of theorizing is not very popular among mathematics educators, and it is not discussed in any of the sections here. However, it is worth noting the quite extensive neuropsychological research on mathematics anxiety (e.g., Moser et al. 2013; Young et al. 2012).

It is inevitable that such a short publication is not complete. Here I will briefly mention two areas of research that have somehow fallen between the sections and deserve greater attention. One such important area of research on mathematics-related affect is the role of emotions and beliefs in problem solving (Goldin 2000; Hannula 2015). First, emotions such as curiosity, frustration, anxiety, surprise, and elation are an important part of the process of attempting to solve a non-routine problem. Such emotions focus attention and bias cognitive processes. Second, general disposition (e.g., confidence) toward mathematics is known to influence the likelihood of succeeding in any given task.

Gender is another area that deserves a few words. Unlike other research on affect, research on affect and gender "has had a recognized and discernible impact on the development and delivery of mathematics instruction" (Leder and Forgasz 2006, p. 412). Perhaps the most robust research finding in mathematics-related affect is that female students have on average lower self-efficacy in mathematics than male students and similar gender differences tend to also be found in other affective variables (Else-Quest et al. 2010).

Taken together, this summary of research shows the richness of research in this area. There are solid findings that allow the building of theoretical foundations about mathematical affect. At the same time, there are open questions and insufficiently explored venues that call for additional research.

2 Surveys of the State of the Art

2.1 Attitude

Pietro Di Martino

2.1.1 The Pioneering Studies About Attitude: The Measurement Era

In mathematics education, early studies about attitude—a construct developed in the context of social psychology—began to appear in the middle of the 20th century (Dutton 1951). The assumption was that not purely cognitive factors play a role in the learning of mathematics.

In these pioneering studies, the definition of attitude is rarely made explicit, and the main goal is to prove causal correlations between attitude and other significant factors (for example, mathematical achievement). Describing the state of the art, Aiken (1970, p. 592) states, "The major topics covered were: methods of measuring attitudes towards arithmetic and mathematics, the distribution and stability of mathematics attitudes, the effects of attitudes on achievement in mathematics, [and] the relationship of mathematics attitudes to ability and personal factors."

Within this context, researchers follow a quantitative and statistical approach that was considered in that period to be a sort of warranty of the scientific nature of mathematics education. Consequently the focus of the research was mainly the development of new scaling methods (such as Thurstone or Likert Scales or questionnaires) to measure attitude and little attention was paid to theoretical aspects, in particular to the definition of attitude, and to the relationship between attitude and other affective constructs. As Leder (1985, p. 21) underlines: “in many cases, authors either implicitly or explicitly define attitude to mathematics in terms of the instrument(s) used in their research.”

At the end of the 1980s, the measurement approach begins to be challenged: several studies show that the correlation between attitude and mathematics achievement is far from being clear. Moreover, the gap between development of instruments and theoretical clarification of the construct began to be considered very problematic, and many scholars explicitly criticized the state, the results, and the *trend* of the research on attitude:

> First, the construct of attitude has been vague, inconsistent, and ambiguous. Second, research has often been conducted without a theoretical model of the relationship of attitude with other variables. Third, the attitude instruments themselves are judged to be immature and inadequate. (Germann 1988, p. 689)

More generally, there has been a gradual affirmation of the interpretive paradigm in mathematics education that has led researchers to try to understand phenomena (“making sense of the world”), abandoning the attempt to explain behavior through measurements or general rules based on a cause-effect scheme (Di Martino and Zan 2015).

As a matter of fact, the shift of perspectives in mathematics education—the movement from a causal-relationship paradigm to an interpretative one—has also deeply influenced research on attitude (Zan et al. 2006) and its methods. The inadequacy of the assumption about cause-effect relationship between attitude and behavior has emerged; attitude is now considered to be an interpretive instrument to understand the reasons for intentional actions: intentional actions involve complex relationships between affective and cognitive aspects; therefore, it is crucial to develop methods able to grasp this complexity.

This shift of perspectives gives new strength to research on attitude that was stuck in the causal-relationship paradigm. In particular, attitude gained renewed popularity in the studies aimed at interpreting the failure in problem-solving activities of students who seem to have the required cognitive resources.

2.1.2 The New Era of Research on Affect (and Attitude)

The beginning of the new era of research on affect in mathematics education can probably be traced to the publication of the book *Affect and Mathematical Problem Solving: A New Perspective* (McLeod and Adams 1989).

This book represents a real turning point in the research on attitude and more generally on affect. Starting with the critique of the research developed on affect until that moment ("This view of beliefs, attitudes, and emotions might be called a black-box approach as opposed to a cognitive approach." Hart 1989, p. 43) and with a shared strong initial assumption ("The initial hypothesis of this project was that affect played an important role in problem solving and that researchers who observed carefully would see the evidence of affect in both students and teachers. That hypothesis has been confirmed." McLeod and Adams 1989, p. 251), the scholars involved in the book highlighted the need to develop a systematic and explicit theoretical framework for dealing with affect (useful to interpret the relationship among affective constructs and between them and cognition). In particular, the need to clearly define the constructs and develop coherent methodologies is stressed:

> There was a lack of definition, lack of clarity, and lack of connections to mathematics. It is possible to avoid making the same mistakes again as new ideas and research methodologies are employed. It is hoped that new researchers on affect will be clear about what is being studied, precise in definition, and respectful of what has been learned previously. (Fennema 1989, p. 209)

A few years after the publication of *Affect and Mathematical Problem Solving*, based on the needs stressed in the book, McLeod (1992) proposed a new framework for research on affect in mathematics education. He identifies three main constructs (emotions, beliefs, and attitudes) and characterizes them. But, as Hannula (2011) underlines:

> Probably the most problematic concept in McLeod's framework is attitudes. Within mathematics attitude research, attitudes have typically been defined as consisting of cognitive (beliefs), affective (emotions), and conative (behavior) dimensions. If we try to combine the tripartite framework with McLeod's, we see that attitude is at the same time a parent and a sibling to emotions and beliefs. (p. 38)

As a matter of fact, in those years researchers provided a variety of definitions of the concept of attitude: all of them involve other factors. In particular, two definitions of attitude are particularly recurrent: a *simple* definition that describes attitude in terms of positive or negative feelings associated with math and a *three-dimensional* definition that recognizes three components in attitude (the emotional disposition, the set of beliefs regarding mathematics, and the behavior related to mathematics). Both the two definitions show enormous theoretical limits (Di Martino and Zan 2001).

The debate about the several definitions of attitude led researchers to consider the *suitableness* of the definition rather than its *correctness*: the adequacy of the definition depends on the issues studied. This was fundamentally the idea of Daskalogianni and Simpson (2000): they suggested considering the definition of attitude to be a working definition: a function of the problems that the researchers pose themselves.

This kind of approach characterizes the new trend of research on attitude as *problem-led*. This view is in line with the very interesting position of Ruffel et al. (1998): “we conjecture that perhaps it [attitude] is not a quality of an individual but rather a construct of an observer’s desire to formulate a story to account for observations” (p. 1).

In relation to the discussion about definition, scholars have debated about the adequacy of methods in research about attitude and their coherence with the definition used.

Within the new interpretative paradigm, the development and use of qualitative methods for research on attitude emerges. In particular, much of the research about attitude has been developed through narratives such as essays, diaries, and interviews (Karsenty and Vinner 2000; Hannula 2002; Kaasila 2007; Di Martino and Zan 2011).

The main strength of this narrative approach that has clearly emerged is the possibility of collecting the aspects and details that respondents consider relevant in the development of their relationship with mathematics. The narrative approach differs from the use of traditional attitude scales—where respondents are requested to express agreement/disagreement on items chosen by others that are sometimes irrelevant for them—in that respondents can specify what they consider crucial and skip what they consider irrelevant. That is, the narrative approach brings out what is central for the respondents.

2.1.3 The TMA Model for Attitude: A Characterization of Attitude Grounded in School Practice

The attitude construct has been widely used by mathematics teachers: often teachers’ diagnosis of “negative attitude” is a causal attribution to students’ failure and perceived as global and uncontrollable rather than an accurate interpretation of students’ behavior that is capable of steering future action. To make this diagnosis useful for dealing with students’ difficulties in mathematics, we conducted a long study based on the collection and analysis of students’ autobiographical narratives (Di Martino and Zan 2011) in order to construct a characterization of attitude strictly linked to students’ experience with mathematics.

An analysis of 1662 anonymous essays entitled “Maths and me: my relationship with maths up to now” written by students of all school levels was conducted. According to a grounded-theory approach to the data (Glaser and Strauss 1967), we used the collected data to discover a set of categories aimed at understanding how students describe their own relationship to mathematics.

At the end of our analysis, we identified three main dimensions in students’ narratives: emotional dispositions towards mathematics, view of mathematics, and perceived competence in mathematics (only 32 essays, 2.1 % of the entire sample, did not refer to at least one of these three dimensions).

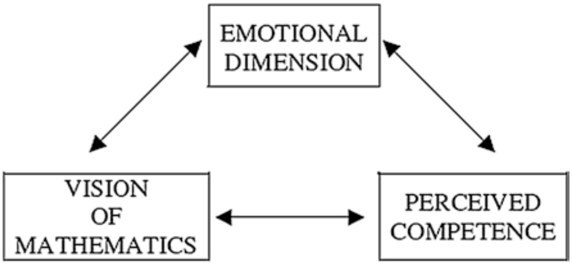

Fig. 1 The TMA model

Therefore we propose a three-dimensional model for attitude (TMA) characterized by the three dimensions that students recognize as crucial in the development of their relationship with mathematics and by their mutual relationships (see Fig. 1).

The arrows in the schema have a crucial role: TMA takes into account the relationship among the three dimensions. These relationships appeared clearly in the students' narratives.

The subjectivity of these relationships among the three dimensions that emerged as one of the results of our research confirms the complexity of the construct:

> The proposed model of attitude acts as a *bridge* [italics in the original] between beliefs and emotions, in that it explicitly takes into account beliefs (about self and mathematics) and emotions, and also the interplay between them. However, in order for it to become an effective theoretical and didactical instrument, the construction and use of consistent instruments for observation, capable of taking into account its complexity, are needed. (Di Martino and Zan 2011, p. 479)

Through the TMA model we have interpreted some recurrent phenomena in the development of attitudes towards mathematics and above all we have given a more sophisticated definition of negative attitude (Di Martino and Zan 2010).

In particular, we have identified different profiles of negative attitude, suggesting implications for teacher practice and for teacher education in order to overcome what we have called the *black box approach*: "that student has a negative attitude toward mathematics" is often the teacher's claim of surrender rather than a precise diagnosis to activate a didactical intervention.

We have briefly described the narrative of the research on attitude towards math (for a more complete report see Di Martino and Zan 2015). This narration must surely be continued—the debate about some critical issues still continues and new issues and new goals have emerged (see Looking Ahead section)—but it is a fact that in the last 25 years the research on attitude in mathematics education has moved ahead in important ways: overcoming a naive approach to the construct, discussing methods, and producing some solid findings (Zan 2013). These solid findings are the significant heritage for those who follow.