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A physically consistent virtual crack closure technique for I/II/III mixed-mode fracture problems

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Abstract

We present a physically consistent virtual crack closure technique (VCCT) that can be applied to three-dimensional problems involving I/II/III mixed-mode fracture. The crack-tip forces exchanged between the nodes of the finite element model along the crack front are decomposed into the sums of three energetically orthogonal systems of forces. The modal contributions to the energy release rate are associated to the corresponding amounts of work done to close the virtually extended crack.

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Keywords: mixed-mode fracture; energy release rate; virtual crack closure technique

1. Introduction

The virtual crack closure technique (VCCT) is widely used to compute the energy release rate, G, in the finite element analysis of fracture mechanics problems (Krueger 2004). The technique has been first proposed for two-dimensional problems by Rybicki and Kanninen (1977), and later extended to three-dimensional problems by Shivakumar et al. (1988). For mixed-mode fracture problems, such as the delamination of composite materials and interfacial fracture, the VCCT furnishes not only the total G, but also the contributions, G_{I}, G_{II}, and G_{III}, associated to the three basic fracture modes (Krueger et al. 2013). Such contributions are associated to the amounts of work done to close the virtually extended crack by the Cartesian components of the nodal forces along the crack front.

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Focusing on I/II mixed-mode fracture problems, Valvo (2012) has demonstrated that the standard VCCT may be inappropriate to analyse problems involving highly asymmetric cracks, for which physically unacceptable, negative values for either G_{I} or G_{II} may be predicted. The origin of this shortcoming has been found to reside in the lack of energetic orthogonality between the Cartesian components of the crack-tip force. Then, a physically consistent VCCT has been proposed, where the crack-tip force is decomposed into the sum of two energetically orthogonal systems of forces, and G_{I} and G_{II} are associated to the corresponding amounts of work done to close the virtually extended crack. As a result, always non-negative G_{I} and G_{II} are obtained (Valvo 2014).

Here, we show how the physically consistent VCCT can be extended to three-dimensional problems involving I/II/III mixed-mode fracture. To this aim, the crack-tip forces – namely, the forces exchanged between the nodes of the finite element model along the crack front – are decomposed into the sums of three energetically orthogonal systems of forces. Accordingly, non-negative G_{I}, G_{II}, and G_{III} are obtained by associating such modal contributions to the corresponding amounts of work done to close the virtually extended crack.

2. Physically consistent VCCT

2.1. Finite element model

Let us consider the three-dimensional problem of a cracked body made of a linearly elastic material with prescribed static and/or kinematic boundary conditions. We set up a finite element model of the problem and restrict our attention to a portion of the mesh in the neighbourhood of the crack front (Fig. 1). Here, we assume a regular mesh made of 8-noded solid elements. A local Cartesian reference system, *Oxyz*, is fixed with the *x*- and *y*-axes respectively orthogonal and tangent to the crack front, and the *z*-axis orthogonal to the crack plane.

The nodes placed on the fracture surface are orderly labelled with the letters *A*, *B*, *C*, … in the direction of crack advance (*x*-axis). Subscripts $j-1$, j, $j+1$, … orderly denote the nodes along the crack front (*y*-axis). Superscripts – and + respectively correspond to the lower and upper crack faces. The facing nodes on the crack surface are initially bonded together by suitable internal constraints, which are progressively released to simulate crack growth. The crack front is initially located on the line connecting nodes C_{j-1}, C_j, C_{j+1}, …

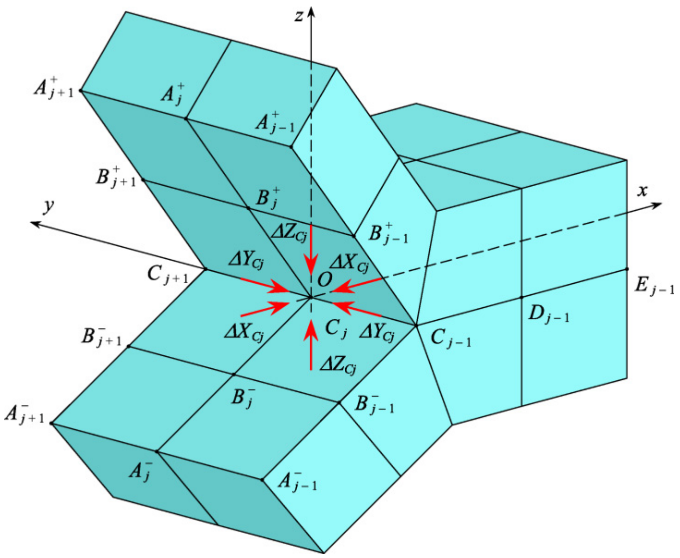

Fig. 1. Finite element mesh at the crack front with crack-tip forces (before crack advance).

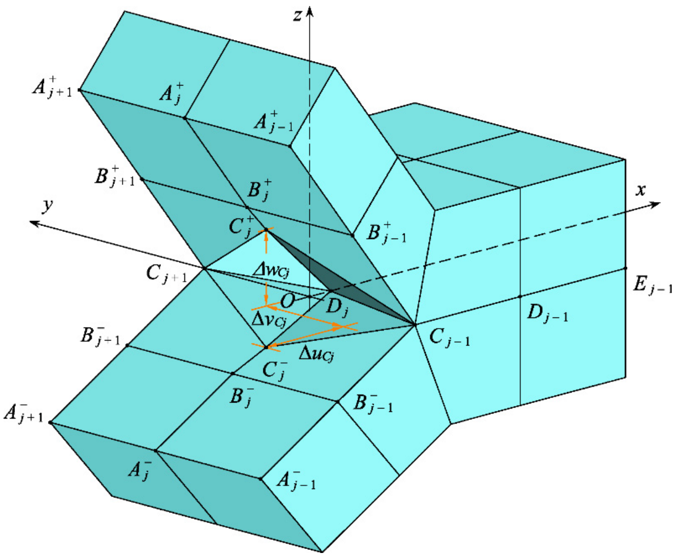

Fig. 2. Finite element mesh at the crack front with crack-tip relative displacements (after crack advance).

2.2. Energy release rate

The energy release rate, G, is defined as the total potential energy of the system spent in the crack growth process, per unit area of new surface created. According to Irwin (1958), the energy spent to extend the crack is equal to the work done to close the crack by the forces that were acting on the crack faces prior to crack extension. Within the adopted finite element model, the energy release rate at the j-th node of the crack front is (Krueger 2004)

$$G = \frac{1}{2\Delta A_j}\left(X_{Cj}\Delta u_{Cj} + Y_{Cj}\Delta v_{Cj} + Z_{Cj}\Delta w_{Cj} \right), \tag{1}$$

where X_{Cj}, Y_{Cj}, and Z_{Cj} are the components of the crack-tip force at node C_j in the $Oxyz$ reference system (Fig. 1). Furthermore, Δu_{Cj}, Δv_{Cj}, and Δw_{Cj} are relative displacements occurring between nodes C_j^- and C_j^+ when the crack front locally advances from node C_j to D_j (Fig. 2). Lastly, ΔA_j is the area of crack surface related to node C_j.

Let us define the crack-tip force vector, $\mathbf{r}_C = (X_{Cj},\ Y_{Cj},\ Z_{Cj})^{\mathrm{T}}$, and crack-tip relative displacement vector, $\Delta\mathbf{s}_C = (\Delta u_{Cj},\ \Delta v_{Cj},\ \Delta w_{Cj})^{\mathrm{T}}$, at node C_j. The relative displacements caused by crack advance are equal in magnitude (and opposite in sign) to the relative displacements produced by application of the crack-tip forces. Thus, for a linearly elastic body we can write

$$\Delta\mathbf{s}_C = \mathbf{F}\mathbf{r}_C, \tag{2}$$

where

$$\mathbf{F} = \begin{bmatrix} f_{xx} & f_{xy} & f_{zx} \\ f_{xy} & f_{yy} & f_{yz} \\ f_{zx} & f_{yz} & f_{zz} \end{bmatrix} \tag{3}$$

is a (symmetric) flexibility matrix, whose elements can be obtained via preliminary analyses (Valvo 2012).

With Eqs. (2) and (3), the energy release rate can be written as follows:

$$G = \frac{1}{2\Delta A_j}\mathbf{r}_C^{\mathrm{T}}\Delta\mathbf{s}_C = \frac{1}{2\Delta A_j}\mathbf{r}_C^{\mathrm{T}}\mathbf{F}\,\mathbf{r}_C = \frac{1}{2\Delta A_j}\left(X_{Cj}, Y_{Cj}, Z_{Cj}\right)\begin{bmatrix} f_{xx} & f_{xy} & f_{zx} \\ f_{xy} & f_{yy} & f_{yz} \\ f_{zx} & f_{yz} & f_{zz}\end{bmatrix}\begin{Bmatrix} X_{Cj} \\ Y_{Cj} \\ Z_{Cj}\end{Bmatrix}. \tag{4}$$

2.3. Mode partitioning

According to the standard VCCT (Krueger 2004), the modal contributions to G simply correspond to the three addends in parenthesis in Eq. (1):

$$G_{\mathrm{I}} = \frac{1}{2\Delta A_j}Z_{Cj}\Delta w_{Cj}, \quad G_{\mathrm{II}} = \frac{1}{2\Delta A_j}X_{Cj}\Delta u_{Cj}, \quad \text{and} \quad G_{\mathrm{III}} = \frac{1}{2\Delta A_j}Y_{Cj}\Delta v_{Cj}. \tag{5}$$

For I/II mixed-mode fracture conditions, Valvo (2014) has shown that physically consistent partitioning of fracture modes is obtained by associating the modal contributions to the amounts of work done by two energetically orthogonal systems of forces. Furthermore, the decomposition of the crack-tip force vector, $\mathbf{r}_C$, into such systems of forces corresponds to the Cholesky decomposition of the flexibility matrix in the form $\mathbf{F} = \mathbf{U}^{\mathrm{T}}\mathbf{D}\mathbf{U}$, where $\mathbf{U}$ is a unit upper triangular matrix and $\mathbf{D}$ is a diagonal matrix. Extending this approach to I/II/III mixed-mode problem yields

$$\mathbf{F} = \begin{bmatrix} f_{xx} & f_{xy} & f_{zx} \\ f_{xy} & f_{yy} & f_{yz} \\ f_{zx} & f_{yz} & f_{zz}\end{bmatrix} = \mathbf{U}^{\mathrm{T}}\mathbf{D}\,\mathbf{U} =$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ \dfrac{f_{xy}}{f_{xx}} & 1 & 0 \\ \dfrac{f_{zx}}{f_{xx}} & \dfrac{f_{xx}f_{yz} - f_{xy}f_{zx}}{f_{xx}f_{yy} - f_{xy}^2} & 1\end{bmatrix}\begin{bmatrix} f_{xx} & 0 & 0 \\ 0 & f_{yy} - \dfrac{f_{xy}^2}{f_{xx}} & 0 \\ 0 & 0 & f_{zz} - \dfrac{f_{zx}^2}{f_{xx}} - \dfrac{1}{f_{xx}}\dfrac{\left(f_{xx}f_{yz} - f_{xy}f_{zx}\right)^2}{f_{xx}f_{yy} - f_{xy}^2}\end{bmatrix}\begin{bmatrix} 1 & \dfrac{f_{xy}}{f_{xx}} & \dfrac{f_{zx}}{f_{xx}} \\ 0 & 1 & \dfrac{f_{xx}f_{yz} - f_{xy}f_{zx}}{f_{xx}f_{yy} - f_{xy}^2} \\ 0 & 0 & 1\end{bmatrix}. \tag{6}$$

The energy release rate becomes

$$G = \frac{1}{2\Delta A_j}\mathbf{r}_C^{\mathrm{T}}\mathbf{U}^{\mathrm{T}}\mathbf{D}\,\mathbf{U}\,\mathbf{r}_C = \frac{1}{2\Delta A_j}(\mathbf{r}_C^*)^{\mathrm{T}}\,\mathbf{D}\,\mathbf{r}_C^*, \tag{7}$$

where

$$\mathbf{r}_C^* = \begin{Bmatrix} r_{\mathrm{II}} \\ r_{\mathrm{III}} \\ r_{\mathrm{I}}\end{Bmatrix} = \mathbf{U}\,\mathbf{r}_C = \begin{Bmatrix} X_{Cj} + \dfrac{f_{xy}}{f_{xx}}Y_{Cj} + \dfrac{f_{zx}}{f_{xx}}Z_{Cj} \\ Y_{Cj} + \dfrac{f_{xx}f_{yz} - f_{xy}f_{zx}}{f_{xx}f_{yy} - f_{xy}^2}Z_{Cj} \\ Z_{Cj}\end{Bmatrix} \tag{8}$$

is a corrected crack-tip force vector, whose components are the energetically orthogonal components of $\mathbf{r}_C$.

The modal contributions to G are

$$G_{\mathrm{I}} = \frac{1}{2\Delta A_j} f_{\mathrm{I}} r_{\mathrm{I}}^2, \quad G_{\mathrm{II}} = \frac{1}{2\Delta A_j} f_{\mathrm{II}} r_{\mathrm{II}}^2, \quad \text{and} \quad G_{\mathrm{III}} = \frac{1}{2\Delta A_j} f_{\mathrm{III}} r_{\mathrm{III}}^2, \tag{9}$$

where

$$f_{\mathrm{I}} = f_{zz} - \frac{f_{zx}^2}{f_{xx}} - \frac{1}{f_{xx}} \frac{\left(f_{xx} f_{yz} - f_{xy} f_{zx} \right)^2}{f_{xx} f_{yy} - f_{xy}^2}, \quad f_{\mathrm{II}} = f_{xx}, \quad \text{and} \quad f_{\mathrm{III}} = f_{yy} - \frac{f_{xy}^2}{f_{xx}} \tag{10}$$

are the (suitably reordered) elements of the diagonal matrix **D**.

Since **F** is a positive definite matrix, it follows that f_{I}, f_{II}, and f_{III} are strictly positive coefficients. Consequently, the modal contributions defined via Eqs. (9) turn out to be non-negative quantities, as required by their physical meaning.

3. Conclusions

We have shown how the physically consistent virtual crack closure technique (VCCT), previously developed for I/II mixed-mode fracture problems, can be extended to three-dimensional problems involving I/II/III mixed-mode fracture. The method has been illustrated with reference to 8-node elements, but can be extended quite simply to other types of solid or plate/shell elements. Examples of applications will be presented in future publications.

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