Chapter

ROUNDNESS EVALUATION BY GENETIC ALGORITHMS

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ABSTRACT

Roundness is one of the most common features in machining, and various criteria may be used for roundness errors evaluation. The minimum zone tolerance (MZT) method produces more accurate solutions than data fitting methods like least squares interpolation. The problem modeling and the application of Genetic Algorithms (GA) for the roundness evaluation is reviewed here. Guidelines for the GA parameters selection are also provided based on computation experiments.

Keywords: Minimum zone tolerance (MZT), roundness error, genetic algorithm, CMM

1. INTRODUCTION

In metrology, the inspection of manufactured parts involves the measurement of dimensions for conformance to product specifications (Figure 1). In series or lot production, feedbacks from statistical analyses performed on multiple products allow process control for quality improvement.

Product specifications are associated with tolerances, which represent the acceptable limits for measured parts. Tolerances come from manufacturing requirements, e.g. assemble parts that fit, or from functional requirements for use or operation of the final product, e.g. rotation of a wheel, power of an engine.

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Metrology involves the acquisition or sampling of individual points by manual instruments, like analog or digital calipers, micrometers and dial gages, or by automated tools like coordinate measuring machines (CMM) or vision systems. Automated tools are equipped with software for post processing of data and are able to measure complex surfaces.

Measurements can be linear, such as size, distance, and depth and in two or three dimensions, such as surfaces and volumes. In addition to dimension tolerance there are form tolerances for two or three dimensional geometric primitives, like straightness (for an edge, an axis), flatness (for a plane), circularity or roundness (for a circle, an arc) or cylindricity (for a peg, a bar, a hole).

The simplest way to assess form tolerances is finding the belonging geometric primitives by interpolation of individual acquired data points. Not always linear regression represents the most accurate estimation of the form error. Overestimates represent a waste for the rejection of acceptable parts, while inversely underestimated form errors may produce defective parts.

The estimation of form errors by non linear methods is an optimization problem where metaheuristics, such as genetic algorithms, ant colony systems or neural networks can provide more accurate results with respect to linear methods, subject to proper modeling of the mathematical problem.

The application of metaheuristics for the estimation of form error is an active research field and final solutions are still far to come, particularly regarding the processing time due to the problem compexity compared to interpolation methods, which provide results in fractions of the second.

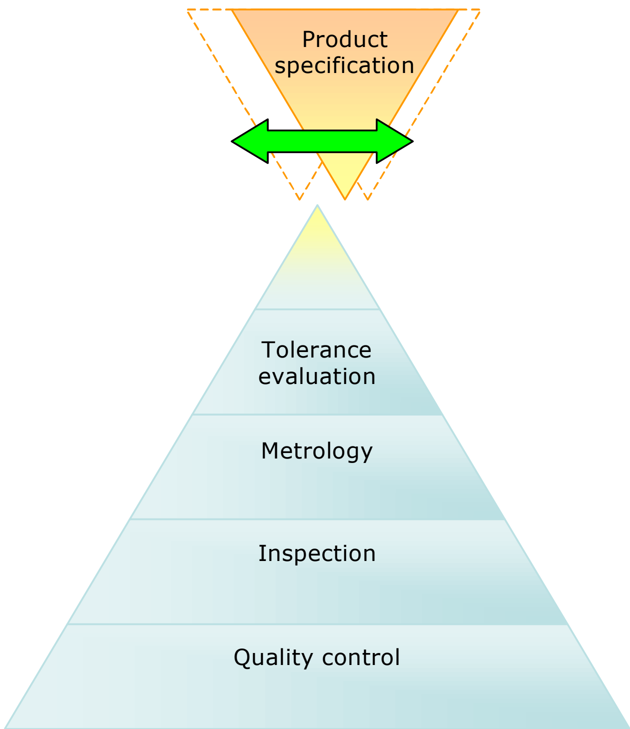

Figure 1. Adjustment between manufacturing tolerances and product specifications.

In the remainder the application of genetic algorithms for roundness evaluation will be discussed. The approach presented can be extended to other types of form errors.

2. Roundness Error

Roundness is the property of being shaped like or approximately like a circle or cylinder. In manufacturing environments, variations on circular features may occur due to: imperfect rotation, erratic cutting action, inadequate lubrication, tool wear, defective machine parts, chatter, misalignment of chuck jaws, etc. The out of roundness of circular and cylindrical parts can prevent insertion, produce vibrations in rotating parts, irregular rotation, noise etc.

Form tolerance is evaluated with reference to an ideal geometric feature, i.e. a circle in the case of roundness.

The most used criteria to establish the reference circle are: the Least- Squares method (LSQ), the Maximum Inscribed Circle (MIC), the Minimum Circumscribed Circle (MCC) and the Minimum Zone Tolerance (MZT). The use of a particular interpolation or data fitting method depends on the required part application, e.g. MIC and MCC can be used when mating a peg into a hole is involved to assess interference. The LSQ is one of the methods used by the coordinate measuring machines for rapidity and because it is efficient in computation with a large number of measured points. The roundness error determined by the LSQ is larger than those determined by other methods, such as the MZT.

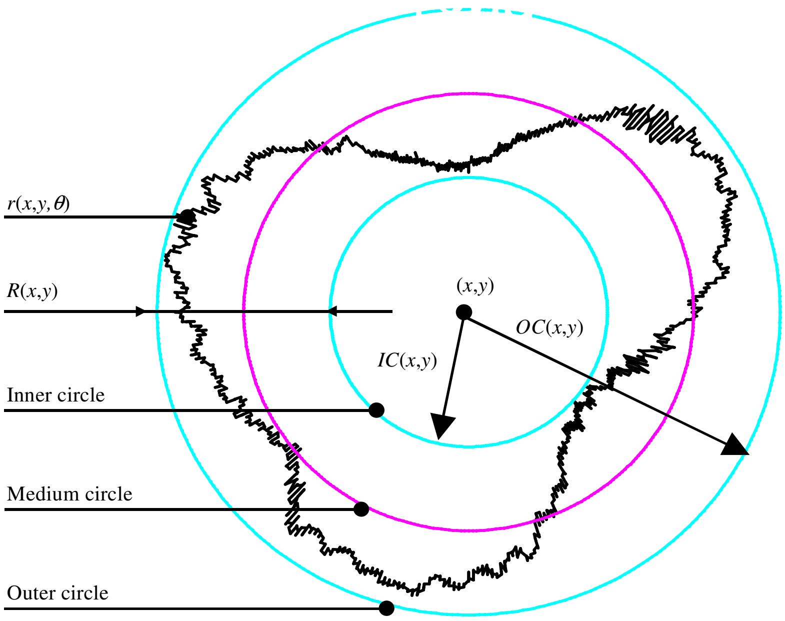

Figure 2. Roundness profile of a real profile with 1800 equally-spaced CMM sample points and reference circles: inner, medium, and outer circles for a given associated derived center (x,y).

3. LITERATURE

The MZT can be considered the best estimation of the roundness error because its definition meets the standard definition of the roundness error, as reported in ISO 1101 [1]. The MZT determines two concentric circles that contain the measured profile and such that the difference in radii is the least possible value as it is shown in Figure 2, where c_1 and c_2 are two possible centers of two concentric circles that include the measured points and where Δr_1 and Δr_2 are their difference in radii. So, once found, the MZ error can be considered the roundness error itself and the related MZ center. However, the MZT is a non linear problem and several methods to solve this problem have been proposed in the literature: computational geometry techniques and the solution of a non linear optimization problem. The first approach is, in general, very computationally expensive, especially, when the number of data points is large. One of these methods is based on the Voronoi diagram [2]. The second approach is based on an optimization function but the inconvenience is that this function has several local minima.

Some examples are: the Chebyshev approximation [3], the simplex search / linear approximation [4] [5], the steepest descent algorithm [6], the particle swarm optimization (PSO) [7] [8], the simulated annealing (SA) [9], and genetic algorithms (GAs) [10] [11] [12] [13].

Xiong [14] develops a general mathematical theory, a model and an algorithm for different kinds of profiles including roundness where the linear programming method and exchange algorithm are used. As limaçon approximation is used to represent the circle, the optimality of the solution is however not guaranteed. Performance of methods have been reviewed in [15].

A strategy based on geometric representation for minimum zone evaluation of circles and cylinders is proposed by Lai and Chen [16]. The strategy employs a non-linear transformation to convert a circle into a line and then uses a straightness evaluation schema to obtain minimum zone deviations for the feature concerned. This is an approximation strategy to minimum zone circles.

M. Wang et al. [17] and Jywe et al. [18] present a generalized non-linear optimization procedure based on the developed necessary and sufficient conditions to evaluate roundness error. To meet the standards the MZ reference circles should pass through at least four points of the sample points. This can occur in two cases: a) when three points lie on a circle and one point lies on the other circle (the 1-3 and the 3-1 criteria); b) when two points lie on each of the concentric circles (the 2-2 criterion). In order to verify these conditions the computation time increases exponentially with the dataset size. Gadelmawla [19] use a heuristic approach to drastically reduce the number of sample points used by the min-max 1-3, 3-1 and 2-2 criteria.

Samuel and Shunmugam [20] establish a minimum zone limaçon based on computational geometry to evaluate roundness error; with geometric methods, global optima are found by exhaustively checking every local minimum candidate. Moroni and Petro [15] propose a technique to speed up the exhaustive generation of solutions (*brute force algorithm*) which starts with a single point and increases one sample point at each step in order to generate all the possible subsets of points, until the tolerance zone of a subset cover the whole dataset (*essential subset*).

A mesh based method with starting center on the LSC, where the convergence depends on the number of mesh cross points, representing a compromise between accuracy and speed, is proposed by Xianqing et al. [21].

The strategy to equally-spaced points sampled on the roundness profile is generally adopted in the literature. Conversely, in previous works the authors developed a cross-validation method for small samples to assess the kind of manufacturing signature on the roundness profile in order to detect critical points such as peaks and valleys [22] [23]. They use a strategy where a next sampling increasing the points near these critical areas of the roundness profile.

In [24], some investigations proved that the increase of the number of sample points is effective only up to a limit number. Recommended dataset sizes are given for different data fitting methods (LSQ, MIC, MCC, MZT) and for three different out-of-roundness types (oval, 3-lobing and 4-lobing). Similar works are [25] and [26] in which substantially the same results are given.

A sampling strategy depends on the optimal number of sample points and the optimum search-space size for best estimation accuracy, particularly with datasets which involve thousands of sample points available by CMM scanning techniques. The sampling strategy tailored for a fast genetic algorithm to solve the MZT problem can be defined as *blind* according to the classification in [27] if it is not manufacturing specific. By sampling strategy not only the number and location of sample points on the roundness profile but also their use by the data fitting algorithm is concerned.

4. MZT MODEL

In the MZT method, the unknown are the (x,y) coordinates of the associated derived center of the minimum zone reference circles of the roundness profile (MZCI [28]). MZCI is formed by two concentric circles enclosing the roundness profile, the inner minimum zone reference circle and the outer minimum zone reference circle, having the least radial separation. The difference between the inner minimum zone reference circle and the outer minimum zone reference circle is the minimum zone error (*MZE*). *MZE* is the target parameter of our optimization algorithm as a function of (x,y).

Given an extracted circumferential line $r(x,y,\theta)$, with $\theta \in (0, 2\pi]$, of a section perpendicular to the axis of a cylindrical feature, the *roundness error* $R(x,y)$ is defined by:

$$R(x,y) = OC(x,y) - IC(x,y),\ (x,y) \in E_{r(x,y,\theta)} \tag{1}$$

where $OC(x,y)$ and $IC(x,y)$ are the radii of the reference circles of center (x,y), and $E_{r(x,y,\theta)}$ is the area enclosed by $r(x,y,\theta)$:

$$OC(x,y) = \max_{\theta \in (0,2\pi]} r(x,y,\theta) \tag{2}$$

$$IC(x,y) = \min_{\theta \in (0,2\pi]} r(x,y,\theta) \tag{3}$$

As a CMM scans the roundness profile by sampling a finite number, n, of equally-spaced points θ_i of the extracted circumferential line ($\theta_i = i\times\frac{2\pi}{n}$, i=1,...,n), the $OC(x,y)$ and $IC(x,y)$ are evaluated by:

$$OC(x,y) = \max_{\theta_i = i\times\frac{2\pi}{n},\, i=1,\ldots,n} r(x,y,\theta_i) \tag{4}$$

$$IC(x,y) = \min_{\theta_i = i\times\frac{2\pi}{n},\, i=1,\ldots,n} r(x,y,\theta_i) \tag{5}$$

Figure 2 shows the mentioned features for a given (x,y).

MZE is evaluated by applying the MZT data-fitting method to solve the following optimization problem:

$$MZE = \min_{(x,y)\in E_{r(x,y,\theta)}} R(x,y) = \begin{cases} \min\left[\max_{\theta_i = i\times\frac{2\pi}{n},\, i=1,\ldots,n} r(x,y,\theta_i) - \min_{\theta_i = i\times\frac{2\pi}{n},\, i=1,\ldots,n} r(x,y,\theta_i)\right] \\ \text{subject to } (x,y)\in E_{r(x,y,\theta_i)} \end{cases} \tag{6}$$

where $E_{r(x,y,\theta_i)}$ is a restricted area in the convex envelopment of the n equally-spaced sample points, i.e. the *search space.*

5. Genetic Algorithms for Roundness Evaluation

GAs were proposed for the first time by Holland [29] and constitute a class of search methods especially suited for solving complex optimization problems [30].

Genetic algorithms are widely used in research for non-linear problems. They are easily implemented and powerful being a general-purpose optimization tool. Many possible solutions are processed at the same time and evolve with both elitist and random rules, so to quickly converge to a local optimum which is very close or coincident to the optimal solution.

Genetic algorithms constitute a class of implicit parallel search methods especially suited for solving complex optimization or non-linear problems. They are easily implemented and powerful being a general-purpose optimization tool. Many possible solutions are processed concurrently and evolve with inheritable rules, e.g. the elitist or the roulette wheel selection, so to quickly converge to a solution which is very close or coincident to the optimal solution.

Genetic algorithms maintain a population of center candidates (the *individuals*), which are the possible solutions of the MZT problem. The center candidates are represented by their *chromosomes*, which are made of pairs of x_i and y_i coordinates. Genetic algorithms operate on the x_i and y_i coordinates, which represent the inheritable properties of the individuals by means of genetic operators. At each generation the genetic operators are applied to the selected center candidates from current population in order to create a new generation. The selection of individuals depends on a fitness function, which reflects how well a solution fulfills the requirements of the MZT problem, e.g. the objective function.

Sharma et al. [2] use a genetic algorithm for MZT of multiple form tolerance classes such as straightness, flatness, roundness, and cylindricity. There is no need to optimize the algorithm performance, choosing the parameters involved in the computation, because of the small dataset size (up to 100 sample points).

Wen et al. [11] implement a genetic algorithm in real-code, with only crossover and reproduction operators applied to the population. Thus in this case mutation operators are not used. The algorithm proposed is robust and effective, but it has only been applied to small samples.

In a genetic algorithm for roundness evaluation the center candidates are the individual of the population (*chromosomes*). The search space is an area enclosed by the roundness profile where the center candidates of the initial population are selected for the data fitting algorithm. The area is rectangular because the crossover operator changes the x_i and y_i coordinates of the parents to generate the offspings [10]. After crossover, the x_i and y_i coordinates of parents and offsprings are located to the rectangle vertexes.

In order to find the MZ error the search space must include the global optima solution i.e. the MZ center. Therefore the centre of the rectangular area is an estimation of the MZ center evaluated as the mean value of the x_i and y_i coordinates of the sampled points [10], [11], [12], [13]. In [11] the search-space is a square of fixed 0.2 mm side, in [13] it is 5% of the circle diameter and center. In [10], the side is determined by the distance of the farthest point and the nearest point from the mean center. In [12] it is the rectangle circumscribed to the sample points.

Optimal sampling and genetic algorithm parameters are listed in Table 1.

Table 1. Algorithm parameters according to [13]

Optimization Geometric and algorithm GA genetic parameters	Symbol	Value	Comment
sample size	n	500	number of equally spaced sample points
search space	E	0.5	initial population randomly selected within
population size	P_s	70	set of chromosomes used in evolving epoch
selection			elitist selection
crossover	P_c	0.7	one point crossover of the $pc \times pop$ parents' genes (i.e. coordinates) at each generation
mutation	P_m	0.07	$pm \times pop$ individuals are modified by changing one gene (i.e. coordinate) with a random value
stop criterion	N	100	the algorithm computes N generations after the last best roundness error evaluated rounded off to the fourth decimal digit (0.1 µm)

6. GA Operation

Selection: during this operation, a solution has a probability of being selected according to its fitness. Some of the common selection mechanisms are: the roulette wheel procedure, the Tournament selection, and the elitist selection. With this latter, the individuals are ordered on the basis of their fitness function; the best individuals produce offspring. The next generation will be composed of the best chromosomes chosen between the set of offspring and the previous population.

Crossover: this operator allows to create new individuals as offspring of two parents by inheriting genes from parents with high fitness. The possibility for this operator to be applied depends on the crossover probability. There are different crossover types: the one point and multiple point crossover, and other sophisticated ones. In the proposed GA was used the arithmetic crossover mechanism, which generates offspring as a component-wise linear combination of the parents.

Mutation: a new individual is created by making modifications to one selected individual. In genetic algorithms, mutation is a source of variability, and is applied in addition to selection and crossover. This method prevents the search to be trapped only in local solutions. The relative parameter is the mutation probability, that is the probability that one individual is mutated.

Stop criterion: the algorithm has an iterative behavior and needs a stop condition to end the computation. Possible criteria include: overcoming a predefined threshold for the fitness function or iteration number or their combinations. In the proposed GA, the stop criterion is controlled by the number N of the iterations if no improvement in the solution occurs.

7. Testing Algorithms

To analyze the behavior of an algorithm with the MZT method, dataset with known MZ error are available. These datasets are generated with NPL Chebyshev best fit circle certified software [31].

The use of certified software has the following benefits

- it produces randomly distributed error makes the results more general, because the results achieved with the genetic algorithm are not manufacturing signature specific;
- the circle center is known, so it allows estimating the circular profile center is computed as an average value of the measuring points coordinates [11] the average MZ center found by the algorithm

Datasets produced by certified software have a user-selected center and radius.

For performance assessment, the algorithm is usually executed on a dataset several tens of times for each test, and the average MZ error and the average computation time are computed and compared with the nominal MZ error.

An example of execution in shown in Figure 3, which displays the average and standard error.

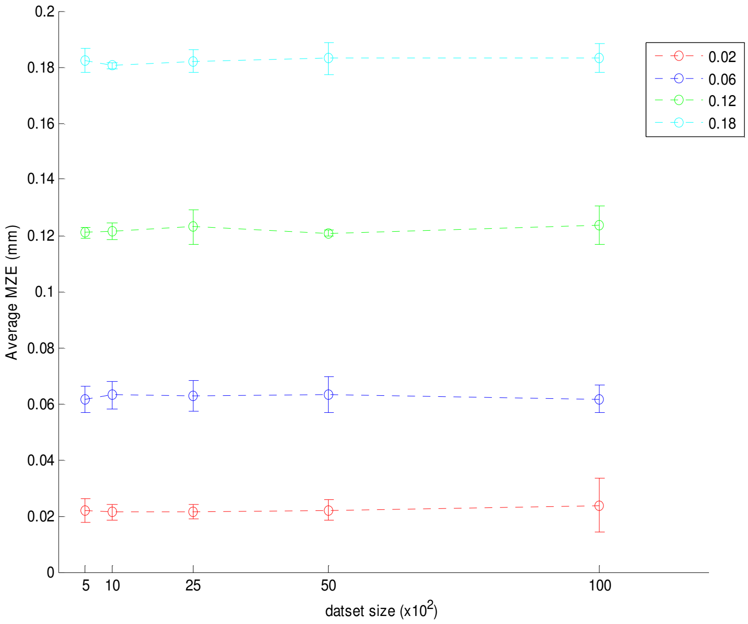

Figure 3. Average MZE with error bars for large datasets and different roundness errors.

In a typical experimental approach, a first raw estimation of the circular profile center is necessary as a starting point for a more accurate roundness evaluation.

A first estimation of the circular profile center is computed as an average value of the measuring points coordinates in expression (1).

SUMMARY

The application of a genetic algorithm for the roundness evaluation of circular profiles using the MZT method has been described.

The GA approach described and the parameters provided may solve most roundness measurement needs, both for small and large datasets (up to several thousands datapoints). The listed literature may serve as a guideline for optimal GA parameters estimation on new problems.

ACKNOWLEDGEMENT

Part of this work has been derived from Alessandro Meo and Luca Profumo's project work for the class of automation of production processes of the master degree in automation engineering at the school of engineering at Pisa University.

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