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Field-strength correlators for QCD in a magnetic background

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We present the results of an exploratory study (by means of Monte Carlo simulations on the lattice) of the properties of the gauge-invariant two-point correlation functions of the gauge-field strengths for $N_f = 2$ QCD at zero temperature and in the presence of a magnetic background field: the analysis provides evidence for the emergence of anisotropies in the nonperturbative part of the correlators and for an increase of the gluon condensate as a function of the external magnetic field.

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1. Introduction

In the last few years, an increasing interest has arisen in the scientific community about the study of the strong interactions in the presence of strong magnetic fields (see, e.g., Ref. [1]).

From a phenomenological point of view, the physics of some compact astrophysical objects, like magnetars, of noncentral heavy ion collisions and of the early Universe involve the properties of quarks and gluons in the presence of magnetic backgrounds going from 10^{10} Tesla up to 10^{16} Tesla (i.e., up to $|e|B \sim 1$ GeV^2).

From a purely theoretical point of view, one emerging feature is that gluon fields, even if not directly coupled to electromagnetic fields, can be significantly affected by them: effective QED-QCD interactions, induced by quark loop contributions, can be important, because of the nonperturbative nature of the theory (see, e.g., Refs. [2, 3, 4, 5, 6]).

It is well known (see Ref. [7] for a complete review on this subject) that many nonperturbative properties of the QCD vacuum can be usefully parametrized in terms of the gauge-invariant two-point field-strength correlators, defined as:

$$\mathscr{D}_{\mu\rho,\nu\sigma}(x) = g^2 \langle \mathrm{Tr}[G_{\mu\rho}(0) S(0,x) G_{\nu\sigma}(x) S^\dagger(0,x)] \rangle, \tag{1.1}$$

where $G_{\mu\rho} = T^a G^a_{\mu\rho}$ is the field-strength tensor (T^a being the $SU(3)$ fundamental generators), and $S(0,x)$ is the parallel transport from 0 to x along a straight line, needed to make the correlators gauge invariant. Such correlators were first considered to take into account the nonuniform distributions of the vacuum condensates and their effects on the levels of the $Q\overline{Q}$ bound states.

Here we present the results of an exploratory lattice study [8] (performed for $N_f = 2$ QCD at zero temperature) of the effects of a magnetic background field on these gluon-field correlators. The analysis is focused on a quantity of phenomenological interest which can be extracted from the correlators, the so-called *gluon condensate*.

2. Field-strength correlators in the absence or presence of external fields

In the vacuum and in the absence of external sources, Lorentz symmetry ($SO(4)$ symmetry in the Euclidean space) implies a simple form for the two-point functions in Eq. (1.1), which can be expressed in terms of two independent scalar functions of x^2, which are usually denoted by $\mathscr{D}(x^2)$ and $\mathscr{D}_1(x^2)$ (see Ref. [7] and references therein):

$$\begin{aligned}\mathscr{D}_{\mu\rho,\nu\sigma}(x) &= (\delta_{\mu\nu}\delta_{\rho\sigma} - \delta_{\mu\sigma}\delta_{\rho\nu}) \left[\mathscr{D}(x^2) + \mathscr{D}_1(x^2)\right] \\ &+ (x_\mu x_\nu \delta_{\rho\sigma} - x_\mu x_\sigma \delta_{\rho\nu} + x_\rho x_\sigma \delta_{\mu\nu} - x_\rho x_\nu \delta_{\mu\sigma}) \frac{\partial \mathscr{D}_1(x^2)}{\partial x^2}.\end{aligned} \tag{2.1}$$

The presence of an external field breaks Lorentz/$SO(4)$ symmetry, so that the most general parametrization is more complex than the one reported in Eq. (2.1). A detailed discussion of this problem can be found in Ref. [8] and will not be reported here.

On the other hand, in our investigation on the lattice we have considered only correlators of the kind $\mathscr{D}_{\mu\nu,\xi}(d) \equiv \mathscr{D}_{\mu\nu,\mu\nu}(x = d\hat{\xi})$, where the two plaquettes are parallel to each other and the separation x is along one ($\hat{\xi}$) of the four basis vectors of the lattice [$\hat{x} = (1,0,0,0)$, $\hat{y} = (0,1,0,0)$, $\hat{z} = (0,0,1,0)$, $\hat{t} = (0,0,0,1)$]. These amount, in general, to 24 different correlation functions.

Without any additional external field, the symmetries of the system group these 24 correlators into two equivalence classes, usually denoted as $\mathscr{D}_{\parallel}$ (when $\xi = \mu$ or $\xi = \nu$) and $\mathscr{D}_{\perp}$ (when $\xi \neq \mu$ and $\xi \neq \nu$), with $\mathscr{D}_{\parallel} = \mathscr{D} + \mathscr{D}_1 + x^2 \frac{\partial \mathscr{D}_1}{\partial x^2}$ and $\mathscr{D}_{\perp} = \mathscr{D} + \mathscr{D}_1$.

In the presence of a constant and uniform magnetic field $\vec{B}$ oriented along the z axis ($\vec{B} = B\hat{z}$), the $SO(4)$ Euclidean symmetry breaks into $SO(2)_{xy} \otimes SO(2)_{zt}$. By virtue of this residual symmetry (which implies two equivalence relations, one between the two *transverse* directions $\hat{x} \sim \hat{y}$ and another between the two *longitudinal* [or: "*parallel*"] directions $\hat{z} \sim \hat{t}$), the 24 correlation functions $\mathscr{D}_{\mu\nu,\xi}$ are grouped into 8 equivalence classes, which can be denoted as:

$$\mathscr{D}_{\parallel}^{tt,t}, \quad \mathscr{D}_{\perp}^{tt,p}, \quad \mathscr{D}_{\parallel}^{tp,t}, \quad \mathscr{D}_{\parallel}^{tp,p}, \quad \mathscr{D}_{\perp}^{tp,t}, \quad \mathscr{D}_{\perp}^{tp,p}, \quad \mathscr{D}_{\perp}^{pp,t}, \quad \mathscr{D}_{\parallel}^{pp,p}, \tag{2.2}$$

where the superscripts t and p stand respectively for the *transverse* $(\hat{x},\hat{y})$ directions and for the "*parallel*" $(\hat{z},\hat{t})$ directions.

In the absence of external field ($B = 0$), the correlators were directly determined by numerical simulations on the lattice in Refs. [9, 10, 11, 12], using the following parametrization vs. the distance d: $\mathscr{D} = A_0 e^{-\mu d} + \frac{a_0}{d^4}$, $\mathscr{D}_1 = A_1 e^{-\mu d} + \frac{a_1}{d^4}$; that is, in terms of $\mathscr{D}_{\parallel}$ and $\mathscr{D}_{\perp}$:

$$\mathscr{D}_{\parallel} = \left[A_0 + A_1\left(1 - \frac{1}{2}\mu d\right)\right] e^{-\mu d} + \frac{a_{\parallel}}{d^4}, \quad \mathscr{D}_{\perp} = (A_0 + A_1)\, e^{-\mu d} + \frac{a_{\perp}}{d^4}, \tag{2.3}$$

where $a_{\parallel} = a_0 - a_1$ and $a_{\perp} = a_0 + a_1$. The terms $\sim 1/d^4$ are of perturbative origin and (according to the *Operator Product Expansion*) are necessary to describe the short-distance behavior of the correlators. The exponential terms represent, instead, the nonperturbative contributions: in particular, the coefficients A_0 and A_1 can be directly linked to the *gluon condensate* of the QCD vacuum (see Eq. (3.3) below).

Inspired by the parametrization (2.3) used in the case $B = 0$, we have used for the eight functions (2.2) in the case $B \neq 0$ the following parametrization:

$$\begin{aligned}
&\mathscr{D}_{\parallel}^{tt,t} = \left[A_0^{tt} + A_1^{tt}\left(1 - \frac{1}{2}\mu^{tt,t} d\right)\right] e^{-\mu^{tt,t} d} + \frac{a_{\parallel}^{tt,t}}{d^4}, \quad \mathscr{D}_{\perp}^{tt,p} = (A_0^{tt} + A_1^{tt}) e^{-\mu^{tt,p} d} + \frac{a_{\perp}^{tt,p}}{d^4}, \\
&\mathscr{D}_{\parallel}^{tp,t} = \left[A_0^{tp} + A_1^{tp}\left(1 - \frac{1}{2}\mu^{tp,t} d\right)\right] e^{-\mu^{tp,t} d} + \frac{a_{\parallel}^{tp,t}}{d^4}, \\
&\mathscr{D}_{\parallel}^{tp,p} = \left[\tilde{A}_0^{tp} + \tilde{A}_1^{tp}\left(1 - \frac{1}{2}\mu^{tp,p} d\right)\right] e^{-\mu^{tp,p} d} + \frac{a_{\parallel}^{tp,p}}{d^4}, \\
&\mathscr{D}_{\perp}^{tp,t} = (A_0^{tp} + A_1^{tp}) e^{-\mu^{tp,t} d} + \frac{a_{\perp}^{tp,t}}{d^4}, \quad \mathscr{D}_{\perp}^{tp,p} = (\tilde{A}_0^{tp} + \tilde{A}_1^{tp}) e^{-\mu^{tp,p} d} + \frac{a_{\perp}^{tp,p}}{d^4}, \\
&\mathscr{D}_{\perp}^{pp,t} = (A_0^{pp} + A_1^{pp}) e^{-\mu^{pp,t} d} + \frac{a_{\perp}^{pp,t}}{d^4}, \\
&\mathscr{D}_{\parallel}^{pp,p} = \left[A_0^{pp} + A_1^{pp}\left(1 - \frac{1}{2}\mu^{pp,p} d\right)\right] e^{-\mu^{pp,p} d} + \frac{a_{\parallel}^{pp,p}}{d^4},
\end{aligned} \tag{2.4}$$

with the constraint $\tilde{A}_0^{tp} + \tilde{A}_1^{tp} = A_0^{tp} + A_1^{tp}$, meaning that, at $d = 0$, the nonperturbative part of the correlation functions $\mathscr{D}^{tp}$ have the same value. The dependence of the various parameters on B is understood and will be discussed in the next section on the basis of the numerical results obtained in Ref. [8] by lattice simulations of $N_f = 2$ QCD (at zero temperature).

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3. Numerical investigation and discussion on the gluon condensate

The correlator $\mathscr{D}_{\mu\nu,\xi}(d) \equiv \mathscr{D}_{\mu\nu,\mu\nu}(x = d\hat{\xi})$ has been discretized through the following lattice observable [9, 10]:

$$\mathscr{D}^L_{\mu\nu,\xi}(d) = \left\langle \mathrm{Tr}\left[\Omega^\dagger_{\mu\nu}(x) S(x, x+d\hat{\xi}) \Omega_{\mu\nu}(x+d\hat{\xi}) S^\dagger(x, x+d\hat{\xi})\right] \right\rangle, \tag{3.1}$$

where $\Omega_{\mu\nu}(x)$ stands for the traceless anti-Hermitian part of the corresponding plaquette, i.e., $\Omega_{\mu\nu} \equiv \frac{1}{2}(\Pi_{\mu\nu} - \Pi^\dagger_{\mu\nu}) - \frac{1}{6}\mathrm{Tr}[\Pi_{\mu\nu} - \Pi^\dagger_{\mu\nu}]\mathbf{I}$. Of course, $\mathscr{D}^L_{\mu\nu,\xi}(d) \to a^4 \mathscr{D}_{\mu\nu,\xi}(d)$ when the lattice spacing $a \to 0$.

We have considered $N_f = 2$ QCD discretized via unimproved rooted *staggered* fermions and the standard plaquette action for the pure-gauge sector. The background magnetic field $\vec{B} = B\hat{z}$ couples to the quark electric charges ($q_u = 2|e|/3$ and $q_d = -|e|/3$, $|e|$ being the elementary charge) and its introduction corresponds to additional $U(1)$ phases entering the elementary parallel transports in the discretized lattice version. Periodicity constraints impose the following condition of quantization on B: $|e|B = 6\pi b/(a^2 L_x L_y)$, $b \in \mathbb{Z}$.

Numerical simulations have been performed on a 24^4 lattice by means of the *Rational Hybrid Monte Carlo* algorithm [13, 14] implemented on GPU cards, with statistics of $O(10^3)$ molecular-dynamics time units for each b, with b ranging from 0 to 18 (corresponding to $0 \le |e|B \le 1.46$ GeV^2). The bare parameters have been set to $\beta = 5.55$ and $am = 0.0125$, corresponding to a lattice spacing $a \simeq 0.125$ fm and to a pseudo-Goldstone pion mass $m_\pi \simeq 480$ MeV.

In order to remove ultraviolet fluctuations, following the previous studies of the gluon-field correlators [9, 10, 11, 12], a *cooling* technique has been used which, acting as a diffusion process, smooths out short-distance fluctuations without touching physics at larger distances: for a correlator at a given distance d, this shows up as an approximate plateau in the dependence of the correlator on the number of cooling steps, whose location defines the value of the correlator.

For each value of $|e|B$, we have fitted the correlators with the parametrization (2.4), including distances in the range $3 \le d/a \le 8$, thus obtaining an estimate for all parameters. From these best fits, it has emerged that the 8 parameters pertaining to the perturbative part of the correlation functions (2.4) satisfy, within the errors, the following equalities:

$$a^{tt,t}_\parallel \simeq a^{tp,t}_\parallel \simeq a^{tp,p}_\parallel \simeq a^{pp,p}_\parallel \equiv a_\parallel, \quad a^{tt,p}_\perp \simeq a^{tp,t}_\perp \simeq a^{tp,p}_\perp \simeq a^{pp,t}_\perp \equiv a_\perp, \tag{3.2}$$

and, moreover, their dependence on B is negligible. In other words, the perturbative part of the correlators shows no significant departure from the case $B = 0$ [see Eq. (2.3)]: therefore, we have fitted again all the data with the parametrization (2.4) together with the assumption (3.2).

Concerning the parameters μ [i.e., the inverse of the *correlation lengths* in the exponential terms in Eq. (2.4)], they show a general tendency for a modest increase, which amounts to about $5-10\%$ for the largest values of $|e|B$ and is slightly more visible for the correlators in the directions orthogonal to $\vec{B}$.

Among the various parameters entering Eq. (2.4), the ones showing the most pronounced variation with $|e|B$ have been the nonperturbative coefficients A_0 and A_1. That implies a significant dependence on the magnetic field of the *gluon condensate*, which is defined as:

$$G_2 = \frac{g^2}{4\pi^2} \sum_{\mu\nu,a} \langle G^a_{\mu\nu} G^a_{\mu\nu} \rangle \tag{3.3}$$

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and is related to the correlator in Eq. (1.1) through an *Operator Product Expansion*. It encodes the main effect of nonperturbative physics to gluon dynamics and its relevance was first pointed out in Ref. [15]. One can extract G_2 from the small-distance limit of the nonperturbative part of the correlator, obtaining, using our parametrization (2.4):

$$G_2 = \frac{1}{\pi^2}\left[\left(A_0^{tt}+A_1^{tt}\right)+4\left(A_0^{tp}+A_1^{tp}\right)+\left(A_0^{pp}+A_1^{pp}\right)\right] \equiv G_2^{tt}+G_2^{tp}+G_2^{pp}, \tag{3.4}$$

where, in the last passage, we have distinguished three contributions, coming from different sets of plaquettes. In Fig. 1 we report the values obtained for G_2 as a function of $|e|B$, normalized to the value of the condensate obtained for $B=0$, where we obtain $G_2 = 3.56(5)\cdot 10^{-2}$ GeV^4, the reported error being just the statistical one.

We notice that G_2 grows as a function of $|e|B$, the increase being of the order of 25% for the largest value of $|e|B$ explored. In the same figure we also report the relative increases in the G_2^{tt}, G_2^{tp} and G_2^{pp} terms. We see that the tt term is the most affected by the magnetic field, whereas the pp contribution shows a really modest dependence on $|e|B$. In Fig. 1, the best fit with a quadratic function $G_2(|e|B)/G_2(0) = 1+K(|e|B)^2$ is also plotted. We obtain $K = 0.164(7)$ GeV^{-4} and $\chi^2/n_{\mathrm{d.o.f.}} = 1.52$, excluding the point at $|e|B = 1.46$ GeV^2.

An increase of the chromomagnetic gluon condensate with $|e|B$ has been also found in Ref. [2], which is in qualitative agreement with the result presented here. A similar behaviour for G_2 has been also predicted making use of QCD sum rules [16].

4. Conclusions

We have found evidence of a significant effect of the external magnetic field on the gluon-field correlation functions. In particular, a large effect (and a significant anisotropy) is observed for the coefficients of the nonperturbative terms in Eq. (2.4), which, on the basis of Eq. (3.4), can be directly related to the gluon condensate. Due to the explicit Lorentz/$SO(4)$ symmetry breaking caused by the magnetic field, we can distinguish among three different contributions to the gluon condensate. An analysis based on Eq. (3.4) shows that each term has a different behaviour as a function of the magnetic field (see Fig. 1). Starting from that, we have observed that the gluon condensate itself increases as a function of B, with the increase being of the order of 20% for $|e|B \sim 1$ GeV^2. Relative differences between the different contributions are of the same order of magnitude, meaning that anisotropies induced by B are significant and comparable to those observed in other pure-gauge quantities (see, e.g., Ref. [6]). The increase of the gluon condensate provides evidence of the phenomenon known as *gluon catalysis*, which had been previously observed analysing the magnetic-field effects on plaquette expectation values [3, 4, 5].

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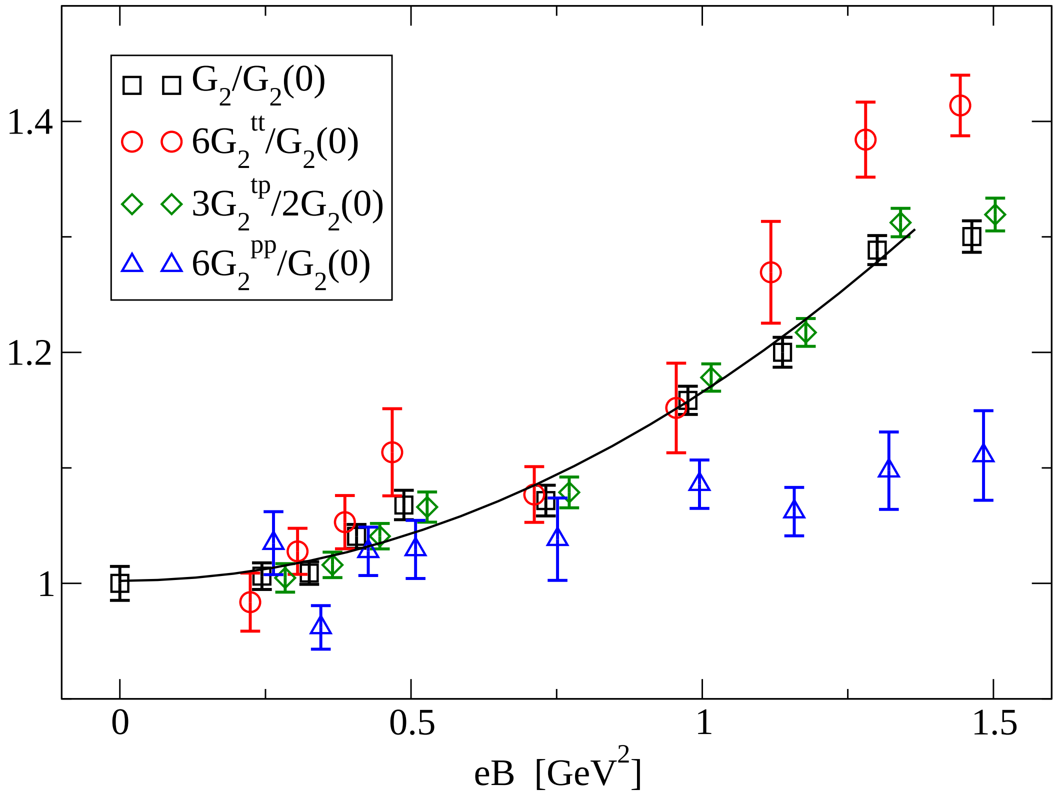

Figure 1: Effects of the magnetic field on the gluon condensate G_2 and on the three different contributions G_2^{tt}, G_2^{tp} and G_2^{pp}. The data points are shifted horizontally for the sake of readability.

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