On sustainable and efficient design of ground-source heat pump systems

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On sustainable and efficient design of ground-source heat pump systems

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Abstract. This paper is mainly aimed at stressing some fundamental features of the GSHP design and is based on a broad research we are performing at the University of Pisa. In particular, we focus the discussion on an environmentally sustainable approach, based on performance optimization during the entire operational life. The proposed methodology aims at investigating design and management strategies to find the optimal level of exploitation of the ground source and refer to other technical means to cover the remaining energy requirements and modulate the power peaks. The method is holistic, considering the system as a whole, rather than focusing only on some components, usually considered as the most important ones. Each subsystem is modeled and coupled to the others in a full set of equations, which is used within an optimization routine to reproduce the operative performances of the overall GSHP system. As a matter of fact, the recommended methodology is a 4-in-1 activity, including sizing of components, lifecycle performance evaluation, optimization process, and feasibility analysis. The paper reviews also some previous works concerning possible applications of the proposed methodology. In conclusion, we describe undergoing research activities and objectives of future works.

1. Introduction
Heat pump (HP) systems are a widely used technology for thermal energy generation in buildings, capable of efficiently supplying heating, cooling, and sanitary hot water. Particularly, ground-source heat pumps (GSHPs) are potentially able to reach higher performances with respect to their traditional alternatives, albeit special attention must be paid to the design of the overall system (heat pump equipment, ground heat exchangers (GHEs), connecting ductwork, and end-user loop). A general layout of a heat pump installation consists of the following main elements: A) external source; B) piping connecting the external source with the external heat pump heat exchanger (i.e. *ground-coupled loop*); C) ground-coupled heat pump (GHP); D) piping connecting the internal heat pump heat exchanger with the internal heating (or cooling) components (radiators, fan coils, …); E) building. Each element is, in turn, made of a number of components interplaying with one another, and directly interacts with the closer elements. This chain of strictly connected components has to be carefully analyzed as a whole. It is absolutely inappropriate to give prominence only to some specific apparatus.

On the contrary, current design methodologies (see, for instance, [1–4]) are typically based on a sequential and hierarchical logic. The first step concerns the analysis of building needs to identify a

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reference thermal power (typically the peak load [3,5]) and select the heat pump capacity. The second step deals with the characterization of the ground source in terms of thermo-physical properties and undisturbed temperature level (i.e. thermal response test or pumping test). Finally, ground heat exchangers (GHEs) are sized to match the thermal requirements of the ground-coupled heat pump unit. In this perspective, each subsystem aims to meet the demands of the previous one. The latter considerations, together with traditional engineering "precautionary principle", often lead to oversized systems, with related energetic and economic disadvantages.

Moreover, as also stressed in [6], most of traditional design standards calculate the number and the size of ground heat exchangers on the basis of several design parameters decided *a priori* (e.g., operative temperatures and flow rates, generators capacity, heating/cooling load share assumed by the ground source, reference design period), without analyzing and simulating the operational life of the global system. Therefore, once the system is operational, the interactions among GSHP subsystems may result in unexpected coupling effects, possibly decreasing the overall operative performance. As a matter of fact, the full design process is mainly based on the personal evaluations of the designer(s) [6,7] and a high level of experience is needed to obtain a sound project. However, there is no guarantee that the final design is the most cost-effective.

Our considerations seem to be confirmed by experimentally monitored seasonal performance factors (SPF): several works have shown that actual SPFs of real GSHP systems vary significantly (from 1 to 6) also for very similar configurations [8–11]. Consequently, despite their theoretical potential in energy and economic savings, high installation costs and the uncertainty on final performances limit GSHPs attractiveness with respect to alternative technologies.

In these circumstances, the establishment of innovative design and management approaches seems necessary to increase the viability of GSHP technology. The installation design must be the product of the complete view of the building needs, the system for energy production (HP and back-up generators), the distribution system and controls, and the characteristics of the ground source. Both equipment size and control have to be optimized according to proper cost-benefit considerations between operative performances and installation costs.

Several works have already dealt with GSHPs simulation and optimization methods (see, for instance, [6,12–17]). However, these performance calculation methodologies have not yet been employed by professionals at the initial design level. It is worth stressing that, in this work, we consider an "optimal design" the one that identifies the best design solution among current market alternatives. We deal with optimal design parameters (e.g., GHEs number and depth, HPs number and capacity) and control variables (temperature set points, speed of compressor, pumps, and fans) with given equipment characteristics. In other words, we do not deal with any technological development.

Herein, the following subjects will be discussed: general behavior of the ground source with regard to the influence of climatic conditions, soil nature, and thermo-physical properties, extraction of energy and degradation on thermal evolution, GSHP equipment modeling, sizing of BHE field, sizing of GSHP unit and back-up generators, and share of the required thermal load between GSHP and back-up generators (control strategy) during the operational life. In order to clarify and exemplify the proposed methodological approach, we built a simple test case, in which we illustrate how to apply the method and obtain positive effects in terms of savings of primary energy and installation and operative costs with respect to traditional design practices. The interconnected effects of BHE sizing and control strategy will be exposed, as evidence of the need for an all-inclusive methodology

2. The design process as an optimization of operative performances

Design methodologies based on matching a reference thermal power demand under nominal conditions are not suggested for a ground-coupled system. As above-mentioned, the final goal of the design method is not to guarantee that a certain peak thermal power is delivered by the GSHP, but that the energetic and economic performances of the overall system (back-up generators included) are maximized with reasonable installation costs. A proper design is based on two main compromises:
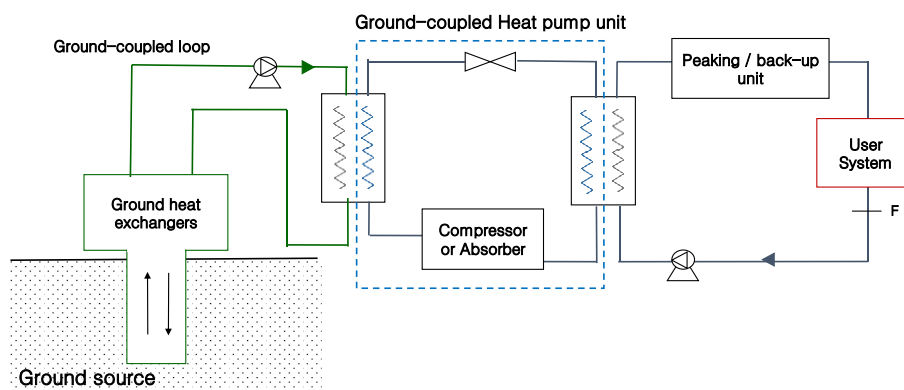
- Depending on local ground source characteristics, GSHP operation alters the initial state of the heat source. In other words, the heat exchanges between soil and GHEs decrease/increase the source temperature, reducing the ground-coupled heat pump efficiency and risking several environmental issues (e.g., ground freezing or overheating).
- Installing large size components (e.g., large GHEs surfaces) limits the alteration of the heat source, as we are reducing the "density" of exploitation. However, this strategy increases investment costs without ensuring sufficient economic savings to repay the initial expenditure. This issue has prompted the development of the so-called *hybrid systems*, which share the thermal load among ground source and other back-up technologies in order to reduce size and cost of ground-coupled equipment [1,18].

Willing to highlight the thermodynamic bases of the above-mentioned trade-off, which affects GSHPs optimal design, we built a simple test case, discussed in the following section

2.1. Thermodynamic features of GSHPs operation: an illustrative analytical example
The existence of an optimal level of exploitation of the geothermal source, corresponding to the best synergy among GSHP unit and back-up generators, can be shown by means of a simple test case. In this section, we will investigate the electrical energy use of a vertical GCHP system, depending on the BHEs size and the share of the building thermal load due to the ground source (control strategy). For the sake of simplicity, we do not consider all the necessary elements of a real design, but we deal with a plain analytical model, in order to highlight the main thermodynamic mechanisms that determine a minimum value of energy consumption.

As already mentioned, GSHPs involve different subsystems, viz. ground reservoir, ground heat exchangers (i.e. vertical BHEs), ground-coupled loop and connecting ductwork, HP unit, back-up generators, and building end-user loop or destination thermal source (see Fig. 1). Each of them operates in strict connection with the others, creating a reciprocal influence on their own performance. Therefore, to apply the proposed design methodology, based on performance simulation, we have to employ a comprehensive set of equations, including at least the physical models of each element involved in the energy conversion process, namely: GSHP unit and back-ups, BHE field, and ground source.

Figure 1. Scheme of the model subsystems.

In this example, we considered a heating system with a GSHP unit and an air heat pump (AHP) as back-up. Thermal performances of the two generators were calculated assuming a constant second-law efficiency value, η^{II}, for each generator (Eqs. 2.a and 2.b): in other words, we took into account only the effects of the different temperature evolution of the two sources. The ground temperature was evaluated by means of the infinite-line source model [19] and the time-superposition technique (i.e.

Duhamel's principle, Eq. 2.e). For sake of simplicity, both external air temperature and building load profiles were assumed sinusoidal (Eqs. 2.c and 2.d).

Similarly to [6], we used a coefficient p_l to represent the fraction of the building heating load delivered by the geothermal heat pump (Eq. 2.g). As above-mentioned, we aimed at analyzing the overall system performance depending on the different load share between air and ground systems: to do that, we performed a sensibility analysis of the overall system energy use (Eq. 1) at various p_l values. In this simple example, p_l was assumed as constant over the system lifetime, leaving a more accurate treatment of the problem to the next section. All the input parameters, including the thermo-physical properties of the soil, are reported in Table 1.

The total electrical energy use, E_{in} (Wh), of the two heat pumps after a period of time τ (yr) was calculated as:

$$E_{in} = \int_0^\tau L(t) \left(\frac{p_l}{COP_{GSHP}(t)} + \frac{1-p_l}{COP_{AHP}(t)} \right) dt \qquad (1)$$

where:

$$COP_{GSHP}(t) = \frac{T_l}{T_l - T_g(t)} \eta_{GSHP}^{II} \qquad (2.a) \qquad \text{(GSHP unit performance)}$$

$$COP_{AHP}(t) = \frac{T_l}{T_l - T_a(t)} \eta_{AHP}^{II} \qquad (2.b) \qquad \text{(Back-up performance)}$$

$$L(t) = \max\left[A_l \cos(2\pi / \omega \cdot t); 0 \right] \qquad (2.c) \qquad \text{(Building thermal load profile)}$$

$$T_a(t) = \langle T_a \rangle - A_a \cos(2\pi / \omega \cdot t) \qquad (2.d) \qquad \text{(External air temperature profile)}$$

$$T_g(t) = T_g^0 + \int_0^\tau W(t-\beta) \frac{d\dot{q}}{dt}(\beta) d\beta \qquad (2.e) \qquad \text{(Ground temperature evolution)}$$

$$W(t) = \frac{1}{2\pi\lambda_g} \int_{\frac{R_{BHE}}{2\sqrt{\alpha_g t}}}^{+\infty} \frac{e^{-\beta^2}}{\beta} d\beta \qquad (2.f) \qquad \text{(Infinite line source model)}$$

$$\dot{q}(t) = \frac{p_l L(t)}{N_{BHE} H} \left(\frac{COP_{GSHP}(t) - 1}{COP_{GSHP}(t)} \right) \qquad (2.g) \qquad \text{(Energy balance of the BHE field)}$$

The set of Eqs. 1 and 2 was solved numerically adopting a time-step of 700 h. The latter value was investigated through a convergence analysis on final E_{in} value: it was seen that a shorter time step provides the very same results in spite of higher computational costs. We can conclude that, for the selected α_g and R_{BHE}, monthly-averaged values of simulation variables (e.g., T_g and COP_{GSHP}) do not differ significantly from an actual integration of the instantaneous values.

The total electric energy consumption after 20 years of operational life is shown in Fig. 2, as a function of p_l and BHEs number. In particular, we used a dimensionless efficiency parameter (ε) to normalize and compare the electric energy consumption of the different cases. In the present work, ε reads:
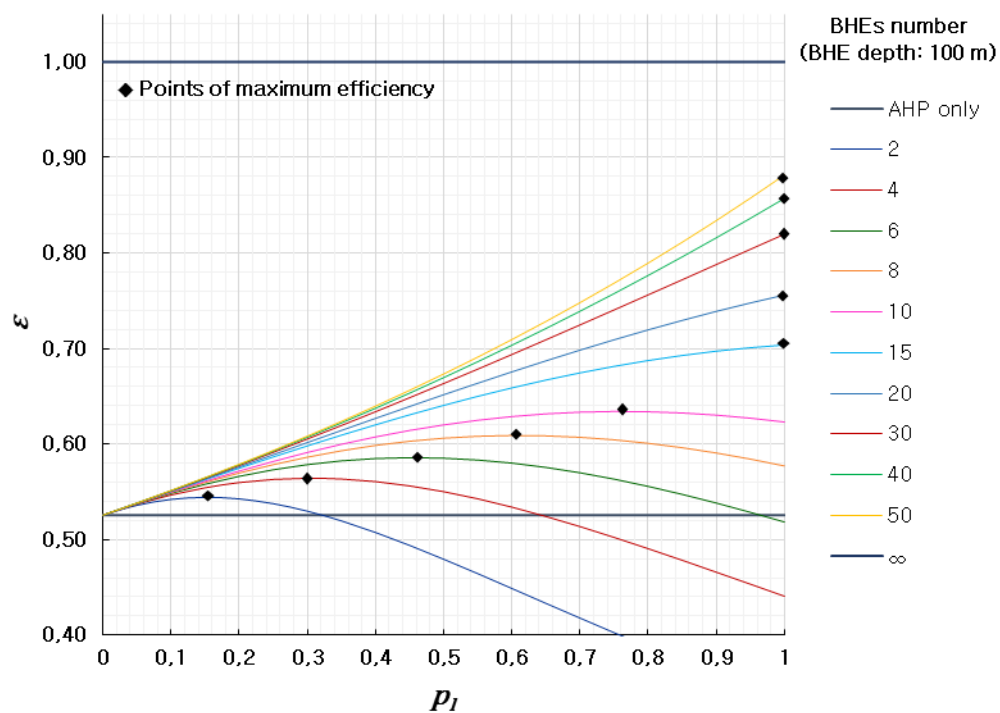
$$\varepsilon = \frac{E_{in}^*}{E_{in}} \qquad (3)$$

where E_{in} is the actual electric energy use calculated through Eqs. 1 and 2 and E_{in}^* is the electric energy use of an ideal case, in which the only GSHP unit is used (i.e., $p_l = 1$) and the ground

temperature remains always constant (i.e., N_{BHE} approaches infinity). In other words, ε evaluates the gap between the actual performance and the theoretical minimum energy consumption (i.e. $\varepsilon = 1$). This latter concept, also known as "task efficiency", has already been discussed and applied in a previous work [20].

Table 1. Input parameters applied to the set of Eqs. 1 and 2.

Parameter	Value	Unit
λ_g	1.7	W/(m·K)
α_g	0.68	mm^2/s
R_{BHE}	7.5	cm
H	100	m
$\langle T_a \rangle$	15	°C
A_a	15	°C
$2\pi / \omega$	1	yr
A_l	20	kW
η_{GSHP}^{II}	0.55	-
η_{AHP}^{II}	0.45	-
τ	20	yr
T_l	45	°C
T_g^0	15	°C

Figure 2. Dimensionless efficiency parameter (ε) as a function of the building load share provided by the GSHP unit p_l and BHEs number.

The results in Fig. 2 show how the ground source is not always convenient with respect to air. Nonetheless, an optimal share of the building load between the two sources can be found. For a given BHEs total depth, the minimum energy use is the result of an optimal compromise between two impairing effects:

- at high p_l, the soil temperature at the borehole surface decreases and can even become lower than air temperature (e.g., when p_l is greater than 0.65 and 4 BHEs are used);

- at low p_l, we are not fully exploiting the ground thermal storage.

We note also that points of maximum efficiency are quite insensitive to a small change in value, which makes the implementation of the control system easier. Regarding BHEs size, we can observe how maximum ε points monotonically increase with borehole number, as a consequence of a reduced alteration of the ground temperature; however, energy savings show a saturation trend, hinting that an oversized system is not going to be cost effective.

The conclusions of this small example confirm our previous considerations for a proper design of GSHP systems. In particular, energetic synergy between ground and back-up technologies can be optimized according to the local external climate, building thermal load, BHEs depth and soil thermo-physical properties. Besides, boreholes number and depth have to be chosen as the optimal tradeoff between savings in operative costs and installation investment.

In this simple test case, we employed a single coefficient η^{II} to take into account all the characteristics and the intrinsic efficiencies of the technological equipment: more complex models are needed to analyze the optimal design of real systems (see section 3). In particular, performance under partial loads strongly depends on the generator capacity, operational limits of components, and on the energy required for the auxiliary systems (e.g., pumping energy in the ground-coupled loop). Besides, significant improvements should be obtained by introducing a time-dependent control law for p_l within the optimization process. To achieve that, a comprehensive physical model of the overall system (including both thermodynamic and technological aspects) has been developed, together with the mathematical formulation of the optimal-design problem (see section 4) and a suitable solution strategy, described in section 5.

3. GSHP model definition and discussion

The previous considerations highlight how the GSHP design process is not a simple calculation of the size of every component, but it consists in a comprehensive procedure based on the evaluation of the overall performance of the system during its operational life and cost-benefit considerations. In other words, the final goal of the designer(s) should be aimed at identifying the best tradeoff among system performance and installation costs.

Current design methodologies do not provide an explicit focus on the optimal share of load delivered by the GSHP with respect to the total thermal need of the building; moreover, the criteria for selecting GHP capacity is not clearly established [5,6].

The optimization of generators capacity and GHEs dimension is a key step for increasing GSHP viability/diffusion. In particular, we are interested in those formulations aimed at investigating the operative performance of the project through the set-up of a physical-based model. This optimization approach is named *"simulation-based optimization procedure"*, as we are dealing with an optimization problem based on the simulation of the operative behavior of the analyzed system [21]. It follows that GSHP design can be seen as an optimization process, in which both design and control variables have to be concurrently evaluated according to their impact on final system performances. With the proposed approach, feasibility study, sizing process, performance analysis and design optimization converge into the very same activity.

As above-mentioned, GSHPs are made of several subsystems (see Fig. 1), with ground source, ground heat exchangers (GHEs) and ground-coupled heat pump unit(s) being the key ones. For each of them, we will discuss the main physical phenomena governing their energetic behavior and a tailored

simulation model will be proposed. These expressions will be finally coupled to a full set of equations to simulate the operative performances of the overall GSHP system (section 3.4).

3.1. External source

The external source of a HP system may be air, surface water, deep water, or ground. The main appeal of the ground source is its rather constant temperature at a certain depth, which should ensure, at least in theory, higher performances with respect to the outdoor air. However, it is worth recalling that transient heat conduction is the main heat transfer mechanism between the ground and GHEs (especially in dry soils); consequently, at equal heat exchangers dimension, the system capacity is lower than air or water systems where the mechanism of heat transfer between the heat source and the heat pump heat exchangers is forced convection (typical thermal conductivity and diffusivity are generally low, varying within a range of $1 - 3$ W/(m K) and $10^{-7} - 10^{-6}$ m^2/s, respectively [1–4]).

Once convection places a role, mass transfer is the main responsible of the heat exchange, jointly with the fluid thermo-physical properties as grouped in Prandtl and Reynolds numbers. The heat transfer coefficient is given by correlations of the form:

$$Nu = C \, \mathrm{Pr}^m \, \mathrm{Re}^n \tag{4}$$

Even if we suppose this correlation (same C, m, and n) to hold both for air and water, for a given heat exchanger geometry, at the same Reynolds number, we get:

$$\frac{Nu_{Air}}{Nu_{Water}} = \left(\frac{\mathrm{Pr}_{Air}}{\mathrm{Pr}_{Water}} \right)^m \tag{5}$$

with the ratio between the two Prandtl numbers roughly around 0.1. In addition, the type of heat exchanger used for air is quite different from that used for water (e.g., cross flow battery for air and plate for water). As very well known, water is a more suitable source than air for two main reasons: it is more effective in terms of heat transfer performances (Eq. 5) and its temperature is higher/lower, on an average basis, and much more constant, thus allowing for a better SCOP and SEER.

With regard to ground source, it is worth stressing that the characterization of the ground source can be divided into two different steps. The first one concerns the characterization of the initial/undisturbed state of the source medium in terms of temperature distribution (varying with time and depth); the second one deals with the evaluation of the ground thermal response when the coupled GSHP system operates.

3.1.1. Characterization of the ground source.

Soil temperature is determined both by the energy exchanges with the above air, which are affected by average outdoor air temperature, solar radiation and surface cover, evapo-transpiration of the soil (both connected to the soil moisture content and surface botany) and those from the surrounding ground [22]. Grossly, we can divide the area affecting the heat transfer to the geothermal exchanger into two main regions: the upper zone, mainly influenced by the climatic conditions above the ground level, and a lower one unaffected by them, but only by the surrounding ground thermo-physical properties.

The typical model to reproduce the underground temperature evolution in undisturbed conditions is the transient heat conduction in a semi-infinite solid. This temperature distribution versus time t and depth z can be also calculated, for instance, by using the model proposed by Labs [23]:

$$T_g(z,t) = \langle T_{air} \rangle - A_s \exp\left(- z \cdot \sqrt{\frac{\pi}{365 \alpha_g}} \right) \cos\left[\frac{2\pi}{365} \left(t - t_0 - \frac{z}{2} \cdot \sqrt{\frac{365}{\pi \alpha_g}} \right) \right] \tag{6}$$

where α_g is the average annual (apparent) thermal diffusivity of undisturbed ground, m²/s; t_0 is the phase of air temperature wave, days; $\langle T_{air} \rangle$ is the average annual air temperature, °C; z is the depth, m; A_s is the amplitude of annual average air temperature wave (i.e. $T(0,t) = A_s \cos\left[2\pi/365 (t - t_0) \right]$), °C. [24,25] take also into account surface conditions through a proper coefficient k_v, which depends on surface vegetation and other correction constants. They adopt the following formula:

$$T_g(z,t) = \left(\langle T_{air} \rangle \pm \Delta T_m \right) - 1,07 \cdot k_v A_s \exp(-0,00031552 z a_g^{-0.5}) \cdot \cos\left[\frac{2\pi}{365}\left(t - t_0 - 0,018335 z a_g^{-0.5} \right) \right]$$

(7)

with the same meaning of symbols. According to Eqs. 6 and 7, it is possible to define a *thermal penetration depth* $d_p \approx \sqrt{2\alpha_g / \omega}$ as the depth at which the amplitude of the temperature oscillation, A_s, decreases of a factor equal to $1/e$. Typical d_p values vary from 1 to 10 meters for annual oscillations.

In addition, during the design process, effective thermo-physical properties of the ground have to be determined. For a preliminary feasibility study, one can use reference values or data from previous nearby projects; however, *in-situ* evaluation methods are generally suggested [1,2]. The main site-investigation technique for closed-loop systems is the thermal response test (TRT), which is able to determine the effective thermal conductivity and diffusivity of the soil, together with the so-called "*borehole thermal resistance*" (see section 3.2). Reference standards, procedures and recommendations for TRTs have been published in literature and engineering handbooks [1,2,26].

3.1.2. Evaluation of ground response during GSHP operation
A first conclusion of the previous discussion is that the effect of the external climate does not propagate below, say, 10 meters from the surface, depending on many factors like: nature of soil (thermo-physical properties), amount of rainfalls and solar radiation, type of soil cover surface, etc. Actually, [27] proved to exert some influence on the outlet temperature of the heat exchanging fluid, also for boreholes 55 m deep, however, since the typical BHEs depth is about 100 m, the temperature oscillation on the top of the borehole is assumed to be negligible on the global heat transfer process during the GSHPs operation [28,29].

Many works present methods for the evaluation of ground temperature evolution at the BHEs surface in a *purely conductive medium* [19,29,30] or in *saturated porous medium* [30–32]. For the sake of simplicity, let us refer to the former case, where Fourier number controls the entire process. According to [6,31], axial effects become significant at very long times (about 1 year); hence, in multi-year simulations, the finite line-source (FLS) [19] model seems the most appropriate one, at least among analytical formulations. The latter reads:

$$\Theta_g(x,t) = \frac{1}{4\pi} \int_0^1 \left[\frac{1}{d/L} erfc\left(\frac{d/L}{2\sqrt{Fo_H}} \right) - \frac{1}{d'/L} erfc\left(\frac{d'/L}{2\sqrt{Fo_H}} \right) \right] dH'$$

(8)

$$Fo_H = \frac{\alpha_g t}{H^2} \qquad R = \frac{r}{H} \qquad Z = \frac{z}{H} \qquad \Theta_g = \frac{\left(T_g - T_g^0\right)\lambda_g}{\dot{q}_{BHE}}$$

$$d/L = \sqrt{R^2 + \left(Z - H'\right)^2} \qquad d'/L = \sqrt{R^2 + \left(Z + H'\right)^2}$$

where H is the borehole depth, m; $\dot{q}_{BHE}$ is the linear heat flux at the BHE surface, W/m.

As heat transfer $\dot{q}_{BHE}$ is not stationary during the operational time, time superposition can be applied to calculate the actual temperature distribution. If we can assume that the radial dimension of the BHE is negligible compared to the size of the geothermal field, also space superposition can be applied to evaluate interferences among the BHEs.

3.2. Ground-coupled borehole heat exchangers model

We propose to analyze the boreholes through the classical heat exchanger theory (see, for instance, [33]). In such a way, we want to ensure that the heat transferred at the evaporator/condenser is coherent with the temperature variation of the fluid in the BHE field. This model can be easily coupled to another concept, widely used in the analysis of BHEs: the *borehole thermal resistance* $\left(R_b\right)$ defined as [34]:

$$R_b = \frac{\bar{T}_w - T_g}{\dot{q}}$$

(9)

where:
- $\bar{T}_w$ is the average temperature of the fluid in the U-loops of the BHE (°C);
- $\dot{q}$ is the linear heat flow through the BHE perimeter in steady-state conditions (W/m).

Following this definition, R_b is a stationary parameter that relates the mean fluid temperature to the BHE average boundary temperature: dynamic and axial effects are thus neglected. Furthermore, R_b conventionally refers to the BHE depth and not to the length of the ducts. Reference formulas to evaluate R_b can be found in [34,35].

We propose to evaluate the fluid outlet temperature from the BHEs through the classical effectiveness method [33] for heat exchanger analysis. Eqs. 10.a and 10.b represent the energy balance and the heat transfer equation for a single BHE

$$\dot{m}_w c_w \left(T_{w,in} - T_{w,out}\right) = \dot{q}H = \dot{Q}_g$$

(10.a)

$$\dot{Q} = KA\left(\bar{T}_w - T_g\right)$$

(10.b)

Combining Eqs. 10.a and 10.b, we obtain:

$$\frac{H}{R_b} = KA$$

(11)

This simple relation illustrates the meaning of R_b within the heat exchanger theory. Again, we stress that the characteristic length is the BHE depth H_{BHE} and not the length of the ducts buried in the borehole. The arrangement and the number of pipes only affect the value of R_b.

Since the borehole surface temperature T_g is assumed to be uniform and constant during each time step, we can write the expression of heat transfer effectiveness, ε_{BHE}, as [33]:

$$\varepsilon_{BHE} = \frac{T_{w,in} - T_{w,out}}{T_{w,in} - T_{w,out}} = 1 - \exp\left(-NTU\right)$$

(12)

where $NTU = \dfrac{KA}{\dot{m}_w c_w}$ is named "Number of Transfer Units". Other ε_{BHE} expressions for BHEs are described in [35]. Similar correlations for energy piles are currently undergoing a research activity through a regression analysis of the results of several transient FEM simulations, in order to identify

the most relevant parameters, characteristic lengths, and time scales [36]. Once $T_{w,out}$ is obtained for every BHE, the overall energy transferred to the ground during the time step Δt is:

$$Q_g = \sum_{N_{BHE}} \dot{m}_w c_w \left(T_{w,in} - T_{w,out} \right) \Delta t \tag{13}$$

3.3. Ground-coupled heat pump unit model and heat generators

Different approaches have been proposed to simulate the heat pump performance as a function of the operative conditions. For design purposes, *black-box* based models seem to be the most effective ones, in terms of quality of result and implementation efforts. The overall performance of the heat pump unit is predicted by appropriate interpolation of manufacturers' data at the operative sources temperature and capacity ratio (CR) [7,12,37,38]. In this work, we refer to CR as the useful thermal output of the HP in heating or cooling mode, divided by its maximum capacity, when operating at the actual temperatures of the thermal source [39]. We stress that CR refers to the maximum thermal output of a HP unit at given source conditions; it should not be confused with the load ratio, which is the ratio between seasonal average thermal power demand and building peak load. The effect of CR on operative coefficients of performance depends on the choice of the HP unit size and on its modulation capability in response to the evolution of the thermal load.

3.4. The full set of equations describing the overall system

In this section, we illustrate the proposed full set of equations necessary for simulating the behavior of a vertical GCHP system during its operational life. The latter one collects the above-discussed modeling formulations and it will be employed in the optimization procedure illustrated in section 4.

Rigorous dynamic simulation methods would need dynamic models of each component and all parameters and boundary conditions should be available with sufficient accuracy at a much shorter time scale. Such a level of detail is often not available, especially at the earliest stages of a design process. Moreover, according to the aims of this work, we need to avoid complex formulations that would be impractical within the above-mentioned optimization procedure.

Consequentially, we decided to adopt a quasi-steady-state method, calculating energy balances over a sufficiently long time step, Δt (i.e. a month), which allows us to neglect internal energy variations (except for the ground) and employ simpler models. All the time-dependent inputs and unknown variables of the system have to be considered as constant for the entire duration of the time step, assuming average values. In section 2, we verified the applicability of this strategy: indeed, we observed that there is a negligible deviation between a monthly-average evaluation and the actual ground temperature evolution.

The choice of a monthly time scale seems appropriate also for BHEs modeling. As above-mentioned, evaluating the heat transfer process within the boreholes through a stationary equivalent thermal resistance (i.e. R_b) is a widely accepted technique (see, for instance, [6,12,34]), as BHEs heat capacity can be neglected after a sufficiently long period of continuous operation [40,41]).

The two coefficients, f_H and f_C, were introduced to represent the control strategy of the system. They are defined as the heat delivered/removed to/from the end-user loop divided by the total energy load during a given time step. In other words, selecting a $f_{H/C}$ value corresponds to change the thermal load at the ground source. Such a control can be achieved, for instance, by varying the CR value of the heat pump unit through its capacity control system. Although their similarity, we stress that f_H and f_C do not represent the integral-average of the instantaneous share of the building thermal load delivered by the ground heat pump (i.e. p_l in Eqs. 2).

The full set of equations reads:

$$Q_{eva/cond} = Q_g \qquad\qquad\qquad\qquad [14.a]$$

$$Q_{eva/cond} = \dot{m}_w c_w \left| T_{w,in} - T_{w,out} \right| \Delta t \qquad [14.b]$$ Energy balance for the fluid of the ground-loop at the evaporator/condenser section

$$Q_{eva/cond} = F\left(T_{w,out}; T_l; CR\right) \qquad [14.c]$$ GHP unit performance

$$Q_g = H\left(\dot{m}_w; c_w; T_{w,in}; T_g; R_b; R_{BHE}; H; N_{BHE}\right) \qquad [14.d]$$ BHE field

$$T_g = S\left(T_g^0; Fo_g; Pe_g; Q_g; R_{BHE}; H; N_{BHE}; D_{BHE}\right) \qquad [14.e]$$ Ground source

$$f_H = \frac{Q_{eva}}{Q_l}\left(\frac{COP}{COP-1}\right) \quad f_C = \frac{Q_{cond}}{Q_l}\left(\frac{EER}{EER+1}\right) \qquad [14.f]$$ Shares of the building load for heating and cooling at GSHP unit

$$Q_{bk} = B\left(T_l; CR_{bk}\right) \qquad [14.g]$$ Back-up generator(s)

$$Q_{bk} = \left(1 - f_{H/C}\right)Q_l \qquad [14.h]$$

Eqs. 14 include both equipment models (Eqs. 14.b-14.g) and two auxiliary equations (Eq. 14.a and Eq. 4.h). The former imposes that heat the exchanged between the BHE field and the ground (Q_g) is equal to the heat transferred in the evaporator/condenser ($Q_{eva/cond}$), in accordance with the quasi-steady-state method); the latter imposes that the building thermal load (Q_l), up to the end-user distribution system, is given by the sum of the thermal energies delivered/removed in heating/cooling mode by GSHP unit and back-up generators.

For the sake of generality, the equations are voluntarily presented in an implicit form, but possible expressions for functions F, H, S and B have been provided in sections 3.1, 3.2 and 3.3. Anyway, any proper formulation can be used accordingly to the specific design project (e.g., Eqs. 2).

4. Statement of the optimal-design problem

As already mentioned, we propose to simulate GSHP systems by means of a quasi-steady-state approach; therefore, our goal is to minimize the finite series of a proper "*return/cost function*" over the operative time under consideration. The decisions about the management of the system (f_H and f_C values) have to be made at each stage of the simulation (i.e. at each time step). This kind of optimization problems are typically called *multistage decision problems* [42]. The mathematical formulation of the problem can be stated as:

find the control sequence $\mathbf{U} = \left\{\mathbf{u}^0, \mathbf{u}^1, \mathbf{u}^2, \mathbf{u}^3 \ldots \mathbf{u}^N\right\}$

that minimizes:

$$J(\mathbf{U}) = \sum_{n=1}^{N} R_{H/C}\left(\mathbf{x}^n, \mathbf{u}^n, n\Delta t\right) \qquad (15.a)$$

subject to:

$$\mathbf{x}^{n+1} = f\left(\mathbf{x}^n, \mathbf{u}^n, n\Delta t\right) \qquad (15.b)$$

$$\mathbf{h}\left(\mathbf{x}^n\right) = 0 \qquad\qquad \mathbf{g}\left(\mathbf{x}^n\right) \le 0 \qquad (15.c)$$

$$u_{p,\min} \le u_p \le u_{p,\max} \qquad (15.d)$$

where:

- $\mathbf{U} = \left\{ \mathbf{u}^0, \mathbf{u}^1, \mathbf{u}^2, \mathbf{u}^3 \ldots \mathbf{u}^N \right\}$ is the set containing all the $\mathbf{u}^n$;

- $\mathbf{u}^0 = \left\{ u_1^0, u_2^0, u_3^0 \ldots u_k^0 \right\}$ is the vector of design variables; the superscript *0* indicates that these variables are not time-dependent;

- $\mathbf{u}^n = \left\{ u_1^n, u_2^n, u_3^n \ldots u_k^n \right\}$ is the vector of control variables at the *n-th* stage;

- $\mathbf{x}^n = \left\{ x_1^n, x_2^n, x_3^n \ldots x_p^n \right\}$ is the vector of state variables at the *n-th* stage;

- $J(\mathbf{U})$ is the so-called performance index;

- $R_{H/C}\left(\mathbf{x}^n, \mathbf{u}^n, n\Delta t \right)$ is the so-called *"return/cost function"*; it represents the contribution of the *n-th* stage to the total performance index;

- $f\left(\mathbf{x}^n, \mathbf{u}^n, n\Delta t \right)$ is the mathematical model of the system, relating the state variables of a stage to the control variables and the state variables of the previous stage (set of Eqs. 14);

- $\mathbf{h}\left(\mathbf{x}^n \right)$ is the vector of the equality constraints;

- $\mathbf{g}\left(\mathbf{x}^n \right)$ is the vector of the inequality constraints.

4.1. Return function

The objective function can be chosen both in thermodynamic, energetic, economical, and thermo-economical terms; multi-objective formulations can be used, too. If we are interested in primary energy uses, we can express $R_{H/C}\left(\mathbf{x}^n, \mathbf{u}^n, n\Delta t \right)$ as:

$$R_H\left(\mathbf{x}^n, \mathbf{u}^n, n\Delta t \right) = \frac{Q_{cond}}{COP_{GSHP}} f_{ep,GSHP} + \frac{Q_{bk}}{\eta_{bk}} f_{ep,bk} \quad \text{(heating mode)} \qquad (16.a)$$

$$R_C\left(\mathbf{x}^n, \mathbf{u}^n, n\Delta t \right) = \frac{Q_{eva}}{EER_{GSHP}} f_{ep,GSHP} + \frac{Q_{bk}}{EER_{bk}} f_{ep,bk} \quad \text{(cooling mode)} \qquad (16.b)$$

For an economically-driven optimization, the return function could be the net present value (NPV), defined as the net cash flow at a given time step, resulting from the algebraic sum of present values of: incentives, savings on operational costs with respect to benchmark technologies, installation costs, and maintenance costs. The operational costs of the energy production in heating $\left(C_H \right)$ and cooling $\left(C_C \right)$ modes are given by:

$$C_H = \frac{Q_{cond}}{COP_{GSHP}} c_{e,GHP} + \frac{Q_{bk}}{\eta_{bk}} c_{e,bk} \quad \text{(heating mode)} \qquad (17.a)$$

$$C_C = \frac{Q_{eva}}{EER_{GSHP}} c_{e,GSHP} + \frac{Q_{bk}}{EER_{bk}} c_{e,bk} \quad \text{(cooling mode)} \qquad (17.b)$$

where $c_{e,GSHP}$ and $c_{e,bk}$ are the unitary costs (possibly undergoing inflation) of the energy needed for running, respectively, GSHP unit and back-up generators. As a matter of fact, the optimal sequence of $\mathbf{u}^n$ depends on the choice of the specific energetic or economic goal; this is particularly true when the ratio between $f_{ep,GSHP}$ and $f_{ep,bk}$ is different from the one between $c_{e,GSHP}$ and $c_{e,bk}$.

4.2. Control variables

The overall optimization problem involves two types of control variables: the variables related to the sizing of the system (design variables) and the ones related to the control strategy. The main control strategy variable is the capacity ratio (CR) of the heat pump unit. At each time step, we can change the thermal output of the GSHP unit and, consequently, the share of the building load delivered by the GSHP system $(f_{H/C})$. At some stages, we can also decide to turn off the heat pump and match the thermal load only with the back-up generators. Hence, the optimization solution will also include the schedule of the HP operation.

The design variables $(\mathbf{u}^0)$ are also "*control variables*" in the terminology common to optimization theory, but they are not time dependent. This notwithstanding, they have to be evaluated within the optimization process. In addition, the design variables can be divided into two sub-groups: the *continuous variables* and the *discrete variables* (also called *design parameters*). The former can assume every value within the allowed range, the latter are instead limited to certain discrete values, such as integer numbers; therefore, different and specific optimization techniques have to be applied. Besides, the possible GSHP system layouts and the size of the components have to be considered as *discrete parameters* for the optimization process, even if they cannot be associated to a numerical variable. The main variables, parameters and constraints for the optimization of GSHP systems are summarized in Table 2.

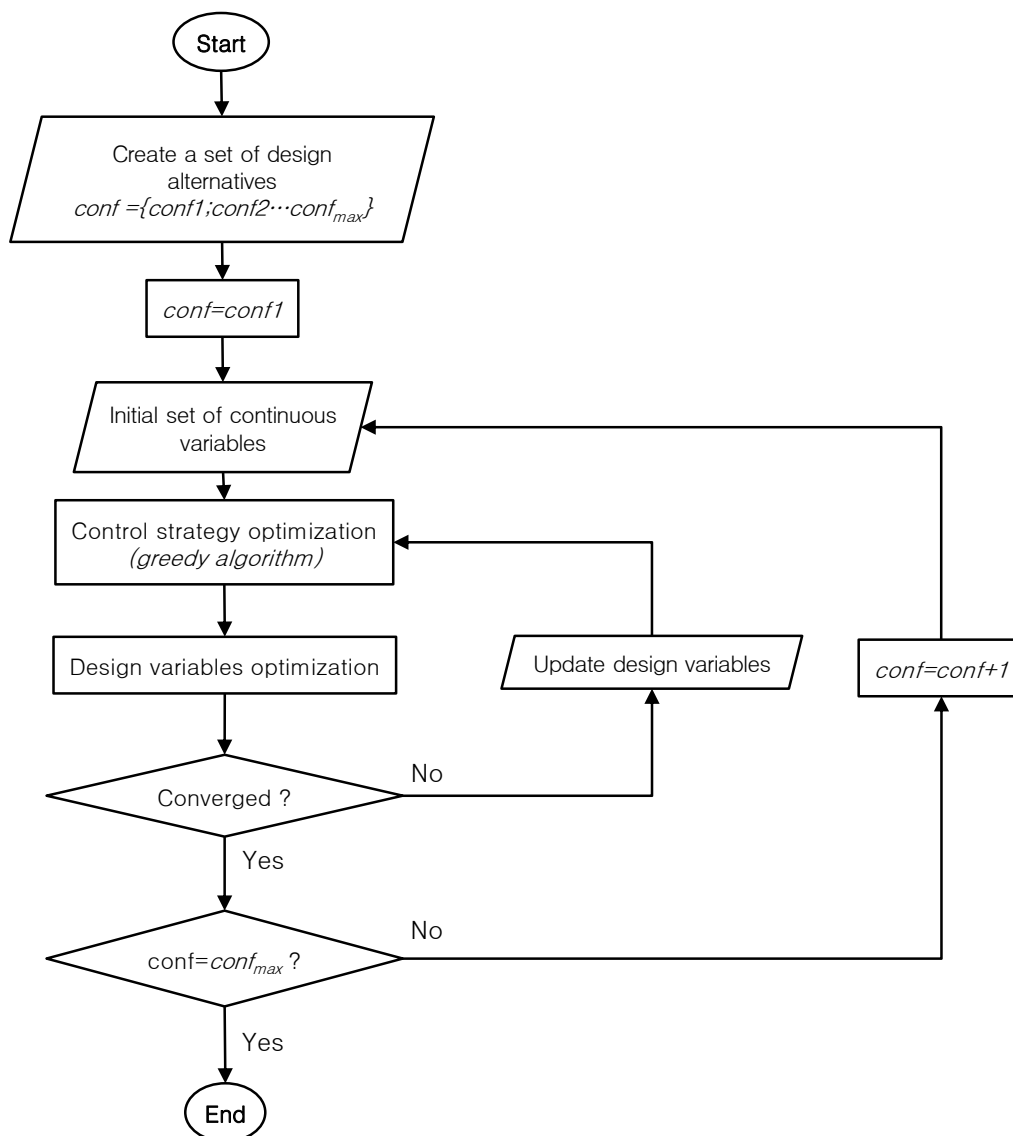**Table 2.** List of variables considered in the proposed optimization procedure.

	Symbol	Name and comments	Lower and upper bounds
Control-strategy variables			
	CR	Capacity Ratio	*Min:* Minimum value specified by manufacturer *Max:* 1
Design variables			
Continuous variables	H	Borehole depth	*Min:* 0 *Max:* 100-150 m (suggested values)
	$\dot{m}_w$	Mass flow rate of ground-coupled loop	*Min:* Hydraulic considerations: e.g. the flow rate must be large enough to guarantee a fluid velocity higher than 0.3 m/s or a turbulent regime within the ducts (suggested values).
Discrete variable or parameters	N_{BHE}	Number of BHEs	Integer value
	$F(\ )$	Function describing HP capacity and performance (Eq. 14.c)	
	$H(\ )$	Function describing BHEs design (Eq. 14.d)	
	$B(\ )$	Function describing back-ups capacity and performance (Eq. 14.g)	

The proposed optimization methodology starts creating a set of possible design alternatives (indicated in Fig. 3 as "*conf*"). As said in section 4.2, the system layout is considered as a discrete variable, thus all the reasonable combinations in terms of BHEs number, generators models and equipment arrangement are included in this "*conf*" set. The latter is investigated through an

"*exhaustive enumeration method*"; in other words, we evaluate the $J(\mathbf{U})$ value of each configuration to find the optimal one. In this context, a physical insight of the problem can significantly reduce the number of tests, saving computational time and efforts.

We investigate continuous design variables and control strategy for each *"conf"* element by an iterative procedure:

- at first, design variables are set to an initial set of guesses and the corresponding control strategy is optimized;
- then, using the previously-found control strategy, design variables are optimized;
- finally, the procedure is iterated, till a convergence criterion is satisfied.

Figure 5. Suggested algorithm for the resolution of the optimization problem.

The optimal control in multistage decision problems is generally determined by means of dynamic programming (DP) techniques (i.e. backward induction). This latter, when applicable, reduces the dimension of the solution space, decomposing the original problem in a sequence of N single decision problems [42]. However, in ground source applications, DP cannot be applied, as both next state

vector, $\mathbf{x}^{n+1}$, and return function, $R\left(\mathbf{x}^n,\mathbf{u}^n,n\Delta t\right)$, depend on all previous steps. Indeed, as above said, the value of the ground temperature is the result of the entire history of heat exchange. The other common strategy is the employment of the so-called evolutionary algorithms (i.e. genetic algorithms), however, the large dimension of the solution space (corresponding to the sum of time steps and design variables number) makes their application impractical and time-consuming.

A *greedy algorithm* is a rapid and straightforward alternative: this resolution strategy finds the optimal choice at each individual time step "…*in the hope that this choice will lead to a globally optimal solution*" [43]. The effectiveness and the limits of this algorithm in GSHP control problems will be discussed in future works, though promising research is still ongoing on the subject.

5. Conclusions and examples of application

In the present work, we proposed an innovative approach to the design of GSHPs, based on energetic/economic optimization on the entire lifecycle of the system. Traditional design methods determine the size of the equipment on the basis of some critical reference conditions; on the contrary, we propose to investigate the possible design alternatives in terms of operative performance. In section 2, we analyzed the main thermodynamics features that affect an optimal design of GSHP systems.

We showed that increasing BHEs number does not always entail a significant increase of system performances: therefore, system oversizing should be avoided. Besides, optimal ground exploitation corresponds to a proper share of the delivered heat among GSHP and back-up system. This optimal share depends on the efficiencies of the involved generation technologies and, for GSHPs, to an optimal thermal level of the ground source.

We propose to evaluate both design and control variables of real GSHPs systems by means of an optimization algorithm, coupled to a comprehensive set of equations representing each subsystem of GSHPs (back-up generators included). The main outputs of the method are: optimal capacity of the ground heat pump unit, optimal number and depth of BHEs, and optimal control strategy.

The proposed set of equations can be promptly adapted to different technological solutions, maintaining the overall structure of the method (e.g., GWHPs, energy piles, solar-assisted HP systems). Besides, the overall approach can be applied in several contexts with different aims: designers can be supported during their professional activity, political authorities can investigate proper assessments and criteria for specific incentives, energy efficiency operators (e.g., energy service companies) can evaluate the investment profitability of any specific GSHP project, industrial operators (e.g., drilling companies and GHP manufacturers) can analyze maximum operative savings to properly decide drilling and equipment prices, researchers can investigate current GSHP systems to seek technological developments and room for improvement. In short, possible applications concern numerous professional, political, economic, and research activities.

For instance, we employed the proposed design methodology in [17], to deal with a typical professional design case: both energetic and economic benefits with respect to traditional design methods were fully illustrated and discussed. In [44], we analyzed a real case study (a primary school located in the north of Italy characterized by high heating loads), to investigate the achievable benefits of possible design alternatives (GS-GAHP and condensing boiler) and different control strategies (lower supply temperature to the end-user loop). In [16], we investigated the optimal BHEs number and the investment profitability as a function of BHEs drilling fees: we showed how an accurate evaluation of maximal GSHPs performances, covering a proper set of benchmark buildings and loads, may help authorities to assess amounts and access criteria for financial incentives, encouraging GSHPs diffusion, but avoiding market distortions or speculations. Finally, in [20], we illustrated as the proposed methodology can be applied to investigate the technological room for improvement of GSHP technology: in other words, we figured out the subsystem on which technological development should be focused, the expected benefits and some hints about a possible strategy for research activities.

Future works will be aimed at improving simulation models and methods, together with optimization algorithms. Besides, we are currently investigating the sensibility of GSHP performances

on a wide range of technological, economic, and environmental factors (e.g., generators technology and efficiencies, thermo-physical properties of the ground, groundwater effects, external air temperature, GHEs number and size, ground-coupled loop design and arrangement, thermal load profile, control strategies, installation costs, and energy prices), in order to investigate the possibility of building reference performance maps relating the mentioned parameters. The final goal is to develop some simplified and straightforward guidelines, in order to extend the practical applicability of the proposed methodology to GSHP operators and stakeholders.

Nomenclature

Symbols and Acronyms

A	Amplitude of a periodic variable
BHE	Borehole heat exchanger
c_e	Energy price per unit (€/Wh)
C	Operational costs (€)
CR	Capacity ratio
D	Dimensionless BHEs distance
d_p	Penetration depth (m)
E_{in}	Energy use (Wh)
E_{in}^*	Energy use of an ideal case, in which the only GSHP unit is used and the ground temperature remains always constant (Wh)
f_{ep}	Primary energy factor
$f_{H/C}$	GSHP share of building load in heating/cooling mode
Fo	Fourier number
GHP	Ground-coupled heat pump unit
H	Borehole depth (m)
$J(\mathbf{U})$	Performance index
K	Overall heat transfer coefficient (W/m²/K)
k_v	Surface vegetation coefficient (Eq. 7)
L	Building thermal load profile (W)
$\dot{m}$	Mass flow rate (kg/s)
N_{BHE}	Number of boreholes
Nu	Nusselt number
Pe	Péclet number
Pr	Prandtl number
p	Fraction of the instantaneous building load delivered by the geothermal heat pump
$\dot{q}$	Heat flow per unit length (W/m)
Q	Thermal energy (Wh)
$\dot{Q}$	Thermal power (W)
R	Dimensionless radius
$R_{H/C}$	Return function in heating or cooling period
R_b	Borehole thermal resistance (m K/W)
R_{BHE}	Borehole radius (m)
Re	Reynolds number
r	Radial coordinate (m)
t_0	Phase of a periodic variable (h)
$\mathbf{U}$	Set containing all the u^n

W	Infinite line source model (Eq. 2.f)
$\mathbf{x}^n$	Vector of state variables
Z	Dimensionless depth
z	Depth (m)

Greek Letters

α	Thermal diffusivity (m²/s)
β	Auxiliary variable
ε	Task efficiency (Eq. 3)
ε_{BHE}	Heat exchanger effectiveness
η	First-law efficiency
η^{II}	Second-law efficiency
Θ	Dimensionless temperature
λ	Thermal conductivity (W/m/K)
τ	Reference operational period (h)
ψ	Exergy efficiency
ω	Angular frequency

Subscripts

AHP	Air HP unit
bk	Back-up generator
C	Cooling mode
$cond$	Condenser
eva	Evaporator
H	Heating mode
g	Ground
in	Inlet/supply
l	Building thermal load
out	Outlet/return
w	Water circulating in the ground-coupled loop

Superscripts

0	Initial time
$*$	Ideal conditions

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