding of its meaning?

Here below is the geometrical figure to be drawn:



Here following are the instructions given by the "instructor":

- *Draw a square!*
- *Halve the square by means of a vertical line!*
- *Draw a horizontal segment that starts from and ends to the half of the vertical sides through the centre!*
- *Connect all points you have found thus getting a rhombus!*
- *Split in two halves all small triangles you have got in the rhombus!*
- *Paint the top left and the lower left parts of the rhombus.*

An activity similar to the previous one was developed in another grade 7 class of the same school. In this case students tackle for the first time such an activity and the teacher decides to choose to describe an elementary figure: the rhombus.

The "instructor's" assignment is to give only "minimal instructions" (e.g.: *draw a segment*, *mark a point …*) without any further explanations to the schoolmates.

Here following are the instructions given by the "instructor":

- *There is a line, yes, … a segment.*
- *With this segment draw an acute angle, …, upward.*
- *Then draw another segment to get an obtuse angle.*
- *Then, attached to this, another segment to get an acute angle.*

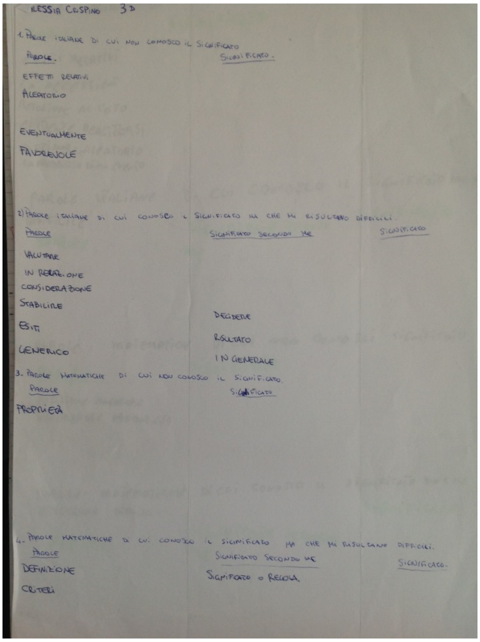
- *Then, attached again, another segment to get an obtuse angle that is equal to the first one.*

During the various steps, the classmates attempt to correct the instructor, trying to adjust and correct his/her instructions.

A posteriori analysis

<u>Activity 1</u>

The assignment is clear to all students, but sometimes they struggle to distinguish between Italian words and mathematical terms. Actually, none of them scans the text, so that they put in the list of unknown words even the vocabulary terms that cannot affect the overall meaning of the text. It may be that they are mislead by the assignment that requires to read the text and identify the unknown words.



ALESSIA CRISPINO 3D

1. PAROLE ITALIANE DI CUI NON CONOSCO IL SIGNIFICATO

PAROLE. SIGNIFICATO.

EFFETTI RELATIVI

ALEATORIO

EVENTUALMENTE

FAVOREVOLE

2) PAROLE ITALIANE DI CUI CONOSCO IL SIGNIFICATO MA CHE MI RISULTANO DIFFICILI.

PAROLE SIGNIFICATO SECONDO ME SIGNIFICATO

VALUTARE

IN RELAZIONE

CONSIDERAZIONE

STABILIRE DECIDERE

ESITI RISULTATO

GENERICO IN GENERALE

3. PAROLE MATEMATICHE DI CUI NON CONOSCO IL SIGNIFICATO.

PAROLE SIGNIFICATO

PROPRIETÀ

4. PAROLE MATEMATICHE DI CUI CONOSCO IL SIGNIFICATO MA CHE MI RISULTANO DIFFICILI.

PAROLE SIGNIFICATO SECONDO ME SIGNIFICATO.

DEFINIZIONE SIGNIFICATO o REGOLA.

CRITERI

The availability of the Italian dictionary proves useful when it is used for a few words; when the number of words to be searched increases , the reading of the text becomes much harder. Moreover, it is to be noted that dictionaries (both on paper and on the network) are mostly addressed to adult users and however, the lexical explanation is often difficult for young learners who therefore need the teacher's mediation.

A similar argument can be made for the use of a dictionary or online translator that appears to be useful only when searching for a single word, but not as effective for whole sentences. The translation in the student's native language is useful in case of words drawn from the natural language, but it is not always the same when the words are drawn from the mathematical language and their meaning is unknown in the language of origin.

All in all, the proposed text on probability does not turn out to be easily understandable and does not allow students to build stable concepts.

Activity 2

In rewriting the text, students move away just a little from the original text, mainly doing a summary. At the end of the activity, the "expert group" says that the task was difficult because of the short time available, as they had to devote considerable time to tidy up the text which, according to them, was confused.

It is noted that, in the rewriting, initially, the mathematical figures were not deemed important, and therefore, were not retrieved from the textbook, although it is a geometry text. The students have put the figures only at the end of the text they have written, making links to them, as if the image interspersed in the text would make reading more difficult.

The rewritten text is understandable, but in some cases an oral explanation by the group of "expert students" is necessary.

The group was in trouble when preparing the check test, because they found it difficult to assess their peers and to design the test. One of the two groups preferred not to write the check test, while the other one initially had some resistance by peers who did not accept to be checked by the peers in the "expert group".

Activity 3

The activity elicited listening skills and attention both from the students who had to draw and the one providing the instructions. It also highlighted the students' difficulty to use the specific subject language. However, no particular difficulties were observed in the foreign students (all second generation immigrants).

As far as both the instructor and the rest of the class were concerned, this first experience pointed out very clearly, the ambiguities that may be originated by inaccurate instructions. Indeed, it was very difficult for the' "instructor" to find a logical order when giving the instructions to draw the desired figure. The difficulty to use a more rigorous and unambiguous linguistic repertoire than the natural language produced slowdowns and continuous adjustments.

In the end, the figures produced by the students were very different and distant from the expected result. This fact has stimulated a plenary discussion based on the following questions: "How do you explain that not all of you have drawn the same figure?"; "Why was the figure that the instructor wanted you to draw properly drawn only in a very limited number of cases?”.

The students quite consciously identified the cause of the failure both in their weak mastering of the language and in the weak sequence of the instructor's commands to draw the figure.

Foreign students did not experience particular difficulties during the activity development, even those who speak a language other than Italian at home, because they all attended primary school in Italy and had therefore already approached the study of mathematics in this country from the very beginning of their schooling.

During the activity, the teacher's role was that of a facilitator that limits the most of his/her speech in the discussion and elicits the concepts as independently as possible within the classroom.

Second piloting

by Marie-Hélène. Le Yaouanq[**] and Brigitte Marin[**]

Training context

As far as writing is concerned, one of the objectives set by the French lower secondary school curriculum for mathematics is “to train students to read and understand a mathematic text better, but also to produce texts which are then subject to a progressive improvement.” Geometry can be seen as afavourable field for different levels of works in either enunciation, understanding of structured texts or in argumentation and validation. The official documents put the emphasis on making the difference between time for research and reasoning, and time for organising and writing this reasoning. They insist on not imposing too early a strict framework or a writing model: “The liberty of writing is one of the main driving forces for learning how to write. This also applies to proving”. (Resource-kit “Raisonnement et démonstration pour le collège”[5]). As for competency-based assessment of the students, it is clearly asked to separate these two elements and examples are given to the trainees to be discussed.

Every year, many sessions in the trainees’ education are dedicated to geometry in lower secondary school as it is an important aspect of the curriculum and the students’ difficulties, as well as the teachers’, are numerous when introducing deductive geometry and thinking organizing. Training is based on various theoretical frameworks like the different geometrical paradigms (Houdement and Kuzniak), the changes of registers of representation (Duval), and the tool-object dialectic (Douady).

[**] ESPE - Université UPEC, Créteil, France.

[5]Titre du document officiel media.education.gouv.fr/file/Programmes/17/7/doc_acc_clg_raisonnement&demonstration_109177.pdf

Building of the sequence

Lower secondary school curriculum offers very precise guidelines:

-"Attention should be paid to the language and to the various meanings of a single word".

- An "efficient way to convince students of the need for an accurate language is the shift from "doing" to "having something done". However, this requirement must not look arbitrary to them. This is when the students have to write instructions to make someone execute them (for example, depicting a complex geometric shape so that it can be represented) or when they have to use a computer for a specific treatment that the need for accuracy appears necessary.

A first lesson will be dedicated to making the students aware of the polysemy of some mathematical terms.

The rest of the lesson will deal with what is called "telephoned figures" (the transmitter dictates instructions to a receiver in order to make him draw a certain geometrical figure) and on the use of a geometry software. Various experiments showed that students could efficiently communicate in *their* language by writing texts which are neither satisfying on the mathematical ground, nor from a language point of view, but they enable the receiver to execute the expected figure. The realization of the expected figure implicitly confirms the transmitter's text. This may question the interest of this work as an isolated work only aiming at working on language accuracy. However, many competences are at stake in this kind of activity: analyzing a figure, distinguishing the figure depiction from its characteristic properties, being able to change the registers of representation, and deconstructing speech into elementary instructions in a coherent and methodical way.

The choice is thus made to implement such a sequence on the «telephoned figures», but focusing on enabling the students to progressively improve their texts. Three sessions were determined in this way:

- a session dedicated to the production and the reception of the telephoned figures. The confrontation between the achieved and the initial figure should lead to an analysis of the problems that might have been encountered, such as missing information or information given in an inadequate order. The absence of language accuracy will probably not systematically be an impediment to the execution of a figure;
- a session designed for executing figures with a geometry software. The algorithmic process needs a list of primary instructions prepared by the work of a previous session. It can reveal potential defects, or implicit elements in the produced text, which would not have been noticed during the manual execution of the figure. The dynamic characteristic of the software also enables to highlight the work on the properties of the figure and not only on the drawing as the figure needs to resist to displacement;

- a last session aiming at improving the first produced text. A work of comparative analysis between “expert texts” extracted from textbooks and the students’ texts should allow the students to identify some differences and then to rewrite their own texts.

This sequence takes place at the end of the school year in a class of year 7. Several allophone students newly arrived in France take part to it with the class while having an extra hour of specific tutoring with their mathematics teacher.

1.1. Execution of the sequence

First session: polysemy of words

The first work relates to the “vertex”. In theory, this term does not seem difficult; it is rather about making the work subject of the session clear.

Then, the term «altitude» is questioned. The notions of measure or size spontaneously come to mind in daily life. The mathematical definition is not mastered and the altitude is confused with the bisection or the median. The students all agree when the teacher asks if an altitude, in mathematics, could be measured: it can’t be since it is a straight line. The calculation formula for the area of a triangle obliges them to reconsider their opinions and to notice the polysemy of the word in mathematics by themselves.

The session ends with a game available online[6] which requires matching a real life definition with a mathematical definition, both referring to the same word.

The students autonomously think on paper before a collective summary shall be made. Participation is important: many proposals are made and are orally defended before checking on the computer with video projection.

The allophone students have been involved in the first part of the session; however the second part was too intense for them. The teacher will go back on some words during the hour of specific tutoring in order to start a glossary.

1.2. Second session: «Telephoned figures»

The instructions to execute the work on the telephoned figures have been practiced beforehand with the allophone students during their tutoring session. The suggested figures are simple and related to the Year 7 curriculum when the parallelogram, the characteristic properties of the square, the rectangle and the rhombus are taught.

The students work in pairs and the teacher dissociates the work by mainly proposing the square as a first figure, the rhombus is only given to four pairs. Two students who very recently arrived to France have to caption a construction video by choosing the terms from a list they were provided with.

Each transmitter receives the name of a figure and gives the instructions to the receiver who writes them down, draws the figure and mentions his doubts, or the pieces information that seem to be missing. Then the students confront the figure they

[6] http://matoumatheux.ac-rennes.fr/tous/vocabulaire/mots2.htm, available address on 2015 may 01

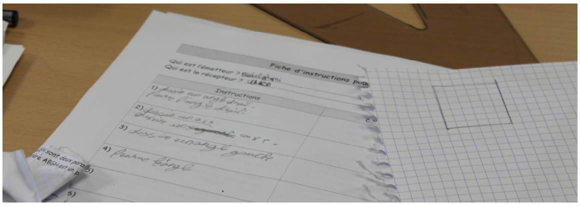
had to describe and the figure they produced. If the square always leads to a square or to a rectangle by lack of mention of the lengths, the rhombus produces a varied range of figures. Only one group ends up with a correct construction program for the rhombus, starting with its diagonals. All the attempts relying on the sides from the other groups lead to wrong figures, and even to unexpected ones such as a hexagon or a triangle attached to a rectangle.

In most of the writings, the vocabulary which is used is not the math vocabulary. For instance, "draw a line" replaces "draw a line segment". Three groups only spontaneously use letters to name points or segment lines.

Many students rely on spatial references, and the terms "horizontal" and "vertical" are often used to obtain perpendicular lines.

Thus, the instructions "draw a horizontal line/draw a vertical line/draw a horizontal line/close it" leads the receiver to the production of a square! The fact that the figure was given to the transmitter by its name and not by a drawing must have given an important indication to the receiver who executed one of the figures that he knows.

The same applies to the list of instructions given by an allophone student: "draw a right angle/draw an axis/draw a *left* angle/close the angle", with which its neighbor produces a square.



One can question the meaning of the "left angle": did the polysemy of the term "right" represent a problem for the allophone student? Is the left angle a right angle to be put on the left-hand side of the drawing? Yet, this student is aware that every angle is not a right angle.

Once work is done, the transmitter and the receiver swap roles and another figure is proposed, but many pairs will not have enough time to do it entirely.

A collective summary is carried out on the case of the rhombus, based on a wrong construction dictated by a group and drawn by the teacher on the blackboard; the word «height» is not used to describe the construction of a height and, when the teacher suggests it, the group does not adopt it. The students who drew a rhombus starting from the diagonals then propose their construction.

Third session: construction with a dynamic geometry software

The students work in pairs on the software GeoGebra and have to create first a square, then a rhombus and eventually a parallelogram. An extra figure is provided for the fastest groups and two groups will be able to deal with it. The students have to write down the functionality of the software they use. All the students draw the square. Some use the "bisection" or "reflection symmetry" tools to create the rhombus faster, making use of the summary of the previous session and to draw the diagonal lines first. The others take a long time to obtain a rhombus, as several constraints had to be taken into account for the same object.

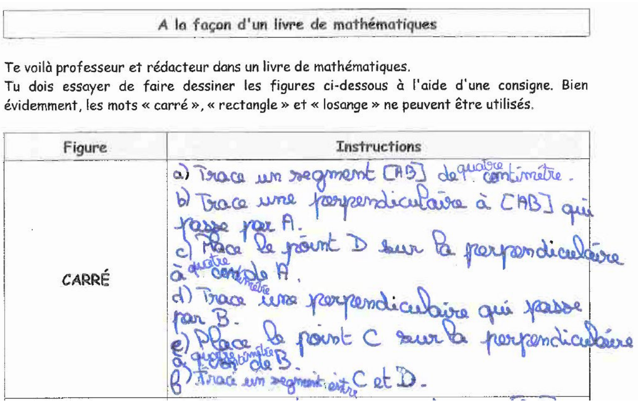
Most of the students declare that the last session helped them do this work. By contrast, some allophone students who kept French as a work language for the software suggest that it would have been easier for them to start working with the software in order to identify the words used and to write the instructions afterwards. The visual aspect of the icons as well as the feedback of the software enable the students to produce a figure autonomously and to learn or find the corresponding mathematical vocabulary.

Fourth session: Improving the texts

The French teacher of the class co-teaches this lesson.

After comparing their texts with an "expert text", the students in priority pick up verb forms, in the imperative, singular or plural, or the infinitive form. Other students focus on the vocabulary that is used and on the use of "mark" or "draw" in the expert texts by questioning the difference. Discussion follows in the class to explain this difference and to show the accuracy of the vocabulary.

The students rewrite their construction program.



A la façon d'un livre de mathématiques

Te voilà professeur et rédacteur dans un livre de mathématiques.
Tu dois essayer de faire dessiner les figures ci-dessous à l'aide d'une consigne. Bien évidemment, les mots « carré », « rectangle » et « losange » ne peuvent être utilisés.

Figure	Instructions
CARRÉ	a) Trace un segment [AB] de quatre centimètre. b) Trace une perpendiculaire à [AB] qui passe par A. c) Place le point D sur la perpendiculaire à quatre centimètre de A. d) Trace une perpendiculaire qui passe par B. e) Place le point C sur la perpendiculaire à quatre centimètre de B. f) Trace un segment entre C et D.

By trying to implement the produced program, the French teacher is used as a guinea pig by some groups both on the mathematical point of view and on the language point of view. Rich interplays allow further improvement.

Post project analysis

"Some inaccuracies still remain but I think that there is a real progression compared to the first session when the writings on telephoned figures were poor", the math teacher declares at the end of this sequence.

The texts are still not perfect but the given pieces of information are more often exhaustive and do not refer to any spatial localization. They begin with clear orders

("Mark", "Draw",…) and the mathematical vocabulary is much more often used. The points and the segment lines are often named in order to simplify and clarify the instructions.

By confronting their texts to expert texts, the students found by themselves the shape that a construction program could have, without the teacher imposing on them an arbitrary and strict framework. Without any constraint, they could picture the type of text expected from them.

The work done on some terms can also help the students improve their understanding of the instructions in the future.

Finally, the requirement to transmit a text to another student or a software and the observation of the obtained result give meaning to this effort of writing and re-writing. However this work does take a long time; the figures have to be simple and the produced texts have to remain short. Yet, it seems essential, as most of the students do not discover by themselves the characteristics of such mathematical texts to re-use them autonomously afterwards. That is shown by the poverty of most of the first produced texts, in the end of Year 7. More sessions would obviously be needed to keep on developing and strengthening their skills to write a mathematical text so that they could be transferred to other productions, different from the construction program ones. This mastery of communication in mathematics also deals with mathematical knowledge and this writing and rewriting work allows working on this mathematical knowledge too.

References

Duval R. (2000), Écriture, raisonnement et découverte de la démonstration en mathématiques, *Recherche en Didactique des Mathématiques*, vol. 20, n° 2.

IREM de Strasbourg (2002), *Ressources pour le programme de sixième*, (brochure avec un CD contenant des activités directement utilisables pour les élèves).

Pluvinage F. (2000), Mathématiques et maîtrise de la langue, *Repères IREM*, n° 39.

Pudelko B. et Legros D. (2000), J'écris donc j'apprends, *Cahiers pédagogiques,* n° 388 – 389.

Third piloting

by Charoula Stathopoulou[***] Eleni Gana[***] and Ioannis Fovos

Introduction

The teaching intervention was implemented in two different educational contexts with corresponding student groups. The first group consisted of 20 students in the 1st class of the 6th Junior High School of Volos (aged 12-13 years old), seven out of whom were Roma people. The second group consisted of detainees (aged 17-21 years old)

[***] Department of Special Education - University of Thessaly, Greece

who attend the 2nd and the 3rd class of a Junior High School that operates inside a detention centre. The majority of the students in this group are students who come from Asian, African and European countries, have lived in Greece for a few to many years and can communicate very little to quite well in the Greek language. The teaching was implemented by the teacher of mathematics of the classes, Mr Ioannis Fovos. The aforementioned teacher taught mathematics in both schools. He had 25 years teaching experience in secondary education, and specifically 12 years of experience in teaching mathematics in a school context that functions inside prisons. In the second context, the piloting of the activities was implemented in cooperation with the teacher of Greek in the school, Mrs Anna Georgiou, who had previously been informed and had prepared for the specific teaching intervention.

Classroom piloting

1st group of piloting: 1st class of the 6th Junior High School

In the 1st teaching hour, the teacher informed the whole class that in the specific teaching context they would explore and reflect on mathematical language (discourse) and its relationship with the language we use in everyday interactions. Then the students were divided into groups of 3 or 4. They were given a worksheet with three activities and a page which contained parts of a text from the school mathematics textbook, from the unit “quadrilateral shapes” (parallelogram, rectangle, rhombus, square, trapezium, isosceles trapezium).

In the first activity, the students referred to their experiences and recalled words we meet in mathematics and our everyday life which have similar or different meanings in each context. All groups contributed with words/word meanings and the teacher wrote the words on the board. After that, mathematical words which are used in everyday life with the same or similar meaning were found in the text (2nd activity). In the *2nd hour of teaching*, the students worked on a word problem given to them and in the end they made one of their own, which they solved. Creating a problem triggered the students' interest, as it was the first time they had to put themselves into the position of making a problem instead of solving it. In the different context of mathematical involvement suggested to them, the students reacted with some insecurity and addressed their teacher more often in order for him to support or check their decisions. At the same time, however, there was more willingness for cooperation in the group and exchange of opinions on the way they would formulate the mathematical concepts and procedures into words.

2nd group of piloting: School inside a detention centre

In the school that functions inside the detention centre, the same steps and teaching tools were used in the implementation of the activity of word problems. There was especially great interest, however, on the part of the students-detainees for the communicative approach of the activity and the participation in the discussions both in the context of the whole class and in the groups was very high.

1st hour of teaching: Recalling words of mathematical language which are also used in everyday life (1st activity of the worksheet), as was expected, was difficult for students with limited resources in the Greek language. Associating the two fields of language use demanded abstract mental processes related to their absolutely framed experience of using Greek. The teacher of mathematics supported the procedure with guided questions and verbal hints and the teacher of language offered corresponding help going back to texts that had been studied in class. The second and the third part of the activity (recognizing words in a written text, association of meaning with everyday language and the equivalent in the students' mother tongue) supported a dynamic interaction among the members of the group. The students interacted in order to locate the words and associate their meanings in the different contexts they are used. Students that had more resources in Greek acted as mediators, translating words of the text into their mother tongue, supporting in this way the access of the weaker students to mathematical language.

2nd hour of teaching: The whole of the class studied a word problem. The discursive traits of the word problem genre were recognized and modelled. After that, groups were formed by the students themselves, based on their common origin and language. Each group undertook to create its own word problem. The students negotiated among themselves not only the mathematical content of the problem they would create, but also the way they were going to phrase it, that is, the choice of mathematical vocabulary and the text organization of the word problem. The discussion in the groups often moved "between" two languages (Greek and the students' mother tongue) and was mainly about their effort to clarify mathematical concepts, but also to choose the appropriate expression in the context of mathematical language. The teacher, when asked to support the students, was informed about alternative suggestions and guided the students efficiently so that they would decide about the final word choice themselves.

Conclusions

The activity of word problems supported processes of conscious reflection on the relationship between mathematical language and everyday language for all the students, native and non-native speakers of Greek. Specifically, the students who were taught mathematics in a second language considered the activity very interesting and productive, because it allowed them to correlate and clarify the meanings the words took depending on the context in which they appear, namely in a mathematical or everyday communication context. Furthermore, those students pointed out that they had also tried on their own to comprehend mathematical concepts based on their first language, but it was not always easy. The validation of translanguaging in the classroom gave them the ability to understand more mathematical terms and concepts, although they had to rely only on the translation given by their classmates who spoke the same language, due to the limitations of the detention centre.[7] The teacher stated

[7] With the activity as a starting point, the class talked about the limitations that the detention system imposed on students' online access to texts in their mother tongue and they wrote a petition asking to be granted permission to use the internet for educational purposes.

that in the future, when designing similar activities, he would provide the students with photocopies containing translations of the terms in their mother tongue.

In conclusion, both the teacher and the students found the construction of a word problem by the students themselves especially interesting and useful. The students pointed out how advantageous and stimulating it was for them to try to “transfer” an occasion and a mathematical problem to appropriate (mathematical) language. According to the teacher, the piloting of the activity was a good initiative for designing corresponding activities with word problems.

Conclusions from the three piloting

by Roberto Peroni****

Language is the most pervasive technological instrument for cultural communication and cognitive development in the human nature evolution, and almost every content and interaction need to be organized linguistically.

The relationship between Mathematics and Language is particularly crucial both because every human language is “un système où tout (ou presque tout) se tient” (de Saussure and Meillet), and because Mathematics is a strictly structured language having not only a specialized lexicon but with a hierarchical syntax, too.

The topic of the Project M3EaL is the relationship in the Maths classroom between Language and three M-factors: Mathematics, Migration, and Multiculturalism This teaching unit proposes a situational context where the relationship (the distance) is between ordinary language and mathematical discourse in a multicultural classroom, with various different native languages (Minorities’ languages) vs. a second language which is the first language of the majority of pupils.

In detail, the three piloting reports current, complex, educational contexts with various migration background students, who are native speakers of minority languages from East Europe, Africa, and Asia, while in the Greek piloting activities, Roma pupils too, and students (17-21 years old) inside a detention centre while attending the 2nd and the 3rd class of Junior High School. Each of them must “learn to learn in a second language (Gibbons, 1993); and if neuro-imaging research has revealed a complex language and mathematics relationship showing left perisylvian language activities in exact calculation (Dehaene et al., 1999) and intraparietal involvements in approximation and quantity comparisons (Dehaene et al., 2004), a recent fMRI study (Wang et al., 2007) on Mandarin Chinese learners of English shows that, compared to L1, calculation in L2, but non parity, can be processed through the L1 system (language dependent) involving however additional neural activation, especially in the left hemisphere including the inferior frontal gyrus (Broca’s area).

**** CAFRE – University of Pisa, Italy

On the teachers' side, the aims are very high and complex: awareness of the positive values of different cultures; creation of conditions for intercultural dialogue in the classroom and an inclusive educational setting; development of critical attitude towards the use of language and its interpretation; and obviously teaching mathematics, i.e. to be able to increase learners' capacity to understand and elaborate the mathematical discourse; and to intervene in the classroom with appropriate and well planned teaching approaches.

The proposed and piloted activities have an explicit multicultural nature. Important common features in each piloting report are the analysis (reading and writing) of part of a textbook, and the analysis (reading and writing) of a "word problem" from a National Standard Assessment Test to search for difficult words meanings and/or difficult sentence meanings to reflect on, list-and translate into the different ordinary languages, and also to search for technical mathematical meaning. In particular, this occurs in the French piloting report in order to avoid misunderstanding from polysemy: for example, "vertex", "height", "right angle".

Reflection activities on geometrical figures ("quadrilateral shapes") are present in each of the three piloting reports, even if more effectively and pragmatically referred in the Italian (Pisa) and French piloting reports through a sort of telephoned figures sequences in order to try to obtain more accurate instructions, a better mastering of language, and figure buildings more similar to the expected ones. In the Paris-Créteil report the students work also in pairs on the software GeoGebra to construct figures, with a little linguistic advantage in French language mastering (work language of the software) by allophone students.

It has been a long term work after which some inaccuracies may remain. Nevertheless, the teaching unit clearly represents a significant progress compared to the first session when the writings on telephoned figures were poor; the texts too may still be not perfect, but they begin with clear orders ("Mark", "Draw", "Connect", "Split" etc.), and the technical mathematical lexicon is more often used, and the points and the segment lines are often named in order to simplify and clarify the instructions.

In all the above mentioned activities the teacher is a facilitator, addressed by each group both for mathematical and language issues: each further interplay allows further improvement, and it is clear that each step in the mastery of communication in mathematics deals with a proportional step in mathematical knowledge, too.

To sum up, the experience from the three piloting shows that:

a. Translanguaging activities on word meaning problems, and on figures meaning support processes of conscious reflection on the relationship between mathematical language and everyday language for all the students, native and non-native speakers (of Italian, French or Greek respectively). Furthermore, the activities on lexical and word-in-text meanings were considered more productive by the students who were taught mathematics in the second language, because it allowed them to correlate the meanings of the words to the different contexts in which they occur: mathematical or ordinary everyday communication language.

b. The figure building activities support not totally conventionalised basic processes, connecting the more abstract symbolic level to the pragmatic movement and gesture on which abstract knowledge is grounded (Bates et al., 1979), (Arzarello et al., 2009), (Alibali et al., 2014), (Novack & Goldin-Meadow, 2015).

c. Well planned and led translanguaging activities, allow better inclusion of students with different linguistic and cultural backgrounds, creating conditions for intercultural dialogue in the classroom, to be further developed out of the school.

References

Alibali, M.W., Nathan, M.J., Wolfgram, M.S., Breckinridge Church, R., Jacobs, S.A., Johnson Martinez, Ch. & Knuth, E.J. (2014). How Teachers Link Ideas in Mathematics Instruction Using Speech and Gesture: a Corpus Analysis**,** *Cognition and Instruction*, 32 (1), pp. 65-100

Arzarello, F., Di Paola, B., Robutti, O. & Sabena, C. (2009). Gestures as Semiotic Resources in the Mathematics Classroom, *Educational Studies in Mathematics*, 70 (2), pp. 97-109.

Bates, E., Benigni, L., Bretherton, I , Camaioni, L. & Volterra,V. (1979). The Emergence of Symbols: Cognition and Communication in Infancy. New York: Academic Press.

Dehaene, S., Spelke, E., Pinel, P, Stanescu, R. & Tsivkin, S. (1999). Sources of Mathematical Thinking: Behavioral and Brain-Imaging Evidence, *Science,* 284, pp. 970-974.

Dehaene,S., Molko, N., Cohen, L. & Wilson, A.J (2004). Arithmetic and the Brain, *Current Opinion in Neurobiology*, 14, pp. 218-224.

Gibbons,P. (1993). Learning to Learn in a Second Language. Portsmouth, NH.: Heinemann.

Novack, M. and Goldin-Meadow, S. (2015). Learning from Gesture: How our Hands Change our Minds, *Educational Psychology Review*. New York: Springer.

Wang, Y., Lin, L., Kuhl, P. & Hirsch, J. (2007). Mathematical and Linguistic Processing Differs between Native and Second Languages: an fMRI Study, *Brain Imaging and Behavior* 1, pp. 68-82.

A FACTORY OF TRIANGLES

by Maria Piccione[*]

INTRODUCTION

The proposal is related to one of the basic themes of geometrical knowledge, precisely it refers to the modality of conceptualization of the triangular shape, with its fundamental properties and relations.

The shape of geometrical figures is one of the cores founding of mathematical thought, and, together with the measure of extensions, it is a characterizing object of the Euclidean Geometry. In particular, the triangular shape is one of the most relevant official topics in Lower Secondary School, the main target of this proposal. The present work introduces the students into a proper field of study for the development of rational thinking, as Mathematics epistemology itself recognizes, and as the discipline teaching requires, in accordance with the statements of the Ministry Guidelines.

Aims

The educational aims of the proposal can be distinguished in cognitive, meta-cognitive and affective goals.

Cognitive aims:

- Improving the visualization process
- Implementing the mathematization process
- Structuring the concept of constant/variable in a class of elements
- Developing graphical representation abilities

[*] Department of Information Engineering and Mathematics, University of Siena, Italy

- Developing language, from common to disciplinary
- Constructing the concept of triangular shape
- Finding the existence condition (“triangular inequality”) and the uniqueness conditions for triangles

Metacognitive aims:

- Developing awareness of the relation acting-understanding
- Developing awareness of the relation knowing-explaining

Affect aims:

- Creating sense of self-efficiency
- Arousing pleasure of doing/observing/discovering.

Outlines of the proposal method and design

Geometrical education has to refer and to emphasize the *subject-real space* and *subject-concrete objects* relations*,* in particular from the beginning to medium educational level. Points, segments, triangles and other entities - with which the discipline deals - are not concrete objects, but entities concretely representable or constructible by drawings or logically structured materials. A methodological perspective based on reality-interaction gives an efficient base for the intuitive stage of cognitive processes and theoretical applications. At present, progresses in neurosciences and cognitive sciences allow to assign scientific value to those statements which still were only intuitions, although experimentally proved.

Moreover, the *enactivism*[8]*,* a general theory of knowledge connected to biology and to philosophies of experience, is influencing education by defining the *enacted model*, which focuses on *sense-making as a situated, embodied activity*. Castelnuovo’s active learning method is consistent with this current framework and it is adopted in this proposal, involving part of its concrete material and manner of use.

The learning context is centered on the use of dynamical models , which are created by the elements of a simple material: a “geometrical Meccano” - consisting of rods, differing in colours and lengths (made of plastic, wood or metal), having holes at their ends or equally spaced holes along them - hooks and strings[9] (Photo 1).

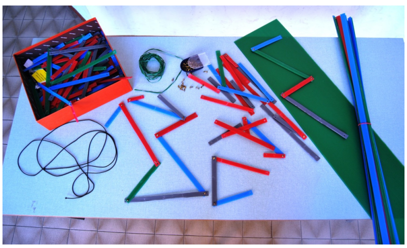


Photo 1. The material for the activity

[8]The theory, starting with the contribution of the biologists Maturana and Varela (1984), is explicitly introduced by Varela, Thompson and Rosch (1991).

[9] It would be better to allow pupils to produce the sticks by themselves, following written instructions.

The work is intended to provide sensory-motor experiences of the object "triangle" and to manage language - in written or oral form - relating to the description of what is done and seen by pupils. Moreover, the topic enables a multicultural view about human knowledge and technology.

How the activity takes place:

- in pairs or in small groups of students, while building flat dynamical figures, analyzing products, drawing, writing observations or comments;
- in class for moments of discussion/comparison of the of results.

The proposal is divided into three sessions: Session 1, with its introductory function; Sessions 2 and 3 providing a natural cognitive path from the idea of three sided figure, to the concept of triangle, as far as to the congruence criteria. Each of them is structured in ordered steps.

A PRIORI ANALYSIS

The proposal is designed to face the present educational problems in a multicultural class, in particular:

- increase of differences among individuals (depending on cultures of origin, previous educational systems and school experiences, levels of relational and cognitive developments, levels of language knowledge as L2 ...)
- forms of unease (proved by behaviors such as isolation, mistrust or indifference to school activities and inability in adequate concentration/participation)

The seminal idea of the work is:

- setting an environment favoring action and individual experience in a social context, providing instruments apt to reduce verbal instructions.

The educational aim consist in:

- allowing everybody to implement cognitive processes, even in presence of linguistic difficulties
- anchoring the language development to experience and peer relations.

The choice of the material and its use aim, , at supporting the mental images generating process, necessary for the conceptual representation. As regards *image-schemata*[10], the great part of the work appeals to the *contact* schema.

Main piloting

by Maria Piccione

This proposal has been piloted in two classes of the first year of the Lower Secondary School "Gandhi" (Firenze). The two samples respectively involved 22 pupils (6 migrant and 5 with learning problems) and 21 pupils (7 migrant and 5 with learning

[10] cfr. *N*úñez et al. (1999)

problems), two teachers (G. Sallustio and A. Scialpi) with the collaboration of the trainer.

It was first described by the trainer and discussed with the two teachers of the classrooms where it had to be piloted.

The work was carried out in three sessions.

Session 1: Becoming familiar with the material *(Duration:* 1 *hour and a half)*

Presentation of the "geometrical meccano"

The pupils of the two classes were gathered in a wide room. As the activity should have been performed with a new structured material, the first step consisted in its presentation and manner of use.

First free compositions

The activity took place in small groups of students (3-4), placed around school desks properly joined.

Each group of students was supplied with a set of sticks (15-20) and hooks: they created a lot of compositions, freely choosing and linking sticks together (figures 1.2, 1.3). Then they were asked to draw the nicest ones, and to reproduce the realized structures, on sheets of paper. These drawings were collected by the teacher at the end of the work.

Session 2: Setting up the motivating context *(Duration:* 4 *hours)*

Thinking/rethinking work*(30 minutes)*

This work was held in the theatre hall - provided with a projector and a screen - with the pupils of the two classes sitting in the platea area.

Firstly, every pupil was given three sticks and hooks for the conjunction and was invited to:

- observe the figure she/he composed;
- look into her/his minds – with her/his eyes closed – searching for images of triangular objects from the real world;
- write down the name and sketch the imagined object on a sheet of paper.

The teacher highlighted the various emerged examples of composed and imagined figures, and collected the sketched ones (Photo 2).

Commented projection*(30 minutes)*

An extensive selection of photos was projected on the screen, in order for everybody to become aware of the amount of triangles present in the real world. (Photo 3)

Subsequently, the activities took place in parallel in the two separated classes.

Photo 2

Photo 3

Internet browsing*(1 hour)*

In the computers' lab, pupils were divided in small groups and, partly guided by the trainer, were asked to search for websites on the internet concerning the triangles; they set the results of the search collecting the preferred representations.

A multicultural overview *(1 hour)*

The widespread use of the "triangulation" in any culture and time of the human history, from the earliest ages to the present days, was pointed out by the teacher by focusing on and by discussing some signifying examples:

- daily uses and applications (arrows' ends, tools …)
- first buildings (tents and bungalows)
- architectural structures (roofing technology, trellis building, network beams…)
- measurements
- mapping (buildings, flats, cities, lands …)
- works in arts and in architecture progressively advanced

The issue related to the successful reasons of the triangle naturally emerged; some of them were indicated as follows:

- stiffness of the triangular structure
- possibility to highlight and fix any point in the mapping process
- esthetic value

Two experiments*(1 hour)*

Two experiments were run in collaboration with the teacher of Technical Education:

- one aimed at checking the stiffness of the triangular structure, proving to compress real models, even the realizations made by means of folded paper;
- the other one consisted in making the classroom mapping by the use of distance-reliever. The room was approximately rectangular in shape and the students could understand and appreciate the necessity to draw a diagonal line in order to achieve the mapping. An assessment on the four sticks connected procedure - by using the rods – clarified the situation and highlighted the sole configuration obtainable by fixing a stick of conjunction of two opposite vertices.

Session 3. Conceptual construction and language development *(Duration: 3 hours)*

The activity was organized in small groups of students (3-4), and was developed through different steps of increasing complexity.

Triangle existence conditions *(1 hour)*

The activity proceeded with the construction of a triangle joining three sticks. Each group of pupils was provided with two different set of triple sticks, in two consecutive times: the task consisted in combining the pieces into figures, in order to realize that:

1. the first triple leads to the production of *just one* "closed" figure (namely a triangle!)
2. the second triple leads to *a lot of* "open" figures …
3. the third triple leads to the overlapping of the sticks (Photo 4).

Graphical representations of the obtained figures in each situation - one pupil in the group may play the role of "reporter" - resolved to be a useful mean for registering the results.

Within each group, the teacher observed the *spontaneous* actions performed by the students during their activity, the verbal communication and the terminology in use (such as "rod" or "side", "open", "closed", "space", "proportion", "sum", "length"…). Progressively, the students of each group were asked to write down a comment on sheets of paper relatively to what they were seeing. The request was just "I see that …".The teacher collected them to analyze the resulting conclusions.

Afterwards, the teacher drew pupils' attention to the condition for triangle construction and to the number of possibilities, by posing the following problem (adapted from "Sticks and Triangles", 21° RMT, and referred to the name of a student):

<Ibrahim puts on his desk five sticks of different length: 4 units; 5 units; 6 units; 9 units; 11 units. How many different triangle could he build? Explain all possibilities.>

One given element (side/angle) triangles' construction*(1 hour)*

After suggesting the students to use sticks or other strings or elastic bands to make triangles, the teacher gave them time to work. Then, he gave rise a discussion within the groups by asking the following question:

- *What elements do you see that may change by movements?*

Two given elements (side&angle / side&side / angle&angle) triangles' construction*(1 hour)*

During this session, the teacher asked the students to set the material fixing one side and one angle and to use strings in order to extend one side and to close the triangle.

They had to build, observe and make graphical reproductions of the obtained figures in various possible configurations (Photo 5).

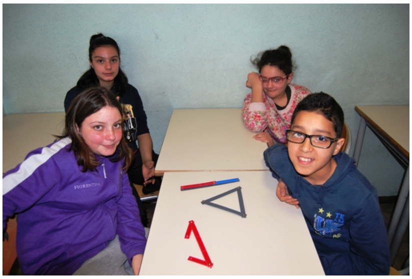


Photo 4

Photo 5

A problematic situation was created by asking the two following questions:

- *Does anything change if we provide another angle?*
- *Does anything change if we provide another side?*

As mentioned above, comments were gathered and organized (see photos).

Definitions

After this activity, pupils were engaged in a verbal description referred to some peculiar triangular shapes and in sharing the related technical terminology (such as isosceles, right, equilateral and scalene triangle). Moreover, pupils were asked again to imagine triangles belonging to the named classes and to describe the possible difference among elements of each class.

Problem solving

The problem solving phase was implemented to verify abilities in identifying triangles (see Annex for the tasks).

A POSTERIORI ANALYSIS

Several results can be derived from the activity; we will focus on the most relevant ones, showing whether and how the implemented proposal accomplish its aims.

- Evidence of the potentialities of "geometrical meccano", within the mathematization process.

This practice has shown that the use of this material - in a socially shared context - favors the individual internal reconstruction of external operations. We will only discuss the role played by the material in discovering the "triangular inequality". The rod has shown to be an *artifact* which can evolve into the concept of "segment", represented by the linguistic *sign* "side". We could observe that, during practice, the three sticks do not play the same role. The pupils tend firstly to consider the rod with fixed ends - that they call "basis" - and therefore they take into account the other two rods, which have still some degrees of freedom - named as "residuals". The role of "residuals" appeared fundamental because it allowed to acknowledge the existence of a relationship among the elements on which the availability of the "closure of the figure" crucially depends. The following comments were obtained by the analysis of protocols related to the Session 2, in particular, by answering the effective request: "I see that …". We could observe that this realization induces a spontaneous cognitive process, traceable by the words used by the pupils –such as *"short/medium/long"*, *"proportionate/un-proportionate"*, *"different/equal"* - and, as a consequence, by expressions referring to comparisons based on a metric interpretation. The fundamental steps of this process appeared to lay on two particular cases: the equilateral and degenerate triangles. The first case is characterized by sentences like *"it closes because the sides are equal"*. The second case – the unexpected one – reveals a powerful function, because it leads the pupils to the mathematization of the closure condition, in terms of "length" and "sum of lengths".

- Instruments for the identification of logical/geometrical competences

For examples:

1. the inability to distinguish the two previous configurations in terms of quantifiers
2. the inability to deduce the corresponding congruence criterion from the possibility to construct a unique triangle - under some conditions.

- Instruments for the evaluation of cognitive discrepancies among pupils within the class

Examples of two borderline cases are:

1. the inability to represent graphically a certain configuration of rods
2. the ability to produce a perfect description of a combinatorial procedure (in the problem "Sticks and triangles").

- Adequacy of the proposal to face the typical problems in a multicultural class.

  Each student took part in the activity at least by touching and seeing real objects.

- Language development

  The transformations of a concrete figure - by handling it - drive the subject to *say/try to say what he is doing* and hence helps language learning and put it through the hoops (presence/lack of words, proper/improper use, vague/rigorous use …).

- Influence on individual emotional components

  The students were really intrigued by the activities.

  They all enjoyed the work and could maintain attention for long times of application. The most striking case refers to a Chinese girl who had previously refused to participate in any school activity, and who finally began to become active.

- Impact on teachers training

At the moment, there are four teachers in the school sharing the will to revise mathematical contents and renewing teaching.

Suggested reading

Bartolini Bussi, M. G., Mariotti M. A. (2008). Semiotic mediation in the mathematics classroom: Artifacts and signs after a Vygotskian perspective, in English L. D. (ed.), *Handbook of International Research in Mathematics Education* (second edition) (746-783). Routledge.

Castelnuovo, E. (2008). *L'officina matematica: ragionare con i materiali*. Lorenzoni F. (ed.). Molfetta: La Meridiana.

Furinghetti, F., Menghini, M. (2014). The role of concrete materials in Emma Castelnuovo's view of mathematics teaching. *Educational Studies in Mathematics* **87** (1-6). Dordrecht: Springer.

*N*úñez, R. E., Edwards, L. D.,Matos, J. F. (1999). Embodied Cognition as Grounding for Situatedness and Context in Mathematics Education. *Educational Studies in Mathematics* **39**: (45-65). Dordrecht: Springer.

Rossi, P. G. (2011), Didattica enattiva. Complessità, teorie dell'azione, professionalità docente. Milano: Franco Angeli.

Second piloting

by H. Moraova[**]

Prague, May 27th 2014, 4 subsequent lessons, 7th grade, CLIL

The Czech team adapted the Italian teaching unit in such a way as to make it meaningful in the context of the school where the unit was piloted. It was piloted in a 7th grade of a lower secondary school ZŠ Fr. Plamínkové in Prague. As there are no migrant pupils in the class, the sequence of lessons was conducted in English language (CLIL – Content and language integrated learning) to simulate the environment where the learners are not fluent in the language of instruction. The sequence of lessons was conducted in one day.

Planning the teaching experiment

The teacher had to analyse the Italian teaching unit and adapt it to the conditions of a Czech school. The first obstacle the teacher encountered was that the construction kit used in Italy is not available in Czech schools. That is why she made the decision to use wooden straws instead.

Also a lot of attention had to be paid to introduction of terminology in the area of triangles in English (obtuse, acute triangles, right angles, equilateral, isosceles, scalene triangles, etc.).

As the teaching experiment was organized in one day, it was also necessary to think of activities that would allow the pupils to have fun. That is why the teacher made the decision to allow pupils to use mobile phones to take pictures of triangles in the school building. This proved to be very motivating.

[**] Faculty of Education - Charles University in Prague, Czech Republic

Course of the teaching experiment

1. *Activity one*: the pupils were given wooden skewers and asked to make sticks 4,5,6, 9 and 11 cm long, then they were asked to make as many triangles from these as possible (Photo 1). The goal of this activity was to make pupils formulate the principle of triangle inequality. The difficulty with using wooden skewers was that the end points were not pointed and thus some pupils managed to construct a triangle even in case that the sum of lengths of two of the skewers equalled the third one. This had to be explained by the teacher as the pupils were lead to formulation of erroneous conclusions.
2. Whole class formulates triangle inequality in English. The teachers checked comprehension.
3. Pupils were given a sheet with a number of exercises whose goal was to practice triangle inequality and make decisions about existence or non-existence of a triangle. This was the part with more calculations and less communication and reasoning, which is also very important in a situation when all pupils are not fluent in the language of instruction.
4. In the next part of the lesson, the pupils were introduced to terminology connected to basic properties of triangles (scalene, equilateral, isosceles, obtuse, acute, right). Using a powerpoint presentation the teacher first introduced the new terms and their illustrations on particular triangles. Then the teacher presented some pictures of triangles and elicited their classification from the class.

   Having finished this activity, the pupils were given sheets of papers and asked to cut out various triangles (Photo 2). The teacher then collected these triangles and distributed them to other pupils at random. They were then asked to take them one by one and write on them the right terms (e.g. acute, isosceles).

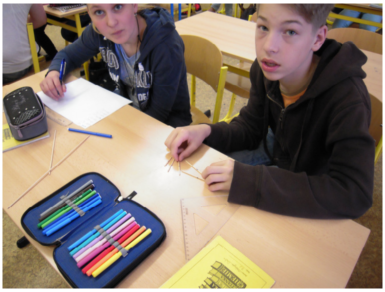


Photo 1

Photo 2

5. Having acquired the basic terminology, the teaching experiment could proceed to the cultural aspect of the teaching unit. First the pictures of triangles in architecture provided by the project team in Siena were presented and the class discussed where they were used (e.g. Opera house in Sydney is world famous, Photo 3). Also use of triangles in various symbols was shown and discussed

(Photo 4). This was followed by the part where the pupils were allowed to take their mobile phones, walk around the school building and take pictures of as many triangles as they can find. This part of the experiment turned out to be very creative and motivating. The pupils were more than willing to search and found triangles even in the most unexpected places (Photo 5 and 6). The pictures from the mobile phones were downloaded to the computer and presented using a projector so that the whole class could share. The pupils were asked to classify the various triangles using the acquired terminology. Also some misconceptions were uncovered (Photo 7 – the pupils made a photo of a pie chart claiming there were triangles despite the fact that one of the sides was an arc, not a straight line).

Photo 3

Photo 4

Photo 5

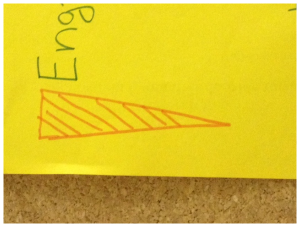


Photo 6

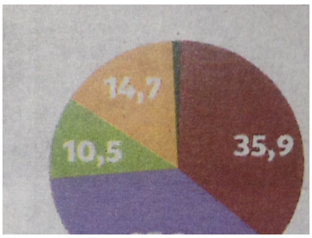


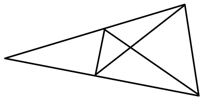
Photo 7

Photo 8

6. In the next part of the teaching experiment, the Italian teaching unit (see Annex) was used again and the pupils were asked to design a triangle tiled floor (Photo 8).

7. In the last part of the teaching experiment the focus was shifted back to culture. The activity started by the problem ***"Well hidden"*** *(question in 21° RMT)*

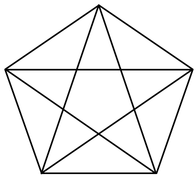
How many different triangles can you see in this figure?



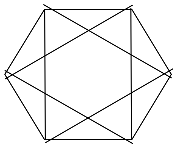
Then attention was moved to five point star, the six point star, the hexagram, Star of David and also to use of triangles on flags of different nations (search on the internet). The pupils started from the task from the Italian unit

How many triangles are there?

(a)



(b)



And then learned how to construct these stars, e.g. by drawing or folding a sheet of paper.

A posteriori analysis of the teaching experiment

The Italian teaching unit proved to be very nicely designed. It combines the traditional with the creative, mathematics with culture, drawing, calculating, manipulating objects, making conjectures and discovering principles. The pupils were motivated and active. They appreciated the use of mobile technology and the creative part of tiling their own floor.

Third piloting

by Marie-Hélène Le Yaouanq[***] and Brigitte Marin[***]

INTRODUCTION

The third piloting was carried out by Mr. Paulou, a mathematics teacher at lower secondary school Roger Martin du Gard, a school located in an Educational Priority Area (in French: ZEP «Zone d'Éducation Prioritaire»), in Year 7.

[***] UPEC University, ESPE, Créteil (France).

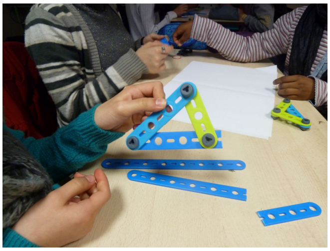
The notions attached to the triangle in the Year 7 curriculum are the triangular inequality and the triangle area. Cases of equal triangles do not appear in the curriculum. However we encourage the students to observe that in the case of a triangle construction, when one side of the triangle is drawn, one can construct several triangles, symmetrical two by two in relation to this side, in relation to bisection and to the middle of this side.

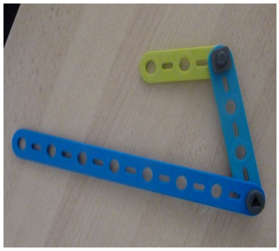
Two phases made up this course sequence.

PHASE 1: TRIANGULAR INEQUALITY

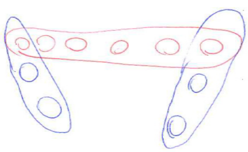
Three steps make up the exploration session.

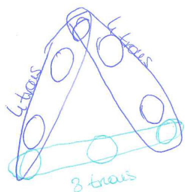
An action phase based on a game called «Meccano» in which plastic rods, with different lengths, can be put together with bolts. The students work in groups. Rods with different lengths are handed out and the students must try to build as many different triangles as they can.





To keep a record of their trials, they have to draw the different results they have obtained on a piece of paper.





Then a collective phase is carried out to conduct an inventory of the triangles that can be built or not. Then the teacher provides a game: the students must guess if someone can build or not a triangle with sides having given length. The teacher intervenes only to regulate, the students must debate orally. This phase shows how difficult it is for the students to find an exact formulation other than “gap not too big”, “close lengths”, etc. (Margolinas,2010).

On the other hand one can notice that, at the end of the session, the activity enabled all the students to develop a method allowing them to decide if a triangle with given lengths could be built or not.

The following sessions will be dedicated to formulate properties and to use them again, to reveal their existence and to show the different possible triangles during construction with instruments and their equality, based on the triangles built with Meccano.

We can notice that students often encountered difficulties with the case of the flattened triangle during construction with ruler and compass (with drawings of no flattened triangles with lengths sides 4, 5, 9), which did not appear during the activity, as the handling led directly in this case to the superposition of rods.

PHASE 2:

The students had to carry out a research phase about countries' flags in which a triangle appears. They had to choose one, to describe it on a geometric point of view, to represent it and to look for the meaning of the different flag elements in relation to the country represented by this flag (shapes, colours).

This work had to be done in pairs outside school, then to be written and presented in class. The students contributed to this work with enthusiasm and gave back particularly careful productions.

This work was completed with area calculations in relation to the triangle area and the expression of proportion with percentages. Many possible openings like construction program writing or interdisciplinary work appeared during this phase.

Reference

Margolinas, 2010, Un point de vue didactique sur la place du langagier dans les pratiques d'enseignement des mathématiques https://hal.archives-ouvertes.fr/halshs-00470238/document (retrieved on 01/05/2015)

Conclusions from the three piloting

by Maria Piccione

Conclusion

The whole experiment combines two components concerning *what* and *how* a certain topic could be conducted in a multicultural class: a teaching model, on the one hand, and a set of practical suggestions, on the other one. The obtained results encourage us to keep going in this work.

Some working choices and pupils' behaviors are to be pointed out from the three piloting, because of their mathematical value and positive feedbacks.

In the second piloting, the material was composed by simple wooden skewers: hence, it allowed the pupils to achieve a sensorial experience of triangular configurations, the basis for the next activity on triangular inequality. Moreover, the experiential training was supported by activities as paper cutting/folding, designing - even complex - "triangulated" figures and tiled floors. The work gives an example of how to reach higher levels of application, concerning the concepts of linear length and angular amplitude. The path can be followed through the questions posed in the worksheets: they require comparative operations and relations between linear length and angular amplitude, as well as the concept of variation range for lengths' values.

In the third piloting, the original Meccano was used. In this case, we find a further explicit reference about the effects of physical action on recognizing the constitutive elements of the model and their possible movements and dispositions. In accordance with the results of other studies, the material also reveals better potentialities than the rule-and-compass, for the understanding of geometrical properties, such as triangular inequality. This evidence can be explained taking into account touch and visual perceptions in a 3-dimensional environment. This benefit is clearly completely missing in a context where figures are already drawn, as the traditional teaching implies.

Useful and contextual suggestions also emerged from this piloting: the equidistant holes, following the length of the Meccano pieces, seem to induce the pupils to express the "closure of the triangle" in terms of holes' number (as a kind of measuring) (see Photo 2). Moreover, as regards the sense of visualization, a point of interest is represented by the discovery of *symmetry* in the distribution of the triangles, realizable under the condition "one fixed side", with respect to the side itself. Some steps towards measuring are done by computing areas and ratios of areas.

In each piloting, the research of triangular representations in the surrounding environment, that is in the natural, as well as in the cultural world, gave rise to a multicultural discussion. In particular, the explanation of triangle' meanings in cultures, and the research in countries' flags – in which a triangle appears – deserve to be emphasized.

In conclusion, by exploring the "wide and deep sea" of triangles, apart from the route, participation and careful production, we can state that interest and even enthusiasm aroused.

Further developments of the proposal

We can give some examples of natural developmental lines of the proposal:

- introducing the similarity criteria and approaching the geometrical concept of shape, differentiating its "gender-sense" and "specie-sense"
- extending the study to quadrilaterals and other polygons classes
- going to measure questions (perimeter/area)
- construction writing program

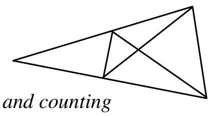
reproducing the proposal by ICTs use, such as GeoGebra , where "sticks" become "segments" in a 2-dimensional space.

ANNEX

Problems

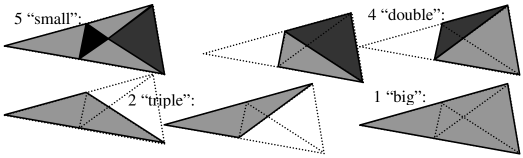
- ***"Well hidden"*** *(question in 21° RMT)*

How many different triangles can you see in this figure?



Involved processes: *visualization, recognition and counting*

Solution

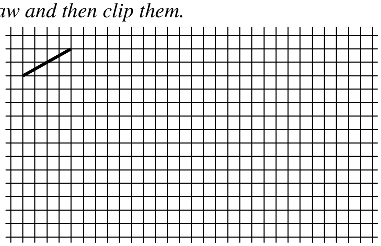


- ***"Triangles clipping"*** (problem in 19° RMT)

In this checkered paper find different triangles (i.e. not-overlapping) following these two instructions:

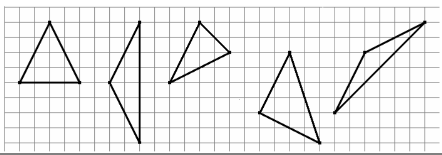
- *they must have two sides with same length of the segment drawn on the down below checker.*
- *they must have their vertexes in the intersection points of the checker.*

How many triangles you can find? Draw and then clip them.



Involved processes*: shape and measure identification; deduction*

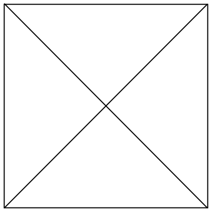
Solution



- ***"The composed figure"***

A figure is drawn on this paper. How many triangles with different shape can you see in it? Can you describe the shape you find?

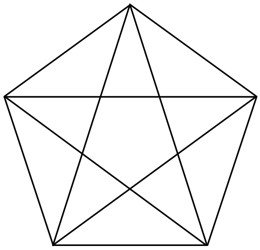
(a)



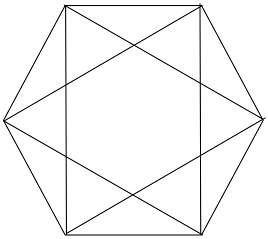
Involved processes: *shape analysis and classification*

Two more difficult version of the problem

(b)



(c)



- ***"A floor made of triangles"***

Create a tiled triangular floor, as you like it better.

Involved processes: *shape analysis and combination*

FINGER MULTIPLICATION

by Barbro Grevholm*

INTRODUCTION

The area of study in the unit is Multiplication from different approaches (history, culture, traditions, use of tools and books), the use of concrete tools in calculations, the use of early algebra for formulation of rules for multiplication and for proving mathematical results, different ways of proving in mathematics and mathematical reasoning.

Piloting with teachers

Anticipated mathematical topics for development are arithmetic of multiplication, operations and rules, factors, products and factorisation, proof and proving, history of mathematics, metacognitive reflections on learning mathematics, and understanding of mathematics in relation to verbatim learning.

Aims of the unit

The aims of the unit is to make pupils reflect about the process of multiplication, realise the properties of multiplication and see links between multiplication and other areas of mathematics. Pupils may also reflect upon what they need to know by heart in mathematics and what can be reproduced with different tools or aids. Pupils may also notice that mathematics is constructed and used by ordinary people in many parts of the world. By interviewing their own family members they can learn about how multiplication is dealt with in their own countries.

Main piloting

by Barbro Grevholm

The proposal: *Original text of the teaching unit Finger multiplication*

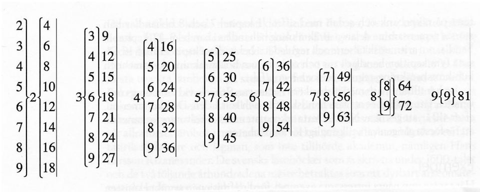
<u>*Start of the unit*</u>:

Session 1

Pupils are given a picture (see below) of a set of numbers in a triangular form from a handwritten book from 1601 (13 years before the first mathematics book was printed in Sweden). The history of the book can be told, see Appendix 2. Group discussion

*Faculty of Engineering and Science, Department of Mathematical Sciences, University of Agder, Norway.

about the picture is initiated by the teacher with the following questions (after pupils had time to study the number pattern):



1. What do you see in this number arrangement? Have you seen something similar before?
2. What could be the reason for presenting this number arrangement in such a way? How did you meet such an arrangement in your mathematics learning?
3. What property for numbers is it that makes the table in the old book possible? What can be the reason for not using such shorter arrangements also today?

Comments for the teacher: We can expect pupils to recognise the row of results in the 2-table and in the 3-table. Maybe they link to the normal multiplication table? Here it is possible for the teacher to ask the class to write down the normal multiplication table with 10 times 10 equalities and in this table colour the products that are present in the triangular pattern. After further investigation and discussion the class may discover that the products that are missing in the triangular pattern are already in some of the present tables. Thus, all copies of calculations are removed in the triangular pattern.

Now the teacher can explain for the pupils that these tables can be used in two ways: to calculate a product of two integers or to find from a number seen as the product what are the factors in that number. That is one can calculate $3 \cdot 4$ or one can ask what are the factors in a given number like 12? Here the teacher may want to repeat the terminology factor and product used in multiplication and make pupils aware of the difference from addition where we talk about terms and sum.

Exercises

1. Use either table to calculate: a) $2 \cdot 9$ and $9 \cdot 2$ b) $8 \cdot 7$ and $7 \cdot 8$ c) $5 \cdot 8$ and $8 \cdot 5$. What do you notice? What is the name for this rule that multiplication of numbers follow? Can you find an easy way to illustrate this law? How about a rectangle with 2 rows and 9 columns? What happens if you look at it from different angles?
2. Find the possible factors in the numbers 18, 27, 42 by using either of the tables. Are there more than one option?
3. In how many ways can you split 48 in integer factors? Is there a simplest way of writing an integer as a product with simple factors? Do you consider $2 \cdot 24$ and $24 \cdot 2$ as two different ways? Or do you see $2 \cdot 3 \cdot 8$ and $8 \cdot 3 \cdot 2$ as two different ways? Why are the two last products equal? What do you name the rules that confirm this to us?
4. Can you think of a situation where it is important to be able to find the factors of an integer?

5. With what number do you need to multiply a) 12 in order to get 36? B) 9 to reach 72? c) 15 to get 90?
6. The number 4 can be factorised as 2·2 and 2+2 is 4 so the sum of the factors is equal to the number. Can you find another number with this property?
7. Now you construct some similar problems and ask you friends in the class to solve them.

Session 2

The teacher introduces the work with a conversation along these lines: Mathematicians often call themselves lazy persons and prefer to do things in the simplest possible way and work as little as possible. One of the things pupils struggle with in school for many years is to drill the multiplication table. How can this work be minimised? What did you find most difficult about the multiplication table? A friend of mine found 7·7 very easy as it was in the year 1949 she learnt the seven-table. Do you have a favourite product that you always know for sure?

Today we have digital tools such as the mobile phone and the calculator or the computer to do the multiplications for us. But what happens if you need to multiply and there be no tools around? Well, people thought about that also in old days and found solutions. One way is to use your fingers for the multiplication.

This is a way of multiplying any numbers between 5 and 10 with your fingers:

Figure source: Grevholm (1988), p. 19
For a translation of the task to English see Appendix 1

Try it and see if it works for you!

This method was once shown to a teacher educator by a mathematics teacher who learnt it from some Romany people who went to adult education in Malmö in Sweden. They asked their teacher to explain to them why this method always works. He was not able to prove that to them so he asked the teacher educator. Can you help the teacher explain why the method always works?

In the Nordic countries the method was also known as the peasant's or farmer's multiplication as it was convenient for anyone who does not have immediate access to paper and pencil.

[Comment for the teacher: The proof can be done by early algebra (see Annex 3) or by convincing oneself that all possible cases are true by a systematic try, as there are only a limited number of cases here.]

Exercises

1. Convince yourself and your worst enemy of the fact that finger multiplication is always giving the correct result if properly done. Explain to your teacher how you did it.
2. Try to make up a mathematical story where you can solve the problem with multiplication. Ask you friend to solve it. Compare each other's methods of thinking. Do you prefer a special one of the methods? Why?

Session 3

There are many different ways of finger multiplication and most of them are known since historical time and come from traditions in different countries. Go to the internet and search for finger multiplication. Explore some other ways of finger multiplication. What is different and what is similar to the way shown here? Is there a proof available? Do the authors explain why the method works? Do you find one method you prefer to the others? Explain why.

Now when you can multiply numbers between 5 and 10 with your fingers how can you reduce the triangular multiplication table to just the cases you need to know by heart? How many different multiplications is that? Write down the reduced table.

With a calculator you can easily find out $12 \cdot 14$. Would it be possible to do that by finger multiplication also? Try to create a way to do that. Is there anything on the internet about multiplying numbers between 11 and 15? Do you find something for even bigger numbers?

What do books about history of mathematics tell you about finger multiplication? See, for example, D. E. Smith, History of mathematics.

In what countries do you find the different methods for multiplication? Think about the way this skill was learnt from one person to another. In old days it was oral communication and you probably showed how to do in a very practical way. You may even describe it for a person who does not speak your language? You can explain by just using gestures and signs. Today if you want to describe the method in written form, as in the internet for example, it takes much more effort than to just show it with your fingers. The skill of finger multiplications seems to be forgotten in many countries today. Why is it so? And why are we asking children in school to learn a table with $10 \cdot 10$ multiplications when we can do with much fewer operations?

Final session

The teacher and the class can have a summarising discussion about what has been learnt.

Here are just some examples of what can be treated:

- What properties of multiplication have we found?
- Can we explain why the table from 1601 looks like it does?
- Can we find the same properties for division of numbers? Why? Find examples.
- How about addition and subtraction? What do you think? Find examples.
- What could be the reason why finger multiplication is not well known any longer?

A fruitful way to handle the summary is to start by letting pupils write down their own view of the learning that took place and then discuss it in the whole class.

Comments for the teacher

Pupils will probably know the commutative law, but they might not have a name for it. The exercises give an opportunity to introduce this terminology and also maybe for pupils to discover other laws for multiplication. We have given some additional material for the teacher, as for example the history of the book from 1601 (in Appendix 2), the algebraic proof (see Appendix 3), the systematic proof, links to good sites about finger multiplication, ways of multiplying numbers larger than 10 (Appendix 4), some texts from history books about finger multiplication (Appendix 2) and so on.

The work is useful from multicultural aspect as we know it has been used in many different groups of people in different countries and over long timespans, and it is a way of working which has been transposed orally or via gestures in old days so it can be made independent of language and is highly concrete. Pupils can interview their parents and grandparents about the table and see if they know the different tables. The triangular table combined with finger multiplication reduces some of the learning by heart of operations in the multiplication table. From research we know that multiplication is an area where learners struggle much.

The main piloting of this teaching unit took place in Norway, in one school in Kristiansand and one in Trondheim. Below are reports and summaries of the results from main the piloting,

General information:

The main piloting was carried out by Kari SofieHolvik and Camilla Normann Justnes and this report builds on the written reports from them and the evaluations made by them. The photos are taken by Camilla Normann Justnes. In a meeting with one of the teachers before the piloting we discussed in what ways the unit would be helpful for the pupils. The teacher saw it as a great help for repeating about multiplication and she also suggested that she could use it to prepare the factorization of numbers which the pupils in her class should learn about later that semester. Thus we included some exercises that deal with factorization. In fact these also make explicit that multiplication and division are inverse operations.

From Karuss school

First half of the class learnt about finger multiplication. The pupils were fascinated and thought it was fun. They were given the task to teach someone else about it, but all of them forgot about that. Thus I, as teacher, had to teach the rest of the class about it later. None of the pupils used the method in the examination as they have the multiplication table in their heads and then that is faster to use. If we repeat the method constantly, it could happen that some of the pupils can use it in a test without aids. There was not time or reason for showing proofs to anyone. But I would like to bring it forward in year 9 when we study algebra.

The final lesson in this project we used to explore other methods for multiplication. The pupils use the internet and searched for other methods, in order to learn at least one of them and then show the class afterwards. They worked in pairs and were very eager. The method that was most fascinating to them was Japanese method, which has been exposed in Facebook lately, where you can count and make crosses (see link).This is some of the methods they found:

- http://vivas.us/i-promise-that-this-japanese-multiplication-technique-will-make-math-way-easier/
- magic math for 7`s
- Multiplication, learning the times table (Mister numbers)
- No.swewe.com
- Guro.sol.no/questions/naturvitenskap/matematikk/hvordan-multiplisere-firesifrede-tall-i-hodet

Reflections on possible changes if I use the unit once more

There was too little time for the pupils to work through all the tasks and make a summary afterwards. Next time I would use a couple of questions, for example 1, 2 and 3 if I was thinking of usefulness for the examination and 5, 6 and 7 if I wanted to focus on investigations. Another option is to differentiate the tasks. Or distribute the tasks randomly and ask pupils to explain to one another in larger groups afterwards. It is in any case important that they have time to talk mathematics together.

I could have used more time (a double lesson) so that more pupils would reach more. The level and type of tasks was well adapted to the year group of pupils. The unit is fine related to repetition of factorization and prime number factorization and other mathematical concepts (factor, multiplication, sum and so on).

The lesson after the final on this unit we had to use for learning of the last subject before examination (volume) and the second lesson this week will be used for exercises on that theme. But later this week we start with repetition and then I will refer to what we worked with in this unit were it fits in and maybe ask if some pupils will use some of the other multiplication tables they got.

I will use finger multiplication when we repeat for examination in week 18 and 19, including task 1. The mathematical proof for why it is correct will be distributed to a few pupils. It is too advanced for most of the pupils in year 8.

The exploration of other multiplication methods must be done after the examination (week 21). They are actually more relevant and suitable in school year 4-6.

From Saupstad School

Here the teacher carried out the teaching unit in a year 5 class and has given us detailed notes of her work. One lesson (45 minutes) was used for the first session. She used the first session starting with telling the story about the old book. She then distributed a copy of the number set to each pupil and gave them 5 minutes to study it. After that the pupils discussed two and two and formulated questions. The teacher took notes on the green board.

Here is the translation of the notes in Photo 1 below:

What do we see?

- *It is times. We can find the answer.*
- *Times tables from 2-9-times.*
- *Times from left to right. Parting from right to left.*
- *It looks like a triangle.*

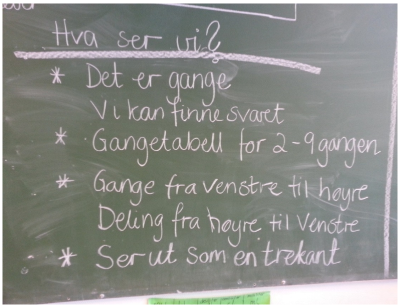


Photo 1: Pupils' answers to "What do we see?"

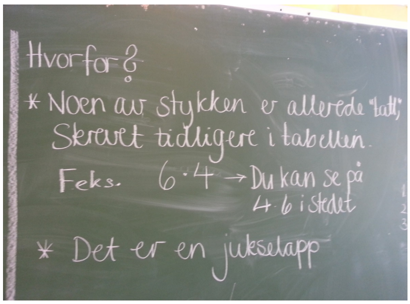


Photo 2: Pupils' answers to "Why?"

Translation of the text in Photo 2:

Why?

- *Some of the tasks are already done. Written earlier in the table. For example 6·4 →You can look at 4·6 instead.*
- *It is a cheating note.*

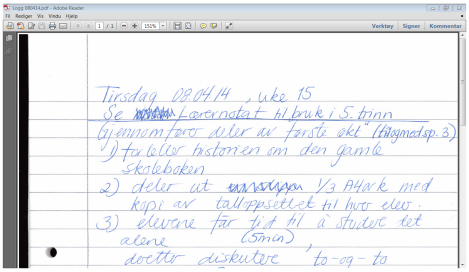


Photo 3. Part of the teacher's notes for her planning of the unit.

The pupils were eager in the discussion and some of them wanted to keep the number set on their desk and use is as a tool. Some of them glued it in their mathematics note book. Pupils then made exercise 1 and used the number set for the calculations.

Conclusions

In Norway teachers are very aware of the need to go through the curriculum and include all parts of it. The examinations are highly important and much time is used to prepare for them. One effect of this is that teachers often feel that there is little freedom and they dare not do other things than what is directly and explicitly in the curriculum. Thus is it is not unexpected that the teachers who are piloting here experience that they cannot use much time on a teaching unit where they do not immediately see how it supports the pupils' learning. This explains why the teacher in Karussschool could not do all parts of the unit in a sequence but has to come back to it later. In the case of Saupstadschool we have unfortunately only got the report from the first session and the rest is missing. We hope to be able to take part of it later.

Both teachers are reporting that pupils were eager to learn and enthusiastic about the triangular shaped number set. Pupils wanted to keep the triangular number set as a tool in the calculations. It is interesting to note that when the pupils see something that can support them and ease the learning they see it as a cheating note. It is as if mathematics has to be hard.

When pupils searched for complementary methods of multiplication they seem to have discovered mathematics from different parts of the world and different cultures.

From the pupils' answers to the questions it is obvious that even in year 8 they use a rather immature terminology like times instead of multiplication and parting instead of division. In teaching sequences like this one there are natural opportunities to learn more about terminology also in different languages.

The teachers are teaching in different age group but still they both found ways to use the unit in fruitful plans. Thus the level of difficulty and the level of choice of subject can be seen as appropriate.

Second piloting

by Andreas Ulovec[**] and Therese Tomiska

General Information

The teaching unit was piloted by a female mathematics teacher with five years teaching experience working in an upper secondary school near Vienna. The Austrian project team sent the material to the teacher approximately 3 weeks before the planned piloting activity. The teacher had a 5th (age 14-15 years), 6th (15-16) and 8th (17-18) grade available for piloting. After a meeting with the project team, she chose to conduct the piloting during a regular mathematics class (50 minutes) in the 6th grade. Eight students (age 17-18), three of which are migrant students, attended the class, which was video recorded and observed by a member of the Austrian project team. After the piloting, an interview was conducted with the teacher.

Classroom piloting

The teacher conducted *Session 1* as described in the Norwegian material by handing out sheets containing a multiplication table from the year 1601 and started a group discussion about it. This discussion lasted about 12 minutes.

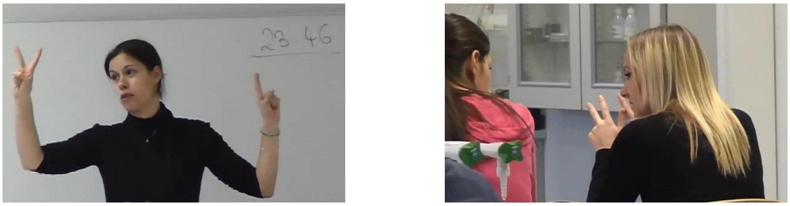
Group discussion about multiplication table

Teacher listening to the students' arguments

The students were particularly interested in the aspects of why there was a need for such tables, whether such tables existed in their own cultures' history, and (mathematically) why these (shortened) tables were sufficient and contain the same basic data as the traditional, square-matrix shaped full multiplication tables they know. The information about the various aspects was partly given by the teacher, partly the students used internet resources to retrieve additional information.

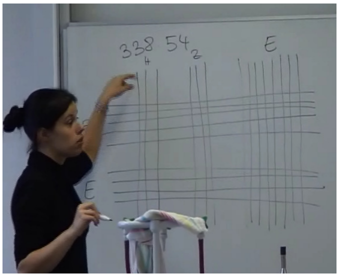
[**]Faculty of Matematics - University of Vienna, Austria.

Session 2 started with the introduction of the method of finger multiplication by the teacher.



Teacher demonstrating and students trying out finger multiplication

Students were then asked to try the method out and – together with the teacher – find an explanation why it works (15 minutes). Students came up with several explanations and wanted to find out whether the method can be extended for larger numbers.



Teacher supporting students' generalisation efforts

They also were interested in whether this or other "unusual" multiplications were used historically. Two of the migrant students (from Turkey) mentioned a geometric-based multiplication method from their own culture, which the teacher then explained further.

As can be seen, the four sessions that were foreseen in the Finger Multiplication material were fitted by the teacher into one lesson. Since there was no timeframe given in the materials, and since the students already knew about multiplication, number systems, and various algebraic methods, a longer duration of the unit was not deemed necessary.

Interview with the teacher

An interview was conducted with the teacher in the afternoon after the piloting took place. The teacher reported that during the break immediately following the class, students were asked by the teacher about their experience with this teaching unit. Both the migrant and non-migrant students responded very positively. The migrant students particularly mentioned the chance of giving background information about their own culture that the other students did not know before. The non-migrant students commented positively on the various historical and cultural references that they not usually get during regular mathematics lessons. The teacher particularly welcomed the possibility of having various anchor points for cultural references, and the opportunity to have the migrant students not only participate, but being a source of information for the other students.

Conclusions

The piloting clearly showed that students are interested in mathematics content from different cultures, and that the active participation of migrant students and the introduction of their cultural backgrounds can enrich the learning situation.

Third piloting

by Hana Moraová[***] and Jarmila Novotná[***]

Place: ZŠ Fr. Plamínkové s RVJ, Prague

Time: 9th September 2014

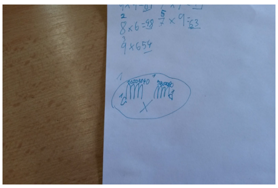
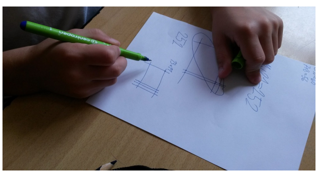
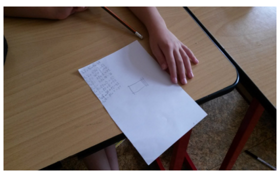
Class: 3rd grade (2 different classes)

Knowledge: knowledge of multiplication tables to 5, one of the classes already started multiplication by 6-9 before holidays, some children remembered it

The lessons were taught in English, videos were prepared to present how to use finger multiplication, but the projector failed to work

The course of both lessons

- Revision of numbers from one to one hundred
- Revision of multiplication 1-5
- Demonstration of finger multiplication (the teacher) on two examples.Using fingers and whiteboard.
- Children asked to do it, very few understood, i.e. two other problems solved in front of the whiteboards, one pupil asked to come, done in cooperation of the teacher, the pupil and the rest of the class who were telling the numbers.
- Then asked to work on their own, the teacher monitoring, helping individually to those, who needed it.
- Then the teacher shows another magic trick, multiplication of two-digit numbers using lines, the children liked this one very much and understood more easily, less counting, no multiplication, i.e. easier.

[***] Faculty of Education - Charles University in Prague, Czech Republic.

Lesson 2: November 2014

One of the two **third grade**s, CLIL lesson.

Warm up – number lines, saying bang instead of numbers divisible by 3, then by 4.

Lead in – revision of finger multiplication (the pupils now have learned multiplication tables, just motivation and fun).

Main activity: *I'll show you a trick* – two digit line multiplication, the pupils were introduced to it in the previous CLIL lesson.

Material – square grid paper (to make it easier to draw the lines and also to make it easier to practice work with units, tens and hundreds that are greater than ten).

The first line multiplication teacher controlled, the teacher draws the model, the children count together the number of intersecting points, numbers that do not exceed ten (in units, tens, hundreds).

Then similar multiplication carried out individually (the same numbers set to everybody).

Then two digit numbers where children will have to put up with addition of units, tens, hundreds exceeding 10, first example done together on the blackboard.

Then pupils work on another pair of numbers on their own, the teacher monitors and checks understanding, works with possible mistakes (the children draw the lines too close and fail to see the border between the first and the second digit, the children forget to add the units etc. higher than ten).

The result is checked and possible mistakes corrected.

Lesson 3: 13th February 2015

Short video recording

Warm up – a song with numbers (only for motivation and as an ice-breaker), number lines as in previous lesson.

Lead-in: revision of multiplication of one-digit numbers; learning the word "divisible"; the children get a card with a number and say I am a number divisible by … and by …, what number am I?", who answers correctly is the next one to go and "be" a number.

Revision of finger multiplication.

Main activity:

1. one multiplication of two two-two digit numbers on a sheet of paper, the teacher monitors and helps where needed.
2. the results is taken and described as units, tens, hundreds.

3. a larger number is written on the blackboard and labelled as tens of thousands, thousands, hundreds, tens, units (starting from units from the right hand side).
4. the teacher instructs the children: *Take 5 pencils with different colours from your pencil case.* The teacher takes 5 different chalks. *Underline tens with one colour, e.g. blue.* The teacher does it on the blackboard. *Take another colour. Underline hundreds.* The teacher underlines hundreds on the blackboard … Until they get to tens of thousands.
5. the teacher instructs: *Now, let us multiply three-digit numbers.* And demonstrates that on the blackboard, slowly, step by step, using the different colours to show units, tens, hundreds and also drawing attention to the difficulty of having more than ten units, tens, hundreds etc. and how to cope.
6. the teacher elicits other two three-digit numbers and asks pupils to work on their own, carefully monitoring the progress, helping out individually; the pupils understand fast, some of them solve the multiplication very quickly, so a new pair of three digit numbers is presented, careful monitoring helps majority of pupils to get into the system.
7. checking the numbers together, language obstacle – some of the children can only say numbers to one hundred in English, the teacher writes a model number in words on the whiteboard, the class practices.

warm up (revision of finger multiplication]

Another piloting: 24th February 2014

4th grade, pictures, use of interactive board, the same school

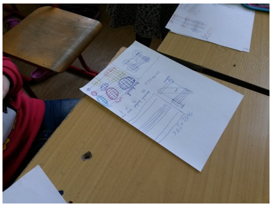
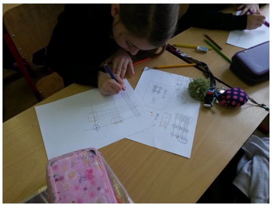
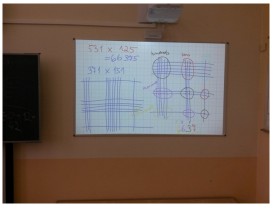
The teacher decided to test the same teaching unit with one year older pupils (slightly higher level of English and more practice in adding higher numbers). The class already know multiplication tables, thus the lesson was not based on finger multiplication but on line multiplications, both two and three digit numbers, use of IT.

Goal of the lesson: practicing recording numbers as units, tens, hundreds thousands, multiplication and addition, motivation, doing mathematics in English.

Warm up – motivation and ice-breaker, showing finger multiplication, trying it out on few numbers.

Main activity – learning new ways of multiplying larger numbers.

1. introduction to basic terminology – units, tens, hundreds …., add, multiply etc.
2. eliciting two 2 digit numbers, showing the principle of line multiplication on the digital whiteboard.
3. eliciting other 2 two-digit numbers, individual work, the teacher monitoring carefully; about half of the pupils discovered how to work with more than 10 units, tens, hundreds and got the right solution, the rest of the pupils were helped individually by the teacher and all was demonstrated on the board – a pupil who finished earlier drew the lines and the rest was done together with attention paid to what to do with units, tens… if there are more then 10.
4. transition to three digit numbers; again making use of 5 different colours for units, tens …; the first example shown on the digital whiteboard using the different colours.
5. other two three-digit numbers elicited, pupils work on their own, the teacher monitors and helps individually, checks that pupils do not mix up units, tens, hundreds; about half of the pupils found it very easy, the rest needed help and support.

No specific conclusions are reported from the Czech teachers.

Conclusions from the three piloting

by Barbro Grevholm

All three piloting reports show that the teaching unit worked well and it could be used in different age groups from year 3 to 8. Pupils are reported to have worked eagerly and with interest and fascination. The teachers seem to have been inspired to carry out both the suggested activities and similar activities found by them about multiplication. The teachers also used the unit to work on language and terminology and they found links and connections between multiplication and other areas of mathematics. The historical aspects were well used and in some cases pupils contributed with experiences from their own culture. The teaching unit seemed to offer something new and unexplored from earlier instruction. Many of the pupils wanted to include the triangular number set as a tool in their mathematics.

A critical aspect could be how much time to use for a unit like this. The answer depends very much on which age group the teacher is working with and from what perspective the unit is used, as repetition, consolidation or exploration.

Another aspect that can be discussed is for which age group a unit like *Finger multiplication* is better used. It probably depends on the level of difficulty that the teacher sets to the questions. It is obviously possible to use it from grade 3 to 8. Some teacher have even used it in algebra I upper secondary school.

The unit can be used to let pupils contribute with experiences and with the creation of tasks. When it comes to multiplication it is generally more difficult to construct your own problems than when it is the operations addition and subtraction, which are intended to be used. Construction of problems can illuminate the different kinds of multiplication tasks that we normally use in school (Verschaffel& De Corte, 1996).

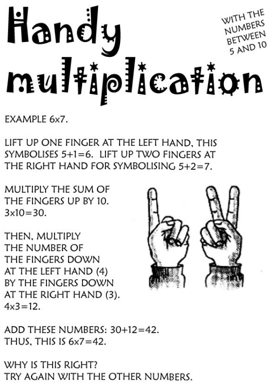
References

Grevholm, B. (1988). *Utmaningen. Problem och tankenötter i matematik.* [The challenge. Problems and mindnuts in Mathematics]. Malmö: Liber.

Verschaffel, L. & De Corte, E. (1996). Number and arithmetic. In *International handbook of mathematics education*, (pp. 99-137). Dordrecht: Kluwer academic Publishers.

Annexes for the teaching unit "Finger multiplication"

Annex 1: Translation of the task Fingerfärdigmultiplikation (Grevholm, 1988)



Annex 2

About the book by Rizanesanders from 1601 (source SharezaHatami, 2014)

Rizanesanders' book was calledRecknekonsten, The art of calculation

About 412 years ago (in 1601) Hans LarssonRizanesander wrote the first Swedish textbook in Arithmetic. One single copy of this handwritten book, *Recknekonsten*, is kept in Uppsala universitetsbibliotek (Uppsala University library).

An interesting question is how much space the multiplication table should be given in the teaching.

Rizanesanders' multiplication-table rests on the commutative law for multiplication; the table demonstrates in all its simplicity use of both mathematical and didactical knowledge.

Rizanesanders' multiplication-table in a simpler version

Below we show Rizanesanders' table in a way which is maybe more adapted to today's teaching. It is clearly visible that the worst table is the simplest one!

Tvåanstabell Table 2	Treanstabell Table 3	Fyranstabell Table 4	Femmanstabell Table 5	Sexanstabell Table 6	Sjuanstabell Table 7	Åttans Tabell Table 8	Nianstabell Table 9
$2 \cdot 2 = 4$							
$2 \cdot 3 = 6$	$3 \cdot 3 = 9$						
$2 \cdot 4 = 8$	$3 \cdot 4 = 12$	$4 \cdot 4 = 16$					
$2 \cdot 5 = 10$	$3 \cdot 5 = 15$	$4 \cdot 5 = 20$	$5 \cdot 5 = 25$				
$2 \cdot 6 = 12$	$3 \cdot 6 = 18$	$4 \cdot 6 = 24$	$5 \cdot 6 = 30$	$6 \cdot 6 = 36$			
$2 \cdot 7 = 14$	$3 \cdot 7 = 21$	$4 \cdot 7 = 28$	$5 \cdot 7 = 35$	$6 \cdot 7 = 42$	$7 \cdot 7 = 49$		
$2 \cdot 8 = 16$	$3 \cdot 8 = 24$	$4 \cdot 8 = 32$	$5 \cdot 8 = 40$	$6 \cdot 8 = 48$	$7 \cdot 8 = 56$	$8 \cdot 8 = 64$	
$2 \cdot 9 = 18$	$3 \cdot 9 = 27$	$4 \cdot 9 = 36$	$5 \cdot 9 = 45$	$6 \cdot 9 = 54$	$7 \cdot 9 = 63$	$8 \cdot 9 = 72$	$9 \cdot 9 = 81$

In Rizanesanders' table the one and ten tables are not used. One can suppose that he wanted the teacher and the pupils to reflect together upon that. He simply uses the commutative law for multiplication to remove all multiplications that exist more than once in the 10 times 10-table.

A multiplication-table, similar to Rizanesanders' multiplication-table, can be found in the first printed textbooks in Sweden in Arithmetic, the one by Aurelius from 1614. Another similar multiplication-table exists in Nils Buddaeus' (1595-1653) textbook. His multiplication-table reminds us about Rizanesanders' multiplication-table.

1

2 4

3 6 9

4 8 12 16

5 10 15 20 25

6 12 18 24 30 36

7 14 21 28 35 42 49

8 16 24 32 40 48 56 64

9 18 27 36 45 54 63 72 81

Below, marked in red, is the 2-table and the following colours show tables for 3, 4, 5, 6 7, 8 and finally the 9 table.

1

1 4

369

4 81216

51015 2025

612 18 243036

714 21 28354249

81624 3240 48 5664

9 1827364554 637281

It is interesting to ask from where the Swedish textbook authors got the idea to present the table in this triangular form. One could hypothesize that it came from Germany as many scholars at that time studied at German universities. And from where did it in such a case come to Germany? We do not know about this at the moment but it would be interesting to search for more information.

Annex 3

Let us think that we want to multiply the two numbers a and b, both are between 5 and 10.

Following the instruction the number of fingers we hold up are (a-5)+(b-5)=a+b-10

This number we multiply by 10 to get 10(a+b-10)

The number of fingers we hold down are (10-a) and (10-b) and we do multiply these numbers to get (10-a)·(10-b)=100-10(a+b)+a·b

The sum of these two numbers is 10(a+b)-100+100-10(a+b)+a·b=a·b, which is the product we wanted to calculate.

Thus we have proven that for any numbers a and b between 5 and 10 the instruction gives the product a·b

Annex 4

Some links about finger multiplication:

http://ncm.gu.se/media/namnaren/npn/arkiv_xtra/09_2/mattfolk.pdf
http://gwydir.demon.co.uk/jo/numbers/finger/multiply.htm
http://scimath.unl.edu/MIM/files/MATExamFiles/WestLynn_Final_070411_LA.pdf
http://threesixty360.wordpress.com/2007/12/31/three-finger-tricks-for-multiplying/
http://www.dccc.edu/sites/default/files/faculty/sid_kolpas/mathteacherfingers.pdf

stampa dicembre 2015

Tipografia Editrice Pisana snc
via Trento, 26 - 56126 Pisa - Italy
Tel. 0039 050 503526
info@tepsnc.it
www.tepsnc.it