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Hadronic total cross sections, Wilson loop correlators and the QCD spectrum

Matteo Giordano[a,1] and Enrico Meggiolaro[b]

[a]*Institute for Nuclear Research of the Hungarian Academy of Sciences (ATOMKI), Bem tér 18/c, H–4026 Debrecen, Hungary*
[b]*Dipartimento di Fisica, Università di Pisa, and INFN, Sezione di Pisa, Largo Pontecorvo 3, I–56127 Pisa, Italy*

Abstract

We show how to obtain rising hadronic total cross sections in QCD, in the framework of the nonperturbative approach to soft high-energy scattering based on Wilson-loop correlators. Total cross sections turn out to be of "Froissart"-type, i.e., the leading energy dependence is of the form $\sigma_{\rm tot} \sim B \log^2 s$, in agreement with experiments. The observed universality of the prefactor B is obtained rather naturally in this framework. In this case, B is entirely determined by the stable spectrum of QCD, and predicted to be $B_{th} = 0.22$ mb, in fair agreement with experiments.

Keywords: total cross sections, QCD, nonperturbative approach

1. Introduction

Explaining the behaviour of hadronic total cross sections at high energy is a very old problem, which is very rarely attacked within the framework of QCD. Most of the approaches to this problem are based on phenomenological models, which are sometimes QCD-inspired, but a full derivation from first principles of QCD is still lacking.

The observed rise of hadronic total cross sections at high energy is well described by a "Froissart-like" behaviour $\sigma_{tot} \sim B \log^2 s$ with a universal prefactor $B \simeq 0.27 \div 0.28$ mb [1]. This behaviour respects unitarity, as encoded in the Froissart-Łukaszuk-Martin bound [2, 3, 4], $\sigma_{tot} \leq B_{\rm FLM} \log^2 \frac{s}{s_0}$, since $B \ll B_{\rm FLM} = \pi/m_\pi^2 \simeq 65$ mb.

Besides deriving σ_{tot} from the first principles of QCD, one should be able to explain the observed universality of B, and also the two orders of magnitude separating B and $B_{\rm FLM}$. A step in this direction has been made in Ref. [5], where we have derived the asymptotic behaviour of hadronic total cross sections in the framework of the nonperturbative approach to soft high-energy scattering [6, 7, 8], finding indeed a "Froissart-like" behaviour, and a theoretical prediction for B in fair agreement with experiments.

2. Nonperturbative approach to soft high-energy scattering

Understanding the rise of σ_{tot} is part of the problem of soft high-energy scattering, characterised by $|t| \leq 1\,{\rm GeV}^2 \ll s$. In this regime perturbation theory is not fully reliable, and a nonperturbative approach is needed. In a nutshell, the nonperturbative approach to *elastic meson-meson scattering* is as follows [6, 7, 8]:

1. mesons are described as wave packets of transverse colourless dipoles;
2. in the soft high-energy regime, the dipoles travel essentially undisturbed on their classical, almost lightlike trajectories;
3. mesonic amplitudes are obtained from the dipole-dipole (dd) amplitudes after folding with the appropriate squared wave functions, $A(s,b) = \langle\langle A^{(dd)}(s,b;\nu)\rangle\rangle$.

Email addresses: giordano@atomki.mta.hu (Matteo Giordano), enrico.meggiolaro@df.unipi.it (Enrico Meggiolaro)

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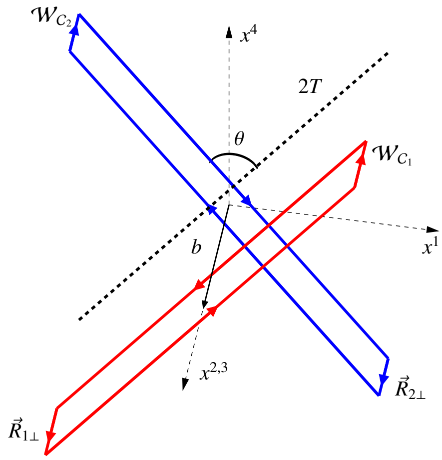

Figure 1: The relevant Euclidean Wilson loops.

Here b is the impact parameter[2], ν denotes collectively the dipole variables (longitudinal momentum fraction, transverse size and orientation), and $\langle\!\langle \ldots \rangle\!\rangle$ stands for integration over the dipole variables with the mesonic wave functions. This approach extends also to processes involving baryons, if one adopts for them a quark-diquark picture [9].

At high energy the dd amplitude is given by the normalised connected correlator C_M of the Wilson loops (WL) running along the classical trajectories of the two dipoles, $A^{(dd)}(s, b; \nu) = -C_M(\chi, b; \nu)$, with $\chi \simeq \log \frac{s}{m^2}$ the hyperbolic angle between the trajectories, and m the mass of the mesons (taken to be equal for simplicity).

The correlator C_M is obtained from the correlator C_E of two Euclidean WL at angle θ (see Fig. 1),

$$C_E(\theta, b; \nu) \equiv \lim_{T\to\infty} \frac{\langle \mathcal{W}_{C_1}\mathcal{W}_{C_2}\rangle}{\langle \mathcal{W}_{C_1}\rangle\langle \mathcal{W}_{C_2}\rangle} - 1\,, \qquad (1)$$

through the analytic continuation (AC) [10, 11]

$$C_M(\chi, b; \nu) = C_E(\theta \to -i\chi, b; \nu)\,. \qquad (2)$$

A more detailed discussion of the approach can be found in Ref. [5] and references therein. The Euclidean formulation has allowed the study of the relevant WL correlator by means of nonperturbative techniques, which include instantons [12, 13], the model of the stochastic vacuum [14], holography [15, 16, 17, 18, 19], and the lattice [13, 20, 21].

[2] We are interested in unpolarised scattering, so only the impact parameter modulus matters; for definiteness, we keep it oriented along direction 2 in the transverse plane.

3. Large-s behaviour of $\sigma_{\rm tot}$

The energy dependence of $\sigma_{\rm tot}$ is determined by the large-s, large-b behaviour of the amplitude through the "effective radius" of interaction $b_c = b_c(s)$, beyond which the amplitude is negligible, as $\sigma_{\rm tot} \propto b_c^2$.

To determine b_c, in Ref. [5] we employed the following strategy. On the Euclidean side, we obtain information on the b- and θ-dependencies of C_E by inserting between the WL operators (for large enough b) a complete set of asymptotic states, characterised by their particle content and by the momenta and spin of each particle. After AC this gives us information on the s- and b-dependencies. This requires two crucial analyticity assumptions:

1. AC can be performed separately for each term in the sum;
2. WL matrix elements are analytic in θ.

A few reasonable finiteness assumptions on the WL matrix elements are also made.

Under these assumptions, at large χ, b, the relevant Minkowskian correlator reads [5]

$$C_M(\chi, b; \nu) \simeq \sum_{\alpha\neq 0} f_\alpha(\nu) \prod_a [w_a(\chi, b)]^{n_a(\alpha)}\,, \qquad (3)$$

where the sum is over (non-vacuum) states α, and $n_a(\alpha)$ is the number of particles of type a in state α. Here

$$w_a(\chi, b) = \frac{e^{[r^{(a)}\chi - b]m^{(a)}}}{\sqrt{2\pi b m^{(a)}}}\,, \quad r^{(a)} \equiv \frac{s^{(a)} - 1}{m^{(a)}}\,, \qquad (4)$$

with $(s^{(a)}, m^{(a)})$ spin and mass of particles of type a, and f_α are functions of the dipole variables only. Particles of type a contribute only for $b \lesssim r^{(a)}\chi$, and so the effective radius of interaction is given by

$$b_c(s) = \left(\max_a r^{(a)}\right) \log\frac{s}{m^2} \equiv \frac{1}{\mu}\log\frac{s}{m^2}\,. \qquad (5)$$

Here we assume the maximum to exist and to be positive. If it were zero or negative, $\sigma_{\rm tot}$ would be constant or vanishing at high energy. The spectrum is supposed to be free of massless states: in case they were present and with spin at most 1, the maximisation should be performed on the massive spectrum only [5].

Using Eq. 3 we find for $\sigma_{\rm tot}$

$$\sigma_{\rm tot} \underset{s\to\infty}{\simeq} 2\pi(1-\kappa)[b_c(s)]^2 \simeq \frac{2\pi}{\mu^2}(1-\kappa)\log^2\frac{s}{m^2}\,, (6)$$

with $|\kappa| \leq 1$ due to unitarity [5]. In general κ depends on the colliding hadrons. Analyticity and crossing symmetry [22, 23] requirements show that universality is most naturally achieved if $\kappa = 0$, corresponding to a vanishing or oscillating correlator as $\chi \to \infty$ at fixed b.

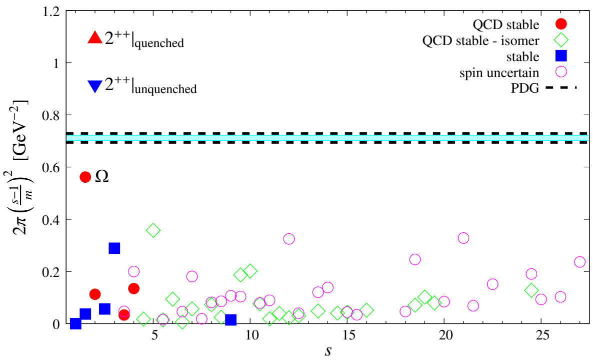

Figure 2: Plot of $2\pi\left(\frac{s-1}{m}\right)^2$ for QCD-stable states. The 2^{++} glueball state [24, 25] and the experimental value for B [1] are also shown. Nuclear data are taken from Ref. [26].

4. Total cross sections from the hadronic spectrum

The total cross section satisfies the bound [5]

$$\sigma_{\rm tot} \underset{s\to\infty}{\lesssim} \frac{4\pi}{\mu^2}\log^2\frac{s}{m^2} = 2B_{\rm th}\log^2\frac{s}{m^2}\,, \qquad (7)$$

with $\mu^{-1} = \max_a r^{(a)}$ determined from the hadronic spectrum by maximising $r^{(a)}$ over the stable states of QCD *in isolation*, as electroweak effects have been neglected from the onset. Only states with $s^{(a)} \geq 1$ are considered, including nuclei (see Fig. 2).

Quite surprisingly, this singles out the $\Omega^{\pm}$ baryon, which yields $B_{\rm th} \simeq 0.56$ GeV^{-2}. The resulting bound on $\sigma_{\rm tot}$ is much more stringent than the original bound, and the prefactor is of the same order of magnitude of the experimental value $B_{\rm exp} \simeq 0.69 \div 0.73$ GeV^{-2} [1]. Furthermore, our bound is not singular in the chiral limit [27].

If universality is achieved as discussed above in Section 3, then $\sigma_{\rm tot}$ is entirely determined by the hadronic spectrum, and reads [5]

$$\sigma_{\rm tot} \underset{s\to\infty}{\simeq} \frac{2\pi}{\mu^2}\log^2\frac{s}{m^2} = B_{\rm th}\log^2\frac{s}{m^2}\,. \qquad (8)$$

This *prediction* for the prefactor is in fair agreement with $B_{\rm exp}$, taking into account a systematic error of order 10% on $B_{\rm exp}$, estimated by comparing the results of different fitting procedures [1, 28, 29, 30]. The same conditions leading to universal total cross sections also give universal, black-disk-like elastic scattering amplitudes [5].

Total cross sections are usually believed to be governed by the gluonic sector of QCD. However, in the *quenched* theory one finds from the glueball spectrum $B_Q \gtrsim 1.6B_{\rm exp}$, suggesting large unquenching effects [5].

5. Conclusions

In Ref. [5] we have derived the asymptotic, high-energy behaviour of hadronic total cross sections in the framework of the nonperturbative approach to soft high-energy scattering [6, 7, 8]. We find a "Froissart-like" behaviour $\sigma_{tot} \sim B\log^2 s$, with B (mainly) determined by the hadronic spectrum (see Eqs. 7 and 8), and in fair agreement with experiments.

Our main results do not depend on the detailed description of the hadrons in terms of partons: adding gluons and sea quarks to the wave functions would lead to more complicated WLs, but since our argument is independent of their detailed form, our conclusions remain unchanged.

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