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Tuning the $3N$ force from $3N$ scattering data

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Abstract. We report on our progress in the inclusion of the subleading $3N$ contact operators for the elastic $p-d$ scattering below the deuteron break-up. We find that, with a nuclear interaction consisting of the AV18 $2N$ potential, supplemented by the leading and subleading $3N$ contact operators, existing discrepancies between theory and experiment, like the well-known A_y puzzle, are substantially reduced.

1 Introduction

Despite long-lasting efforts in the determination of a realistic three-nucleon ($3N$) force, none of the presently available models leads to a satisfactory description of bound and scattering states of the $A = 3$ system [1]. It seems natural to ascribe the above situation to the fact that these models include a very small number of adjustable parameters, compared to the two-nucleon ($2N$) interaction case. For example, in the framework of the chiral expansion, only 2 low-energy constants (LECs) enter up to and including N3LO [2].

In Ref. [3] we have classified all subleading $3N$ contact operators, which involve two powers of nucleon momenta. These terms would show up at the N4LO of the low-energy expansion, but they represent the first correction to the $3N$ force in the pionless version of the effective theory. A matching procedure to the pionful theory at N3LO allows to express them in terms of pion-nucleon LECs [4]. The resulting three-nucleon potential, which depends on 10 LECs ($E_{1,...,10}$) whose values should be fitted to experimental data, can be cast in a local form in coordinate space,

$$\begin{aligned} V = \sum_{i\neq j\neq k} \quad & (E_1 + E_2\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j + E_3\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j + E_4\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j\boldsymbol{\sigma}_i\cdot\boldsymbol{\sigma}_j)\left[Z_0''(r_{ij}) + 2\frac{Z_0'(r_{ij})}{r_{ij}}\right]Z_0(r_{ik}) \\ & +(E_5 + E_6\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_j)S_{ij}\left[Z_0''(r_{ij}) - \frac{Z_0'(r_{ij})}{r_{ij}}\right]Z_0(r_{ik}) \\ & +(E_7 + E_8\boldsymbol{\tau}_i\cdot\boldsymbol{\tau}_k)(\mathbf{L}\cdot\mathbf{S})_{ij}\frac{Z_0'(r_{ij})}{r_{ij}}Z_0(r_{ik}) \\ & +(E_9 + E_{10}\boldsymbol{\tau}_j\cdot\boldsymbol{\tau}_k)\boldsymbol{\sigma}_j\cdot\hat{\mathbf{r}}_{ij}\boldsymbol{\sigma}_k\cdot\hat{\mathbf{r}}_{ik}Z_0'(r_{ij})Z_0'(r_{ik}) \end{aligned} \tag{1}$$

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where S_{ij} and $(\mathbf{L}\cdot\boldsymbol{S})_{ij}$ are respectively the tensor and spin-orbit operators for particles i and j, and the function $Z_0(r)$, which also depends on the cutoff Λ, is the Fourier transform of the cutoff function,

$$Z_0(r) = \int \frac{d\mathbf{p}}{(2\pi)^3} e^{i\mathbf{p}\cdot\mathbf{r}} F(\mathbf{p}^2;\Lambda). \tag{2}$$

chosen of gaussian type, $F(\mathbf{p}^2;\Lambda) = \exp -(\mathbf{p}^2/\Lambda^2)$. The 10 additional LECs parametrize the short-range component of the $3N$ force, and are unconstrained by chiral symmetry. As such, they could provide the necessary flexibility to arrive at a truly realistic model for the $3N$ interaction.

2 Numerical procedure

In order to put such conjecture to quantitative scrutiny, we examine to which extent the AV18 $2N$ interaction [5], supplemented by the leading and subleading contact-range $3N$ force, provides a satisfactory fit to the high-precision low-energy $p-d$ scattering data of Ref. [7].

We performed the calculation at $E = 3$ MeV proton energy. The three-body problem was solved with the Hyperspherical Harmonics method, reviewed in Ref. [6]. The $p-d$ scattering wave function with relative orbital angular momentum L , total spin S and total angular momentum J, is written as the sum of an internal and an asymptotic part

$$\Psi_{LSJJ_z} = \Psi_{\mathrm{C}} + \Psi_{\mathrm{A}}, \tag{3}$$

where the internal part is expanded on the Hyperspherical Harmonics basis,

$$\Psi_C = \sum_\mu c_\mu \Phi_\mu \tag{4}$$

μ denoting a set of quantum numbers necessary to completely specify the basis element, while the asymptotic part describes the relative motion between the proton and the deuteron at large separation, which takes the form of a linear combination of the regular and irregular solutions of the Coulomb $p-d$ Schroedinger equation at relative momentum q, duly regulated at small distance, $\Omega^\lambda_{LSJJ_z}$ with $\lambda = R, I$ respectively,

$$\Psi_A^{LSJJ_z} = \Omega^R_{LSJJ_z} + \sum_{L'S'} \mathcal{R}^J_{LS,L'S'}(q)\Omega^I_{L'S'JJ_z}. \tag{5}$$

The weights $\mathcal{R}^J_{LS,L'S'}$ of the irregular solution relative to the regular one are related to the K matrix whence the scattering phaseshifts and mixing parameters can be determined. They can be found, together with the coefficient c_μ in Eq. (4), from the Kohn variational principle, which requires the functional

$$\left[\mathcal{R}^J_{LS,L'S'}(q)\right] = \mathcal{R}^J_{LS,L'S'}(q) - \langle \Psi_{L'S'JJ_z}|H-E|\Psi_{LSJJ_z}\rangle \tag{6}$$

to be stationary under changes of the variational parameters in Ψ_{LSJJ_z}, normalized such that

$$\langle \Omega^R_{LSJJ_z}|H-E|\Omega^I_{LSJJ_Z}\rangle - \langle \Omega^I_{LSJJ_z}|H-E|\Omega^R_{LSJJ_Z}\rangle = 1. \tag{7}$$

This leads to a linear system

$$\sum_{L''S''} \mathcal{R}^J_{LS,L''S''}(q) X_{L'S',L''S''} = Y_{LS,L'S'}, \tag{8}$$

with the matrices

$$X_{LS,L'S'} = \langle \Omega^I_{LS} + \Psi^I_C|H-E|\Omega^I_{L'S'}\rangle, \quad Y_{LS,L'S'} = \langle \Omega^R_{LS} + \Psi^R_C|H-E|\Omega^I_{L'S'}\rangle, \tag{9}$$

and the $\Psi_C^{R/I}$ solutions of

$$\sum_{\mu'} c_\mu \langle \Phi_\mu | H - E | \Phi_{\mu'} \rangle = -\langle \Phi_\mu | H - E | \Omega_{LS}^{R/I} \rangle. \tag{10}$$

The interest of such a procedure lies in that the matrices can be computed once and for all, for each one of the operators entering the contact potential, and kept on disk. Then, linear combinations depending on the actual values of the LECs E_i can be formed to obtain the corresponding phaseshifts and mixing angles. From the phaseshifts we compute the observables by adding all partial waves up to $J = 15/2$ and $L = 6$. We checked that such truncation does not lead to appreciable differences.

3 Results

We performed multiparameter fits of the $3N$ LECs to the ^{3}H binding energy, the doublet and quartet $N-d$ scattering lengths and to $p-d$ data on the total differential cross section and vector and tensor polarization observables reported in Ref. [7]. We used the POUNDerS algorithm [8] developed at Argonne. It should be noticed that, in the isospin limit, $N-d$ scattering is only sensitive to the isospin $T = 1/2$ component of the $3N$ force. There exists only one combination of the subleading $3N$ contact operators which projects on isospin $T = 3/2$ [1], therefore we can at most determine 9 out of the 10 subleading contact LECs. It could be, however, that $p-d$ scattering alone cannot determine all of them, and only entails correlations among them. This would be apparent if no sensible improvement in the χ^2 is obtained when a parameter is added. This starts to be the case from more than 6 parameters. In Fig. 1 we show the results of a 6-parameter fit for the cutoff $\Lambda = 300$ MeV, showing that the χ^2 can be lowered to 2.0 per degree of freedom.

The description of the polarization observables is much improved, in particular the well-known A_y and T_{11} discrepancies are much decreased. The LECs have been rescaled by inserting appropriate powers of F_π and Λ according to naive dimensional analysis, so that they become adimensional and should be of order 1, if natural. This turns out to be the case for most of them. Similar results are obtained for other values of the cutoff $\Lambda = 200$, 400 and 500 MeV. The exploration of the parameter space is not complete yet, and there is room for a further decrease of the reduced χ^2. Work is under way to check whether the improved description is mantained at different energies.

We also checked, along the lines of Ref. [9], the constraints from the large-N_c limit of QCD. Excluding one of the LECs (e.g. E_8) to account for the $T = 1/2$ projection, and absorbing its effect in the remaining LECs, these constraints take the form

$$E_2 = \frac{2}{3}(E_3 - E_5) + O(1/N_c), \quad E_9 = 3E_5 + O(1/N_c). \tag{11}$$

It is interesting to notice that these constraints are qualitatively satisfied by the fit outcome, while a strict application of them would conflict with the aimed accuracy.

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1. Strictly speaking this is only true at the level of the momentum space operators, neglecting cutoff effects.

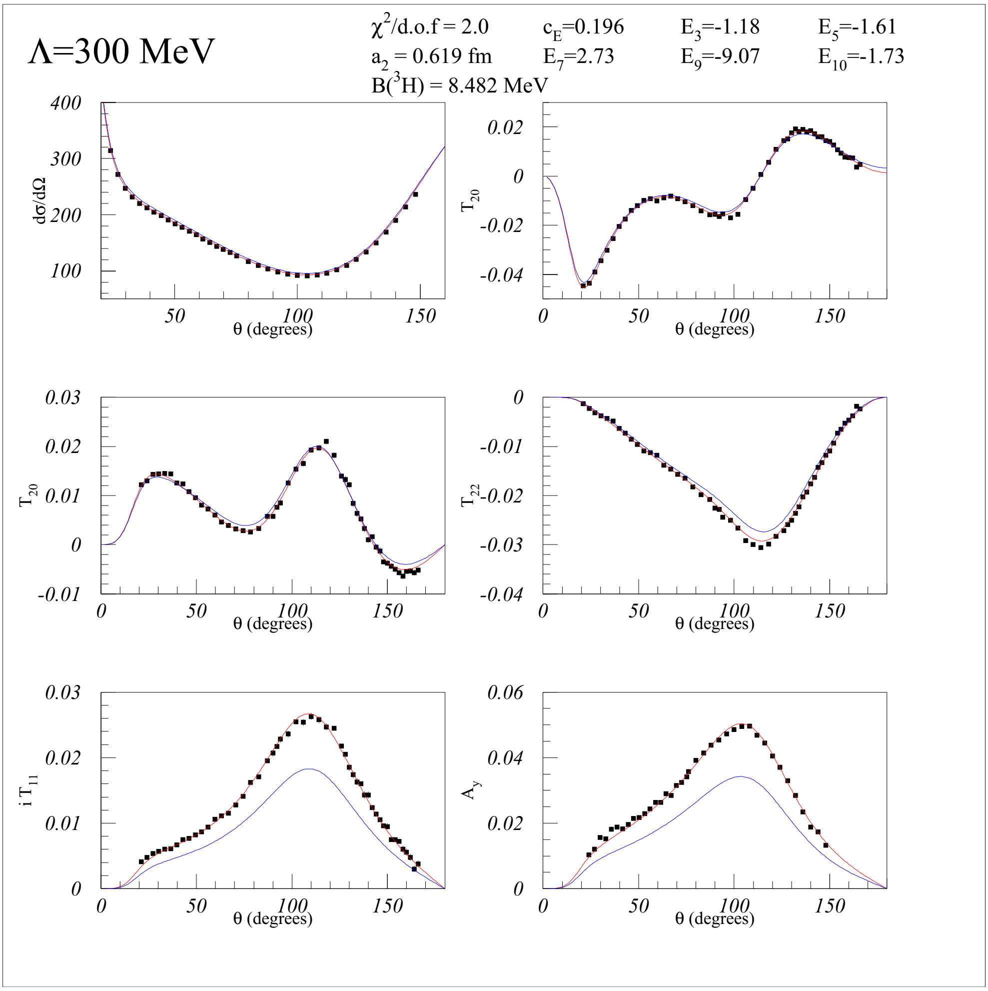

FIGURE 1. Results of a 6-parameter fit to $p-d$ scattering observables at 3 MeV laboratory energy. Blue lines denote the AV18 prediction, while the red lines include a subset of the contact-range 3N force, with the cutoff $\Lambda = 300$ MeV. Also fitted are the triton binding energy $B(^3\mathrm{H})$ and the doublet scattering length a_2.

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