# A stepwise transmission/reflection multiline-based algorithm for broadband permittivity measurements of dielectric materials

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Abstract— Transmission/reflection (T/R) techniques for measuring dielectric material's complex permittivity are broadband but have usually problems when the length of the tested sample is a multiple of  $\lambda/2$ . In this paper, we apply a stepwise scheme to a multiline T/R measurement method that solve those ambiguity problems thus allowing wideband and accurate permittivity estimation.

Keywords—wideband automated measurement; propagation constant; complex permittivity; multiline technique

### I. INTRODUCTION

Knowledge of materials dielectric constants, namely the complex equivalent permittivity and permeability, is fundamental to understand their electromagnetic behavior and to predict the operation mode of antennas and microwave devices. Several techniques have been proposed to measure permittivity and permeability of dielectric materials and, among the others, transmission/reflection (T/R) methods are commonly used for broadband measurements [1].

The majority of T/R techniques requires knowledge of the calibrated scattering parameters of the guiding structure hosting the material under test (MUT). Unfortunately, an accurate and broadband calibration procedure is not trivial for some of T/R devices.

A measurement technique that completely circumvent the device calibration is presented in [2]. It starts from the transmission and reflection measured data of two lines, which are filled with dielectric material. It has to be pointed out that the two guiding structures must have the same geometrical parameters, but different length (Fig. 1). The complex propagation constant is than determined from the measured scattering parameters and finally the complex permittivity is derived [2], [3]. However, there is an intrinsic problem of ambiguity due to the periodicity of the propagation constant in frequency, which makes broadband measurements troublesome.

In order to solve the aforementioned limitation, we propose here the application of a different calculation scheme of the phase factor. The proposed approach is inspired by the procedure proposed [4] for the Nicholson-Ross-Weir [5], [6] inversion method but it is adapted to the specific needs of the analyzed multiline TR method. The algorithm starts from a reasonable solution of the phase factor at the lowest frequency of interest and extrapolate the others values by a finitedifference scheme at the remaining frequencies. A new condition to select the correct propagation constant from the roots of a quadratic equation has been set.

# II. THEORY

According to multiline method [2], [3], to calculate the propagation constant in the MUT transmission and reflection data are arranged in wave cascade matrices as follows:

$$[\mathbf{M}_{i}] = [\mathbf{A}] \cdot [\mathbf{L}_{i}] \cdot [\mathbf{B}] \tag{1}$$

where  $M_i$  is the matrix of the whole line, A and B represent the input and output connector, which are not explicitly calculated, and  $L_i$  is the matrix of the line hosting the sample of material:

$$[\mathbf{L}_i] = \begin{bmatrix} e^{-\gamma l_i} & 0\\ 0 & e^{\gamma l_i} \end{bmatrix} \tag{2}$$

By defining the matrices:

$$[\mathbf{M}^{(1,2)}] = [\mathbf{M}_2] \cdot [\mathbf{M}_1]^{-1} \tag{3}$$

$$[L^{(1,2)}] = [L_2] \cdot [L_1]^{-1} \tag{4}$$

we can obtain the equation:

$$[\mathbf{M}^{(1,2)}] \cdot [\mathbf{A}] = [A] \cdot [L^{(1,2)}]$$
 (5)

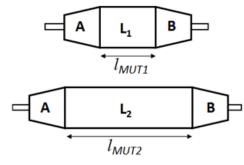


Fig. 1. Schematic of the device for the material's permittivity measurement: the two lines are the same except for the length of the samples of material under test.

Diagonal elements of the  $L^{(l,2)}$  matrix are the eigenvalues of (5):

$$\psi_{+}^{(1,2)} = e^{\mp \gamma (l_2 - l_1)} \tag{6}$$

and the propagation constant can be calculated as the their average:

$$\psi = \frac{1}{2} \left( \psi_{+}^{(1,2)} + \frac{1}{\psi_{-}^{(1,2)}} \right) = e^{-\gamma (l_2 - l_1)}$$
 (7)

The inversion of (7), needed to calculate the dielectric permittivity of the sample is not trivial because of the infinite number of branch points of the logarithmic operator:

$$\gamma = -\frac{\ln\left(e^{-\gamma(l_2 - l_1)}\right)}{l_2 - l_1} = -\frac{\ln\left|e^{-\gamma(l_2 - l_1)}\right| - \arg\left(e^{-\gamma(l_2 - l_1)}\right) - 2\pi n}{l_2 - l_1}$$
(8)

We apply a finite-difference scheme similar to [4] in order to avoid the problem concerning the branch point selection. The procedure can be summarized with the following steps:

1. we assume a reasonable solution of (8) at the lowest frequency:

$$\ln \psi(\mathbf{f}_0) = -\ln |\psi(\mathbf{f}_0)| - j\phi_0$$

$$\phi_0 = \arg \left(\frac{\psi(\mathbf{f}_0)}{|\psi(\mathbf{f}_0)|}\right)$$
(9)

2. assuming the phase difference at two consecutive measurements frequencies is lower than  $2\pi$ , we set the phase constant and the propagation factor as:

$$\ln \psi(\mathbf{f}_{i}) = -\ln |\psi(\mathbf{f}_{i})| - j\phi_{i}$$

$$\phi_{i} = \phi_{i-1} + \arg \left(\frac{\psi(\mathbf{f}_{i})}{|\psi(\mathbf{f}_{i})|} \cdot \frac{|\psi(\mathbf{f}_{i-1})|}{|\psi(\mathbf{f}_{i-1})|}\right)$$
(10)

It is important to correctly identify and assign the eigenvalues in (6) in order to choose the sign of the relation in (8). To this aim, we apply the following rule:

- 1. we assign the eigenvalues to  $\psi_{+}^{(1,2)}$ ;
- 2. we calculate the inequality:

$$\left| \psi(\mathbf{f}_i) - \psi(\mathbf{f}_{i-1}) \right| \le \left| \frac{1}{\psi(\mathbf{f}_i)} - \psi(\mathbf{f}_{i-1}) \right| \tag{11}$$

3. we maintain the assignments if it is fulfilled, or change the assignment if it is not.

# III. RESULTS

As an example, we report the results obtained by simulating two blocks of material with a complex permittivity  $\epsilon_r = 25$ ,  $\sigma$ =0.01S/m in a coaxial line. The simulations have been performed with Ansys HFSS. The obtained scattering parameters have been perturbed with an uncorrelated Gaussian

noise (zero mean and variance equal to 0.001). Fig. 2 shows the calculation of the equivalent complex permittivity of the unknown material extracted by applying both the multiline algorithm inn its original form and our improved version. The proposed method always succeeded in extracting the correct phase factor from the propagation constant and therefore the correct values of permittivity. Differently from the standard procedure, the method allows removing the  $\lambda/2$  ambiguities.

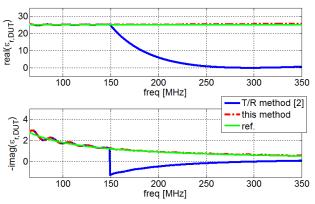


Fig. 2. Complex permittivity of a simulated material ( $\epsilon_r = 25$ ,  $\sigma$ =0.01S/m): method in [2] diverges at about 150MHz because it does not take into account the branch point selection (8).

## IV. CONCLUSIONS

An efficient automated procedure applied to the multiline method to retrieve the dielectric permittivity of materials has been presented. The procedure, differently from the original formulation of the method, allows removing the typical  $\lambda/2$  ambiguities. It is now under investigation the extension of this method to a simultaneous use of three or more standards of MUT of different length to enlarge the measurements bandwidth and also improving the accuracy of the estimation.

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